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Method study of parameter choice for a circular proton–proton collider *

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Abstract: In this paper we show a systematic method of appropriate parameter choice for a circular proton–proton collider by using an analytical expression for the beam–beam tune shift limit, starting from a given design goal and technical limitations. A suitable parameter space has been explored. Based on the parameter scan, sets of appropriate parameters designed for a 50 km and 100 km circular proton–proton collider are proposed.

Keywords: circular proton–proton collider, parameter choice, beam–beam tune shift limit

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1 Introduction

With the discovery of the Higgs boson at the LHC, the world high-energy physics community is investigating the feasibility of a Higgs Factory as a complement to the LHC for studying the Higgs and pushing the high energy frontier. CERN physicists are busy planning the LHC upgrade program, including HL-LHC and HE-LHC. They also plan a more inspiring program called FCC, including FCC-ee and FCC-hh. Both the HE-LHC and the FCC-hh are proton–proton (pp) colliders aiming to explore the high energy frontier and expecting to find new physics [1, 2]. Chinese accelerator physicists also plan to design an ambitious machine called CEPC-SPPC (Circular Electron–Positron Collider-Super Proton–Proton Collider). The CEPC-SPPC program contains two stages. The first stage is an electron–positron collider with center-of-mass energy 240 GeV to study the Higgs properties carefully. The second stage is a proton–proton collider at center-of-mass energy of more than 70 TeV [3]. The SPPC design is just starting, and so we have developed a systematic method of how to make an appropriate parameter choice for a circular pp collider by using an analytical expression of beam–beam tune shift, starting from the required luminosity goal, beam energy, physical constraints at the interaction point (IP) and some technical limitations.

2 Beam–beam tune shift limit

In storage ring colliders, the physical ingredients for the theoretical explanation of the beam–beam tune shift are the stochastic heating, the plasma pinch at the interaction points and the synchrotron radiation damping effect. In e^+e^- colliders, the quantum excitation is very strong, the position of each particle is random and the state of the particles can be regarded as a gas, where the positions of the particles follow statistical laws. Synchrotron radiation is the main source of heating. Besides, when two bunches undergo collision at an interaction point, every particle in each bunch will feel the deflected electromagnetic field of the opposite bunch and the particles will suffer from additional heating. With the increase of the bunch particle population N_e, this kind of heating effect will get stronger. There is a limit condition beyond which the beam emittance will blow up. This emittance blow-up mechanism introduces a limit for beam–beam tune shift which is thoroughly discussed in Ref. [4]:

$$\xi_{y,\max,\mathrm{ee}} \leqslant 2845\gamma\sqrt{\frac{r_e}{6\pi R N_{\mathrm{IP}}}} = \frac{2845}{2\pi}\sqrt{\frac{T_0}{\tau_y \gamma N_{\mathrm{IP}}}}, \qquad (1)$$

where N_{IP} is the number of interaction points (when there are N_{IP} interaction points, the independent heating effects have to be added in a statistical way), R is the dipole radius, r_e is the classical radius of the electron, τ_y is the transverse damping time and T_0 is the revolution time.

For hadron circular colliders (taking a proton–proton collider as an example), simply substituting r_e in Equation (1) with the proton classical radius r_p will not give meaningful results. The physical reason is that when we calculate the stochastic heating effect due to beam–beam interaction in e^+e^- storage rings, all the particles in each colliding bunch participate in this physical process due to the mixture of strong synchrotron radiation-induced

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random motion and motion due to the strong nonlinear beam–beam transverse forces. But in a hadron collider, the synchrotron radiation effect is very weak, and it is the transverse nonlinear beam–beam forces which are responsible for the stochastical motion. The particles inside a bunch, before suffering from the nonlinear beam–beam effect, are very cold and we can trace each particle without missing it. In the presence of the beam–beam interaction, due to the strong nonlinear beam–beam forces, some particles located in the outer part of the bunch will undergo nonlinear beam–beam force-induced stochastical motion. Assuming a round colliding bunch of Gaussian transverse distribution, the number of these heated particles, $N_{\mathrm{p,h}}$ can be estimated by $N_{\mathrm{p,h}}=f(x)N_{\mathrm{p}}$ [5], with

$$f(x)=1-\frac{2}{\sqrt{2\pi}}\int_0^x \mathrm{e}^{-\frac{t^2}{2}}\,\mathrm{d}t, \tag{2}$$

