Spectrum and Bethe-Salpeter amplitudes of Ω baryons from lattice QCD *

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Abstract: The Ω baryons with $J^P = 3/2^{\pm}, 1/2^{\pm}$ are studied on the lattice in the quenched approximation. Their mass levels are ordered as $M_{3/2^+} < M_{3/2^-} \approx M_{1/2^-} < M_{1/2^+}$, as is expected from the constituent quark model. The mass values are also close to those of the four Ω states observed in experiments. We calculate the Bethe-Salpeter amplitudes of $\Omega(3/2^+)$ and $\Omega(1/2^+)$ and find there is a radial node for the $\Omega(1/2^+)$ Bethe-Salpeter amplitude, which may imply that $\Omega(1/2^+)$ is an orbital excitation of Ω baryons as a member of the $(D, L_N^P) = (70, 0_2^+)$ supermultiplet in the $SU(6) \bigotimes O(3)$ quark model description. Our results are helpful for identifying the quantum numbers of experimentally observed Ω states.

Keywords: Ω baryon, excited state, BS amplitude

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1 Introduction

There are four Ω baryon states (strange number $S = -3$) observed from experiments [1]. Except for the lowest-lying one, Ω(1672), which is well known as a member of the $J^P = 3/2^+$ baryon decuplet, the J^P quantum numbers of the other states, namely, Ω(2250), Ω(2380), and Ω(2470), have not been completely determined from experiments. If they are dominated by the three-quark components, the conventional $SU(6) \bigotimes O(3)$ quark model with a harmonic oscillator confining potential can be used to give them a qualitative description. In this picture, the baryons made up of u,d,s quarks can be classified into energy bands that have the same number N of the excitation quanta in the harmonic oscillator potential [2]. Each band consists of a number of supermultiplets, specified by (D, L_N^P), where D stands for the irreducible representation of the flavor-spin $SU(6)$ group, L is the total orbital angular momentum, and P is the parity of the supermultiplet. For Ω baryons whose flavor wave functions are totally symmetric, the ground state of Ω baryons should be in the $(56, 0_0^+)$ supermultiplet with the quantum number $J^P = 3/2^+$, namely the Ω(1672) state. The states in the $(70, 1_1^-)$ supermultiplet should have a total spin $S = 1/2$ and a unit of the orbital excitation, such that their J^P quantum number can be either $3/2^-$ or $1/2^-$. Therefore $\Omega_{3/2^-}$ and $\Omega_{1/2^-}$ are expected to be approximately degenerate in mass up to a small splitting due to the different spin wave functions. The $J^P = \frac{1}{2}^+$ Ω baryons should be in either the $(56, 2_2^+)$ or $(70, 0_2^+)$ multiplets. Therefore the several lowest Ω states should have the energy levels ordered as

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$M_{3/2^+} < M_{3/2^-} \approx M_{1/2^-} < M_{1/2^+}$. On the other hand, for the $(56, 2_2^+)$ and $(70, 0_2^+)$ multiplets, since they belong to the different $SU(6)$ representations, their spatial wave functions can be different and can serve as a criterion to distinguish them from each other.

However, the quark model is not an ab-initio method and can only give qualitative results, so studies from first principles are desired, such as the lattice QCD method. Early lattice QCD studies can be found in [3, 4]. The most recent systematic study with unquenched configurations was carried out in Ref. [5] where the authors found 11 strangeness -3 states with energies near or below 2.5 GeV using sophisticated smearing schemes for operators and a variational method for the extraction of energy levels, but found it difficult to distinguish the single Ω states from possible scattering states. In this work, we explore the excited states of Ω baryons in the quenched approximation, whose advantage in this topic is that the excited states are free from the contamination of scattering states. We focus on the several lowest-lying Ω states with $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}$. In addition to their spectrum, we also investigate the Bethe-Salpeter amplitudes of these states through spatially extended operators, which may shed light on the internal structure of these Ω states.

This paper is organized as follows: Section 2 contains our calculation method including the operator constructions, fermion contractions and wave function definitions. The numerical results of the spectrum and the wave functions are presented in Section 3. The conclusions and a summary can be found in Section 4.

