

Study of fusion Q -value rule in sub-barrier fusion of heavy ions^{*}

LIU Xing-Xing(刘星星)^{1,2} ZHANG Gao-Long(张高龙)^{1,2;1)} ZHANG Huan-Qiao(张焕乔)³

¹ School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China

² International Research Center for Nuclei and Particles in the Cosmos, Beihang University, Beijing 100191, China

³ China Institute of Atomic Energy, Beijing 102413, China

Abstract: A vast body of fusion data has been analyzed for different projectiles and target nuclei. It is indicated that the sub-barrier fusion depends on the fusion Q -value. In terms of a recently introduced fusion Q -value rule and an energy scaling reduction procedure, the experimental fusion excitation functions are reduced and compared with each other. It is found that the reduced fusion excitations of selected fusion systems show a similar trend. The fusion data for massive nuclei are in agreement with the Q -value rule. In the fusion process, the Q contribution should be considered. Within this approach, the sub-barrier fusion cross sections of most fusion systems can be predicted without involving any structure effects of colliding nuclei. Instances of disagreement are presented in a few fusion systems. The use of the energy scaling as a criterion of possible experimental data inconsistency is discussed. More precise experimental fusion data need to be measured.

Key words: compound nuclei, sub-barrier fusion, new energy scaling

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1 Introduction

Heavy-ion fusion is one of the most interesting topics in nuclear physics [1]. Many experiments and theoretical calculations have provided rich information on heavy-ion sub-barrier fusion [2–4]. Study of fusion reaction mechanisms can improve the estimate of the product yield of heavy elements by modelling calculations, especially for synthesis of superheavy elements as well as the recent development of radioactive ion beams [5].

There are many experimental data on sub-barrier fusion. For example, in the fusion of $^{16}\text{O}+^{144,148,154}\text{Sm}$ [6] it was shown that nuclear deformation changes the barrier height of the fusion system. However, for $^{40,48}\text{Ca}+^{90,96}\text{Zr}$ systems, all with almost spherical nuclei, the measured fusion excitation functions and the shapes of deduced fusion barrier distributions show different behaviors [7]. When fitting the fusion excitation functions for these systems, if we not only consider the inelastic excitations of projectile and target nuclei but also insert the neutron transfer degree of freedom into the model, then the results of theoretical calculation can agree well with the experimental data. The fusion data for two symmetric calcium systems $^{40}\text{Ca}+^{40}\text{Ca}$ and $^{48}\text{Ca}+^{48}\text{Ca}$ were explained by coupled-channel calculations which include

coupling to one and two phonons, and mutual excitations [8]. However, for the asymmetric $^{40}\text{Ca}+^{48}\text{Ca}$ system it was necessary to include coupling to neutron transfer channels which have positive Q -values. Therefore, dynamic effects play an important role in the sub-barrier fusion process. Such fusion reactions have been explored in many experiments [9–13].

Theoretically, many models have been used to reproduce the fusion excitation functions and the barrier distributions such as the quantum molecular dynamic model [14], the time-dependent Hartree-Fock method [15], simplified semiclassical model [16] and the quantum diffusion approach model [17]. It has been pointed out in Ref. [18] that although the presented approaches succeed in reproducing fusion experiment data, all of them suffer from an apparent non-energy conservation paradox. The reason is that the energy balance is neglected in these studies which is equivalent to a tacit assumption of the fusion Q -value being zero. However, the fusion Q -value seems to be indispensable for sub-barrier fusion considerations. Fusion excitation functions for two hypothetical fusion systems with similar barrier heights but different fusion Q -values are schematically shown in Fig. 1. It is obvious that the excitation functions converge to zero at the energy threshold equal to $-Q$. It can be seen in Fig. 1

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1) E-mail: zgl@buaa.edu.cn

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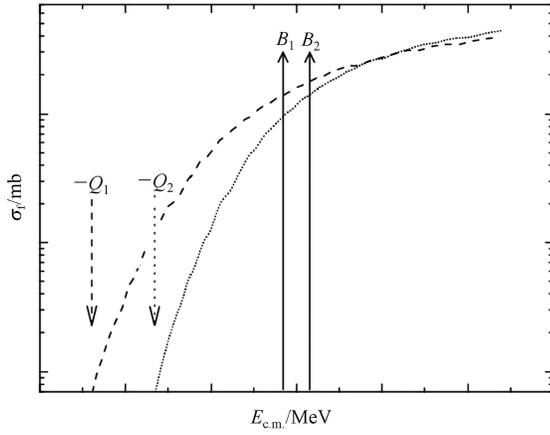


Fig. 1. General shapes of the fusion excitation functions for two hypothetical fusion systems with similar barrier heights B_1 and B_2 , and different fusion energy thresholds $-Q_1$ and $-Q_2$. Q stands for fusion Q -value.

that at enough low energy values the fusion cross section, due to its general properties, will always be larger for systems with lower thresholds. Here we neglect rare instances of a resonance-like shape of the fusion excitation function. So the influence of the Q -value on sub-barrier fusion needs to be systematically examined for

different fusion systems. In this paper we try to examine the behavior of maximally rich experimental fusion data in terms of a fusion Q -value rule and an associated energy parametrization to observe if there is a systematic trend that can describe the sub-barrier fusion process.

