A realistic model for charged strange quark stars

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Abstract: We report a general approach to solve an Einstein-Maxwell system to describe a static spherically symmetric anisotropic strange matter distribution with linear equation of state in terms of two generating functions. It is examined by choosing Tolmann IV type potential for one of the gravitational potentials and a physically reasonable choice for the electric field. Hence, the generated model satisfies all the required major physical properties of a realistic star. The effect of electric charge on physical properties is highlighted.

Keywords: Einstein-Maxwell system, equation of state, strange charged matter.

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1 Introduction

The modeling and description of compact astrophysical objects has been of fundamental importance in astrophysics and has attracted much attention in the relativistic community. There has been a substantial effort on experimental searches for strangelets ("strange-matter nuggets") at leading laboratories and these terrestrial experiments are being complemented by observational searches for strange quark stars (SQS) [1–5]. The models developed in these studies predict at least twice the equilibrium density of nuclear matter exotica, $\rho_s \simeq 2.7\times10^{14}$ g cm^{-3} (or $n_s \simeq 0.16$ baryons fm^{-3}) that could eventually appear in the form of hyperons, a Bose condensate of pions (or kaons), or deconfined quark matter. Measurements of rotational periods of stellar objects is an experimental signature for the mass-radius relationship, and the rotational periods of such self-bound SQS are considerably shorter than those predicted for gravitationally bound neutron stars. The pulsar SAX J1808.4-3658, with a rotation period of 2.5 ms, is one of the fastest spinning X-ray pulsar observed and its higher mass-radius ratio suggests a likely SQS candidate[4, 5]. Moreover, recent discoveries of very heavy pulsars with mass around two solar masses [6, 7] has motivated the study within the conventional MIT bag model of charged strange quark stars. Studies show that the presence of electromagnetic field modifies the upper bound to mass-to-radius ratio set by Buchdahl limit for static neutral spheres [8]. Weber et al. [9–11] and Negreiros et al. [12] show that electric fields generated by charge distributions in SQS increase the stellar mass by up to 30% depending on the strength of the electric field. Bare strange quark matter (SQM) may produce ultra-strong electric fields of the order of 10^{18} V/cm [13] and 10^{20} V/cm for color superconducting SQM [14–16]. Hence, the study of Einstein-Maxwell systems in static spherically symmetric spacetime are of vital importance to construct fluid models of superdense matter and study the role played by electromagnetic field in the stability of charged compact objects. If a sphere carries a certain modest electric charge density, then it can remain in equilibrium under its own gravitation and electric repulsion [17], and be more stable than the same system without charge. The electric charge in fact counterbalances the gravitational attraction by repulsive Coulombian force in addition to the pressure gradient [18], and hence the gravitational collapse of a spherically symmetric distribution of matter to a point singularity may be avoided during the gravitational collapse or during an accretion process [19].

A less extreme high density scenario is the formation of more stable quark matter in the centre of a neutron star. In such hybrid stars when SQM is created, an interesting possibility is that the energy released by the conversion may actually trigger the whole star to convert to SQM [20]. This deconfined state of noninteracting quark matter, consisting of almost equal amounts of up, down and strange quarks confined inside a bag, is speculated to be absolutely stable and forms the true ground state of hadronic matter [1, 20, 21]. In this regime, when matter becomes dense enough to overcome the potential barrier, the exothermic energy of the reaction then may ensure the conversion of all matter to SQM, which is referred to as a SQS. On the assumption that SQM

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make up the whole star, the MIT bag model [22] simulates confinement by the introduction of a vacuum pressure, that acts as a force on the quarks confining them to a region in space. In the simplified version of the bag model, one gets the equation of state (EOS) for SQM as $p=(1/3)(\rho-4B)$ [23, 24], where B is known as the bag constant, which is the universal pressure on the surface of any region containing quarks. The stability of the SQS is due to the long-range effects of confinement of quarks, represented by the bag constant B [13, 22, 25–32]. Here we assume that quarks are massless and non-interacting, with pressure $p_{\rm q}=(1/3)\rho_{\rm q}$, total energy density $\rho=\rho_{\rm q}+B$ and the total pressure $p=p_{\rm q}-B$, where $\rho_{\rm q}$ is the quark energy density. The EOS formulated by Dey et al. [3] can also be approximated to a linear form [33] that describes the quark interaction in a quark star by an interquark vector potential originating from gluon exchange and a density-dependent scalar potential which restores chiral symmetry at high density. Moreover, the linearity in EOS found in color-superconducting matter (e.g. Lugones and Horvath [34]) and the tentative parameterizations done by Alford et al. [35], for example, further motivate the linearity advocated by the MIT bag model in studying ultra-compact stellar objects such as SQS [25–31]. There have also been studies to model ultra-compact SQS using a Van der Waals type EoS [36, 37]. Quark matter theory consists of too many unsolved puzzles which prohibits considering all physical and astrophysical properties simultaneously.