2 Operators and correlation functions

2.1 Interpolating operators for Ω baryons

The interpolating operator for Ω baryons can be expressed as

$$\mathcal{O}^{\mu} = \epsilon^{abc}(s_a^{\mathrm{T}}\mathcal{C}\gamma^{\mu}s_b)s_c, \tag{1}$$

where $\mathcal{C} = \gamma_2\gamma_4$ is the C-parity operator, a, b, c are color indices, and s^{T} means the transpose of the Dirac spinor of the strange quark field s. However, $\mathcal{O}^{\mu}$ has no definite spin and can couple to the $J = 3/2$ and $J = 1/2$ states [6]. The $J = 3/2$ and $J = 1/2$ components of $\mathcal{O}^{\mu}_{\Omega}$ can be disentangled by introducing the following projectors [4]

$$\begin{aligned} \mathcal{P}^{\mu\nu}_{3/2} &= \delta^{\mu\nu} - \frac{1}{3}\gamma^{\mu}\gamma^{\nu} - \frac{1}{3p^2}(\not{p}\gamma^{\mu}p^{\nu} + p^{\mu}\gamma^{\nu}\not{p}), \\ \mathcal{P}^{\mu\nu}_{1/2} &= \delta^{\mu\nu} - \mathcal{P}^{\mu\nu}_{3/2}. \end{aligned} \tag{2}$$

In the lattice studies, only the spatial components of $\mathcal{O}^{\mu}$ are implemented. If we consider the Ω baryons in their rest frames, the projectors above can be simplified as

$$\begin{aligned} \mathcal{P}^{ij}_{3/2} &= \delta^{ij} - \frac{1}{3}\gamma^i\gamma^j, \\ \mathcal{P}^{ij}_{1/2} &= \frac{1}{3}\gamma^i\gamma^j. \end{aligned} \tag{3}$$

Thus the spin projected operators with definite spin quantum number can be obtained as

$$\begin{aligned} \mathcal{O}^i_{3/2} &= \sum_j \mathcal{P}^{ij}_{3/2}\mathcal{O}^j_{\Omega}, \\ \mathcal{O}^i_{1/2} &= \sum_j \mathcal{P}^{ij}_{1/2}\mathcal{O}^j_{\Omega}. \end{aligned} \tag{4}$$

Furthermore, one can also use the parity projectors $P^{\pm} = \frac{1}{2}(1 \pm \gamma_4)$ to ensure the definite parities of baryon states.

It should be noted that for now all the operators are considered in the continuum case. On a finite lattice, the spatial symmetry group $SO(3)$ breaks down to the octahedral point group O, whose irreducible representations corresponding to $J = 1/2$ and $J = 3/2$ are the two-dimensional G_1 representation and the four-dimensional H representation, respectively. Generally, there exist subduction matrices to project the continuum operators to octahedral point group operators [7], say,

$$\mathcal{O}(J, \Lambda)_r = \sum_m S(J, \Lambda)^m_r \mathcal{O}(J)_m, \tag{5}$$

where $\mathcal{O}(J)_m$ is the continuum operator with total spin J and the third component of spin m, $\mathcal{O}(J, \Lambda)_r$ is the r-th component of the octahedral point group operator under irreducible representation Λ, $S(J, \Lambda)^m_r$ is the subduction matrices. In our case, $S\left(\frac{1}{2}, G_1\right)$ and $S\left(\frac{3}{2}, H\right)$ are both unit matrices, so that the operators in Eq. 4, which are actully used in this study, are already the irreducible representations of the lattice symmetry group O.

We also consider the spatially extended interpolation operators by splitting $\mathcal{O}^{\mu}$ into two parts with spatial separations. The expressions are written explicitly as

$$\begin{aligned} \mathcal{O}^{\mu}_1(r) &= \sum_{|\vec{r}|} \epsilon^{abc}[s_a^{\mathrm{T}}(x + \vec{r})\mathcal{C}\gamma^{\mu}s_b(x)]s_c(x), \\ \mathcal{O}^{\mu}_2(r) &= \sum_{|\vec{r}|} \epsilon^{abc}[s_a^{\mathrm{T}}(x)\mathcal{C}\gamma^{\mu}s_b(x + \vec{r})]s_c(x), \\ \mathcal{O}^{\mu}_3(r) &= \sum_{|\vec{r}|} \epsilon^{abc}[s_a^{\mathrm{T}}(x)\mathcal{C}\gamma^{\mu}s_b(x)]s_c(x + \vec{r}), \end{aligned} \tag{6}$$

where the summations are over $\vec{r}$'s with the same $r = |\vec{r}|$ in order to guarantee the same quantum number as the case of $r = 0$. These three splitting procedures have been verified to be numerically equivalent, so we make use of the third type, $\mathcal{O}_3(r)$, in the practical study. These operators are obviously gauge variant, so we carry out the

lattice calculation by fixing all the gauge configurations to the Coulomb gauge first.

The general form of the two-point function of a baryon of quantum number J^P with $P=\pm$ is

$$C_J^{\pm,i}(r,t)=\mathrm{Tr}\left[(1\pm\gamma_4)\sum_{\vec{x},j}\langle\mathcal{P}_J^{ij}\mathcal{O}_3^j(r,x)\bar{\mathcal{O}}_3^j(0)P_J^{ji}\rangle\right]. \quad (7)$$

The summation on $\vec{x}$ ensures a zero momentum. For Ω baryons, there exist six different Wick contractions as shown in Fig. 1.

