

# Predicting lepton flavor mixing from $\Delta(48)$ and generalized $CP$ symmetries\*

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**Abstract:** We propose to understand the mixing angles and  $CP$ -violating phases from the  $\Delta(48)$  family symmetry combined with the generalized  $CP$  symmetry. A model-independent analysis is performed by scanning all the possible symmetry breaking chains. We find a new mixing pattern with only one free parameter, excellent agreement with the observed mixing angles can be achieved and all the  $CP$ -violating phases are predicted to take nontrivial values. This mixing pattern is testable in the near future neutrino oscillation and neutrinoless double-beta decay experiments. Finally, a flavor model is constructed to realize this mixing pattern.

**Key words:** family symmetries, generalized  $CP$ , lepton mixing,  $CP$  violation

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## 1 Introduction

Discrete family symmetry has been widely used to explain the lepton flavor mixing [1] in the past years. The discovery of a sizable value of  $\theta_{13}$  by reactor experiments [2] excludes many neutrino mixing models, and opens the possibility of measuring the Dirac  $CP$ -violating phase in the next generation neutrino experiments. The underlying physics of flavor mixing and  $CP$  violation is still an open question. The history of physics tells us that symmetry always plays a crucial role in understanding the natural world. Inspired by the success of the family symmetry paradigms, it is natural to extend the family symmetry to include a generalized  $CP$  (GCP) symmetry  $H_{CP}$  [3–6], to predict both flavor mixing angles and  $CP$  phases. This idea has been implemented within  $S_4$  [4, 6, 7],  $A_4$  [8] and  $T'$  [9] family symmetries, where the lepton mixing matrix is found to depend on one single parameter  $\vartheta$ , which can be fixed by the measurement of the mixing angle  $\theta_{13}$ .

In this paper, we propose to impose the  $\Delta(48)$  family symmetry together with GCP symmetry on the theory. Compared with the well-known  $S_4$  and  $A_4$  family symmetries,  $\Delta(48)$  provides many candidates for GCP transformations which could lead to new mixing patterns. After a brief introduction to the GCP symmetry and the group

theory of  $\Delta(48)$ , we derive all the GCP transformations which are consistent with  $\Delta(48)$ . Then we present possible lepton mixing patterns derived from different symmetry breaking chains in a model-independent way. Finally we focus on phenomenological implications and model building aspect of a new pattern that has not been discussed in the literature. A longer and more complete version of this paper has been presented in Ref. [10].

## 2 $\Delta(48)$ and the consistent generalized $CP$ transformations

A field multiplet  $\phi$  transforms under the family and GCP symmetries as

$$\phi \xrightarrow{g} \rho(g)\phi \quad \text{and} \quad \phi \xrightarrow{CP} X\phi^*, \quad (1)$$

respectively, where  $\rho(g)$  is a representation matrix of the group element  $g \in G_f$ , and  $X \in H_{CP}$  is the GCP transformation matrix. Both of them are unitary matrices. It is nontrivial to combine the family symmetry with the GCP symmetry. The so-called consistence equation has to be satisfied:  $X\rho^*(g)X^{-1} = \rho(g')$  for  $g, g' \in G_f$  [3–5]. Furthermore,  $X$  maps one group element  $g$  into another element  $g'$ , consequently  $X$  corresponds to an automorphism of  $G_f$ . It has been established that there is a one-to-one correspondence between the GCP transformation

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and the automorphism of  $G_f$  [11].

In the present work, the family symmetry is chosen to be  $G_f = \Delta(48) \cong (Z_4 \times Z_4) \rtimes Z_3$ , which is a finite subgroup of  $SU(3)$  of order 48 with generators  $a, c$  and  $d$  satisfying

$$\begin{aligned} a^3 &= c^4 = d^4 = 1, & cd &= dc, \\ aca^{-1} &= c^{-1}d^{-1}, & ada^{-1} &= c, \end{aligned} \quad (2)$$

where  $a$  generates  $Z_3$  and  $c, d$  are generators of  $Z_4 \times Z_4$ . It belongs to the  $\Delta(3n^2)$  series [12] with  $n=4$ . Any group element  $g \in \Delta(48)$  can be expressed as  $g = a^k c^m d^n$  with  $k=0, 1, 2$  and  $m, n=0, 1, 2, 3$ .

