Particle acceleration by stimulated emission of radiation in cylindrical waveguide *

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Abstract: In particle acceleration by stimulated emission of radiation (PASER), efficient interaction occurs when a train of micro-bunches has periodicity identical to the resonance frequency of the medium. Previous theoretical calculations based on the simplified model have only considered the energy exchange in the boundless condition. Under experimental conditions, however, the gas active medium must be guided by the metal waveguide. In this paper, we have developed a model of the energy exchange between a train of micro-bunches and a gas mixture active medium in a waveguide boundary for the first time, based on the theory of electromagnetic fields, and made detailed analysis and calculations with MathCAD. The results show that energy density can be optimized to a certain value to get the maximum energy exchange.

Key words: PASER, train of micro-bunches, active gas mixture medium, cylinder waveguide

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1 Introduction

The great demands of high-energy physics, lasers, medicine and material science have spurred research into developing a new generation of compact low-cost table-top particle accelerators with widespread applications in various fields. Particle acceleration by stimulated emission of radiation (PASER) is one of the most promising ways to achieve this goal. In PASER, energy stored in an active medium is transferred directly to the electrons traversing the medium, and therefore, accelerating the former. PASER does not need a high power beam driver, and thus neither phase matching nor compensating for phase slippage are required. Moreover, an electron gun is also not needed in PASER because of its capability of generating its own micro-bunched electron beam source in a Penning trap. Especially, with this low-cost, highly compacted and simple structure, a PASER acceleration gradient of the order of 1 GV/m could be feasibly obtained.

Since 1995, Levi Schachter and his coworkers have made a series of theory analyses and calculations about the PASER process. In 2006 [1], a proof of the principle experiment was carried out at the Brookhaven National Laboratory Accelerator Test Facility, with a photocathode-driven microwave linear accelerator and a high peak power CO_2 laser. This was the first experimental result which showed the feasibility of the PASER. Recently [2, 3], Miron Voin studied the wake generated by electron bunches in a CO_2 gas mixture active medium, and the results show that a wake accelerating gradient can be achieved of the order of GV/m.

In the PASER process, efficient interaction occurs only under the resonance condition. In 2006, Samer Banna [4] and coworkers made a theoretical calculation with the PASER process based on a simplified model in which they only considered the energy exchange between the train of electrons and the gas mixture active medium in the boundless condition. Considering the experimental situation, however, the gas active medium needs to be guided by a cylindrical waveguide. In the following section, we develop a 2D model, deduce the formula of the energy exchange between the train of micro-bunches and the gas mixture active medium in the waveguide boundary, and make a series of analyses and calculations with MathCAD [5] on the influence of various parameters on the gain of the micro-bunches.

2 Model description

We consider an electron bunch which consists of M micro-bunches [6], and assume that each micro-bunch is azimuthally symmetric and has a radius R_b and length Δ, carries charge -Q and moves at a constant velocity v_0.

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The distance between two adjacent micro-bunches is λ_0. The radius of the waveguide is $R_{\rm w}$. We denote by z_μ the longitudinal coordinate of the center of the μth micro-bunch at t=0, as illustrated in Fig. 1. In the framework of this study, the medium is assumed to remain in the linear regime throughout the interaction region.

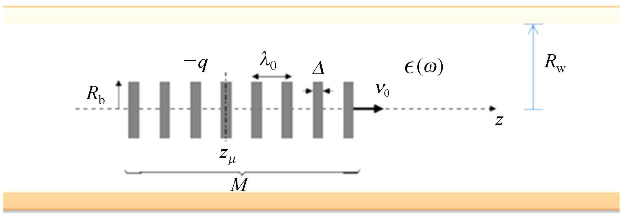

Fig. 1. (color online) Schematic of beam-active medium interaction in a waveguide.

The moving train of micro-bunches generates a current density in the longitudinal direction z-axis that is given in the time domain by

$$J_z(r,z;t) = -qv_0\sum_{\mu=1}^{M}\frac{1}{2\pi}\left[\frac{2}{R_{\rm b}^2}h(R_{\rm b}-r)\right] \times\left\{\frac{1}{\Delta}h\left[\frac{\Delta}{2}[z-(z_\mu+v_0t)]\right]\right\}. \quad (1)$$

For a conceptual description, we consider the wake generated by a thin charged loop ($Q_{\rm b}$) of radius R_σ, located at t=0 at z_σ with velocity υ_0. The current generated by this loop is

$$J_z(r,z;t)=\frac{-Q_{\rm b}v_0}{2\pi r}\delta(r-R_\sigma)\delta(z-z_\sigma-v_0t). \quad (2)$$

