Thermodynamic surface system of static black holes and area law *

Ming Chen(陈明)[1;1)] Yong-Chang Huang(黄永畅)[1,2;2)]

[1]Institute of Theoretical Physics, Beijing University of Technology, Beijing 100124, China

[2]CCAST (World Lab.), P.O. Box 8730, Beijing 100080, China

Abstract: We propose a new picture of black holes through a special holographic screen. This holographic screen contains all the degrees of freedom of a black hole. We find that this holographic screen is similar to the ordinary thermodynamic surface system. Meanwhile, through the "white-wall box" and the formula of sound velocity, we find some similarities between gravitons and photons. We further assume that such a holographic screen is a kind of Bose-Einstein condensate of gravitons. Through this assumption and those similarities, we finally get the area law of static black holes.

Keywords: black holes, holographic screen, surface system, "white-wall box", Bose-Einstein condensates, area law

PACS: 04.20.-q, 04.70.Dy, 05.70.-a **DOI:** 10.1088/1674-1137/40/11/113101

1 Introduction

It is well known that the sum of the areas of black hole horizons cannot decrease [1]. As a result, a black hole horizon can be attributed to its entropy ($\mathcal{S}$), which is proportional to its area ($\mathcal{A}$). Their relation can be expressed through the Bekenstein-Hawking formula [2–4]:

$$\mathcal{S}=\frac{\mathcal{A}}{4}\left(\frac{c^3k_B}{G\hbar}\right)\propto\mathcal{A}. \tag{1}$$

At the same time, the four laws of black hole mechanics are analogous to the laws of thermodynamics, and they are originally derived from the classical Einstein equation [5]. With the discovery of quantum Hawking radiation [6], it turns out that such analogy is actually an identity.

From the other side, Einstein equation is somewhat an equation of state, and we can reconstruct general relativity as the thermodynamic limit of a more fundamental theory of gravity [7].

These investigations are essential in understanding the AdS/CFT correspondence [8, 9] and firewalls [10]. Furthermore, Eq. (1) relates the area of a horizon and its entropy. This relation between a geometric quantity and microscopic data is the key concept of holography. There are many works about holography motivated by Eq. (1), including Refs. [11–13].

Therefore, gravity has many similarities to other known emergent phenomena, such as thermodynamics and hydrodynamics. These similarities imply that gravity is much like an emergent phenomenon instead of a fundamental force [14, 15].

Meanwhile, across a horizon, only a region of Planck length ($L_{\rm p}$) contributes to the microstates [12, 16]. Similarly, if we want to declare that the particle and the black hole have become one system, we have to choose a location away from the black hole horizon at a distance of the order of the Compton wavelength [2].

In the context of holography, these arguments imply that the information of a black hole can be stored in some special location away from the horizon. This location can correspond to a holographic screen, and we can project all the degrees of freedom (DOF) of a black hole onto it.

This kind of holographic screen is very similar to the thermodynamic surface system, which is described by the surface tension parameter (ξ) and the surface area ($\mathcal{A}$). The state equation (or the free energy $\mathcal{F}$) of the system is described as follows:

$$\mathcal{F}=\xi\mathcal{A}+\text{const}. \tag{2}$$

Meanwhile, in any static spacetime with a bifurcation horizon, the action functional for gravity can be interpreted as the free energy of spacetime [12, 17]. If we just care about static spacetime, the similarity between this holographic screen and the thermodynamic system will be much more obvious.

Moreover, because equilibrium only exists in the

Received 13 May 2016, Revised 3 June 2016

* Supported by National Natural Science Foundation of China (11275017, 11173028)

1) E-mail: saintiming@emails.bjut.edu.cn

2) E-mail: ychuang@bjut.edu.cn

static condition, this special thermodynamic system can stay in the equilibrium state. Meanwhile, since static horizons have well defined temperatures, they are surely in thermal equilibrium. As a result, we can build an equilibrium correspondence between this thermodynamic system and static horizons. Such a correspondence provides another support for this similarity.