$\Delta(48)$  has eight irreducible representations:

- 1) Three 1-dimensional (1d) representations  $1, 1', 1''$ ;
- 2) Five 3d representations  $3, \bar{3}, 3', \bar{3}'$  and  $\tilde{3}$ , where  $\tilde{3}(\bar{3}')$  is the complex conjugate of  $3(3')$ . The former four are the faithful representations of  $\Delta(48)$ , while the last one is not.

We shall work in the generator  $a$  diagonal basis. For the representation  $3$ , we choose:

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad c = \frac{1}{3} \begin{pmatrix} 1 & 1-\sqrt{3} & 1+\sqrt{3} \\ 1+\sqrt{3} & 1 & 1-\sqrt{3} \\ 1-\sqrt{3} & 1+\sqrt{3} & 1 \end{pmatrix}, \quad (3)$$

with  $\omega = e^{i2\pi/3}$ , and the representation matrix of  $d$  is given by  $d = a^{-1}ca$ . Some Kronecker products that will be used later are presented here:  $3 \otimes \bar{3} = 1 \oplus 1' \oplus 1'' \oplus 3' \oplus \bar{3}'$ ,  $\bar{3} \otimes \bar{3} = 3_S \oplus 3_A \oplus \bar{3}, 3' \otimes 3' = \bar{3}'_S \oplus \bar{3}'_A \oplus \bar{3}$ ,  $3 \otimes 3' = 3 \oplus 3' \oplus \bar{3}$ ,  $3 \otimes \bar{3}' = 3 \oplus \bar{3}' \oplus \bar{3}, \bar{3}' \otimes \bar{3} = 3 \oplus \bar{3} \oplus 3'$ .

The basic paradigm is that the symmetry  $\Delta(48) \rtimes H_{CP}$  is respected at high energy scales, and is then spontaneously broken to different subgroups  $G_\nu \rtimes H_{CP}^\nu$  and  $G_l \rtimes H_{CP}^l$  in the neutrino and charged lepton sectors by flavon fields. This misalignment between the symmetry breaking patterns leads to particular predictions for mixing angles and  $CP$  phases. Without loss of generality, three generations of the left-handed lepton doublets are assigned to  $\Delta(48)$  triplet  $3$ . The invariance of the Lagrangian under residual family symmetries and residual GCP symmetries implies that the neutrino mass matrix  $m_\nu$  and the charged lepton mass matrix  $m_l$  should satisfy

$$\rho^\dagger(g_\nu) m_\nu \rho^*(g_\nu) = m_\nu, \quad \rho^\dagger(g_l) m_l m_l^\dagger \rho(g_l) = m_l m_l^\dagger, \quad (4a)$$

$$X_\nu^\dagger m_\nu X_\nu^* = m_\nu^*, \quad X_l^\dagger m_l m_l^\dagger X_l = (m_l m_l^\dagger)^*, \quad (4b)$$

where neutrinos are assumed to be Majorana particles,  $g_\nu, g_l$  denote the group elements of the residual family symmetries  $G_\nu, G_l$ , and  $X_\nu, X_l$  denote the elements of the remnant GCP symmetries  $H_{CP}^\nu, H_{CP}^l$ , respectively. By systematically scanning all the possible remnant family subgroups  $G_\nu$  and  $G_l$ , we find that only the case  $G_\nu = Z_2$  and  $G_l = Z_3$  can lead to viable phenomenology. One can choose  $G_\nu = \{1, c^2\}$  and  $G_l = \{1, a, a^2\}$  without loss of generality, since all the possible choices are related by group conjugation. From the constraint of Eq. (4a), we find that the charged lepton mass matrix  $m_l$  is diag-

onal in the chosen basis, and the neutrino mass matrix  $m_\nu$  takes the form:

$$\begin{aligned} m_\nu &= \alpha \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + \beta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \\ &+ \gamma \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + \epsilon \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \end{aligned} \quad (5)$$

where  $\alpha, \beta, \gamma$  and  $\epsilon$  are complex parameters, and they are further constrained by the neutrino residual GCP symmetry  $H_{CP}^\nu$ , as shown in Eq. (4b).