2 Methods

Figure 2 shows the fusion excitation functions for the $^{32}\text{S}+^{90,94,96}\text{Zr}$ [13, 19], $^{33}\text{S}+^{90,91,92}\text{Zr}$ [20], $^{36}\text{S}+^{90,96}\text{Zr}$ [21], $^{40}\text{Ca}+^{90,96}\text{Zr}$ [9], $^{48}\text{Ca}+^{90,96}\text{Zr}$ [7], $^{40}\text{Ca}+^{124,132}\text{Sn}$ [11, 12], $^{48}\text{Ca}+^{124,132}\text{Sn}$ [11], $^{16}\text{O}+^{58,62}\text{Ni}$ [22], $^{16,18}\text{O}+^{76,74}\text{Ge}$ [23], $^{16}\text{O}+^{144,148,154}\text{Sm}$ [6], $^{17}\text{O}+^{144}\text{Sm}$ [6] and $^{58,64}\text{Ni}+^{58,64}\text{Ni}$ [24, 25] systems. The corresponding values of the fusion energy balance $Q = M_c - M_p - M_t$ are shown in the picture, where M_c , M_p and M_t denote the masses of compound nucleus, projectile and target nuclei, respectively. In Fig. 2(a), we can see that, at sub-barrier energies, the fusion cross sections are very different among $^{40,48}\text{Ca}+^{90,96}\text{Zr}$. The fusion cross sections are enhanced with the increase of Q -values. That of $^{40}\text{Ca}+^{96}\text{Zr}$, with the largest Q -value, has the largest enhancement among all systems considered. That of $^{40}\text{Ca}+^{90}\text{Zr}$, with the smallest Q -value, is the weakest among them. This phenomenon can also be