However, if a black hole can be described by such a thermodynamic surface system, what are the ingredients of this special "surface"?

We know that Bose-Einstein condensates (BECs) can be analogue models of gravity [18–20]. We can also take general relativity as a quantum theory which propagates a unique kind of weakly coupled quantum particle with zero mass and spin-2 [21], then assume black holes can achieve BECs.

On the other hand, although blackbody radiation (photon gas) exhibits no BECs at low temperatures, the "white-wall box" provides a possible way for photons to achieve BECs [22, 23]. Moreover, photon gas is one kind of equilibrium radiation, which is consistent with our previous equilibrium premise.

These investigations inspire us to think about the relationship between photons and gravitons [24]. If there are some similarities between them, gravitons may exist like photon gas and then form that special thermodynamic surface.

That is, the ingredients of this special "surface" can be a BECs of gravitons. Our following work will deal with such a possibility and try to get some useful results.

The arrangement of this paper is as follows. In Section 2, through the properties of photon gas in the "white-wall box" and the formula of sound velocity, we discuss the similarities between gravitons and photons. In Section 3, we use these similarities to get the area law of a static black hole with a bifurcation horizon, and get its negative heat capacity. Section 4 gives a summary and outlook.

2 Similarities between photons and gravitons

We know that there are many similarities between gravitation and electromagnetism. Therefore, like Maxwell's equations, Einstein's equations have radiative solutions. According to the approximated linear expressions of Einstein's equations, it is natural to consider the corresponding relationship between classical propagating waves and gravitational waves. This implies the similarities between them.

Furthermore, there are different understandings of gravitational waves [7]. Without caring about the geometrical background, the system is based on any causal horizon. The local equilibrium conditions used in Ref. [7] are somehow equivalent to the weak-field limit. So we can have a system of local partial differential equations that are time reversal invariant, and its solutions include propagating waves.

In other words, this new insight into Einstein's equations gives the thermodynamic property of the wave solution. This allows us to think of the propagating wave as analogous to sound in gas. Because what we care about here is the energy transferred, such a propagating wave will propagate as an adiabatic compression wave no matter what kind of variations they might consist of.

Combining the above arguments, we have enough reasons to conclude that there are similarities between gravitational waves and classical propagating waves (e.g. sound waves).

Meanwhile, we have learned to describe the fundamental microscopic phenomena in terms of elementary particles and their collisions. In classic electrodynamics, it is the plane wave solutions of Maxwell's equations that naturally lead to an interpretation in terms of photons. Similarly, it is the radiative solutions of Einstein's equations that lead to the concept of a particle of gravitational radiation, i.e., the graviton [25]. So, these similarities are equivalent to similarities between gravitons and gas molecules.

However, this kind of similarity makes little sense. We can neither determine the speed of gravitons nor any meaningful properties they might have. As a result, we have to find another way to meet our interests.

As argued in Section 1, both gravitons and photons can achieve BECs, so we should build a relationship between them. If such a relationship is really established, we can naturally get the light speed of gravitons.

According to Refs. [22, 23], one can get a 2-dimensional "white-wall box" for photon gas. In this system, thermalization is achieved in a photon-number-conserving way by photon-scattering off dye molecules, and the cavity mirrors provide both an effective photon mass and a confining potential.

The Bose-Einstein distribution factor for this type of photon gas is:

$$f(\mu,T,\varepsilon)=\frac{1}{\exp\left(\dfrac{\varepsilon-\mu}{k_{\rm B}T}\right)-1}, \tag{3}$$

where the transversal energy (effective energy) is $\varepsilon=\varepsilon'-\hbar\omega_{\rm cutoff}=\hbar\omega$, and μ is the chemical potential.