This current density excites the z component of the magnetic vector potential which, in turn, satisfies the inhomogeneous wave equation. Hence, in the frequency domain, the magnetic vector potential is given by

$$A_z(r,z;\omega) = -\frac{\mu_0Q_{\rm b}}{(2\pi)^2}\sum_s\frac{J_0\left(p_s\dfrac{r}{R_{\rm w}}\right)J_0\left(p_s\dfrac{R_\sigma}{R_{\rm w}}\right)}{\dfrac{1}{2}R_{\rm w}^2J_1^2(p_s)} \times\int_{-\infty}^{\infty}{\rm d}\omega\frac{{\rm e}^{-{\rm j}\left(\dfrac{\omega}{v_0}\right)(z-z_\sigma)}}{-\dfrac{p_s^2}{R_{\rm w}^2}-\dfrac{\omega^2}{\upsilon_0^2}+\dfrac{\omega^2}{c^2}\epsilon(\omega)}. \quad (3)$$

So the longitudinal electric field in the frequency domain is

$$E_z(r,z;\omega) = -\frac{\mu_0Q_{\rm b}}{(2\pi)^2}\sum_s\frac{J_0\left(p_s\dfrac{r}{R_{\rm w}}\right)J_0\left(p_s\dfrac{R_\sigma}{R_{\rm w}}\right)}{\dfrac{1}{2}R_{\rm w}^2J_1^2(p_s)} \times\int_{-\infty}^{\infty}{\rm d}\omega\frac{-{\rm j}\omega\left[1-\dfrac{1}{\beta^2\epsilon(\omega)}\right]{\rm e}^{-{\rm j}\left(\dfrac{\omega}{v_0}\right)(z-z_\sigma)}}{-\dfrac{p_s^2}{R_{\rm w}^2}-\dfrac{\omega^2}{\upsilon_0^2}+\dfrac{\omega^2}{c^2}\epsilon(\omega)}. \quad (4)$$

Based on the Green theory, the electric field generated by the train of micro-bunches is expressed by

$$E_z(r,z;\omega) = -\frac{\mu_0q}{(2\pi)^2}\sum_{\mu=1}^{M}\sum_s\frac{J_{\rm c}^2\left(p_s\dfrac{R_{\rm b}}{R_{\rm w}}\right)}{\dfrac{1}{2}R_{\rm w}^2J_1^2(p_s)}{\rm sinc}\left(\frac{\omega_\Delta}{2v_0}\right) \times\int_{-\infty}^{\infty}{\rm d}\omega\frac{\dfrac{c^2}{{\rm j}\omega\epsilon(\omega)}\left[\dfrac{\omega^2}{c^2}\epsilon(\omega)-\dfrac{\omega^2}{\upsilon^2}\right]}{-\dfrac{p_s^2}{R_{\rm w}^2}-\dfrac{\omega^2}{\upsilon^2}+\dfrac{\omega^2}{c^2}\epsilon(\omega)}{\rm e}^{-{\rm j}\left(\frac{\omega}{v_0}\right)(z-z_\mu)}. \quad (5)$$

Using the Fourier transform, we can get the electric field in the time domain

$$E_z(r,z;t) = -\frac{\mu_0q}{(2\pi)^2}\sum_{\mu=1}^{M}\sum_s\frac{J_{\rm c}^2\left(p_s\dfrac{R_{\rm b}}{R_{\rm w}}\right)}{\dfrac{1}{2}R_{\rm w}^2J_1^2(p_s)}{\rm sinc}\left(\frac{\omega_\Delta}{2v_0}\right) \times\int_{-\infty}^{\infty}{\rm d}\omega\frac{\dfrac{c^2}{{\rm j}\omega\epsilon(\omega)}\left[\dfrac{\omega^2}{c^2}\epsilon(\omega)-\dfrac{\omega^2}{\upsilon^2}\right]}{-\dfrac{p_s^2}{R_{\rm w}^2}-\dfrac{\omega^2}{\upsilon^2}+\dfrac{\omega^2}{c^2}\epsilon(\omega)}{\rm e}^{\omega\left(t-\frac{z-z_\mu}{v_0}\right)}. \quad (6)$$

Using the term for the longitudinal electric field as determined in Eq. (6), and the expression for the current density Eq. (1), and integrating the power density over the beam volume, we get the total power exchange in the interaction process.