where N_{p} is the number of particles inside a bunch, and x is the limit between the cold core and the heated region. With this condition, the limit for beam–beam tune shift can be expressed as [5]:

$$\xi_{y,\mathrm{max,pp}}=\frac{2845\gamma}{f(x)}\sqrt{\frac{r_p}{6\pi R N_{\mathrm{IP}}}}=\frac{2845}{2\pi f(x)}\sqrt{\frac{T_0}{\tau_y\gamma N_{\mathrm{IP}}}} \tag{3}$$

and x in Eq. (2,3) can be solved by the following equation [5, 6]:

$$x^2=\frac{4f(x)}{\pi\xi_{y,\mathrm{max,pp}}N_{\mathrm{IP}}}=\frac{4f(x)^2}{2845\pi\gamma}\sqrt{\frac{6\pi R}{r_{\mathrm{p}}N_{\mathrm{IP}}}}, \tag{4}$$

where N_{IP} is the number of interaction points, R is the dipole radius, r_{p} is the classical radius of the proton, τ_y is the transverse damping time and T_0 is the revolution time.

3 Machine parameter choice

The energy design goal of the SPPC is about 70–100 TeV, using the same tunnel as the CEPC, which is about 50 km in circumference. A larger circumference for the SPPC, like 100 km, is also being considered. It is planned to use superconducting magnets of about 20 T [7]. We can develop a systematic way to calculate the parameters starting from the maximum beam–beam tune shift limit and the design goal. Our design goal is: luminosity L, beam energy E_0, ring circumference C_0 and IP numbers N_{IP}. Table 1 shows the goals, known quantities and constants.

Table 1. The design goal and known quantities.

circumference	$C_0=54.7$ km
beam energy	$E_0=35$ TeV
IP numbers	$N_{\mathrm{IP}}=2$
luminosity	$L=1.0\times10^{35}$ cm^{-2}s^{-1}
dipole magnets	$B=20$ T
arc filling factor	$f_1=0.79$
bunch filling factor	$f_2=0.80$

The luminosity of a pp collider can be expressed as a function of the beam current I_{b}, the beam-beam tune shift ξ_y, the beta-function at IP β^*, the classical proton radius r_{p}, its charge e, and the luminosity reduction factor due to the crossing angle and hourglass effect [8]:

$$L=\frac{I_{\mathrm{b}}}{e}\frac{\xi_y}{\beta^*}\frac{\gamma}{r_{\mathrm{p}}}F_{\mathrm{ca}}F_{\mathrm{h}}, \tag{5}$$

where F_{ca} F_{h} can be expressed as [9, 10]:

$$F_{\mathrm{ca}}=\frac{1}{\sqrt{1+\left(\frac{\sigma_z\theta_c}{2\sigma^*}\right)^2}}, \tag{6}$$

$$F_{\mathrm{h}}=\frac{\beta^*}{\sqrt{\pi}\sigma_z}\exp\left(\frac{\beta^{*2}}{2\sigma_z^2}\right)K_0\left(\frac{\beta^{*2}}{2\sigma_z^2}\right). \tag{7}$$

From Eqs. (3) and (5) and $\tau_y=\frac{2E_0T_0}{J_yU_0}$, one finds a limit for the luminosity by

$$\begin{aligned}L_0 &= \frac{I_{\mathrm{b}}}{e}\frac{\xi_{y,\mathrm{max,pp}}}{\beta^*}\frac{\gamma}{r_{\mathrm{p}}}=\frac{I_{\mathrm{b}}}{e}\frac{2845}{\beta^*2\pi f(x)}\frac{\gamma}{r_{\mathrm{p}}}\sqrt{\frac{T_0}{\tau_y\gamma N_{\mathrm{IP}}}}\\ &= \frac{2845}{2\pi r_{\mathrm{p}}ef(x)}\frac{1}{\beta^*}\sqrt{\frac{I_{\mathrm{b}}P_{\mathrm{SR}}\gamma}{2E_0N_{\mathrm{IP}}}},\end{aligned} \tag{8}$$

giving

$$L_{\max}=L_0F_{\mathrm{ca}}F_{\mathrm{h}}. \tag{9}$$

The beta function at an IP can then be written as:

$$\beta^*=\frac{2845}{2\pi r_{\mathrm{p}}ef(x)}\frac{1}{L_0}\sqrt{\frac{I_{\mathrm{b}}P_{\mathrm{SR}}\gamma}{2E_0N_{\mathrm{IP}}}}, \tag{10}$$

where P_{SR} is the synchrotron radiation power per ring:

$$P_{\mathrm{SR}}=U_0I_{\mathrm{b}}. \tag{11}$$

U_0 is the energy loss per turn [9]:

$$U_0=0.00778[\mathrm{MeV}]\frac{(E_0[\mathrm{TeV}])^4}{\rho[\mathrm{m}]} \tag{12}$$

and the critical photon energy is [9]:

$$E_{\mathrm{c}}[\mathrm{keV}]=1.077\times10^{-4}(E_0[\mathrm{TeV}])^2B[\mathrm{T}]. \tag{13}$$

The proton beam is a round beam ($\sigma_x=\sigma_y=\sigma^*$), so the RMS IP spot size is:

$$\sigma^*=\sqrt{\beta^*\epsilon}=\sqrt{\beta^*\frac{\epsilon_{\mathrm{n}}}{\gamma}}. \tag{14}$$

Beta at the 1st parasitic encounter with bunch separation Δt is:

$$l_1=c\times\Delta t, \tag{15}$$

$$\beta_1=\beta^*+\frac{(l_1/2)^2}{\beta^*}. \tag{16}$$

RMS spot size at the 1st parasitic encounter is:

$$\sigma_1=\sqrt{\beta_1\epsilon}=\sqrt{\beta_1\frac{\epsilon_{\rm n}}{\gamma}}. \qquad (17)$$

Many circular colliders are being designed to reach high luminosity through using a large number of closely spaced bunches. This introduces the potential problem of parasitic encounters near the interaction point. To avoid unwanted parasitic encounters, we use a crossing angle scheme. Before the two beams enter separate beam pipes, they travel in the same vacuum chamber where parasitic 'long range' collisions can occur. Although these are rather weak due to the separation of the bunches, their large number makes their effect very important and will lower the luminosity. In LHC the two beams share an approximately 130 m long common beam pipe along the interaction regions (IR). The exact length is 126 m in IR2 and IR8, which feature superconducting separation dipole magnets next to the triplet assemblies, and 140 m in IR1 and IR5, which feature normal conducting and therefore longer separation dipole magnets next to the triplet assemblies. Together with the large number of bunches (2808 for each proton beam), and a nominal bunch spacing of 25 ns, the long common beam pipe implies 34 parasitic collision points for each experimental insertion region (for four experimental IRs this implies a total of 136 unwanted collision points) [9]. In the SPPC the length of the common beam pipe is decided by the IR optics design. From the preliminary design, there are 2 common beam pipes and each is about 300 m. As we use the same bunch spacing (25 ns) as the LHC, the number of parasitic collision points is about 160. These long range collisions may cause tune shift spread. The spread must be limited to those values that avoid crossing the dangerous resonance in the operation tune space of the machine. This crossing may also produce instabilities in the beam, limiting the performance of the machine (beam lifetime and luminosity) and creating radiation problems in the detectors (beam–halo) [11, 12]. To obtain a high beam–beam parameter, with a small effect on the luminosity, and considering the experience gained from experiments at the LHC and Tevatron [13–15], the full crossing angle $\theta_{\rm c}$ is chosen to provide a beam–beam separation of $n_{\rm s}=$ 10–12 RMS beam sizes for the parasitic crossings [8, 16]:

$$\theta_{\rm c}=\frac{2\times\frac{n_{\rm s}}{2}\times\sigma_1}{l_1/2}=\frac{2n_{\rm s}\sigma_1}{l_1}. \qquad (18)$$

Then we can rewrite $F_{\rm ca}$ as:

$$F_{\rm ca}=\frac{1}{\sqrt{1+\Phi^2}}, \qquad (19)$$

where

$$\begin{aligned}\Phi &= \frac{\sigma_z\theta_{\rm c}}{2\sigma^*}=\frac{n_{\rm s}\sigma_z\sigma_1}{l_1\sigma^*}=\frac{n_{\rm s}\sigma_z\sqrt{\beta_1\frac{\epsilon_{\rm n}}{\gamma}}}{l_1\sqrt{\beta^*\frac{\epsilon_{\rm n}}{\gamma}}}=\frac{n_{\rm s}\sigma_z}{l_1}\sqrt{\frac{\beta_1}{\beta^*}}\\ &= n_{\rm s}\sqrt{\frac{\sigma_z^2}{(c\Delta t)^2}+\frac{1}{4(\beta^*/\sigma_z)^2}}.\end{aligned} \qquad (20)$$

Φ is the Piwinski angle, β^* is the beta function at IP, σ_z is the bunch length and Δt is the bunch separation.