In the 2-dimensional "box", the number of states between energy $\varepsilon\to\varepsilon+{\rm d}\varepsilon$ is:

$${\rm d}n=\frac{\mathcal{A}}{h^2}2\pi p{\rm d}p=\frac{2\pi\mathcal{A}}{c^2h^2}\varepsilon{\rm d}\varepsilon, \tag{4}$$

where the symbol $\mathcal{A}$ denotes the area of the "box" . So, the number of photons is:

$$\mathrm{d}N(\varepsilon)=\frac{4\pi\mathcal{A}\varepsilon\mathrm{d}\varepsilon}{c^2h^2\exp\left(\dfrac{\varepsilon-\mu}{k_{\mathrm{B}}T}\right)-1}, \tag{5}$$

and the energy is:

$$U(\varepsilon,\mu,T)\mathrm{d}\varepsilon=\varepsilon\mathrm{d}N(\varepsilon)=\frac{4\pi\mathcal{A}\varepsilon^2\mathrm{d}\varepsilon}{c^2h^2\exp\left(\dfrac{\varepsilon-\mu}{k_{\mathrm{B}}T}\right)-1}, \tag{6}$$

thus the total energy of all the frequencies is:

$$U(\varepsilon,\mu,T)=\frac{4\pi\mathcal{A}}{c^2h^2}\int_0^\infty\frac{\varepsilon^2\mathrm{d}\varepsilon}{\exp\left(\dfrac{\varepsilon-\mu}{k_{\mathrm{B}}T}\right)-1}. \tag{7}$$

Using the following formula:

$$\frac{1}{\exp(x)-1}=\sum_{j=1}^{\infty}\exp(-jx), \tag{8}$$

we can get:

$$U(\mu,T)=\frac{8\pi\mathcal{A}k_{\mathrm{B}}^3T^3}{c^2h^2}\sum_{j=1}^{\infty}\exp\left(\frac{j\mu}{k_{\mathrm{B}}T}\right)\frac{1}{j^3}. \tag{9}$$

If $\exp\left(\dfrac{j\mu}{k_{\mathrm{B}}T}\right)$ is very small, we can make the suitable approximation by keeping the first two orders of the expansion:

$$\begin{aligned}U(\mu,T)&\cong\frac{8\pi\mathcal{A}k_{\mathrm{B}}^3T^3}{c^2h^2}\sum_{j=1}^{\infty}\left(\frac{1}{j^3}+\frac{1}{j^2}\frac{\mu}{k_{\mathrm{B}}T}\right)\\&=\frac{2.404k_{\mathrm{B}}^3}{\pi c^2\hbar^2}\mathcal{A}T^3+\frac{\pi k_{\mathrm{B}}^2}{3c^2\hbar^2}\mu\mathcal{A}T^2.\end{aligned} \tag{10}$$

According to Ref. [22], when $\mu\to 0$ and T is a limited value, we can neglect the term including the factor μ and get:

$$U(\mu,T)\cong\bar{\sigma}\mathcal{A}T^3, \tag{11}$$

where $\bar{\sigma}=\dfrac{2.404k_{\mathrm{B}}^3}{\pi c^2\hbar^2}$ is the Stefan-Boltzmann constant in 2-dimensional space.

Therefore, we can get the pressure and the entropy as follows:

$$P=\frac{1}{2}\bar{\sigma}T^3, \tag{12}$$

$$S=\frac{3}{2}\bar{\sigma}\mathcal{A}T^2. \tag{13}$$

By rewriting Eq. (13) as $S=3\left(\dfrac{\bar{\sigma}}{2}\right)^{\frac{1}{3}}P^{\frac{2}{3}}\mathcal{A}$, then we will have:

$$\mathrm{d}S=3\left(\frac{\bar{\sigma}}{2}\right)^{\frac{1}{3}}\left(\frac{2}{3}P^{-\frac{1}{3}}\mathcal{A}\mathrm{d}P+P^{\frac{2}{3}}\mathrm{d}\mathcal{A}\right), \tag{14}$$

thus, the adiabatic compression coefficient is:

$$\kappa_{\mathrm{s}}=-\frac{1}{\mathcal{A}}\left(\frac{\partial\mathcal{A}}{\partial P}\right)_{\mathrm{s}}=\frac{2}{3P}. \tag{15}$$