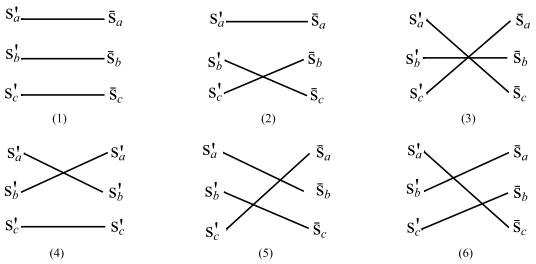

Fig. 1. Six ways of contraction. We use color indices to label the three s quarks. Each solid line stands for a quark propagator.

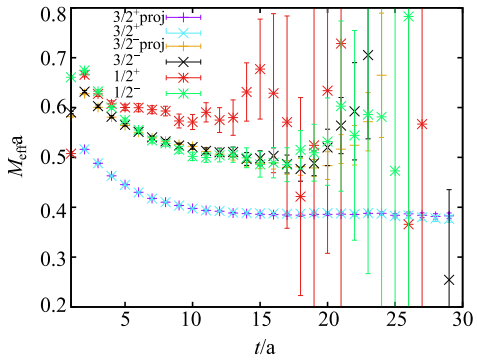

Fig. 2. (color online) The effective mass plateaus of Ω baryons using a point source with/without projections. Except for $J=\dfrac{3}{2}$ states, the point-source two-point functions have no good plateaus at the short time range.

2.2 Source technique

In principle, all states with the same quantum number J^P contribute to the two-point functions $C_J^{P,i}(r,t)$. For baryons, it is known that the the signal-to-noise ratio of the two-points damps very quickly since the noise decreases as $\sim \mathrm{e}^{-3/2m_\pi t}$ in t, which is much slower than the decay of the signal $\mathrm{e}^{-M_B t}$, where M_B is the baryon mass. Therefore, in order to obtain clear and reliable signals of the ground state from two-point functions in the available early time range, some source techniques are implemented by replacing the local operator $O_3^j(\mathbf{0},0)$ by some versions of spatially extended source operators $O_3^{j,(s)}(0)$ which enhance the contribution of the ground state and suppress that from excited states. The extended source operator $O_3^{j,(s)}$ is usually realized by calculating the quark propagators through a source vector with a spatial distribution $\phi(\boldsymbol{x})$,

$$M(x;y)S_F^{(s)}(y;t_0)=\sum_{\boldsymbol{z}}\delta(\boldsymbol{x}-\boldsymbol{z})\delta(t-t_0)\phi(\boldsymbol{z}), \quad (8)$$

thus the effective propagator $S_F^{(s)}(y;t=0)$ relates to the normal point source propagator $S_F(y;\boldsymbol{z},t_0)$ as

$$S_F^{(s)}(y;t_0)=\sum_{\boldsymbol{z}}\phi(\boldsymbol{z})S_F(y;\boldsymbol{z},t_0). \quad (9)$$

When one calculates a baryon two-point function using the same Wick contraction by replacing the point-source propagators with the effective propagators, it is equivalent to using the spatially extended source operator

$$O^{(s)}(t_0)=\sum_{\boldsymbol{z},\boldsymbol{w},\boldsymbol{v}}\phi(\boldsymbol{z})\phi(\boldsymbol{w})\phi(\boldsymbol{v})\psi(\boldsymbol{z},t_0)\psi(\boldsymbol{w},t_0)\psi(\boldsymbol{v},t_0), \quad (10)$$

where $\psi\psi\psi$ stands for the original baryon operator (the color indices and corresponding γ matrices are omitted for simplicity. Note that gauge links should be considered if one requires the gauge invariance of spatially extended operators). The matrix element of $O^{(s)}$ between the vacuum and the baryon state $|B\rangle$, which manifests the coupling of this operator to the state, can be expressed as,

$$\langle 0|O^{(s)}|B\rangle=\sum_{\boldsymbol{z},\boldsymbol{w},\boldsymbol{v}}\phi(\boldsymbol{z})\phi(\boldsymbol{w})\phi(\boldsymbol{v})\Phi_B(\boldsymbol{z},\boldsymbol{w},\boldsymbol{v})\zeta_B, \quad (11)$$

where ζ_B is the spinor reflecting the spin of $|B\rangle$, and $\Phi_B(\boldsymbol{z},\boldsymbol{w},\boldsymbol{v})$ is its Bethe-Salpeter amplitude, which is defined as the corresponding matrix element of the original operator,

$$\langle 0|\psi(\boldsymbol{z})\psi(\boldsymbol{w})\psi(\boldsymbol{v})|B\rangle\equiv\Phi_B(\boldsymbol{z},\boldsymbol{w},\boldsymbol{v})\zeta_B. \quad (12)$$