As the early work of Ruderman [38] and others [39, 40] shows, the pressure anisotropy $\Delta=p_{\rm t}-p_{\rm r}$ plays a crucial role in modelling SQS relativistically and small anisotropies might in principle drastically change the stability of the system. Various factors may contribute to pressure anisotropy [41] and there are two particularly related to our primary interest. The first one is strong electromagnetic fields [14, 42, 43]. The second source of anisotropy expected to be present in neutron stars and, in general, in highly dense matter, is the viscosity [44–46].

Static spherically symmetric spacetimes with anisotropic distribution are widely used in describing compact objects and with strange matter. Such models produce solutions which may be related to strange matter equation of state [37, 47–53]. Hence, in this work, we develop a general approach to solve the Einstein-Maxwell system to describe a static spherically symmetric anisotropic SQS with linear equation of state in terms of two generating functions. The matter properties may be investigated by an appropriate choice of the two generating functions. In this work, to signify the importance of this approach, particular choices of a Tolmann IV type function for one of the gravitational potentials, and a physically reasonable function for the electric field E, are made to model SQS. This enables comparison with previous investigations of the uncharged ($E=0$) scenario.

2 Charged sphere

The gravitational field for static, spherically symmetric spacetime can be describe by the line element

$$\mathrm{d}s^2=-\mathrm{e}^{2\nu(r)}\mathrm{d}t^2+\mathrm{e}^{2\lambda(r)}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2) \quad (1)$$

in Schwarzschild coordinates $(x^a)=(t,r,\theta,\phi)$.

$$T_{ij}=\mathrm{diag}(-\rho-E^2,p_{\rm r}-E^2,p_{\rm t}+E^2,p_{\rm t}+E^2), \quad (2)$$

where $\rho=\rho(r)$ is the energy density, $p_{\rm r}=p_{\rm r}(r)$ is the radial pressure, $p_{\rm t}=p_{\rm t}(r)$ is the tangential pressure and $E=E(r)$ is the electric field. These quantities are measured relative to the comoving fluid velocity $u^i=\mathrm{e}^{-\nu}\delta^i_0$. For the line element (1) and matter distribution (2) the Einstein-Maxwell system of field equations can be expressed as

$$\rho=\frac{1}{r^2}\left[r(1-\mathrm{e}^{-2\lambda})\right]'-E^2, \quad (3)$$

$$p_r=-\frac{1}{r^2}\left(1-\mathrm{e}^{-2\lambda}\right)+\frac{2\nu'}{r}\mathrm{e}^{-2\lambda}+E^2, \quad (4)$$

$$p_t=\mathrm{e}^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right)-E^2, \quad (5)$$

$$E=\frac{1}{2r^2}\int_0^r r^2\sigma\mathrm{e}^{\lambda}\mathrm{d}r \ = \ \frac{q}{r^2}, \quad (6)$$

where $\sigma=\sigma(r)$ is the proper charge density, $q=q(r)$ is the total charge within a sphere of radius r and primes denote differentiation with respect to r. For detailed derivation of the field equation see Maurya et al [54]. In the field equations (3)–(6), we employ the geometrized units $G=c=8\pi=1$. The system of equations (3)–(6) governs the behaviour of the gravitational field for an isotropic charged imperfect fluid.