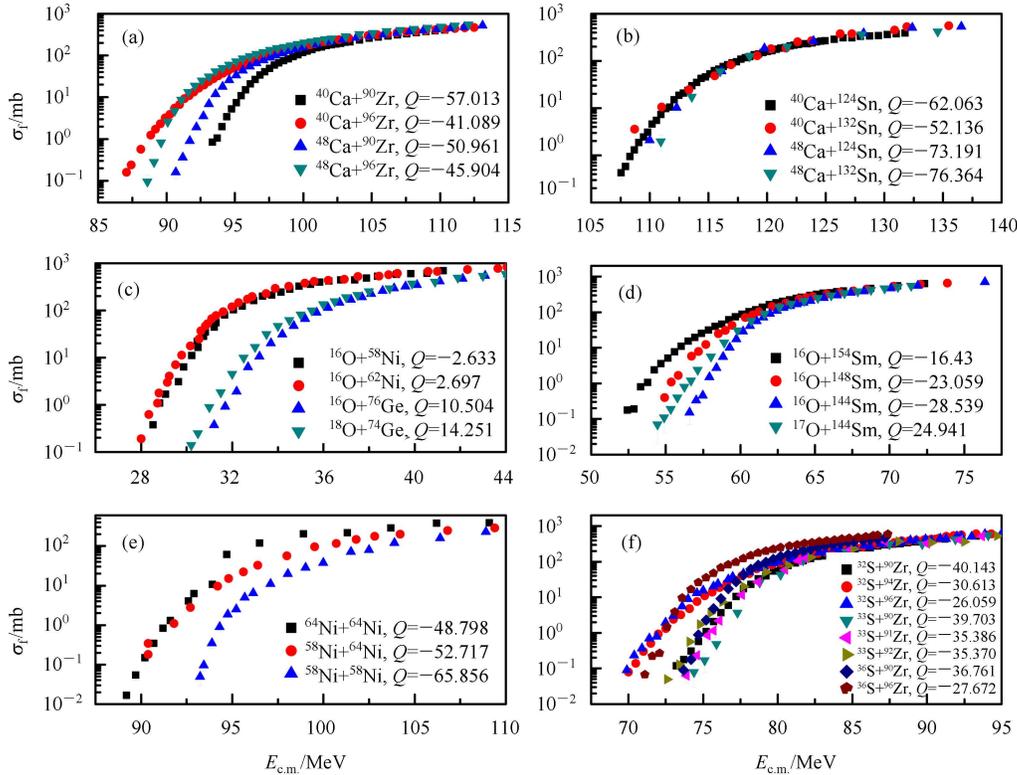


Fig. 2. (color online) Comparison of the fusion excitation functions with the different Q -values. (a) $^{40,48}\text{Ca}$ on $^{90,96}\text{Zr}$; (b) $^{40,48}\text{Ca}$ on $^{124,132}\text{Sn}$; (c) ^{16}O on $^{58,62}\text{Ni}$ and $^{16,18}\text{O}$ respectively on $^{76,74}\text{Ge}$; (d) ^{16}O on $^{144,148,154}\text{Sm}$ and ^{17}O on ^{144}Sm ; (e) $^{58,64}\text{Ni}$ on $^{58,64}\text{Ni}$; (f) $^{32,36}\text{S}$ on $^{90,96}\text{Zr}$ and ^{33}S on $^{90,91,92}\text{Zr}$.

clearly observed from the other fusion systems shown in Figs. 2(b), (d) and (e). In Fig. 2(f) there is a large difference in Q -values between $^{32}\text{S}+^{90}\text{Zr}$ and $^{32}\text{S}+^{96}\text{Zr}$. As a result, at sub-barrier energies their fusion cross sections have a large difference. That of $^{32}\text{S}+^{96}\text{Zr}$, which has the largest Q -value among them, shows the largest enhancement. The Q -value of $^{32}\text{S}+^{90}\text{Zr}$ is smallest, so the fusion cross-section of $^{32}\text{S}+^{90}\text{Zr}$ is also smallest. Q -value of $^{32}\text{S}+^{94}\text{Zr}$ lies between those of $^{32}\text{S}+^{96}\text{Zr}$ and $^{32}\text{S}+^{90}\text{Zr}$. The fusion cross sections of $^{32}\text{S}+^{94}\text{Zr}$ also lie between those of $^{32}\text{S}+^{96}\text{Zr}$ and $^{32}\text{S}+^{90}\text{Zr}$. The same phenomenon occurs for $^{33}\text{S}+^{90,91,92}\text{Zr}$ as well as $^{36}\text{S}+^{90}\text{Zr}$ and $^{36}\text{S}+^{96}\text{Zr}$. In addition, the difference of Q -values between $^{32}\text{S}+^{90}\text{Zr}$ and $^{32}\text{S}+^{96}\text{Zr}$ is larger than that between $^{36}\text{S}+^{90}\text{Zr}$ and $^{36}\text{S}+^{96}\text{Zr}$. As a result, at the sub-barrier energies, the difference of fusion cross sections between $^{32}\text{S}+^{90}\text{Zr}$ and $^{32}\text{S}+^{96}\text{Zr}$ is larger than that between $^{36}\text{S}+^{90}\text{Zr}$ and $^{36}\text{S}+^{96}\text{Zr}$. In Fig. 2(c) we can also see that the fusion cross sections are enhanced with the increase of Q -values for $^{16}\text{O}+^{58,62}\text{Ni}$, and between $^{16}\text{O}+^{76}\text{Ge}$ and $^{18}\text{O}+^{74}\text{Ge}$. From the above examples, we know that the larger the Q -value is, the larger the sub-barrier fusion cross section is for similar systems. The system with the largest Q -value shows the largest sub-barrier fusion cross section. Moreover, the larger the difference in Q -values between the similar systems is, the larger the difference of sub-barrier fusion cross sections is. Therefore, we think that the sub-barrier fusion depends on the Q -value. In the fusion process the Q contribution should be considered.

However, in Fig. 3 we show the fusion excitation functions of S+Ni systems with their Q -values. It is shown that $^{32}\text{S}+^{64}\text{Ni}$ has three sets of experimental data [26–28]. $^{32}\text{S}+^{58}\text{Ni}$ [26, 27], $^{34}\text{S}+^{64}\text{Ni}$ [26, 27] and $^{36}\text{S}+^{64}\text{Ni}$ [26, 29] have two sets of experimental data, respectively. The experimental data of $^{34}\text{S}+^{58}\text{Ni}$ and $^{36}\text{S}+^{58}\text{Ni}$ are from Ref. [26]. In the figure, the data from the first $^{32}\text{S}+^{58}\text{Ni}$ to the last $^{36}\text{S}+^{64}\text{Ni}$ [26, 29] and from the second $^{32}\text{S}+^{58}\text{Ni}$ to the last $^{34}\text{S}+^{64}\text{Ni}$ [27] were obtained at the same laboratory. The data of the last $^{32}\text{S}+^{64}\text{Ni}$ are from another laboratory [28]. In comparison with data, for the same fusion system, not only the experimental data but also the curvatures are different. Since the S+Ni data are obviously inconsistent, they do not obey the fusion Q -value rule. However, any set of data for S+Ni systems obtained at the same laboratory follow the fusion Q -value rule. Therefore, we think that this rule could serve as a criterion of experimental data consistency. The situation on the experimental data for S+Ni fusion has to be clarified.