In order to enhance the coupling $\langle 0|O^{(s)}|B\rangle$ and suppress the related coupling of excited states, the essence is to tune the parameters in $\phi(\boldsymbol{x})$ such that $\phi(\boldsymbol{z})\phi(\boldsymbol{w})\phi(\boldsymbol{v})$ resembles $\Phi_B(\boldsymbol{z},\boldsymbol{w},\boldsymbol{v})$ as closely as possible and the overlap integration in Eq. (11) (actually summations over the spatial lattice sites) can be maximized. If the BS amplitudes can be approximately interpreted to be the spatial wave function of a state, the coupling of this operator to excited states can be subsequently minimized according to the orthogonality of the wave functions. Commonly used source techniques include the Gaussian smeared source [8,9] and the wall source in a fixed gauge. The Gaussian smeared source corresponds to the function $\phi(\boldsymbol{x})\sim\mathrm{e}^{-\sigma^2|\boldsymbol{x}|^2}$ with σ^2 a tunable parameter, while the wall source in a fixed gauge is the extreme situation

of the Gaussian smeared source when $\sigma \to \infty$. The Gaussian smeared source usually works well for states whose BS amplitude has no radial nodes. This is similar to the case in quantum mechanics where a Gaussian-like function serves as a good trial wave function of the ground state in solving a bound state problem using the variational method with σ the variational parameter.

For this work, we first try the Gaussian smeared source for Ω baryons and find it works well for $\Omega_{\frac{3}{2}^+}$. This is not surprising since the $\Omega_{\frac{3}{2}^+}$ is the ground state whose spatial wave functions is $(1s)(1s)(1s)$ in the standard quark model with a harmonic oscillator potential. However for other states, especially for $\Omega_{\frac{1}{2}^+}$, we cannot get a good effective mass plateau before the signals are overwhelmed by noise. Similar phenomena were observed in previous works (see Ref. [4] for example). Inspired by the quark model description that the $J^P = \frac{1}{2}^+$ decuplet baryons belong to the higher excitation energy bands, we conjecture that the BS amplitude of $\Omega_{\frac{1}{2}^+}$ has radial node(s), and thereby propose a new type of source which reflects some node structure, say,

$$\phi(\boldsymbol{x}) = (1 - A|\boldsymbol{x}|^2)\mathrm{e}^{-\sigma^2|\boldsymbol{x}|^2}, \tag{13}$$

where σ and A are parameters to be tuned to give a good effective mass plateau in the early time range. The effects of the extended source operator on the effective masses of different states are illustrated in Fig. 3. For $J^P = \frac{3}{2}^{\pm}, \frac{1}{2}^-$ states, we use the Gaussian smeared sources which improve the qualities of the effective mass plateaus as expected. For the $J^P = \frac{1}{2}^+$ state, the new type of source operators with the nodal structure makes the effective mass plateaus fairly satisfactory, in contrast to the case of a point source. We advocate that this new type of source operator can be potentially applied to other studies on radial excited states of hadrons.

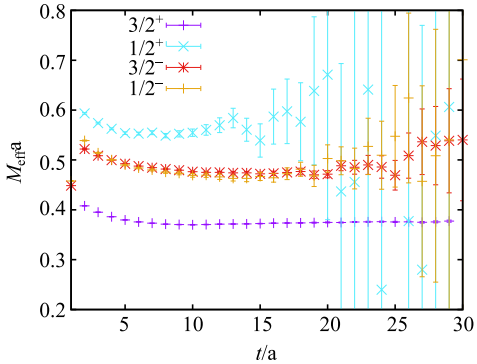

Fig. 3. (color online) Smeared source Ω spectrum. For $J^P = \frac{3}{2}^{\pm}$ and $J^P = \frac{1}{2}^-$, we use a common Gaussian smeared source, while for $J^P = \frac{1}{2}^+$, we use a novel smeared source with a radial node.

3 Numerical details and simulation results

The gauge configurations used in this work were generated on two anisotropic ensembles with tadpole-improved gauge action [10]. The anisotropy $\xi \equiv a_s/a_t = 5$ and the lattice sizes are $L^3 \times T = 16^3 \times 96$ and $24^3 \times 144$, respectively. The relevant input parameters are listed in Table. 1, where the a_s values are determined through the static potential with the scale parameter $r_0^{-1} = 410(20)$ MeV. The spatial extensions of the two lattices are larger than 3 fm, which are expected to be large enough for Ω baryons such that the finite volume effects can be neglected. We use the tadpole improved Wilson clover action [11] to calculate the quark propagators with the bare strange quark mass parameter being tuned to reproduce the physical ϕ meson mass value. (In the calculation of the two-point function of the ϕ meson, we ignore the $s\bar{s}$ annihilation diagram, which contributes little to the two-point function. This can be understood qualitatively through the OZI rule). We used a modified version of a GPU inverter [12] to calculate all the inversions in this work.