In the meantime, the two versions of the sound velocity formula are:

$$v=\sqrt{\left(\frac{\partial P}{\partial\rho}\right)_{\mathrm{s}}}=\sqrt{\gamma\frac{P}{\rho}}, \tag{16}$$

$$v=\sqrt{\frac{1}{\rho\kappa_{\mathrm{s}}}}, \tag{17}$$

where $\rho=\dfrac{m}{V}$ is the density， m is the mass, V is the volume, and P is the pressure of the medium. The subscript "s" indicates the adiabatic progress and $\gamma=\dfrac{C_{P,m}}{C_{V,m}}$ is the ratio of specific heat capacity.

If we put Eq. (15) into Eq. (17), we can get:

$$v=\sqrt{\frac{3}{2}\frac{P}{\rho}}. \tag{18}$$

Considering the photon gas in n-dimensional space, the relationship between the angular frequency (ω) and the number of quantum states is:

$$D(\omega)\propto\omega^{n-1}. \tag{19}$$

So the total energy of the n-dimensional photon gas is,

$$U=\int_0^\infty\varepsilon f(\omega,T)D(\omega)\mathrm{d}\omega\propto T^{n+1}. \tag{20}$$

From the Stefan-Boltzmann law, the relationship between the temperature and the energy of the photon gas is $U\propto T^{\gamma}$. Comparing it with Eq. (20), we can get $\gamma=n+1$.

This reminds us that Eq. (18) is the special form in the 2-dimensional space, and the general formula should be:

$$v=\sqrt{\frac{\gamma}{n}\frac{P}{\rho}}. \tag{21}$$

If we define $c=\sqrt{n}v$, then:

$$c=\sqrt{\gamma\frac{P}{\rho}}. \tag{22}$$

Comparing Eq. (22) with Eq. (16), when we get Eq. (16), we have used the equation of adiabatic progress, i.e., the Poisson formula $PV^{\gamma}=\text{const}$.

The photon gas satisfies the equation of sound velocity. This further reminds us that the photon gas is a kind of sound gas. Under compression, it can propagate in the form of adiabatic compression just like sound.

We have argued at the beginning of this section that there are similarities between gravitational waves and sound waves. By now, the above conclusion implies that there are also similarities between sound waves and photon gas.

Furthermore, from Section 1, we know that the universal nature of gravity is also demonstrated by the fact that its basic equations closely resemble the laws of thermodynamics and hydrodynamics.

Complying with this, it is valid to generalize the similarities between gravitational waves and adiabatic compression waves to the similarities between photon gas and gravitational waves. Because we can take these similarities equally as similarities between photons and gravitons, we hope to get some properties of gravitons from the properties of photons.

In fact, when the fields are strong, we can also get the plane wave solution of the gravitational field. The Rindler transformations work for a wide variety of spherically symmetric solutions to gravitational field equations. The condition $-g_{00}=N^2\to 0$ on the horizon can generate out the plane wave solutions of any classical (or quantum) field equations near the horizon [24]. This is consistent with the plane wave solution of the gravitational field in the weak-field limit.

In the strong field situation, the chemical potential will be large, and we cannot just make the expansion like Eq. (10). However, we can put the changes of μ into Eq. (9). As we deal with static black holes, we can just leave the sum part of Eq. (9) as a big number which is denoted as M.

Now, the only changes are Eq. (11) and those which follow, so we can get Eq. (11) directly from Eq. (9) by replacing its sum part as M, and Eq. (11) changes to:

$$U(\mu,T)\cong\frac{8\pi k_{\mathrm{B}}^3 M}{c^2h^2}\mathcal{A}T^3=\bar{\sigma}^*\mathcal{A}T^3, \tag{23}$$

where $\bar{\sigma}^*=\dfrac{8\pi k_{\mathrm{B}}^3 M}{c^2h^2}=\dfrac{2k_{\mathrm{B}}^3 M}{\pi c^2\hbar^2}$.