$$\begin{aligned}P(t) &= -2\pi\int_0^\infty r{\rm d}r\int_{-\infty}^{\infty}J_z(r,z;t)E_z(r,z;t){\rm d}z\\ &= -2\pi\int_0^\infty r{\rm d}r\int_{-\infty}^{\infty}{\rm d}zqv_0\sum_{\mu'=1}^{M}\frac{1}{2\pi}\left[\frac{2}{R_{\rm b}^2}h(R_{\rm b}-r)\right]\\ &\quad\times\left\{\frac{1}{\Delta}h\left[\frac{\Delta}{2}[z-(z_{\mu'}+v_0t)]\right]\right\}\\ &\quad\times\frac{\mu_0q}{(2\pi)^2}\sum_{\mu=1}^{M}\sum_s\frac{J_{\rm c}^2\left(p_s\dfrac{R_{\rm b}}{R_{\rm w}}\right)}{\dfrac{1}{2}R_{\rm w}^2J_1^2(p_s)}{\rm sinc}\left(\frac{\omega_\Delta}{2v_0}\right)\\ &\quad\times\int_{-\infty}^{\infty}{\rm d}\omega\frac{\dfrac{c^2}{{\rm j}\omega\epsilon(\omega)}\left[\dfrac{\omega^2}{c^2}\epsilon(\omega)-\dfrac{\omega^2}{\upsilon^2}\right]}{-\dfrac{p_s^2}{R_{\rm w}^2}-\dfrac{\omega^2}{\upsilon_0^2}+\dfrac{\omega^2}{c^2}\epsilon(\omega)}{\rm e}^{{\rm j}\omega\left(t-\frac{z-z_\mu}{v_0}\right)}. \quad (7)\end{aligned}$$

Integrating on the horizontal and vertical respectively, we can obtain the simplified expression as follows

$$P(t) = -2\pi q v_0 \frac{1}{2\pi^2} \frac{\mu_0 q}{(2\pi)^2} \sum_{\mu'=1}^{M} \sum_{\mu=1}^{M} \sum_{s} \frac{J_c^2\left(p_s \frac{R_b}{R_w}\right)}{\frac{1}{2} R_w^2 J_1^2(p_s)} \times \text{sinc}\left(\frac{\omega_\Delta}{2v_0}\right) \int_{-\infty}^{\infty} d\omega \frac{\frac{c^2}{j\omega\epsilon(\omega)}\left[\frac{\omega^2}{c^2}\epsilon(\omega) - \frac{\omega^2}{v^2}\right]}{-\frac{p_s^2}{R_w^2} - \frac{\omega^2}{v_0^2} + \frac{\omega^2}{c^2}\epsilon(\omega)} \times e^{j\omega \frac{z_{\mu'} - z_\mu}{v_0}} \text{sinc}\left(\frac{\omega \varDelta}{2v_0}\right), \tag{8}$$

where $z_\mu = z_1 + (\mu - 1)\lambda_0$, $\mu = 1, 2, \cdots, M$, $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the free space impedence. The double averaging over the longitudinal initial location of the micro-bunches may be simplified, to read

$$\sum_{\mu'=1}^{M} \sum_{\mu=1}^{M} e^{-j\left(\frac{\omega}{v_0}\right)(z_{\mu'} - z_\mu)} = M^2 \frac{\text{sinc}^2\left(\frac{\omega\lambda_0 M}{2v_0}\right)}{\text{sinc}^2\left(\frac{\omega\lambda_0}{2v_0}\right)}. \tag{9}$$

So the total power exchange is

$$P(t) = -\frac{Q^2 v_0 \mu_0}{2\pi^2 R_w^2} \sum_s \frac{J_c^2\left(p_s \frac{R_b}{R_w}\right)}{J_1^2(p_s)} \times \int_{-\infty}^{\infty} d\omega \frac{\frac{c^2}{j\omega\epsilon(\omega)}\left[\frac{\omega^2}{c^2}\epsilon(\omega) - \frac{\omega^2}{v^2}\right]}{-\frac{p_s^2}{R_w^2} - \frac{\omega^2}{v_0^2} + \frac{\omega^2}{c^2}\epsilon(\omega)} \times M^2 \frac{\text{sinc}^2\left(\frac{\omega\lambda_0 M}{2v_0}\right)}{\text{sinc}^2\left(\frac{\omega\lambda_0}{2v_0}\right)} \text{sinc}^2\left(\frac{\omega\varDelta}{2v_0}\right). \tag{10}$$