Each GCP transformation corresponds to an automorphism of the family symmetry  $G_f$ . The automorphism group of  $\Delta(48)$  is  $\text{Aut}(\Delta(48)) \cong \Delta(48) \rtimes D_8$  with 384 group elements. Its outer automorphism group is proven to be a dihedral group  $\text{Out}(\Delta(48)) \cong D_8$ , with generators  $u_1$  and  $u_2$  defined as

$$\left\{ \begin{array}{l} a \xrightarrow{u_1} a^2 \\ c \xrightarrow{u_1} cd^2 \end{array} \right\}, \quad \left\{ \begin{array}{l} a \xrightarrow{u_2} a \\ c \xrightarrow{u_2} cd^2 \end{array} \right\}. \quad (6)$$

The following multiplication rules are fulfilled

$$u_1^4 = u_2^2 = (u_1 u_2)^2 = id. \quad (7)$$

Each group element in  $\text{Out}(\Delta(48))$  can be expressed as  $u_1^\mu u_2^\nu$  for  $\mu=0, 1, 2, 3$  and  $\nu=0, 1$ . The generators  $u_1$  and  $u_2$  act on the irreducible representation of  $\Delta(48)$  as

$$\begin{aligned} 1' &\xleftarrow{u_1} 1'', & 3 &\xrightarrow{u_1} 3', & 3 &\xrightarrow{u_1} \bar{3}, & 3 &\xrightarrow{u_1} \bar{3}', & 3 &\xrightarrow{u_1} 3, & \bar{3} &\xrightarrow{u_1} \bar{3}, \\ 1' &\xrightarrow{u_2} 1', & 1'' &\xrightarrow{u_2} 1'', & 3 &\xleftarrow{u_2} 3', & \bar{3} &\xleftarrow{u_2} \bar{3}', & \bar{3} &\xrightarrow{u_2} \bar{3}. \end{aligned} \quad (8)$$

The 8 outer automorphisms generated by  $u_1$  and  $u_2$  lead to different  $CP$  transformations and should have distinct physical implications. In the present work, we minimally extend the  $\Delta(48)$  family symmetry to include only those nontrivial  $CP$  transformations which map one irreducible representation into its complex conjugate. We find that there are three outer automorphisms,  $u_1^2, u_1 u_2$ , and  $u_1^3 u_2$ , satisfying this requirement.

1) The first automorphism  $u_1^2$  interchanges all 3d irreducible representations with their complex conjugate representations. The corresponding GCP matrix in each 3d irreducible representation is determined to be just a permutation  $X(u_1^2) = P_{23}$ . Thus, the GCP transformation acts on a 3d field  $\phi = (\phi_1, \phi_2, \phi_3)^T$  as

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \xrightarrow{CP} P_{23} \begin{pmatrix} \phi_1^* \\ \phi_2^* \\ \phi_3^* \end{pmatrix} = \begin{pmatrix} \phi_1^* \\ \phi_3^* \\ \phi_2^* \end{pmatrix}. \quad (9)$$

This is the so-called  $\mu$ - $\tau$  reflection symmetry [13].

2) The second automorphism  $u_1 u_2$  interchanges  $3'$  with  $\bar{3}'$  but maps  $3$  and  $\bar{3}$  into themselves. In  $3'$  and

$\bar{3}'$  representation space, we find it to be  $X(u_1 u_2) = \mathbb{1}_3$ , i.e., the conventional  $CP$  transformation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} \xrightarrow{CP} \mathbb{1}_3 \begin{pmatrix} \phi_1^* \\ \phi_2^* \\ \phi_3^* \end{pmatrix} = \begin{pmatrix} \phi_1^* \\ \phi_2^* \\ \phi_3^* \end{pmatrix}. \quad (10)$$

3) The third one  $u_1^3 u_2$  exchanges 3 with  $\bar{3}$  while maps  $3'$  and  $\bar{3}'$  into themselves. In 3 and  $\bar{3}$ , the corresponding GCP matrix is  $X(u_1^3 u_2) = \mathbb{1}_3$  as well.