These differences will not change the final result (Eq. (22)) because $\bar{\sigma}^*$ does not get into the relevant calculations, and this tells us how photons act in the strong field.

Furthermore, from the Rindler horizon in static spacetimes, we know that the vacuum state defined in a coordinate system covers the full manifold, and it appears as a thermal state to an observer who is confined to part of the manifold partitioned by a horizon. This result will hold for any static spacetime with a bifurcation horizon, such as the Schwarzschild spacetime and de Sitter spacetime.

All these cases describe a situation in thermal equilibrium at a temperature $k_{\mathrm{B}}T=\dfrac{\hbar c\kappa}{2\pi}$ (where κ is the surface gravity of the horizon) [12]. In all, we can generalize these similarities from weak field to strong field directly as long as we consider the static bifurcation horizons.

Apparently, photons are different from gravitons in many ways. They are particles transferred between different interactions, and they have different coupling constants, etc. However, they do indeed have some similarities.

Anyway, we know that Bekenstein-Hawking entropy (Eq. (1)) is a robust prediction of a yet unknown quantum theory of gravity. Any theory which fails to reproduce this prediction is certainly incorrect. We can check these similarities with this criterion.

3 Area law of static black holes

We will use the similarities proposed in the last two sections, and assume that gravitons can achieve BECs, and such BECs act as the thermodynamic surface (or the holographic screen) of black holes. This means that gravitons exist like a photon gas on the surface. Or equally, the surface acts like the "white-wall box" for gravitons.

This is consistent with the Unruh effect, because the gravitons provide the source of the photon-like particles radiated by this thermodynamic system, like the radiation of the "white-wall box".

From Eq. (23), by using the area heat capacity $C_{\mathcal{A}}=\left(\dfrac{\partial U}{\partial T}\right)_{\mathcal{A}}$, the entropy $S=\displaystyle\int_0^T\frac{C_{\mathcal{A}}}{T}\mathrm{d}T$, and the free energy $\mathcal{F}=U-TS$, we can get:

$$S=\frac{3}{2}\bar{\sigma}^*\mathcal{A}T^2, \tag{24}$$

$$\mathcal{F}=-\frac{1}{2}\bar{\sigma}^*\mathcal{A}T^3, \tag{25}$$

where $T=\dfrac{\hbar c\kappa}{2\pi k_{\mathrm{B}}}$, and $\mathcal{A}$ is the area of horizon of the static black hole [12].

We can simplify Eq. (25) by using the property of an adiabatic system whose entropy is unchanged, so we have:

$$\mathcal{A}T^2 = \text{const.} = \mathcal{A}_{\text{eff}}. \tag{26}$$

Thus, we can reduce the order of the black hole temperature T, and Eq. (25) changes to:

$$\mathcal{F} = -\frac{1}{2}\bar{\sigma}^*\mathcal{A}_{\text{eff}}T = -\frac{k_{\text{B}}^3 M}{\pi c^2\hbar^2}\mathcal{A}_{\text{eff}}T. \tag{27}$$

This means that there is a minimum quantum for length or area, of the order of $\frac{\pi c^2\hbar^2}{k_{\text{B}}^2 M}$ (compared to Ref. [12]). Then the horizon area can be divided into $N = \frac{\mathcal{A}_{\text{eff}}k_{\text{B}}^2 M}{\pi c^2\hbar^2}$ patches (compared to Ref. [14]). If we take the surface of the black hole as the ground state, and the gravitons achieve the BECs here, every unit of the area preserves a graviton with a photon-like average energy of $\bar{\varepsilon} = -k_{\text{B}}T$ (notably, we can see that the minus sign is consistent with the negative heat capacity of a black hole; this is a natural requirement of this thermodynamic surface system). Then, from the surface system:

$$\xi\mathcal{A}_{\text{eff}} = -Nk_{\text{B}}T, \tag{28}$$

i.e.,

$$\xi = \frac{k_{\text{B}}^3 M}{\pi c^2\hbar^2}T. \tag{29}$$

And Eq. (2) becomes:

$$\mathcal{F} = -\frac{k_{\text{B}}^3 M}{\pi c^2\hbar^2}\mathcal{A}_{\text{eff}}T + \text{const.}, \tag{30}$$

i.e., we can get the same result as Eq. (27) simply from the surface system.

Because the static condition always requires constant energy, we can set the constant of Eq. (30) to be U_0 as the self-sustained energy of gravitons, i.e.,

$$\mathcal{F} = -\frac{k_{\text{B}}^3 M}{\pi c^2\hbar^2}\mathcal{A}_{\text{eff}}T + U_0. \tag{31}$$

Finally:

$$\mathcal{S} = \frac{k_{\text{B}}^3 M}{\pi c^2\hbar^2}\mathcal{A}_{\text{eff}} = \left(\frac{k_{\text{B}}^3 M T^2}{\pi c^2\hbar^2}\right)\mathcal{A} \propto \mathcal{A}. \tag{32}$$

This is just like the Bekenstein-Hawking entropy (Eq. (1)). Because the assumption about the minimum quanta for length or area is different from Ref. [12], Eq. (32) and Eq. (1) are not exactly the same.

4 Summary and outlook

Gravity is a kind of emergent phenomenon. This result pulls gravity out of the shield of Einstein's geometric background, and it changes the former picture of black holes. We propose another picture of black holes through a special holographic screen. This screen is located slightly away from the static bifurcation horizon and contains all of its DOF. This is reminiscent of the ordinary thermodynamic surface system. Indeed, they have some similarities to each other. And if we consider static spacetime with a bifurcation horizon, these similarities are much more reasonable.

We then think about the ingredients of this special surface. Inspired by the BEC model of gravity and Dvali's work, we assume that this surface is a kind of BEC of gravitons. At the same time, we know photons can also achieve BECs. The gravitons' speed of light motivates us to build a relationship between gravitons and photons. This was done through the "white-wall box". We first point out that gravitational waves are similar to classic propagating waves. Then we get the similarities between sound waves and photon gas. Finally, we arrive at the similarities between gravitational waves and photon gas. Notably, these similarities can naturally lead to the light speed of gravitons.

Although we build these similarities from the weak-field limit, we can still generalize it to the strong-field situation with small changes. This justifies our work by using this similarity to get the area law of static black holes. We also get a natural reason for their negative heat capacity.

Meanwhile, we already know that Eq. (2) is a very important characteristic for the normal surface system whose constant is exactly zero. As we are now dealing with a black hole, the constant in Eq. (2) cannot be determined in the same way as a normal surface system. We have achieved:

$$U = (\mathcal{A}\xi + U_0) - \mathcal{A}T\frac{\mathrm{d}\xi}{\mathrm{d}T}. \tag{33}$$

As long as ξ is linear with T , we can always make sure that U is a constant (U_0). Meanwhile, it is exactly consistent with our static background. Maybe the linear relationship between ξ and T can be the discriminant of the black hole surface system.

Finally, if gravitational waves are really adiabatic compression waves, what is their medium?

Topological order points out that the vacuum is a kind of string-net condensate [26, 27], similar to BECs. Inspired by this theory, we can take the medium of the gravitational waves as the vacuum or the string-net condensate.

Furthermore, topological order theory cannot include gravity. Through the similarities between gravitons and photons, we hope to deal with gravity within topological order theory[1)] This will be done in the future.

The authors appreciate M. Wasay Abdul so much for his precious advice on this paper.