So the energy exchange during the passage d is

$$W = -\frac{Q^2 d \eta_0^2}{2\pi^2 R_w^2} \sum_s \frac{J_c^2\left(p_s \frac{R_b}{R_w}\right)}{J_1^2(p_s)} \times \int_{-\infty}^{\infty} d\omega \frac{j\omega\left[\epsilon_r(\omega) - \frac{1}{\beta^2}\right]}{\epsilon_r(\omega)\left[-\frac{p_s^2}{R_w^2} - \frac{\omega^2}{v_0^2} + \frac{\omega^2}{c^2}\epsilon_r(\omega)\right]} \times M^2 \frac{\text{sinc}^2\left(\frac{\omega\lambda_0 M}{2v_0}\right)}{\text{sinc}^2\left(\frac{\omega\lambda_0}{2v_0}\right)} \text{sinc}^2\left(\frac{\omega\varDelta}{2v_0}\right). \tag{11}$$

Considering the gas mixture active medium which is represented by a dielectric function

$$\epsilon_r(\omega > 0) \equiv 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2 + 2j\omega/T_2}. \tag{12}$$

It is convenient to introduce a set of normalized quantities as follows:

$$\bar{R}_b = \frac{R_b}{\lambda_0},\ \bar{\varDelta} = \frac{\varDelta}{\lambda_0}, \varOmega = \omega\frac{\lambda_0}{c},\ \varphi = \frac{\varOmega}{2\beta},\ Q_{\text{total}} = MQ$$

$$F_\parallel(\varphi, \bar{\varDelta}, M) = \text{sinc}^2(\varphi\bar{\varDelta}) \frac{\text{sinc}^2(\varphi M)}{\text{sinc}^2(\varphi)}$$

$$F_\perp(\varOmega) = \frac{1}{-\frac{p_s^2}{R_w^2} - \frac{\varOmega^2}{\lambda_0^2\beta^2} + \frac{\omega^2}{c^2}\epsilon_r(\varOmega)}.$$

Then the normalized dielectric function is:

$$\epsilon_r(\varOmega > 0) = 1 + \frac{\varOmega_p^2}{(2\pi)^2 - \varOmega^2 + 2j\alpha\varOmega},$$

in which

$$\alpha = \lambda_0 / cT_2,\quad \varOmega_p^2 = \omega_p^2 \lambda_0^2 / c^2.$$

Evaluating the integral only at the poles of the dielectric function

$$\epsilon_r(\varOmega) = 0, \tag{13}$$

we obtain

$$\varOmega = \varOmega_\pm = j\alpha \pm \sqrt{(2\pi)^2 + \varOmega_p^2 - \alpha^2} = j\alpha \pm \varOmega_R, \tag{14}$$

where

$$\varOmega_R = \sqrt{(2\pi)^2 + \varOmega_p^2 - \alpha^2}.$$

Using the Cauchy residue theorem, the energy exchange reads

$$W = -\frac{Q_{\text{total}}^2 d \eta_0^2 c}{\pi R_w^2 \lambda_0 \beta^2} \frac{\varOmega_p^2}{\varOmega_R} \sum_s \frac{J_c^2\left(p_s \frac{R_b}{R_w}\right)}{J_1^2(p_s)}. \times F_\parallel\left(\frac{\varOmega_R}{2\beta}, \bar{\varDelta}, M\right) \text{Re}[\varOmega_+ F_\perp(\varOmega_+)]. \tag{15}$$

So the energy obtained by the electron micro-bunches $\Delta E_k = -W$ is

$$\Delta E_k = -\frac{Q_{\text{total}}^2 d \eta_0^2 c}{\pi R_w^2 \lambda_0 \beta^2} \frac{\varOmega_p^2}{\varOmega_R} \sum_s \frac{J_c^2\left(p_s \frac{R_b}{R_w}\right)}{J_1^2(p_s)} \times F_\parallel\left(\frac{\varOmega_R}{2\beta}, \bar{\varDelta}, M\right) \text{Re}[\varOmega_+ F_\perp(\varOmega_+)], \tag{16}$$

and the relative energy change is expressed by

$$\Delta\bar{E}_k = -\frac{\Delta E_k}{N_{\text{el}} m c^2 (\gamma - 1)}. \tag{17}$$

The expression in Eq. (16) shows that the energy exchange between the train of electron micro-bunches and the gas active medium is related to the number of modes and is linearly proportional to interaction length d.