Table 1. The input parameters for the calculation. Values of the coupling β, anisotropy ξ, the lattice size, and the number of measurements are listed. a_s/r_0 is determined by the static potential, the first error of a_s is the statistical error and the second comes from the uncertainty of the scale parameter $r_0^{-1} = 410(20)$ MeV.

β	ξ	a_s	La_s/fm	$L^3 \times T$	$N_{\rm conf}$
2.4	5	0.222(2)(11)	$\sim$3.55	$16^3 \times 96$	1000
2.8	5	0.138(1)(7)	$\sim$3.31	$24^3 \times 144$	1000

As mentioned before, the spatially extended operators we use for Ω baryons are not gauge invariant, so we calculate the corresponding two-point functions in the Coulomb gauge by first carrying out gauge fixing to the gauge configurations. By use of source vectors with properly tuned parameters, we generate the quark propagators in this gauge, from which the two-point functions in different channels are obtained. Since we focus on the ground states in each channel, the related two-point functions are analyzed with the single-exponential function form in properly chosen time windows,

$$C_2^J(r,t) \overset{t\to\infty}{\sim} N^J \Phi^J(r) \mathrm{e}^{-m^J t}, \tag{14}$$

where J denotes different quantum numbers, N^J stands for an irrelevant normalization constant, $\Phi^J(r)$ is the BS amplitude and m^J is the mass. In order to take care of the possible correlation, we fit $C_2^J(r,t)$ with different r

simultaneously through a correlated miminal-χ^2 fit procedure, where the covariance matrix is calculated by the bootstrap method. As such, in addition to the masses m_J, we can also obtain the r-dependence of the the BS amplitudes $\Phi^J(r)$. Figure 4 shows the effective mass plateaus for $C^J(r=0,t)$ and the fit range. We quote the bootstrap errors as the statistical ones for masses and BS amplitudes.

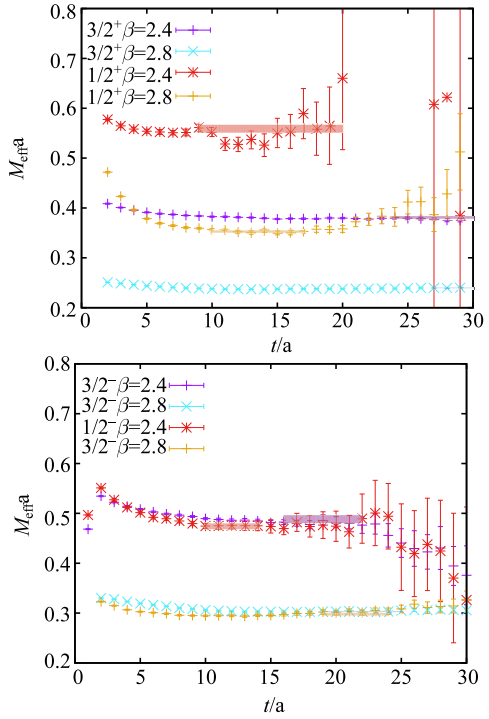

Fig. 4. (color online) Effective mass plots of the Ω system. The two ensembles are both included. The points with error bars are lattice data, while the colored bands are fit results indicating both the fit range and the fit error.

Table 2. The spectrum of the $J^P=\frac{3}{2}^{\pm},\frac{1}{2}^{\pm}$ Ω baryons on the two lattices. The errors of masses are all statistical (we do not include the error owing to the uncertainty of $r_0^{-1}=410(20)$ MeV here). On the last line, except for the well established Ω(1672), we tentatively assign the other three Ω baryons in experiments to the states we observe in this study (labelled with question marks) .

β	$m_{\Omega_{3/2^+}}$ /GeV	$m_{\Omega_{3/2^-}}$ /GeV	$m_{\Omega_{1/2^-}}$ /GeV	$m_{\Omega_{1/2^+}}$ /GeV
2.4	1.668(9)	2.176(26)	2.189(13)	2.464(26)
2.8	1.695(4)	2.153(5)	2.125(14)	2.492(14)
Exp.	Ω(1672)	Ω(2250)?	Ω(2380)?	Ω(2470)?