When the luminosity is reduced by less than 10% due to the crossing angle effect, we have $F_{\rm ca}\geqslant 0.9$. From Eq. (19) we get:

$$\Phi\leqslant 0.484322(\text{rad}). \qquad (21)$$

We know the number of bunches:

$$n_{\rm b}=\frac{T_0 f_2}{\Delta t} \qquad (22)$$

and the bunch population:

$$N_{\rm p}=\frac{I_{\rm b}}{n_{\rm b}f_{\rm rev}e}. \qquad (23)$$

Combining Equations (10), (20), (21), (22) and (23), we can get reasonable values of β^* $I_{\rm b}$ Δt $n_{\rm b}$ $N_{\rm p}$ and the ratio β^*/σ_z, where we should also consider the instability influence and the technical constraints. From the definition of beam–beam tune shift [9]:

$$\xi_y=\frac{N_{\rm p}r_{\rm p}}{4\pi\epsilon_{\rm n}}, \qquad (24)$$

we can get the normalized emittance:

$$\epsilon_{\rm n}=\frac{N_{\rm p}r_{\rm p}}{4\pi\xi_{y,\max,\rm pp}}. \qquad (25)$$

We can then calculate σ^*, β_1, σ_1, $\theta_{\rm c}$, $F_{\rm ca}$ and $F_{\rm h}$. Finally, we get the final value of the luminosity:

$$L=L_0F_{\rm ca}F_{\rm h}. \qquad (26)$$

4 Comparison of LHC parameter list with parameters obtained by our method

To check our method, we use it to choose and calculate the LHC parameters and compare them with the actual LHC parameter list [17].

When CERN accelerator physicists chose and designed the LHC parameters, they also started from the beam–beam tune shift parameter. Their beam–beam parameter, however, was chosen from experience and assumed to be a constant number. In the LHC Design Report, they wrote "Experience from the SPS and the Tevatron shows that the total tune spread including all other sources and the beam–beam effect, should not exceed 0.015. This allows about 0.01 for the overall beam–beam tune spread $\Delta Q_{\rm bb}$. The tune spread from a head-on collision is ξ, which is the maximum tune shift. With three proton experiments requiring head-on collisions implies that the linear beam–beam tune shift for each IP should satisfy $\xi \leqslant 0.0033$" [18]. They chose $\xi_y = 0.0033$, and put it into the luminosity formula (Eq. (5)) to calculate the parameters. Many of the formulae we use to calculate the parameters are the same as theirs, but our starting point is a little different from them. We find the maximum beam–beam tune shift limit is related to the number of IPs $N_{\rm IP}$, the transverse damping time τ_y, the revolution time T_0 and the beam energy γ. This relationship is described by Eq. (3), which was discussed in Ref. [5]. The conclusion is that this analytical formula can give good predictions of maximum beam–beam tune shift by comparing the calculated value with existing machines and some machines under design.

Using Eqs. (2) and (4), we obtain a quadratic equation about x. Putting the value of R, $N_{\rm IP}$, $r_{\rm p}$ and γ into this equation, we can get the numerical and reasonable solution $x = 2.238$ and $f(x) = 0.02523$. Putting $f(x) = 0.02523$ into Equation (3), we obtain the maximum beam–beam tune shift as 0.0032, which is quite close to the parameter list choice 0.0033 and the experimental value of beam–beam tune shift 0.0034 in the LHC. This indicates that our method is reasonable to estimate the maximum beam–beam tune shift limit. The second column in Table 2 shows the other LHC parameters obtained using our method. Most of the parameters are close to the parameters in the real LHC parameter list, except that the crossing angle is a little larger. This is because the full crossing angle in the LHC parameter list is chosen to provide a beam–beam separation of 9.325 RMS beam sizes for the parasitic crossings, while ours is 10 RMS beam sizes. The value of $F_{\rm ca}$ obtained by our method is 0.8303. This is because we want to keep the β^* and bunch length σ_z the same as the LHC parameter list. Under this condition, the ratio of β^* and σ_z is only β^*/σ_z=7.5; from Eqs. (19) and (20) and parameter scan in Fig. 3 and Fig. 4, we can easily find that if we want $F_{\rm ca} \geqslant 0.9$, the ratio of β^* and σ_z should be $\beta^*/\sigma_z \geqslant 13$. This ratio for FCC-hh is 14.57, and we choose 15 for SPPC. From the comparison, we find that the method starting from beam–beam tune shift to choose a set of parameters for a circular proton–proton collider is reasonable and can be used.