Taking account of the inner automorphisms, we find that residual GCP transformations compatible with the remnant family symmetry  $G_\nu = \{1, c^2\}$  can be expressed as

$$X_\nu = \rho(c^m d^n) P_{23} \text{ or } \rho(c^m d^n) \quad (11)$$

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \quad (13)$$

in which  $c_{ij} = \cos\theta_{ij}$ ,  $s_{ij} = \sin\theta_{ij}$ ,  $\delta$  is the Dirac  $CP$ -violating phase and  $\alpha_{21}$ ,  $\alpha_{31}$  are the Majorana  $CP$ -violating phases. It is more convenient to redefine the Majorana phase  $\alpha'_{31} \equiv \alpha_{31} - 2\delta$  during the analysis of the neutrinoless double-beta decay.

With different choices of remnant GCP transformations in Eq. (11) and abandoning the cases which predict degenerate neutrino masses, we obtain 4 kinds of mixing patterns, denoted by patterns A, B, C and D. Each mix-

ing pattern depends on one free parameter  $\vartheta$  and predicts  $\sin^2\theta_{12} = 1/(3\cos^2\theta_{13})$  since the structure of  $m_\nu$  in Eq. (5) preserves the second column of the PMNS matrix as  $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})^T$ . As a consequence, mixing angles as well as  $CP$  phases are strongly correlated, as shown in Table 1. For proper values of  $\vartheta$ , all cases are compatible with the present neutrino oscillation data [15] within  $3\sigma$  range, except  $\theta_{13}$  in pattern C.

### 3 Model-independent results of the PMNS matrix

We shall further investigate the constraints on the neutrino mass matrix  $m_\nu$  in Eq. (5) by applying the residual GCP symmetry and then derive the PMNS matrix. In the PDG convention [14], the PMNS matrix is cast in the form

$$U_{\text{PMNS}} = V \text{diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}), \quad (12)$$

with

Table 1. The predictions for lepton mixing patterns and the associated mixing parameters for all possible choices of residual GCP symmetries in the neutrino sector, and all the mixing patterns are found to depend on only one parameter  $\vartheta$  varying from 0 to  $2\pi$ . The related GCP matrices hold for all faithful 3d representations 3,  $\bar{3}$ ,  $3'$  and  $\bar{3}'$  with  $k_1, k_2=0, 1, m=0, 1, 2, 3$ . The sign “ $+\infty$ ” for  $|\tan\delta|$  implies that the corresponding Dirac  $CP$ -violating phase is  $\pm\pi/2$ .

	pattern A	pattern B	pattern C	pattern D
GCP matrix $X$	$\rho(c^{2k_1+k_2} d^{2k_2}) P_{23}$	$\rho(c^{2k_1+k_2} d^{2k_2})$	$\rho(c^{2k_1+k_2+1} d^{2k_2}) P_{23}$	$\rho(c^m d^{2k_2+1}) P_{23}$
$\sin^2\theta_{13}$	$\frac{1}{3} - \frac{1}{3} \cos\vartheta$	$\frac{1}{3} - \frac{1}{3} \cos\vartheta$	$\frac{1}{3} - \frac{1}{2\sqrt{3}} \cos\vartheta$	$\frac{1}{3} - \frac{1+\sqrt{3}}{6\sqrt{2}} \cos\vartheta$
$\sin^2\theta_{12}$	$\frac{1}{2+\cos\vartheta}$	$\frac{1}{2+\cos\vartheta}$	$\frac{2}{4+\sqrt{3}\cos\vartheta}$	$\frac{2\sqrt{2}}{4\sqrt{2}+(1+\sqrt{3})\cos\vartheta}$
$\sin^2\theta_{23}$	$\frac{1}{2}$	$\frac{1}{2} \mp \frac{\sqrt{3}\sin\vartheta}{4+2\cos\vartheta}$	$\frac{1}{2} \mp \frac{\sqrt{3}\cos\vartheta}{8+2\sqrt{3}\cos\vartheta}$	$\frac{1}{2} \mp \frac{(3-\sqrt{3})\cos\vartheta}{8\sqrt{2}+2(1+\sqrt{3})\cos\vartheta}$
$J_{CP}$	$-\frac{\sin\vartheta}{6\sqrt{3}}$	0	$\mp \frac{\sin\vartheta}{6\sqrt{3}}$	$\mp \frac{\sin\vartheta}{6\sqrt{3}}$
$ \tan\delta $	$+\infty$	0	$\left  \frac{4+\sqrt{3}\cos\vartheta}{1+\sqrt{3}\cos\vartheta} \tan\vartheta \right $	$\left  \frac{4\sqrt{2}+(1+\sqrt{3})\cos\vartheta}{1-\sqrt{3}-\sqrt{2}\cos\vartheta} \tan\vartheta \right $
$ \tan\alpha_{21} $ or $ \cot\alpha_{21} $	0	0	$\left  \frac{\sqrt{3}+2\cos\vartheta}{\sin\vartheta} \right $	$\left  \frac{1+\sqrt{3}+2\sqrt{2}\cos\vartheta+(1-\sqrt{3})\sin\vartheta}{1+\sqrt{3}+2\sqrt{2}\cos\vartheta-(1-\sqrt{3})\sin\vartheta} \right $
$ \tan\alpha'_{31} $	0	0	$\left  \frac{4\sqrt{3}\sin\vartheta}{1-3\cos2\vartheta} \right $	$\left  \frac{4\sin\vartheta}{2-3\sqrt{3}+(2+\sqrt{3})\cos2\vartheta} \right $