The masses for different Ω states on the two lattices are listed in Table 2, where the mass values are expressed in physical units using the lattice spacings in Table 1. The masses of these states are insensitive to the lattice spacings, which implies that the discretization uncertainty is small for these states. It is seen that the mass of the $J^P=\frac{3}{2}^+$ Ω we obtain is consistent with the physical mass of Ω(1672), and the masses of $J^P=\frac{3}{2}^-$ and $\frac{1}{2}^-$ are almost degenerate, as expected from the quark model, but lower than the experimental states Ω(2250) and Ω(2380). For the $J^P=\frac{1}{2}^+$ state, we get a mass of 2.464(26) GeV on the coarse lattice and 2.492(14) GeV on the fine lattice, which is in agreement with the mass of Ω(2470).

The BS amplitudes for the $\frac{3}{2}^+$ and $\frac{1}{2}^+$ states are plotted in Fig. 5 (normalized as $\Phi_J(r=0)=1$). In order to compare the results from different lattices, we plot the x-axis in physical units. From the figure one can see that the discretization artifacts are also small for BS amplitudes. We do observe a radial node in the BS amplitude of $\frac{1}{2}^+$ state. We use the following functions

$$\Phi_{\frac{3}{2}^+}(r)=\mathrm{e}^{-(r/r_0)^\kappa},$$
$$\Phi_{\frac{1}{2}^+}(r)=(1-b\ r^\kappa)\mathrm{e}^{-(r/r_0)^\kappa}, \quad (15)$$

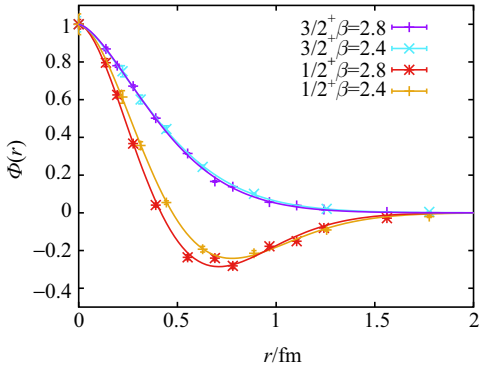

Fig. 5. (color online) The BS amplitudes of $\Omega_{\frac{3}{2}^+}$ and $\Omega_{\frac{1}{2}^+}$. The dots are lattice results while the lines are the fitting functions in Eq. (15). A radial node of the BS amplitude of the $\Omega_{\frac{1}{2}^+}$ is observed.

to fit the data points, which are also plotted in curves in the figure. The fit results are summarized in Table 3.

Now we resort to the non-relativistic quark model to understand the radial behavior of the BS amplitude of $J^P=1/2^+$ Ω. In the non-relativistic approximation, the relativistic quark field ψ can be expressed in terms of its non-relativistic components through the Foldi-Wouthuysen-Tani transformation

$$\psi=\exp\left(\frac{\gamma\cdot\boldsymbol{D}}{2m_s}\right)\begin{pmatrix}\chi\\ \eta\end{pmatrix}, \quad (16)$$

Table 3. Fit results of the BS amplitudes for $\Omega_{\frac{3}{2}^+}$ and $\Omega_{\frac{1}{2}^+}$.

J^p	β	r_0/fm	κ	b
$\frac{3}{2}^+$	2.4	0.504(3)	1.49(2)	
$\frac{3}{2}^+$	2.8	0.494(4)	1.55(2)	
$\frac{1}{2}^+$	2.4	0.568(5)	1.74(3)	3.7(1)
$\frac{1}{2}^+$	2.8	0.529(8)	1.73(4)	4.5(2)

where the Pauli spinor χ annihilates a quark and η creates an anti-quark, and $\boldsymbol{D}$ is the covariant derivative operator. η and χ satisfy the conditions

$$\chi|0\rangle=0,\ \langle 0|\chi^\dagger=0,\ \eta^\dagger|0\rangle=0,\ \langle 0|\eta=0. \tag{17}$$

With this expansion, the operator $\mathcal{O}^i_\Omega$ can be expressed as

$$\mathcal{O}^i_\Omega\sim\epsilon^{abc}\left[\chi^{aT}\left(1+\frac{(\sigma\cdot\overleftarrow{\boldsymbol{D}})^2}{4m_s^2}\right)\sigma_2\sigma_i\left(1+\frac{(\sigma\cdot\overrightarrow{\boldsymbol{D}})^2}{4m_s^2}\right)\chi^b - \chi^{aT}\frac{\sigma\cdot\overleftarrow{\boldsymbol{D}}}{2m_s}\sigma_2\sigma_i\frac{\sigma\cdot\overrightarrow{\boldsymbol{D}}}{2m_s}\chi^b\right]\begin{pmatrix}\left(1+\frac{(\sigma\cdot\overrightarrow{\boldsymbol{D}})^2}{4m_s^2}\right)\chi^c\\ \frac{\sigma\cdot\overrightarrow{\mathbf{D}}}{2m_s}\chi^c\end{pmatrix} + \ldots. \tag{18}$$