Using Eq. (3) in the luminosity formulae can make the parameters more systematic. We also take account of the influence of crossing angle effect. We use the luminosity reduction factor due to crossing angle as a limitation to optimize the parameters.

Table 2. Comparison of LHC parameter list with parameters obtained by our method.

	LHC-list	LHC-new
main parameters and geometrical aspects		
beam energy $[E_0]$/TeV	7	7
circumference $[C_0]$/km	26.7	26.7
dipole field $[B]$/T	8.33	8.33
dipole curvature radius $[\rho]$/m	2801	2801
bunch filling factor $[f_2]$	0.78	0.79
arc filling factor $[f_1]$	0.79	0.783
total dipole length $[L_{\rm Dipole}]$/m	17599	17599
arc length $[L_{\rm ARC}]$/m	22476	22476
total straight section length $[L_{\rm ss}]$/m	4224	4224
physics performance and beam parameters		
peak luminosity per IP $[L]$/cm^{-2}s^{-1}	1.0E+34	0.99E+34
beta function at collision $[\beta^*]$/m	0.55	0.56
max B-B tune shift perIP $[\xi y]$	0.0033	0.0032
number of IPs contributing to ΔQ	3	3
max total beam–beam tune shift	0.01	0.0096
circulating beam current $[I_{\rm b}]$/A	0.5805	0.5814
bunch separation $[\Delta t]$/ns	25/5	25/5
number of bunches $[n_{\rm b}]$	2808	2812
bunch population $[N_{\rm p}](10^{11})$	1.15	1.15
normalized RMS transverse emittance $[\varepsilon]$/μm	3.75	4.37
RMS IP spot size $[\sigma^*]$/μm	16.7	18.2
beta at 1st parasitic encounter $[\beta_1]$/m	26.12	25.38
RMS spot size at the 1st parasitic encounter $[\sigma_1]$/μm	114.6	122.0
RMS bunch length $[\sigma_z]$/mm	75.5	75.1
full crossing angle $[\theta_{\rm c}]$/μrad	285	325
reduction factor according to cross angle $[F_{\rm ca}]$	0.8391	0.8303
reduction factor according to hour glass effect $[F_h]$	0.9954	0.9956
energy loss per turn $[U_0]$/MeV	0.0067	0.0067
critical photon energy $[E_{\rm c}]$/keV	0.044	0.044
SR power per ring $[P_0]$/MW	0.0038	0.0039

5 Parameter choice for SPPC

5.1 Parameter scan

Using the method above, we scan the goal luminosity L with different bending radii ρ, IP numbers $N_{\rm IP}$ and different ratios of β^*/σ_z. Table 3 shows the input parameters. We get some meaningful results which are shown in Figs. 1 to 6. These results tell us the relationships between different parameters and the variation of the parameters.

Table 3. Input parameters for machine design.

energy E_0	35.0 TeV
circumference C_0	54.7 km
goal luminosity L	$(1-4)\times10^{35}$ cm^{-2}s^{-1}
IP numbers $N_{\rm IP}$	$2-4$
bending radius ρ	$5.9-6.5$ km
ratio of β^*/σ_z	10–20

Figure 1 shows that larger luminosity needs a smaller vertical IP beta function. Larger bending radius and more interaction points require smaller β^* at the same goal luminosity.

Figure 2 shows that smaller bending radius and less interaction points give larger vertical beam–beam tune shift, while the parameter ξ_y has no relationship with peak luminosity.