From the above examples the fusion cross sections of most of the fusion systems are related to the Q -value. In order to take into account the Q -value explicitly, an energy reduced parameter E_r is introduced. As a result,

the experimental energy $E_{c.m.}$ is replaced by a reduced energy parameter E_r [18] given by

$$E_r = \frac{E_{c.m.} + Q}{V_c + Q}, \quad (1)$$

where Q is the reaction energy of the fusion reaction as defined before, $E_{c.m.}$ is the incident energy of the fusion system in the center of mass frame. V_c is the Coulomb barrier defined as $V_c = \frac{Z_p Z_t e^2}{R}$, where Z_p and Z_t are the charge numbers of projectile and target nuclei, respectively, $e^2 = 1.44 \text{ MeV}\cdot\text{fm}$, $R = r_0 (A_p^{1/3} + A_t^{1/3})$, where A_p and A_t are the mass numbers of projectile and target nuclei, respectively, and r_0 is the reduced radius. In Eq. (1) the only free parameter is r_0 . We use Eq. (1) to calculate E_r of the different fusion systems, and then compare their fusion cross sections as a function of E_r .

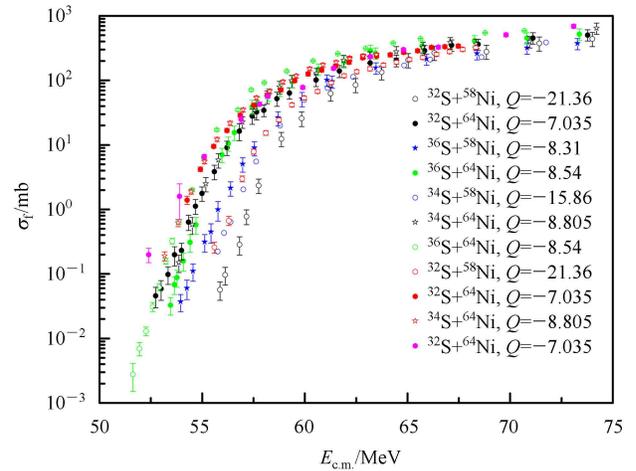


Fig. 3. (color online) Comparison of the fusion excitation functions for $^{32}\text{S}+^{58}\text{Ni}$, $^{32}\text{S}+^{64}\text{Ni}$, $^{34}\text{S}+^{58}\text{Ni}$, $^{34}\text{S}+^{64}\text{Ni}$, $^{36}\text{S}+^{58}\text{Ni}$ and $^{36}\text{S}+^{64}\text{Ni}$.

3 Results and discussions

According to Eq. (1), we calculate E_r for the systems shown in Fig. 2 and in addition $^{32,36}\text{S}+^{110}\text{Pd}$ [10], $^{16}\text{O}+^{60,62,64}\text{Ni}$ [22, 30, 31], $^{18}\text{O}+^{58,60,64}\text{Ni}$ [30, 31] and ^{118}Sn [32], $^{16}\text{O}+^{186}\text{W}$ [6], $^{58,64}\text{Ni}+^{124,132}\text{Sn}$ [33] as well as $^{64}\text{Ni}+^{118}\text{Sn}$ [33]. Here the fusion systems from light to heavy were included. Fig. 4 shows their reduced fusion cross sections $\sigma_F / ((A_p)^{1/3} + (A_t)^{1/3})^2$ as a function of E_r . In Fig. 4(a) when r_0 is changed from $0.92 \times 1.44 \text{ fm}$ to $0.95 \times 1.44 \text{ fm}$, the fusion excitation function can be adjusted to a similar band except those of $^{36}\text{S}+^{90,96}\text{Zr}$. Under this condition, the reduced fusion cross sections of $^{36}\text{S}+^{90,96}\text{Zr}$ are lower than those of the others at sub-barrier energies and are a little higher than those of the others above the barrier energies. Is the ^{36}S nucleus much different from ^{32}S to cause less fusion below the barrier