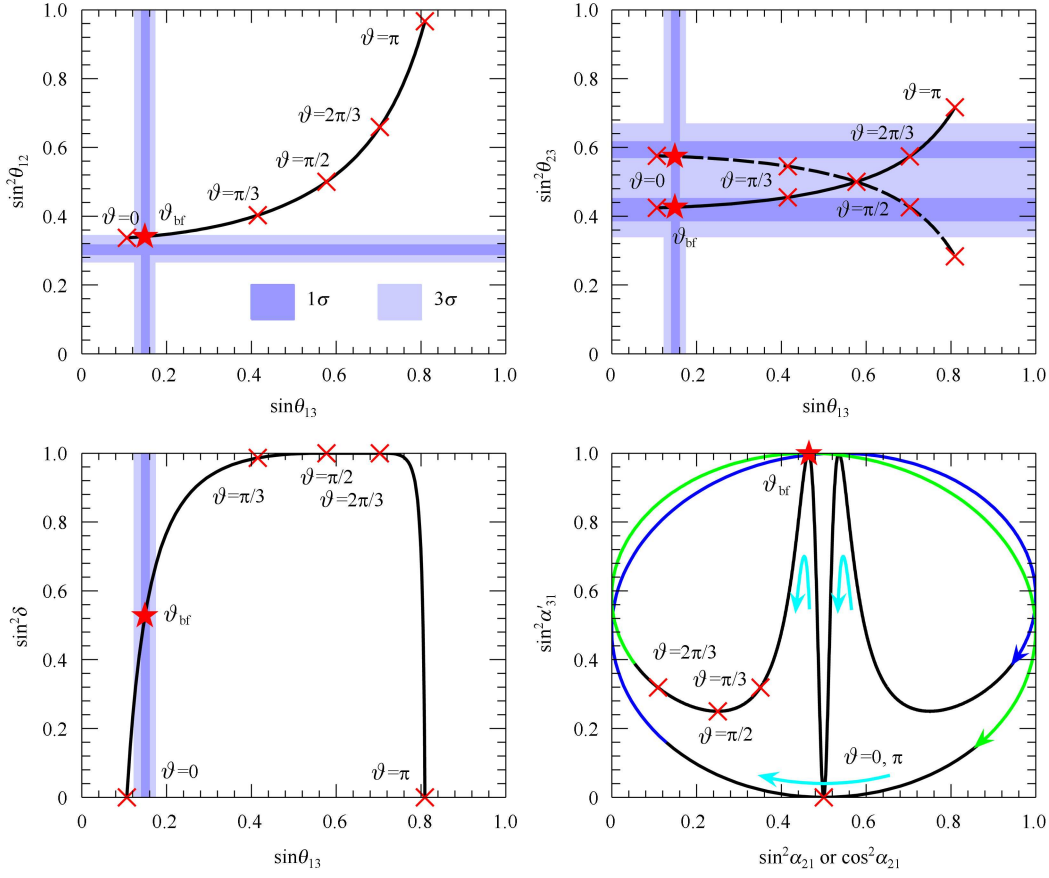


Fig. 1. Correlations among the mixing angles and  $CP$ -violating phases in pattern D. We mark the best-fit value  $\theta_{\text{bf}}$  of the parameter  $\vartheta$  with a red star, and also label  $\vartheta=0, \pi/3, \pi/2, 2\pi/3, \pi$  with a red cross on the curve. In the top right panel, the results of  $\sin^2\theta_{23}$  for the first octant and the second octant of  $\theta_{23}$ , are shown in a solid line and dashed line, respectively. The  $1\sigma$  and  $3\sigma$  ranges of the mixing angles are taken from Ref. [15].

In the following, we will focus on pattern D which is completely new as far as we know. In this case, the GCP symmetry corresponding to the outer automorphism  $u_1^2$  should be implemented. All the mixing parameters, in particular the  $CP$  phases are nontrivially dependent on  $\vartheta$ , and the correlations between the mixing parameters are plotted in Fig. 1. Excellent agreement with the present global-fitting data of mixing angles can be achieved. It is interesting to note that the relation between  $\sin^2\alpha_{21}$  (or  $\cos^2\alpha_{21}$ ) and  $\sin^2\alpha'_{31}$ , shown in low right panel, looks like the “compound eyes” of an insect. Taking the  $3\sigma$  ranges of mixing angles from [15], we obtain

$$0.162 \leq |\vartheta| \leq 0.341, \quad (14)$$

and the  $CP$ -violating phases are constrained to lie in the following intervals:

$$\begin{aligned} 0.292 \leq \sin^2\delta \leq 0.667, \quad 0.781 \leq \sin^2\alpha'_{31} \leq 1, \\ 0.455 \leq \sin^2\alpha_{21} \text{ or } \cos^2\alpha_{21} \leq 0.478. \end{aligned} \quad (15)$$

The quadrants of  $CP$ -violating phases cannot be de-

termined in the present model-independent approach. Notice that the Dirac phase  $\delta$  is large but not maximal, and this prediction could be tested in the next generation neutrino oscillation experiments LBNE and Hyper-K.

This pattern is also testable in the future neutrinoless double-beta ( $0\nu\beta\beta$ ) decay experiments. The rate of  $0\nu\beta\beta$  decay is determined by the nuclear matrix element and the effective parameter  $\langle m \rangle_{ee} = |m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{i\alpha_{21}} + m_3 s_{13}^2 e^{i\alpha'_{31}}|$ . In Fig. 2, we show the prediction for  $\langle m \rangle_{ee}$  as a function of the lightest neutrino mass, where the constraint in Eq. (14) has been taken into account. The upper bounds from cosmology (the sum of neutrino masses  $\sum m_i < 0.23$  eV) [16] and the current  $0\nu\beta\beta$  bound ( $\langle m \rangle_{ee} < 0.32$  eV) [17] are also included in the figure. The next generation  $0\nu\beta\beta$  decay experiments will reach the sensitivity of  $\langle m \rangle_{ee} \approx (0.01-0.05)$  eV after 5 years of data taking [17]. As a consequence, if the signal of  $0\nu\beta\beta$  would not be observed, the inverted mass ordering scenario of this pattern would be excluded, since we have  $\langle m \rangle_{ee} > 0.02$  eV in this case as shown in Fig. 2.