3 Influence of the parameters on energy exchange

Similar to the boundless condition, we consider the influence of various parameters on the energy exchange in the waveguide.

Figure 2 illustrates the relative change in the kinetic energy of the macro-bunch versus the energy density stored in the excited CO_2 gas mixture at the resonance frequency when considering 500 modes. $R_w=120\lambda_0$, $R_b=10\lambda_0$, $\Delta=0.1\lambda_0$, $M=150$. Fig. 2(a) shows the result for the waveguide boundary condition, while Fig. 2(b) shows that for the boundless condition.

From the simulation result in Fig. 2, we can see that the kinetic energy of the train of micro-bunches in the boundary condition is smaller than that in the boundless condition [7], and energy gain also oscillates as a function of w_{act}. When the energy density stored in the excited CO_2 gas mixture medium is w_{act}=1550 J/m^3, the largest energy gain can be obtained. So the optimum energy density is w_{act}=1550 J/m^3.

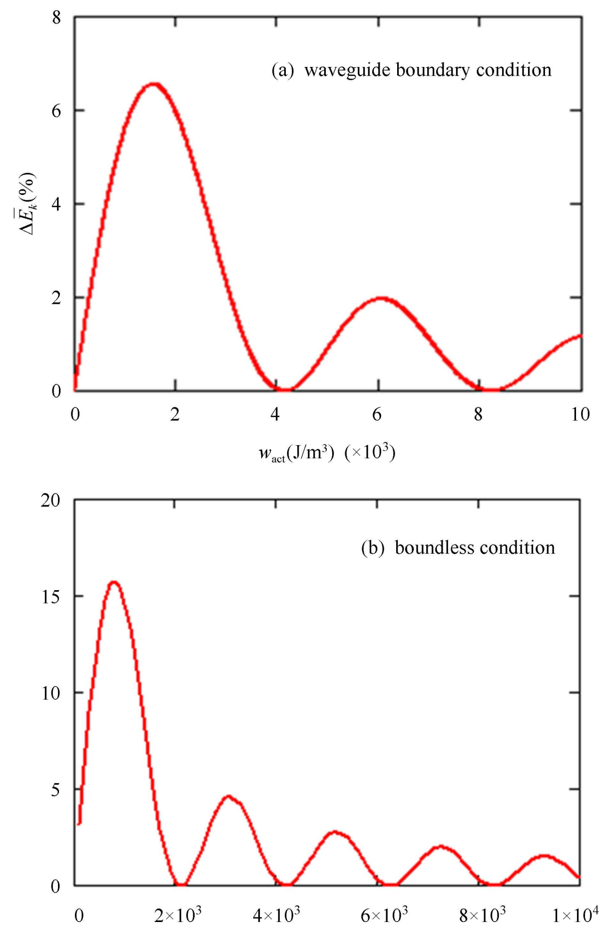

Fig. 2. (color online) Relative change in the kinetic energy of the macro-bunch versus the energy density stored in the excited CO_2 gas mixture.

Figure 3 shows the influence of the radius of the waveguide on the energy exchange.

In Fig. 3, the result illustrates that with the increase of the radius of the waveguide, the effect of the radius on the energy exchange is small. When the radius is large enough, the energy exchange is no longer exchanged to the radius, which corresponds to the boundless condition.

In Fig. 4, we show the influence of the average initial kinetic energy in the macro-bunch on energy exchange.

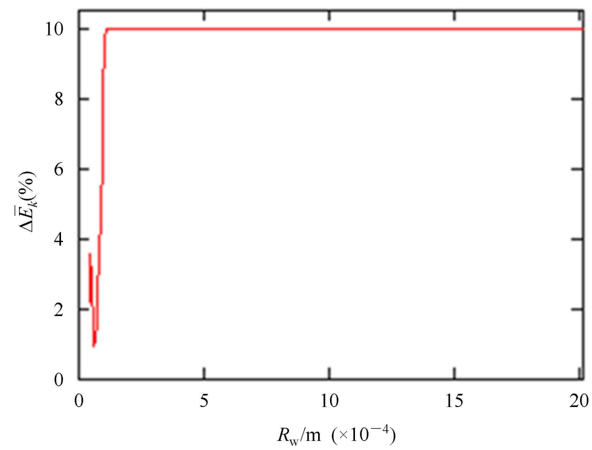

Fig. 3. (color online) Relative change in the kinetic energy of the macro-bunch versus the radius of the waveguide.