If we introduce the transformation

$$x=r^2, \quad Z(x)=\mathrm{e}^{-2\lambda(r)} \text{ and } y^2(x)=\mathrm{e}^{2\nu(r)}, \quad (7)$$

the system (3)–(5) takes the form

$$\rho=\frac{1-Z}{x}-2\dot{Z}-E^2, \quad (8)$$

$$p_{\rm r}=4Z\frac{\dot{y}}{y}+\frac{Z-1}{x}+E^2, \quad (9)$$

$$p_{\rm t}=4xZ\frac{\ddot{y}}{y}+(4Z+2x\dot{Z})\frac{\dot{y}}{y}+\dot{Z}-E^2, \quad (10)$$

$$E=\frac{1}{4\sqrt{x}}\int_0^x\sqrt{\frac{x}{Z}}\sigma\mathrm{d}x \ = \ \frac{q}{x}, \quad (11)$$

where dots denote differentiation with respect to the variable x.

For a physically realistic relativistic star we expect that the matter distribution should satisfy a barotropic equation of state $p_{\rm r}=p_{\rm r}(\rho)$. As motivated in the introduction, we assume a linear EOS to represent SQM:

$$p_{\rm r}=\alpha\rho-\beta, \tag{12}$$

where α and β are constants. With the inclusion of (12) the general solution to the Einstein system (3)–(5) can be expressed as

$$\mathrm{e}^{2\lambda}=\frac{1}{Z} \tag{13}$$

$$\mathrm{e}^{2\nu}=y^2 \tag{14}$$

$$\rho=\frac{1-Z}{x}-2\dot{Z}-E^2, \tag{15}$$

$$p_{\rm r}=\alpha\rho-\beta, \tag{16}$$

$$p_{\rm t}=p_{\rm r}+\Delta, \tag{17}$$

$$\Delta=4xZ\frac{\ddot{y}}{y}+\dot{Z}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1-Z}{x}-2E^2 \tag{18}$$

$$\sigma=4\sqrt{\frac{Z}{x}}(x\dot{E}+E) \tag{19}$$

in terms of the gravitational potential Z and the electric field E, where the quantity $\Delta=p_{\rm t}-p_{\rm r}$ is the measure of anisotropy,

$$y=\mathrm{d}x^{\frac{-(1+\alpha)}{4}}Z^{\frac{-\alpha}{2}}\exp\left[\int\left(\frac{(1+\alpha)(1-E^2x)-\beta x)}{4xZ}\right)\mathrm{d}x\right] \tag{20}$$

and d is the constant of integration. Therefore the line element (1) takes the form

$$\begin{aligned}\mathrm{d}s^2=&-\mathrm{d}^2r^{-(1+\alpha)}Z^{-\alpha}\\&\times\exp\left[\int\left(\frac{(1+\alpha)(1-r^2E^2)-\beta r^2}{rZ}\right)\mathrm{d}r\right]\mathrm{d}t^2\\&+Z^{-1}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2).\end{aligned} \tag{21}$$

Hence, any solution describing static spherically symmetric anisotropic charged matter distribution with a linear equation of state can be easily determined by two generating functions $Z(r)$ and $E(r)$. It is noted that when $E(r)=0$ the line element (21) describes the anisotropic uncharged matter distribution with a linear equation of state [55].

3 Particular model

To investigate the physical significance of the matter quantities, the solution must be given in simple elementary functions without an integral. A variety of choices can be made for the gravitational potential Z and the electric field E. However, the choices must be physically reasonable to model a realistic stellar object. To investigate the physical significance of the matter quantities, we choose a particular form:

$$Z=\frac{(1+ax)(1-bx)}{(1+2ax)}, \tag{22}$$

$$E^2=\frac{kx}{(1+2ax)}, \tag{23}$$

where a,b and k are real constants. A similar form of the gravitational potential (22) has been used to study a relativistic compact sphere with isotropic matter distribution by Tolman [56] and satisfy all the physical requirements: regularity of the gravitational potentials at the origin, positive definiteness of the energy density and the isotropic pressure at the origin, vanishing of radial pressure at some finite radius, monotonic degrease of the energy density and the isotropic pressure with increasing radius and satisfy the casuality condition [57]. The electric field vanishes at the origin and is continuous for a wide range of the parameters a and k. For the choices (22) and (23) the solution (13)–(19) takes the form