References

1 S.W Hawking, Phys. Rev. Lett., **70**: 1344 (1971)
2 J.D Bekenstein, Nuovo Cimento Lett., **4**: 737–740 (1972)
3 J.D Bekenstein, Phys. Rew. D, **7**: 2333–2346 (1973)
4 J.D Bekenstein, Phys. Rew. D, **9**: 3292–3300 (1974)
5 J. M. Bardeen, B. Carter, and S. W. Hawking, Commun. Math. Phys., **31**: 161–170 (1973)
6 S.W Hawking, Commun. Math. Phys., **43**: 199 (1975)
7 T.Jacobson, Phy. Rev. Lett., **75**: 7 (1995)
8 G.t Hooft [arXiv:gr-qc/9310026]; L.Susskind, J. Math. Phys, **36**: 6377 (1995)
9 J. M. Maldacena, Adv. Theor .Math. Phys., **2**: 231 (1998); S.S.Gubser, I.R.Klebanov, and A.M.Polyyakov, Phys. Lett.B, **428**: 105 (1998); E.Witten, Adv. Theor. Math. Phys., **2**: 253 (1998); O.Aharony, S.S. Gubser, J. Maldacena, H.Ooguri, and Y.Oz, Phys. Rep., **323**: 183 (2000)
10 S. L. Braunstein, S. Pirandola, and K. Zyczkowski, Phys.Rev. Lett., **110**: 101301 (2013); A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, J. High Energy Phys., **02**: 062 (2013); S. L. Braunstein [arXiv:0907.1190]
11 J. M. Maldacena, Int. J. Theor. Phys., **38**: 1113 (1990)
12 T. Padmanabhan, Rep. Prog. Phys., **73**: 046901 (2010)
13 Tadashi Takayanagi [arXiv: 1204.2450v1]
14 E. P. Verlinde, J. High Energy Phys., **1104**: 029 (2011)
15 Yu-Xiao Liu, Yong-Qiang Wang, and Shao-Wen Wei, Class. Quant. Grav., **27**: 185002 (2010) [arXiv:1002.1062]; Rong-Gen Cai, Li-Ming Cao, and Nobuyoshi Ohta, Phys. Rev. D, **81**: 084012 (2010), [arXiv:1002.1136]; Shao-Wen Wei, Yu-Xiao Liu, and Yong-Qiang Wang, Commun. Theor. Phys., **56**: 455-458 (2011) [arXiv:1001.5238]
16 A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, J. High Energy Phys., **1302**: 062 (2013)
17 T. Padmanabhan [arXiv:0308070v2]
18 W. G. Unruh, Phys. Rev. Lett., **46**: 1351 (1981)
19 M. Visser [arXiv:gr-qc/9311028]
20 C. Barceló, S. Liberati, and M. Visser, Living Rev.Relativity, **8**: 12 (2005)
21 G. Dvali and C. Gomez, Fortschr. Phys, **61**: 742 (2013); Eur. Phys. J. C, **74**: 2752 (2014); [arXiv:1212.0765]; [arXiv: 1307.7630]; J. Cosmol. Astropart. Phys., **01**: 023 (2014); G. Dvali and C. Gomez, Phys. Lett. B, **719**: 419 (2013); [arXiv:1403.6850]
22 J. Klaers, F. Vewinger and M. Weitz, Nature Physics, **06**: 512 (2010)
23 J. Klaers, J. Schmitt, F. Vewinger, and M. Weitz, Nature, **468**: 545–548 (2010)
24 G. Dvali and C. Gomez [arXiv:1207.4059v1]
25 S. Weinberg, Gravitation and Cosmology (Sons, Inc: John Wiley, 1972), p. 1–20
26 Xiao-Gang Wen, [arXiv:1210.1281v2].
27 Xiao-Gang Wen, Phys. Rev. D, **68**: 065003 (2003)

1)Ming Chen and Yong-Chang Huang, Unification of some elementary particles based on topological order theory.