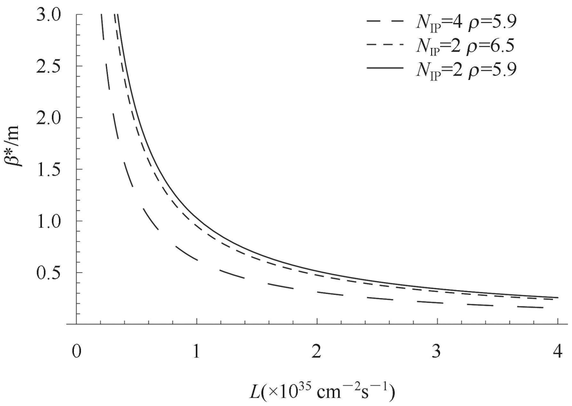

Fig. 1. Vertical beta at IP as the function of goal luminosity. (equation(10)).

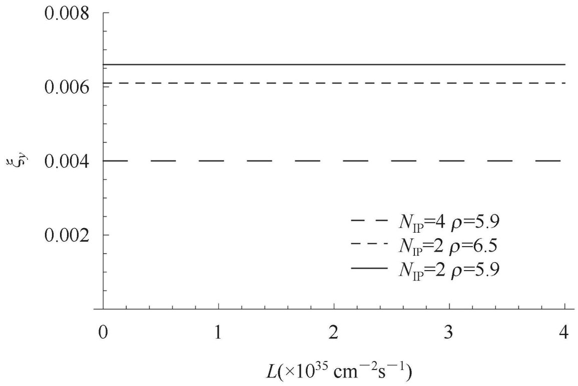

Fig. 2. Vertical beam–beam tune shift as the function of peak luminosity. (equation(3)).

Figures 3 and 4 tell us that the reduction factor from the crossing angle is related to bunch separation (Δt) and the ratio of IP beta and RMS bunch length (β^*/σ_z). The maximum value of this factor is 1, and larger β^*/σ_z brings this value nearer to 1. If we want this effect to reduce the luminosity by less than 10%, we should have $F_{\rm ca}\geqslant0.9$. The dashed line in Fig. 3 and Fig. 4 is the value equal to 0.9, and we can easily get important information from the figures. We should choose a larger β^*/σ_z - about 15 is quite reasonable, giving a bunch separation of 25 ns. If we want to choose a smaller bunch separation, like 5 ns, the ratio of β^* and σ_z should be more than 20. We should consider both of these and choose the most suitable values. Figure 5 shows a 3D diagram of the relationship of $F_{\rm ca}$, Δt and β^*/σ_z.

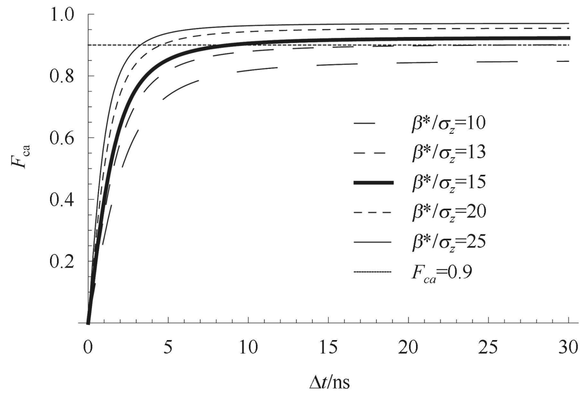

Fig. 3. $F_{\rm ca}$ as the function of Δt. (equation(19)).

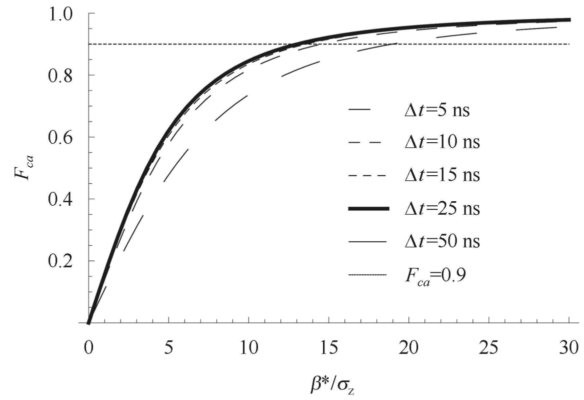

Fig. 4. $F_{\rm ca}$ as the function of the ratio of β^* and σ_z. (equation(19)).