$$\mathrm{e}^{2\lambda}=\frac{(1+2ax)}{(1+ax)(1-bx)} \tag{24}$$

$$\begin{aligned}\mathrm{e}^{2\nu}=&\,\mathrm{d}^2(1+ax)^{\frac{a[a(1-\alpha)-2b\alpha+\beta]+k(1+\alpha)}{2a(a+b)}}(1+2ax)^{\alpha}\\&\times(1-bx)^{\frac{-b[b(1+3\alpha)+2a(1+2\alpha)+\beta]+2a\beta+k(1+\alpha)}{2b(a+b)}}\end{aligned} \tag{25}$$

$$\begin{aligned}\rho=&\frac{3(a+b)+(2a+7b)ax+6a^2bx^2}{(1+2ax)^2}\\&-\frac{kx}{(1+2ax)}\end{aligned} \tag{26}$$

$$p_{\rm r}=\alpha\rho-\beta, \tag{27}$$

$$p_{\rm t}=p_{\rm r}+\Delta, \tag{28}$$

$$\begin{aligned}\Delta=&\frac{4x(1+ax)(1-bx)}{(1+2ax)}\frac{\ddot{y}}{y}\\&-\left(\frac{a+b+2abx(1+ax)}{(1+2ax)^2}\right)\left(1+2x\frac{\dot{y}}{y}\right)\\&+\frac{a+b(1+ax)-2kx}{1+2ax}\end{aligned} \tag{29}$$

$$\sigma^2=\frac{4k(1+ax)(1-bx)(3+4ax)^2}{(1+2ax)^4} \tag{30}$$

The solution (24)–(30) is given in simple elementary functions so that it is more convenient to study the physical behaviour of SQS. It is interesting to note that the solution (24)–(30) reduces to the uncharged model of Thirukkanesh and Ragel [55] when $k=0$ (i.e., $E=0$). Thirukkanesh and Ragel [55] showed that their model satisfies all the physical requirements to describe a realistic star with anisotropic matter distribution. They have also shown that their model contains the dark energy star model of Lobo [58] and isotropic models of Einstein and de Sitter.

4 Junction condition

The interior solution generated in Section 3 must match smoothly with the exterior Reissnar-Nordstrom metric:

$$\mathrm{d}s^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 +r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2) \tag{31}$$

across the boundary $r=s$, where M is the gravitational mass of the distribution such that

$$M=m(s)+\varepsilon(s) \tag{32}$$

while

$$m(s)=\frac{1}{2}\int_0^s \rho r^2\mathrm{d}r, \quad \varepsilon(s)=\frac{1}{2}\int_0^s r\sigma q\mathrm{e}^{\lambda}\mathrm{d}r \tag{33}$$

$$\text{and} \quad Q=q(s) \tag{34}$$

In the above $m(s)$ is the total mass inside the sphere while $\varepsilon(s)$ is the mass equivalence of the electromagnetic energy distribution and Q is the total charge inside the sphere [59]. This imposes the conditions:

$$\left(1-\frac{2M}{s}+\frac{Q^2}{s^2}\right) = \mathrm{d}^2(1+as^2)^{\frac{a[a(1-\alpha)-2b\alpha+\beta]+k(1+\alpha)}{2a(a+b)}}(1+2as^2)^{\alpha} \times(1-bs^2)^{\frac{-b[b(1+3\alpha)+2a(1+2\alpha)+\beta]+2a\beta+k(1+\alpha)}{2b(a+b)}} \tag{35}$$

$$\left(1-\frac{2M}{s}+\frac{Q^2}{s^2}\right)=\frac{(1+as^2)(1-bs^2)}{(1+2as^2)}. \tag{36}$$

Condition (36) is satisfied automatically due to the preposition (32) while condition (35) restricts the constant d as

$$d^2=(1+as^2)^{\frac{(1+\alpha)[a^2+2ab-k]-a\beta}{2a(a+b)}}(1+2as^2)^{-(1+\alpha)} \times(1-bs^2)^{\frac{(1+\alpha)(4ab+3b^2-k)-(2a+b)\beta}{2b(a+b)}}$$

5 Physical analysis

In this section, we show that the solutions generated in Section 3 are physically viable for modelling a charged anisotropic strange star

i. *Regularity of the gravitational potentials at the origin.*
Since $\mathrm{e}^{2\nu(0)}=\mathrm{d}^2, \mathrm{e}^{2\lambda(0)}=1$, which are constants and $(\mathrm{e}^{2\nu(r)})'=(\mathrm{e}^{2\lambda(r)})'=0$ at the origin ($r=0$), the gravitational potentials are regular at the origin.