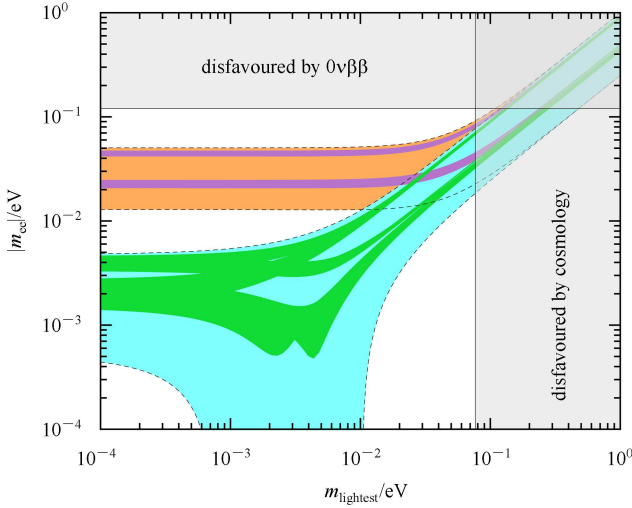


Fig. 2. The prediction of the effective neutrino mass  $\langle m \rangle_{ee}$  as a function of the lightest neutrino mass  $m_1$  in normal ordering (NO) or  $m_3$  in inverted ordering (IO) in pattern D. The splits within each mass ordering come from the uncertainties of the quadrants of  $CP$ -violating phases.

## 4 A simplified model

Finally, we shall construct a simple flavor model in which pattern D is realized. The field arrangement is listed in Table 2, where  $\ell_L, e_R, \mu_R, \tau_R$  denote the left-handed and right-handed lepton fields,  $H$  represents the Higgs field, and  $\phi_1, \varphi_1, \rho_1, \varphi, \xi$  are the gauge-singlet flavon fields. The additional  $Z_3 \times Z_4 \times Z_5$  symmetry is used to eliminate undesired dangerous operators and derive suitable vacuum alignments. Yukawa couplings invariant under  $\Delta(48) \times Z_3 \times Z_4 \times Z_5$  are

$$\begin{aligned}
 -\mathcal{L}_1 = & \frac{y_\tau}{\Lambda} \phi_1 \bar{\ell}_L H \tau_R + \frac{y_\mu}{\Lambda^2} (\phi_1 \varphi_1)_3 \bar{\ell}_L H \mu_R \\
 & + \frac{y_{e_1}}{\Lambda^3} (\rho_1 (\phi_1 \phi_1)_{\bar{3}})_3 \bar{\ell}_L H e_R \\
 & + \frac{y_{e_2}}{\Lambda^3} (\phi_1 (\varphi_1 \varphi_1)_{\bar{3}'})_3 \bar{\ell}_L H e_R \\
 & + \frac{y_\varphi}{\Lambda^2} \varphi (\bar{\ell}_L \tilde{H} \tilde{H}^T \ell_L^c)_{\mathbf{3}_S} \\
 & + \frac{y_\xi}{\Lambda^2} \xi (\bar{\ell}_L \tilde{H} \tilde{H}^T \ell_L^c)_{\bar{\mathbf{3}}} + \text{h.c.}, \quad (16)
 \end{aligned}$$

in which  $\tilde{H} = i\sigma_2 H^*$ , and  $\Lambda$  is the cut-off scale. Moreover, all coupling coefficients are real since the GCP symmetry is imposed. The flavon vacuum expectation values can be realized by using the supersymmetric driving field method. Here we directly list them as

$$\begin{aligned}
 \langle \phi_1 \rangle &= v_{\phi_1} (1, 0, 0)^T, \langle \varphi_1 \rangle = v_{\varphi_1} (0, 1, 0)^T, \langle \rho_1 \rangle = v_{\rho_1} (0, 1, 0)^T, \\
 \langle \varphi \rangle &= e^{i\frac{\pi}{4}} v_\varphi (1, 1, 1)^T, \quad \langle \xi \rangle = v_\xi (0, -\omega^2, \omega)^T, \quad (17)
 \end{aligned}$$

where  $v_{\phi_1}, v_{\varphi_1}$  and  $v_{\rho_1}$  are generally complex, while  $v_\varphi$  and  $v_\xi$  are real. Notice that the vacuum of the neutrino flavons  $\varphi$  and  $\xi$  preserve  $Z_2 \times H_{CP}^\gamma$  symmetry, where the residual GCP matrix is  $X_\nu = \rho(d) P_{23}$ . As a result, pattern D is naturally produced.