and more fusion above the barrier? We do not know what the reason is for $^{36}\text{S}+^{90,96}\text{Zr}$. Maybe it is due to the experiment or other effects. If all data sets for $^{32,36}\text{S}+\text{Zr}$ fusion are consistent, then one has to suggest that the ^{36}S nucleus is much different from ^{32}S to cause less fusion below the barrier and more fusion above the barrier. Meanwhile, in Fig. 4(c), (d) and (e), when r_0 is approximately changed from 0.92×1.44 fm to 0.95×1.44 fm, all of the fusion systems can be adjusted to the similar band. In particular, for $^{16}\text{O}+^{144,148,154}\text{Sm}$ and $^{17}\text{O}+^{144}\text{Sm}$ systems r_0 is only fixed to 0.95×1.44 fm. However, for the light systems shown in Fig. 4(b), in order for different systems to follow a similar band, r_0 is adjusted from 0.85×1.44 fm to 0.90×1.44 fm except $^{16}\text{O}+^{58}\text{Ni}$ and $^{18}\text{O}+^{118}\text{Sn}$ which have negative Q -values, with other O+A systems having positive Q -values. In Fig. 4(b), at sub-barrier energies, the fusion excitation functions of $^{16}\text{O}+^{58,62}\text{Ni}$, $^{16,18}\text{O}+^{76,74}\text{Ge}$ and $^{18}\text{O}+^{118}\text{Sn}$ have almost no difference. The experimental data of several other systems are only available above the Coulomb barrier. In the present survey $^{18}\text{O}+\text{Ni}$ data are included. One should bear in mind that these data demonstrate the opposite trend to the Q -value rule for the fusion cross

sections below the barrier. The fusion cross sections are lower for systems of higher Q -values. The data seem to follow positive Q -value of 2n transfer with the highest cross section for the highest 2n Q -value. It is an open question whether this behavior is due to a limitation of fusion Q -value rule for light systems or due to the quality of the data. Therefore, it would be desirable to improve the quality of $^{18}\text{O}+\text{Ni}$ fusion data by extending appropriate measurements at the sub-barrier energy region. In Fig. 4, we can see that the E_r values of all the systems start from almost 0.7 except that of Fig. 4(b), which is from about 0.75.

In order to further explore the light fusion systems, we select $^{28}\text{Si}+^{24,26}\text{Mg}$ [34], $^{58,62,64}\text{Ni}$ [35, 36] and ^{30}Si [37] fusion systems to obtain the reduced excitation functions shown in Fig. 5(a). The r_0 values of $^{28}\text{Si}+^{24,26}\text{Mg}$, $^{62,64}\text{Ni}$ are 0.95×1.44 fm. Those of $^{28}\text{Si}+^{58}\text{Ni}$ and $^{28}\text{Si}+^{30}\text{Si}$ are 0.97×1.44 fm and 0.93×1.44 fm, respectively. It is indicated that after using the new scaling method, the reduced fusion excitation functions can almost all keep the similar band except the $^{28}\text{Si}+^{26}\text{Mg}$ system which shows a deviation from the others. The Q -value of the $^{28}\text{Si}+^{24}\text{Mg}$ fusion system, at 12.905 MeV, is less than that of the