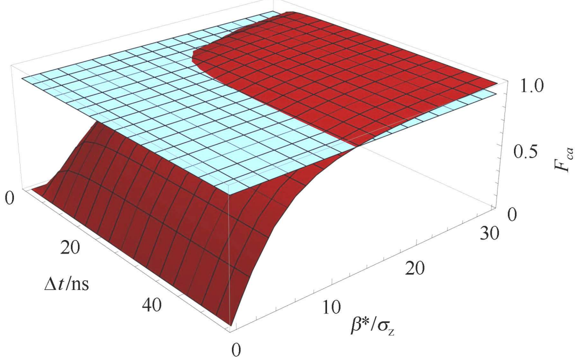

Fig. 5. (color online) The 3D diagram of the relationship of $F_{\rm ca}$ Δt and β^*/σ_z. (equation(19)).

Table 4. Parameter lists for LHC, HL-LHC, HE-LHC, FCC-hh and SPPC.

	LHC	HL-LHC	HE-LHC	FCC-hh	SPPC (Pre-CDR)	SPPC-54.7 km	SPPC-100 km	SPPC-100 km	SPPC-78 km
main parameters and geometrical aspects									
beam energy[E_0]/TeV	7	7	16.5	50	35.6	35.0	50.0	68.0	50.0
circumference[C_0]/km	26.7	26.7	26.7	100(83)	54.7	54.7	100	100	78
dipole field[B]/T	8.33	8.33	20	16(20)	20	19.69	14.73	20.03	19.49
dipole curvature radius[ρ]/m	2801	2801	2250	10416 (8333.3)	5928	5922.6	11315.9	11315.9	8549.8
bunch filling factor[f_2]	0.78	0.78	0.63	0.79	0.8	0.8	0.8	0.8	0.8
arc filling factor[f_1]	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79	0.79
total dipole length [L_{Dipole}]/m	17599	17599	14062	65412 (52333)	37246	37213	71100	71100	53720
arc length[L_{ARC}]/m	22476	22476	22476	83200 (66200)	47146	47105	90000	90000	68000
straight section length[L_{ss}]/m	4224	4224	4224	16800	7554	7595	10000	10000	10000
physics performance and beam parameters									
peak luminosity per IP[L]/cm^{-2}s^{-1}	1.0E+34	5.0E+34	5.0E+34	5.0E+34	1.1E+35	1.2E+35	1.52E+35	1.02E+36	1.52E+35
beta function at collision[β^*]/m	0.55	0.15(min)	0.35	1.1	0.75	0.85	0.97	0.24	1.06
max beam–beam tune shift perIP [ξ_y]	0.0033	0.0075	0.005	0.005	0.006	0.0065	0.0067	0.008	0.0073
number of IPs contributing to ΔQ	3	2	2	2	2	2	2	2	2
max total B-B tune shift	0.01	0.015	0.01	0.01	0.012	0.013	0.0134	0.016	0.0146
circulating beam current[I_{b}]/A	0.5805	1.12	0.478	0.5	1.0	1.024	1.024	1.024	1.024
bunch separation[Δt]/ns	25 / 5	25 / 5	25 / 5	25 / 5	25	25	25	25	25
number of bunches[n_{b}]	2808	2808	2808	10600(8900) 53000(44500)	5835	5835	10667	10667	8320
bunch population[N_{p}](10^{11})	1.15	2.2	1	1.0/0.2	2.0	2.0	2.0	2.0	2.0
normalized RMS transverse emittance[ε]/μm	3.75	2.5	1.38	2.2/0.44	4.10	3.72	3.65	3.05	3.36
RMS IP spot size[σ^*]/μm	16.7	7.1	5.2	6.8	9.0	8.85	7.85	3.04	7.86
beta at the 1st parasitic encounter[β_1]/m	26.12	93.9	40.53	13.88	19.5	18.70	16.51	64.1	15.36
RMS spot size at the 1st parasitic encounter[σ_1]/μm	114.6	177.4	62.3	23.9	45.9	43.2	33.6	51.9	31.14
RMS bunch length[σ_z]/mm	75.5	75.5	75.5	80(75.5)	75.5	56.5	65	15.8	70.6
full crossing angle[θ_{c}]/μrad	285	590	185	74	146	138	108	166	99
reduction factor according to cross angle[F_{ca}]	0.8391	0.314	0.608	0.910	0.8514	0.9257	0.9248	0.9283	0.9248
reduction factor according to hour glass effect[F_{h}]	0.9954	0.9491	0.9889	0.9987	0.9975	0.9989	0.9989	0.9989	0.9989
energy loss per turn[U_0]/MeV	0.0067	0.0067	0.201	4.6(5.86)	2.10	1.97	4.30	14.7	5.69
critical photon energy[E_{c}]/keV	0.044	0.044	0.575	4.3(5.5)	2.73	2.60	3.97	9.96	5.25
SR power per ring[P_0]/MW	0.0038	0.0073	0.0962	2.4(2.9)	2.1	2.0	4.4	15.1	5.82
transverse damping time [τ_x]/h	25.8	25.8	2.0	1.08(0.64)	1.71	1.80	2.15	0.86	1.27
longitudinal damping time [τ_ε]/h	12.9	12.9	1.0	0.54(0.32)	0.85	0.90	1.08	0.43	0.635