Leptons acquire masses after symmetry breaking. The charged lepton mass matrix is found to be diagonal with

$$\begin{aligned}
 m_e &= \left| y_{e_1} \frac{v_{\rho_1} v_{\phi_1}^2}{\Lambda^3} v + 2\omega y_{e_2} \frac{v_{\phi_1} v_{\varphi_1}^2}{\Lambda^3} v \right|, \\
 m_\mu &= \left| y_\mu \frac{v_{\phi_1} v_{\varphi_1}}{\Lambda^2} v \right|, \quad m_\tau = \left| y_\tau \frac{v_{\varphi_1}}{\Lambda} v \right|, \quad (18)
 \end{aligned}$$

in which  $v = \langle H \rangle = 175$  GeV. The neutrino mass matrix is of the form of Eq. (5) with

$$\alpha = e^{i\frac{\pi}{4}} \frac{y_\varphi v_\varphi v^2}{\Lambda^2}, \quad \beta = -\omega^2 \frac{y_\xi v_\xi v^2}{\Lambda^2}, \quad \gamma = -\epsilon = \omega \frac{y_\xi v_\xi v^2}{2\Lambda^2}. \quad (19)$$

The PMNS matrix is exactly pattern D, and the parameter  $\vartheta$  fulfills  $\tan \vartheta = y_\xi v_\xi / (2\sqrt{3} y_\varphi v_\varphi)$ . For the neutrino masses, we find that the neutrino mass spectrum can only be NO. A detailed calculation shows that

$$\frac{\Delta m_{21}^2}{\Delta m_{31}^2} = \frac{1}{2} - \frac{3 \sin^2 \vartheta - 1}{4 \sin \vartheta}. \quad (20)$$

To be compatible with data  $|\Delta m_{21}^2 / \Delta m_{31}^2| \approx 0.03$ , we find  $\vartheta \approx -0.351$ , or equivalently,  $y_\xi v_\xi \approx -1.27 y_\varphi v_\varphi$ , which leads to the predictions:

$$\begin{aligned}
 \theta_{13} &\approx 10.14^\circ, \quad \theta_{12} \approx 35.9^\circ, \quad \theta_{23} \approx 40.8^\circ, \\
 \delta &\approx 304.4^\circ, \quad \alpha_{21} \approx 222.3^\circ, \quad \alpha_{31} \approx 352.9^\circ. \quad (21)
 \end{aligned}$$

The lightest neutrino mass  $m_1$  and the effective mass  $\langle m \rangle_{ee}$  is fixed in this model,

$$m_1 \approx 0.0278 \text{ eV}, \quad \langle m \rangle_{ee} \approx 0.0112 \text{ eV}. \quad (22)$$

Table 2. Fields and their transformation properties under  $\Delta(48) \times Z_3 \times Z_4 \times Z_5$ , where  $\omega_5 = e^{i2\pi/5}$  and  $\ell_L = (\ell_{\tau L}, \ell_{\mu L}, \ell_{e L})^T$ .

fields	$\ell_L$	$e_R$	$\mu_R$	$\tau_R$	$H$	$\phi_1$	$\varphi_1$	$\rho_1$	$\varphi$	$\xi$
$\Delta(48)$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}'$	$\bar{\mathbf{3}}'$	$\mathbf{3}$	$\bar{\mathbf{3}}$
$Z_3$	$\omega^2$	$\omega$	1	$\omega^2$	1	1	$\omega^2$	$\omega$	$\omega$	$\omega$
$Z_4$	$-i$	1	$i$	$-1$	1	$i$	$i$	$i$	$-1$	$-1$
$Z_5$	1	$\omega_5^2$	1	$\omega_5^3$	1	$\omega_5^2$	$\omega_5^3$	$\omega_5^4$	1	1

## 5 Summary

In summary, we have proposed the discrete group  $\Delta(48)$  to explain lepton mixing angles and predict  $CP$ -violating phases in the framework of generalized  $CP$  symmetries.  $\Delta(48)$  has a large automorphism group and thus provides rich choices for GCP transformations. By systematically scanning all the possible symmetry breaking chains, we find 4 different mixing patterns compatible with experimental data. Among them, pattern D is a completely new mixing pattern that has not

been discussed in the literature. It predicts nontrivial  $CP$ -violating phases, which can be tested in the future neutrino oscillation and neutrinoless double-beta decay experiments. We have realized this pattern in an effective flavor model, where all the neutrino flavor mixing parameters, the absolute scale of neutrino masses and  $\langle m \rangle_{ee}$  are fixed.

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