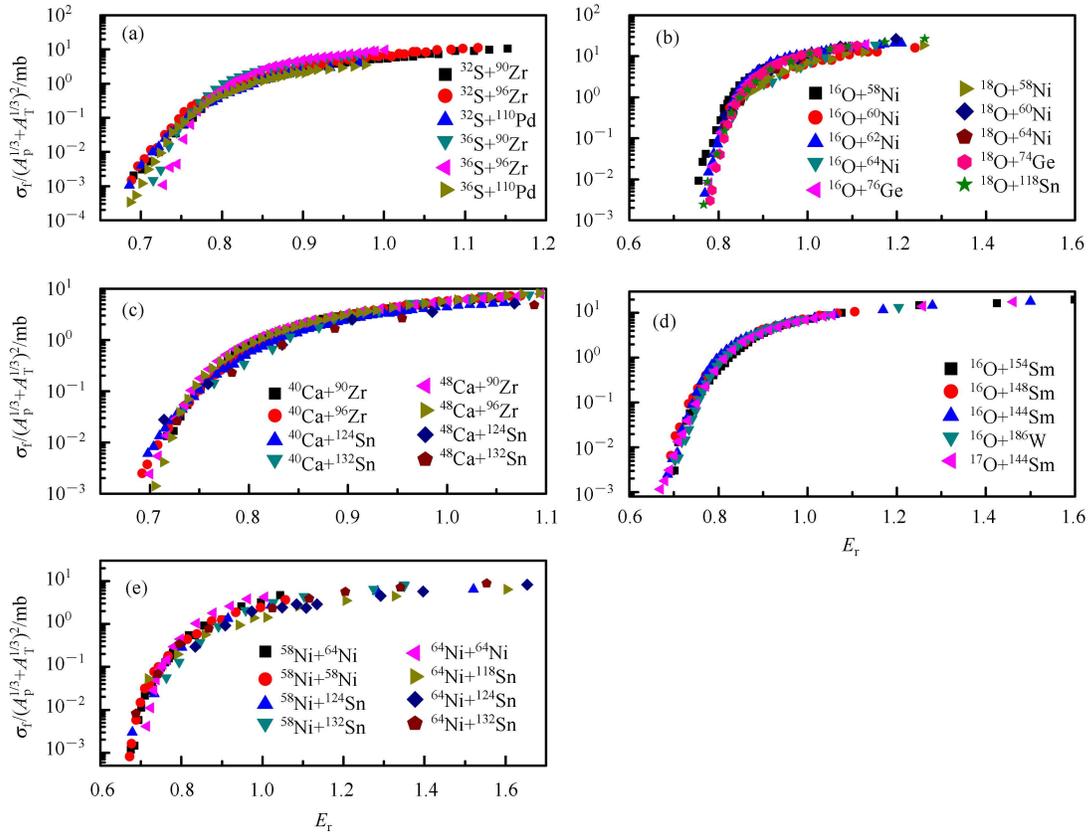


Fig. 4. (color online) The reduced fusion excitation functions in terms of the reduced energy parameter E_r . (a) $^{32,36}\text{S}$ on $^{90,96}\text{Zr}$ and ^{110}Pd ; (b) $^{16,18}\text{O}$ on $^{58,60,62,64}\text{Ni}$ and $^{76,74}\text{Ge}$ as well as ^{18}O on ^{118}Sn ; (c) $^{40,48}\text{Ca}$ on $^{90,96}\text{Zr}$ and $^{124,132}\text{Sn}$; (d) ^{16}O on $^{144,148,154}\text{Sm}$ and ^{17}O on ^{144}Sm ; (e) $^{58,64}\text{Ni}$ on $^{58,64}\text{Ni}$ and $^{58,64}\text{Ni}$ on $^{124,132}\text{Sn}$ as well as ^{64}Ni on ^{118}Sn .

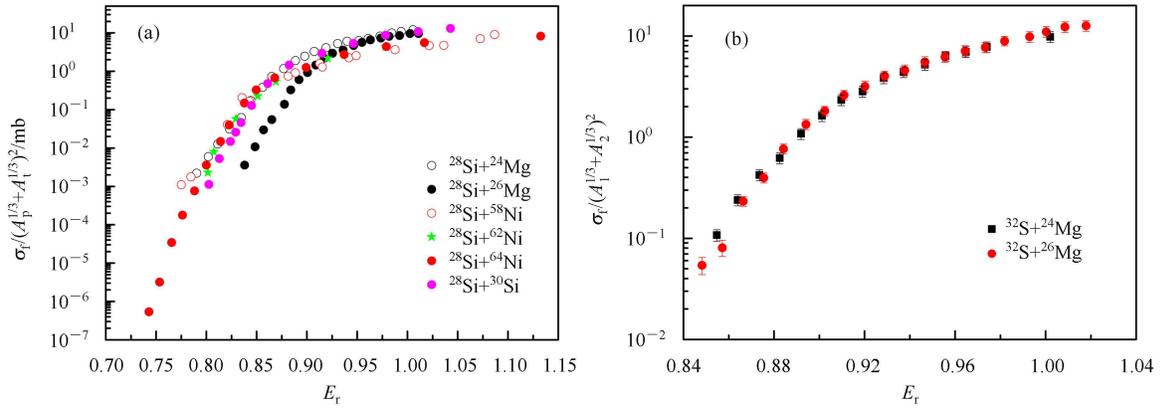


Fig. 5. (color online) Comparison of the reduced fusion excitation functions via a variable of the reduced c.m. energies E_r . (a) Among $^{28}\text{Si}+^{24,26}\text{Mg}$ [34], $^{58,62,64}\text{Ni}$ [35, 36] and ^{30}Si [37]; (b) between $^{32}\text{S}+^{24}\text{Mg}$ and $^{32}\text{S}+^{26}\text{Mg}$ [38].

$^{28}\text{Si}+^{26}\text{Mg}$ fusion system at 18.544 MeV. However, the fusion cross sections of $^{28}\text{Si}+^{24}\text{Mg}$ are larger than those of $^{28}\text{Si}+^{26}\text{Mg}$. It is opposite to those of Fig. 2. Is the structure of ^{26}Mg different from that of ^{24}Mg ? However, in Fig. 5(b) the reduced excitation functions of $^{32}\text{S}+^{24,26}\text{Mg}$ [38] are similar when their r_0 values are 0.955×1.44 fm and 0.95×1.44 fm, respectively. Moreover, there is little difference between their r_0 values. Therefore, $^{32}\text{S}+^{24,26}\text{Mg}$ fusion systems obey the fusion Q -value rule. We can also see that the E_r values of several systems are more than 0.75. Thus, when considering the condition of Fig. 4(b), we think that, when keeping the above r_0 values, the trend of the fusion excitation function for light fusion systems is different from that of heavy fusion systems. If one wants to keep a similar band between them, for light fusion systems the change range of r_0 is larger with respect to that of heavy fusion systems. There is no regularity. For heavy fusion systems the change of r_0 value is generally from 0.93×1.44 fm to 0.97×1.44 fm, which fluctuates around 0.95×1.44 fm. Of course, in the light mass region, many experiments need to be done to clearly describe this process. There is a need for more precise experimental fusion cross section data for light nuclear systems.