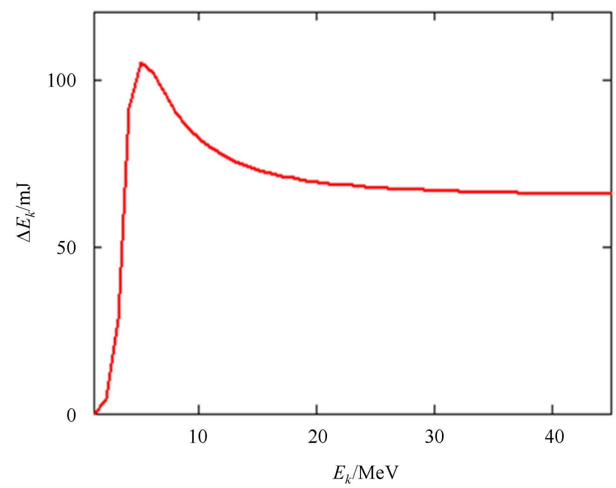

Fig. 4. (color online) Macro-bunch kinetic energy increase versus the average kinetic energy of the electrons.

For relativistic electrons, Fig. 4 illustrates that the kinetic energy increase of the electron beam is γ-independent, as any viable acceleration structure must exhibit. In the former calculation, we let E_k=45 MeV, which satisfies the conditions theory of relativity, so the kinetic energy increase does not change with the initial kinetic energy.

In Fig. 5 we show the kinetic energy gain versus the length of a single micro-bunch. The energy gain vanishes as the length of the micro-bunch approaches the modulation wavelength, and it reaches a maximum for the shortest bunch. In practice, the length of the micro-bunch is limited primarily by the modulation process and by the space-charge effects within the micro-bunch.

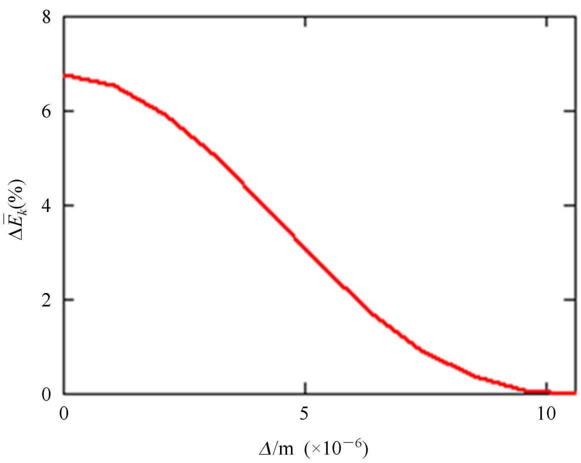

Fig. 5. (color online) Macro-bunch kinetic energy increase versus the length of the micro-bunch.

The kinetic energy gain for a given amount of charge in the micro-bunch versus the number of micro-bunches was examined, and the result is presented in Fig. 6.

When the charge in the macro-bunch is fixed, with the increase of the number of micro-bunches, the total charge in the macro-bunch decreases, and so does the energy gain.

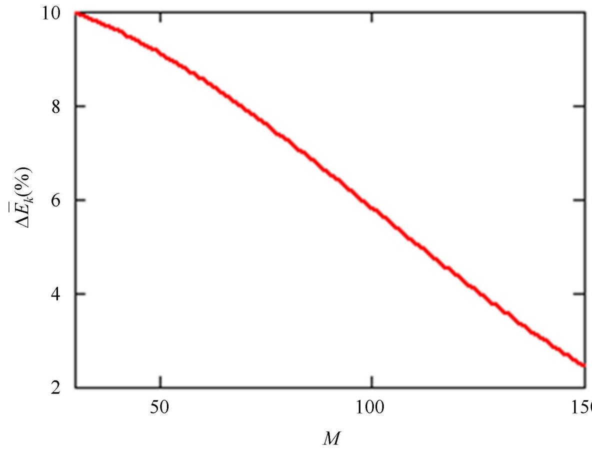

Fig. 6. (color online) Macro-bunch kinetic energy increase versus the number of micro-bunches.

4 Conclusion

The theory and simulation results on the energy exchange between the train of micro-bunches and the gas mixture active medium in a waveguide show that the train of electrons moving in an active medium in the waveguide condition can get energy from the active medium. Because the PASER process does not need phase matching or compensation of phase slipping, we can extend the interaction length at will to get as much energy as we need.

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