Figure 6 shows the reduction factor from the hourglass effect as a function of the ratio of IP β function and RMS bunch length. A larger ratio gives a larger F_{h} value. To reduce the reduction of luminosity from the hourglass effect, we should choose a reasonably large ratio of β^* and σ_z.

Overall, we should decrease the number of IPs and increase the bending radius in order to achieve higher luminosity. $N_{\mathrm{IP}}=2$ is a reasonable minimum value for the numbers of IPs. Assuming the maximum dipole arc filling factor is 80%, a 5.9 km bending radius will be a limit for the 54.7 km ring.

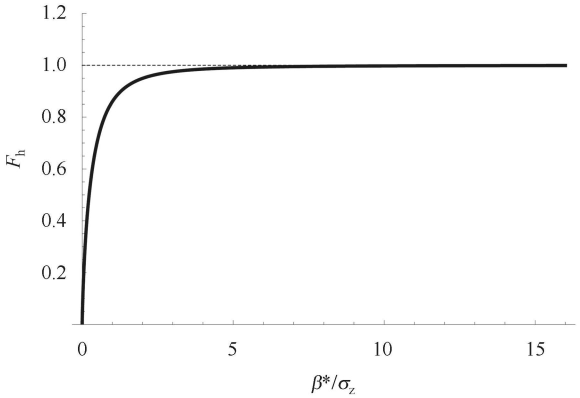

Fig. 6. $F_{\rm h}$ as the function of the ratio of β^* and σ_z. (equation(7)).

5.2 Machine parameter choice for SPPC

Combining the discussions above, we obtain a new set of parameters for the 54.7 km SPPC. In this set of parameters, the relationships between the parameters are more systematic. The full crossing angle $\theta_{\rm c}$ keeps the separation of 12 RMS beam sizes for the parasitic crossings. The luminosity reduction factor due to the crossing angle is larger than 0.9 as the bunch length is a little shorter, and the ratio of β^* and σ_z is about 15. We also give a set of parameters for the larger circumference SPPC, considering both 78 km and 100 km. Table 4 is the parameter list for the SPPC. As a comparison, we put the parameters for LHC, HL-LHC, HE-LHC, and FCC-hh together in Table 4 [7, 8]. The first plan for SPPC uses the same tunnel as the CEPC. The circumference is 54.7 km, which is determined by CEPC. We choose the dipole field as 20 T and get a center-of-mass energy of 70 TeV. If we want to explore the higher energy, we should make the circumference larger. To explore a center-of-mass energy of 100 TeV while keeping the dipole field at 20 T, the circumference should be 78 km at least. With this condition, there is hardly any space to upgrade, so a 100 km SPPC is much better because the dipole field is then only 14.7 T. If the dipole field is kept at 20 T in a 100 km SPPC, we can get a center-of-mass energy as high as 136 TeV.

6 Conclusion

In this paper, a systematic method was developed for making an appropriate parameter choice for a circular pp collider using an analytical expression for beam–beam tune shift limit, starting from a given luminosity goal, beam energy and technical limitations. Using this method, we have clearly shown the relations of machine parameters with the goal luminosity and hence give a parameter choice in an efficient way. We also show the parameters chosen for a 50 km SPPC and larger circumference SPPC, including both 78 km SPPC and 100 km SPPC.

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