ii. *Positive definiteness of the energy density and the radial pressure at the origin.*
At the origin, the energy density $\rho(0)=3(a+b)$ and the radial pressure $p_{\mathrm{r}}(0)=3\alpha(a+b)-\beta$. Hence, it is possible to make both the energy density and radial pressure be positive at the origin for appropriate choices of the parameters a,b,α and β.

iii. *Vanishing of radial pressure at some finite radius.*
At $r=s$, the radial pressure $p_{\mathrm{r}}(s)=0$ gives

$$s=\frac{1}{2}\left\{\frac{1}{a[a(3b\alpha-2\beta)-k\alpha]}\left[a(4\beta-\alpha(2a+7b)+k\alpha)-\sqrt{\alpha[a^2(2a+b)[\alpha(2a-23b)+16\beta]+k\alpha[10a(2a+b)+k]]}\right]\right\}^{1/2}$$

which is finite for appropriate choices of the parameters a,b,α,β and k.

iv. *Monotonic degrease of the energy density and the radial pressure with increasing radius.*
As $\frac{\mathrm{d}\rho}{\mathrm{d}r} = -\left[\frac{2ar(2a+b)(5+2ar^2)}{(1+2ar^2)^3}+\frac{k}{(1+2ar^2)^2}\right]$ which is negative for suitable choices of parameters a,b and k, the energy density is a decreasing function of r. Therefore, the radial pressure p_{r} is also a decreasing function for the same choice of parameters because $\frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}r}=\alpha\frac{\mathrm{d}\rho}{\mathrm{d}r}$.

v. *Continuity of the electric field across the boundary.*
From (11), it is clear that $E(s)=\frac{Q}{s^2}$. Therefore, the electric field is continuous across the boundary.

vi. *Casuality condition: speed of sound should be less than the speed of light.*
The square of the speed of sound in the interior of the star is given by $\frac{\mathrm{d}p_{\mathrm{r}}}{\mathrm{d}\rho}=\alpha$. To maintain casuality throughout the interior of star we must choose values of α such that $0<\alpha<1$.

Recent discoveries of very heavy pulsars with mass around two times the solar mass [6, 7] motivates seeking clarification within the framework of the conventional MIT bag model. As motivated in the introduction, we fix $\alpha=1/3$ in the bag model. Many studies have used this value to model a static relativistic strange star [24, 32, 48, 60]. The significance of the equation of state and interactions for the maximum mass is that if the pressure for a given energy density is large, the system is able to sustain a large gravitational force. Hence, as given by equation (4) and illustrated in Table 1, net electric charge distribution inside the SQS can significantly enhance the star mass and thus support experimental observations of very heavy stars. However, it is of interest to verify the chemical equilibrium among quarks and electrons in the quark star matter with respect to the weak interactions and thus ascertain the validity within the bag model. In this regard, including the effect of the strange quark mass and expressing the quark chemical potential as an average of up, down and strange quark chemical potentials, the chemical potential of electrons may be expressed as $\mu_{\rm e} \simeq m_{\rm s}^2/4\mu$, where m_s is the strange quark mass; and hence, β in the equation of state (12) becomes in natural units

$$\frac{1}{8\pi G}\left[\frac{4B}{3}+\frac{\mu^2 m_{\rm s}^2}{2\pi^2}\right]$$

to represent non-interacting unpaired SQM within the bag model with strange quark mass corrections to lowest order [61].

Table 1. Density (central and surface), Electric field (surface), Total charge, gravitational mass and chemical potential for different choice of parameter k.