In Fig. 4 it is shown that at sub-barrier energies, the reduced fusion excitation functions of similar systems can be adjusted to a similar band by changing r_0 . The large enhancement of the sub-barrier fusion cross sections which is shown in Fig. 2 is reduced, in some cases even by one or two orders of magnitude. As a result, according to the above analysis, when considering the Q dependence of the fusion cross section, the new energy scaling method can keep the similarity of fusion excitation function for similar systems, especially at sub-barrier energies. It further systematically shows that the characteristic of compound nucleus nature mainly affects

the sub-barrier fusion reaction. A little fluctuation of fusion cross sections comes from other mechanisms which are left in a small space from the compound nucleus. From the compound nucleus nature it means that, during the contact time between the colliding nuclei, they have enough interaction opportunity for mutual excitation and exchanging of nucleons to keep the balance of the collective degree of freedom. Eventually the system of colliding nuclei reaches a balance to fuse into a compound nucleus. Therefore, the fusion is a long-lasting process. Due to the effect of the Q -value of the fusion reaction, the barrier height and the incident energy may be changed during the fusion process. When the ratios of the change of the incident energies to that of the barrier heights for the different fusion systems are similar, the sub-barrier fusion cross sections as a function of E_r follow the same band. Hence, the Q -value effect cannot be omitted in the sub-barrier fusion process.

In order to check if the heavy fusion systems shown in Fig. 4 show a systematic behavior, we select several systems from Fig. 4(a),(c),(d) and (e) to compare their reduced fusion excitation function, which is shown in Fig. 6. It is obvious that at sub-barrier energies they are almost the same. This indicates that all other heavy fusion systems also follow a similar band. A systematic trend can describe the sub-barrier fusion cross sections of heavy fusion systems. However, above the Coulomb barrier all systems follow the same band except $^{58}\text{Ni}+^{64}\text{Ni}$. Therefore, we think that the other Ni+A systems also cannot follow the same band with the other systems. Then we select the systems of Fig. 4 (a), (c) and (d) to plot the reduced fusion cross section as a function of the variable E_r , which is shown in Fig. 7. It can be seen that all of the new fusion excitation functions follow a common trend with little difference between each other. It means that a characteristic of compound nucleus nature lies in

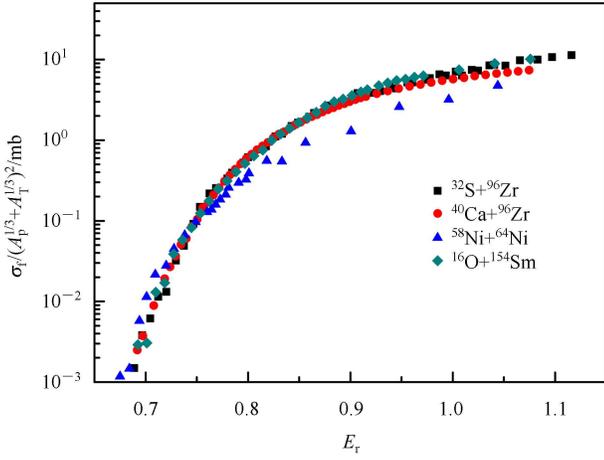


Fig. 6. (color online) Comparison of the reduced fusion excitation functions via a variable of the reduced c.m. energies E_r among ^{32}S on ^{96}Zr , ^{40}Ca on ^{96}Zr , ^{58}Ni on ^{64}Ni and ^{16}O on ^{154}Sm .

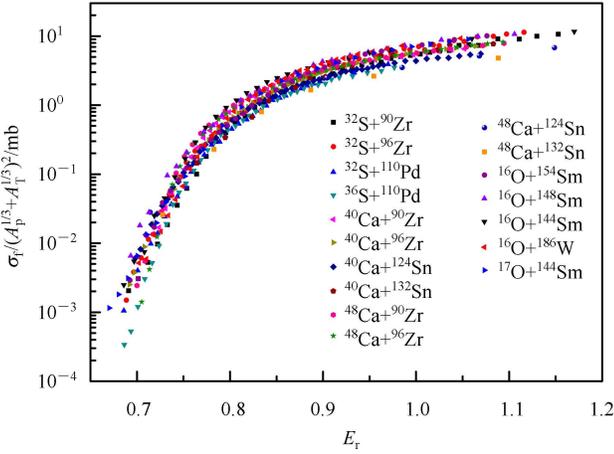


Fig. 7. (color online) Comparison between the present experiment data and normalized previous data in the laboratory system.