We would like to caution that this expansion is not justified rigorously for the strange quark since its relativistic effect in the hadron might be important. However, the non-relativistic quark model usually gives reasonable descriptions of hadron spectra, so we tentatively follow this direction to make the following discussion. The non-relativistic wave function for a baryon state in its rest frame is defined in principle as

$$\Psi_J(\boldsymbol{x_1},\boldsymbol{x_2},\boldsymbol{x_3})\zeta\sim\langle 0|\epsilon^{abc}\chi^{aT}(\boldsymbol{x_1})\chi^b(\boldsymbol{x_2})\chi^c(\boldsymbol{x_3})|\Omega_J\rangle, \tag{19}$$

where ζ stands for the spin wave function for Ω_J. If we introduce the Jacobi coordinates,

$$\begin{aligned}\boldsymbol{R}&=\frac{1}{3}(\boldsymbol{x}_1+\boldsymbol{x}_2+\boldsymbol{x}_3)\\ \boldsymbol{\rho}&=\frac{1}{\sqrt{2}}(\boldsymbol{x}_1-\boldsymbol{x}_2)\\ \boldsymbol{\lambda}&=\frac{1}{\sqrt{6}}(\boldsymbol{x}_1+\boldsymbol{x}_2-2\boldsymbol{x}_3),\end{aligned} \tag{20}$$

as is usually done in the non-relativistic quark model study of baryons, in the rest frame of $\Omega_{1/2^+}$ ($\boldsymbol{R}=0$), the matrix element of $\mathcal{O}^i_J(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x_3})$ between the vacuum and the Ω state can be written qualitatively as

$$\begin{aligned}&\langle 0|\mathcal{O}^i_J(\boldsymbol{x}_1,\boldsymbol{x}_2,\boldsymbol{x_3})|\Omega_J\rangle\\ &\sim\left(D^i+A^i\frac{\partial^2}{\partial\rho^2}+B^i\frac{\partial^2}{\partial\rho\partial\lambda}+C^i\frac{\partial^2}{\partial\lambda^2}\right)\Psi_J(\boldsymbol{\rho},\boldsymbol{\lambda})\zeta,\end{aligned} \tag{21}$$

where we approximate the covariant derivative $\boldsymbol{D}$ by the spatial derivative $\boldsymbol{\nabla}$.

In the standard non-relativistic quark model with harmonic oscillator potentials for baryons, baryons can be sorted into energy bands of the the two independent oscillators, the so-called ρ-oscillator and λ-oscillator, which are depicted by the radial and orbital quantum numbers (n_λ,l_λ) and (n_ρ,l_ρ) [2]. For baryons made up of u,d,s quarks, these energy bands are labelled as (D,L^P_N), where D is the irreducible representation of the flavor-spin $SU(6)$ group, $L=|l_\rho-l_\lambda|,|l_\rho-l_\lambda|+1,\ldots,l_\rho+l_\lambda$ is the total orbital angular momentum, $N=2(n_\rho+n_\lambda)+(l_\rho+l_\lambda)$ is the total number of the excited quanta of the harmonic oscillators, and P is the parity of baryons. For the flavor symmetric Ω baryons, the lowest $J^P=\frac{1}{2}^+$ states can be found in the supermultiplets $(56,2^+_2)$, and $(70,0^+_2)$. $(56,2^+_2)$ has the excitation mode $(n_\lambda,n_\rho)=(0,0)$ and $(l_\lambda,l_\rho)=(2,0)$ or $(0,2)$ with the total spin $S=\frac{3}{2}$, and gives the quantum number $J^P=\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+,\frac{7}{2}^+$. $(70,0^+_2)$ has the excitation mode $(n_\lambda,n_\rho)=(0,0)$ and $(l_\lambda,l_\rho)=(1,1)$ with $S=\frac{1}{2}$, which corresponds to the quantum number $J^P=\frac{1}{2}^+$. In this picture, the spatial wave function of the $(56,2^+_2)$ multiplet can be written qualitatively (here we ignore the angular part) [13,14]

$$\Psi(\rho,\lambda)\sim(\rho^2+\lambda^2)\mathrm{e}^{-\alpha(\rho^2+\lambda^2)}, \tag{22}$$

while the spatial wave function of the $(70,0^+_2)$ multiplet is either

$$\Psi(\rho,\lambda)\sim(\rho^2-\lambda^2)\mathrm{e}^{-\alpha(\rho^2+\lambda^2)}, \tag{23}$$

or

$$\Psi(\rho,\lambda)\sim\rho\lambda\mathrm{e}^{-\alpha(\rho^2+\lambda^2)}, \tag{24}$$