Radii $(s)=10$ km $a=0.02, \alpha=1/3$		$E=0$		$E\neq 0$					
$b\times 10^{-3}$	$\rho(0)/(10^{15}$ gcm$^{-3})$	$\rho(s)/(10^{15}$ gcm$^{-3})$	$M/M_\odot$	k	$E(s)/$ $(V{\rm cm}^{-1})$	$Q/$ C	$\rho(s)/(10^{15}$ gcm$^{-3})$	$M/M_\odot$	$\mu/$ MeV
2.624	3.638	0.531	1.8863	5×10^{-5}	3.303×10^{20}	3.675×10^{20}	0.477	2.431	402.8
2.136	3.560	0.488	1.7873	1×10^{-5}	1.477×10^{20}	1.643×10^{20}	0.488	1.896	396.2
2.075	3.550	0.483	1.7749	5×10^{-6}	1.044×10^{20}	1.162×10^{19}	0.477	1.829	395.3
2.026	3.542	0.478	1.7649	1×10^{-6}	4.671×10^{19}	5.197×10^{19}	0.477	1.776	387.7
2.020	3.541	0.478	1.7637	5×10^{-7}	3.303×10^{19}	3.675×10^{19}	0.477	1.769	391.1

We choose a value for the bag constant $B^{1/4}=150$ MeV according to the SQM hypothesis, and with $m_{\rm s}\simeq 90$ MeV [61], values for the physical quantities in Table 1 have been calculated for a pulsar of radius 10 km by choosing the values for model parameters a, b and k such that the conditions [i]–[vi] are satisfied to correspond to a realistic star. In Table 1, we clarify that the inclusion of net electric charge distribution inside the SQS can significantly enhance the star mass and thus support experimental observations of very heavy pulsars of about two solar masses, such as PSR J 1614-2230 [6] and PSR J0348+0432 [7]. We observe from Table 1 that μ remains within the acceptable limit of 300-500 MeV [61], and hence chemical equilibrium among quarks and electrons in the SQM is maintained. Within these stability limits, the energy-density associated with electric field of the order 10^{19} to 10^{20} Vcm^{-1} contributes to about 0.3% to 22% to the gravitational mass of the SQS, and these numerical results are in the order of magnitude reported on charged SQS by Negreiros et al [12], which also supports massive two-solar-mass pulsars. However, Madsen's work [62] shows that electron-positron pair creation in supercritical electric fields limits the net charge Q of a static, spherically symmetric SQS to the order of 10^{15} C, which is below the values generated in Table 1. At present there is no generally accepted unified theory of electromagnetism and gravitation, and hence the upper limit for net charge of strange stars might still be an open question.

Figures 1–10 illustrate graphically the physical features of realistic SQS that may correspond to pulsars that have a radius of around 10 km. These graphs are plotted for the parameter values correspond to the first row of Table 1. Figure 1 shows that the electric field E^2 is zero at the centre and increases monotonically and reaches a maximum value at the stellar boundary. Figure 2 shows that the charge density is non-singular at the center of the star, decreases monotonically from the center to the surface of the star, and attains a non-zero value at the surface. Figures 3–5 show the radial dependence of $\rho, p_{\rm r}$ and $p_{\rm t}$ for charged (solid curve) and uncharged (dashed curve) cases of the model. Figures 3 and 4 show that the energy density and radial pressure are continuous throughout the interior and monotonically decreasing towards the surface. There is a clear surface boundary with non-zero energy density with zero radial pressure at $r=10$ km. The scenario of non-zero tangential pressure at the surface boundary has been interpreted as due to particle movement in circular orbits [63, 64], and is related to the surface tension [65]. It is also seen that the presence of electric field reduces the interior energy density and enhances the radial pressure. Figure 5 illustrates the continuous radial dependence of the tangential pressure, but with a pronounced effect from the electric field; $p_{\rm t}$ becomes zero around $r=6.4$ km and then changes direction and increases for $r>6.4$ km, showing a non-zero value at the boundary, which is physically viable [41]. As a result the pressure anisotropy is directed inwards $(\Delta<0)$ for charged case. For the uncharged case there is a turbulence in the pressure anisotropy, directing inward for $r<6.2$ km and then directing outward in the region $r>6.2$ km, which indicates instabilities in the internal substance along the radius [40]. Figure 7 illustrates the stability condition $\left(-1 \leqslant \frac{\mathrm{d}p_\mathrm{t}}{\mathrm{d}\rho} - \frac{\mathrm{d}p_\mathrm{r}}{\mathrm{d}\rho} \leqslant 0\right)$ [66] for the same parameter values in Table 1, which shows that the outer region of the modeled compact star is potentially stable ($r > 4.0$ km for the charged case and $r > 2.5$ for the uncharged case). It is clear from Figure 8 and Figure 9 that in both charged and uncharged cases the variation $\rho - p_\mathrm{r} - 2p_\mathrm{t}$ and $\rho + p_\mathrm{r} + 2p_\mathrm{t}$ are positive throughout the interior of the star and hence satisfy the energy conditions [47, 53]. Figure 10 illustrates the relation between radius and mass for different values of Q.