the fusion. The Q -value effect should be considered in the fusion process. However, for much heavier fusion systems, for example $^{16}\text{O} + ^{208}\text{Pb}$ [39], $^{36}\text{S} + ^{204,208}\text{Pb}$ [40, 41], $^{40}\text{Ar} + ^{144,154}\text{Sm}$ [42], $^{40}\text{Ca} + ^{124}\text{Sn}$ [43] and $^{48}\text{Ca} + ^{154}\text{Sm}$ [44], their reduced fusion excitation functions do not keep the similar band when r_0 is changed from $0.93 \times 1.44\text{fm}$ to $0.95 \times 1.44\text{fm}$, which is shown in Fig. 8. The present one-parameter reduced method is not enough to describe data for much heavier fusion systems. However, we can see that the reduced fusion excitation functions of $^{48}\text{Ca} + ^{154}\text{Sm}$ and $^{40}\text{Ar} + ^{144,154}\text{Sm}$ keep the same band. Those of $^{36}\text{S} + ^{204,208}\text{Pb}$ keep the same band, also those of $^{16}\text{O} + ^{208}\text{Pb}$ and $^{40}\text{Ca} + ^{124}\text{Sn}$ keep the same band. It is like a total mass curvature dependence for heavier fusion systems. Therefore, for very heavy systems the application of this scaling cannot be straightforward since

the drop model limitation of angular momentum for a compound nucleus exists and it strongly depends on the mass of the compound nucleus. According to this trend, in addition to the behavior of Ni+A systems at the sub-barrier energy region, this new energy scaling method can be used to predict sub-barrier fusion cross sections for most heavy fusion systems. For light fusion systems it also seems to show a systematic trend. However, more experimental data are needed.

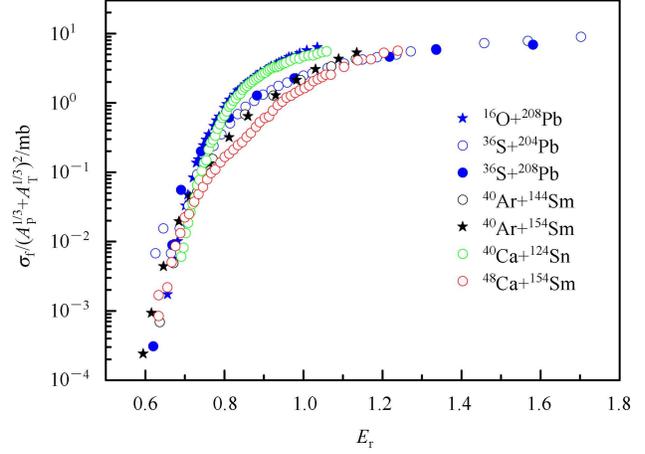


Fig. 8. (color online) The reduced fusion excitation functions via a variable of the reduced c.m. energies E_r for $^{16}\text{O} + ^{208}\text{Pb}$ [39], $^{36}\text{S} + ^{204,208}\text{Pb}$ [40, 41], $^{40}\text{Ar} + ^{144,154}\text{Sm}$ [42], $^{40}\text{Ca} + ^{124}\text{Sn}$ [43] and $^{48}\text{Ca} + ^{154}\text{Sm}$ [44].

4 Summary

The experimental fusion excitation functions are analyzed for thirty-eight fusion systems which have very different fusion energy Q -values. It has been shown that fusion cross sections depend generally on the Q -value at colliding energies below the Coulomb barriers. According to the new energy scaling method, the incident energy $E_{c.m.}$ is replaced by E_r defined in Eq. (1). The rescaled fusion cross sections as a function of variable E_r are systematically explored. The scaling allows one to reduce fusion experimental data to a quasi-universal curve for similar systems. The behavior of the new fusion excitation function shows that the characteristic of compound nucleus nature mainly affects the sub-barrier fusion process. It has been found that the fusion excitation functions of a few fusion systems do not obey the proposed Q -value rule. Two cases are discussed: the $^{18}\text{O} + \text{Ni}$ data, showing an anti-trend with respect to the fusion Q -value rule, and the data of $^{28}\text{Si} + ^{24,26}\text{Mg}$. If the situation with $^{18}\text{O} + \text{Ni}$ data can be clarified by improving the experimental data, the latter case demonstrates a drastic breaking of the Q -value rule. It is interesting

that very similar systems $^{32}\text{S}+^{24,26}\text{Mg}$ obey the Q -value rule. For much heavier fusion systems, the new scaling method is not enough to describe the fusion data. It seems to show a total mass curvature dependence. The fusion Q -value rule and the associated energy scaling

can serve as a hint of possible data inconsistency due to its predictive power.

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