where the parameter α depends on the constituent quark mass and the parameters in the potential. Obviously, the local operators correspond to $\lambda=\rho=0$, such that their coupling to the $J^P=\frac{1}{2}^+$ state can be largely suppressed. Recall that the interpolation operator we use for Ω baryons is $\mathcal{O}_3(r)$, which corresponds to $\rho=0$ and $\lambda\propto r$. As such, we have the qualitatively radial behaviour of the Bethe-Salpeter amplitudes

$$\langle 0|\mathcal{O}^i_{3,\frac{1}{2}^+}(r)|\Omega_{\frac{1}{2}^+}\rangle\sim(A'+B'r^2+C'r^4)\mathrm{e}^{-\alpha r^2}\zeta^i, \tag{25}$$

if we use the wave functions in Eq. (22) and Eq. (23), and

$$0|\mathcal{O}^i_{3,\frac{1}{2}^+}(r)|\Omega_{\frac{1}{2}^+};(70,0^+_2)\rangle\sim(A''+B''r^2)\mathrm{e}^{-\alpha r^2}\zeta^i \tag{26}$$

for the wave function form in Eq. (24). Obviously, the former may have two nodes in the r direction, while the latter has only one. In this sense, the radial behavior of the BS amplitudes in Fig. 5 may imply that the $J^P = \frac{1}{2}^+$ Ω baryon we have observed is possibly mainly the $(70, 0_2^+)$ state, whose spatial wave function may have the qualitative form in Eq. (24). It should be noted that these discussions are very tentative and the the reality can be much more complicated. This can be seen in Table 3 where the parameters κ deviate substantially from $\kappa = 2$ which corresponds to the harmonic oscillator potential.

4 Summary

We have carried out a lattice study of the spectrum and the Bethe-Salpeter amplitudes of Ω baryons in the quenched approximation. In the Coulomb gauge, we propose a new type of source vector for the calculation of quark propagators, which is similar in spirit to the conventionally used Gaussian smearing source technique, but is oriented to increase the coupling to the states whose Bethe-Salpeter amplitude may have more complicated nodal behavior than that of the ground state. As for excited states, whether orbital excitation or radial excitation, it is expected that their BS amplitude may have radial nodes, so we use source vectors with nodal structures, which resemble the node structure of its BS amplitude. This technique works in practice, since we can obtain fairly good effective mass plateaus for $J^P = \frac{1}{2}^+$ at the early time slices.

With the quark mass parameter tuned to be at the strange quark mass using the physical mass of the ϕ meson, we calculate the spectrum of Ω baryons with the quantum number $J^P = \frac{3}{2}^\pm, \frac{1}{2}^\pm$ on two anisotropic lattices with the spatial lattice spacing set at $a_s = 0.222(2)$ fm and $a_s = 0.138(1)$ fm, respectively. On both lattices, the $J^P = \frac{3}{2}^-$ and $\frac{1}{2}^-$ Ω baryons have almost degenerate masses in the range from 2100 MeV to 2200 MeV. This is compatible with the expectation of the non-relativistic quark model that they are in the same supermultiplet $(70, 1_1^-)$ with the same excitation mode, say, $(n_\lambda, n_\rho) = (0,0)$ and $(l_\rho, l_\lambda) = (1,0)$ or $(0,1)$, and the same total quark spin $S = \frac{1}{2}$. For the $\frac{1}{2}^+$ Ω baryon, we obtain its mass at roughly 2400–2500 MeV. Furthermore, we also calculate the BS amplitude of the $\frac{1}{2}^+$ Ω baryon in the Coulomb gauge and observe a radial node, which can be qualitatively understood as the reflection of the second order differential of the non-relativistic wave function of $(70, 0_2^+)$ baryons. Therefore it is preferable to assign the $\frac{1}{2}^+$ Ω state we observe to be a member of the $(70, 0_2^+)$ supermultiplet instead of that of $(56, 2_2^+)$.

We notice that the latest $N_f = 2+1$ full-QCD lattice calculation has obtained 11 energy levels of the Ω spectrum around and below 2500 MeV, but has difficulties in the assignment of their states because there is no reliable criterion to distinguish single particle states from the would-be scattering states. Fortunately we are free of this kind of trouble with the quenched approximation, so that the masses we obtain can be taken as those of the bare Ω baryon states before their hadronic decays are switched on. In comparison with the experiments, our predicted masses of $J^P = \frac{3}{2}^-$ and $\frac{1}{2}^-$ Ω baryons are close to that of $\Omega(2250)$, and the mass of $J^P = \frac{1}{2}^+$ is consistent with $\Omega(2470)$. This observation may be helpful in determining their J^P quantum numbers.

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