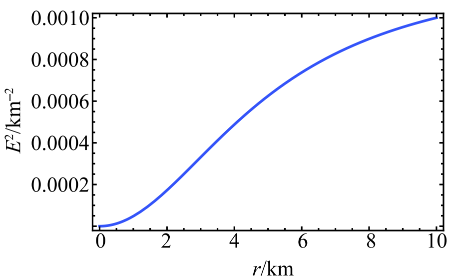

Fig. 1. Electric field.

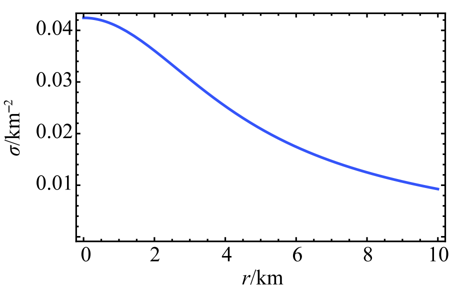

Fig. 2. Charge density.

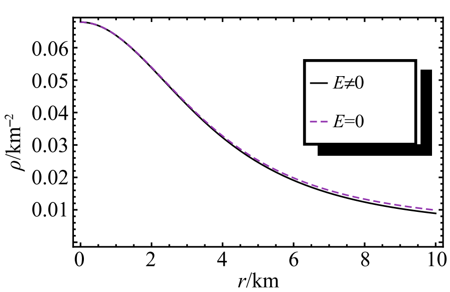

Fig. 3. Energy density.

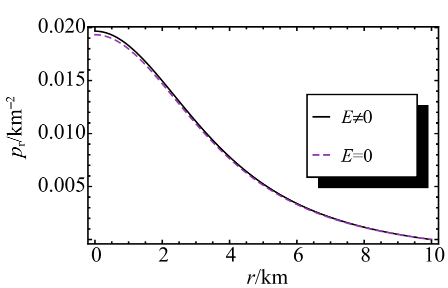

Fig. 4. Radial pressure.

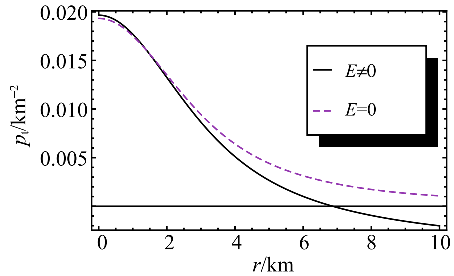

Fig. 5. Tangential pressure.

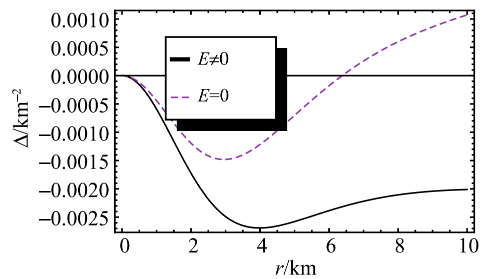

Fig. 6. Measure of anisotropy.

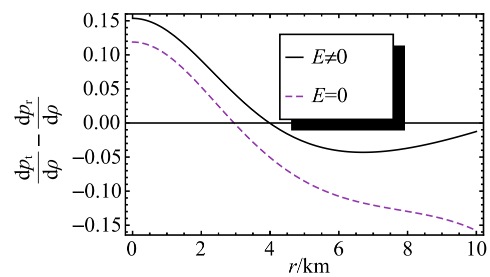

Fig. 7. Stability condition.

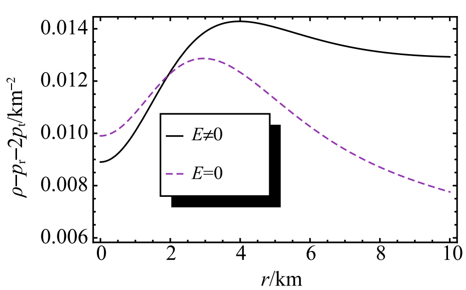

Fig. 8. Energy condition.

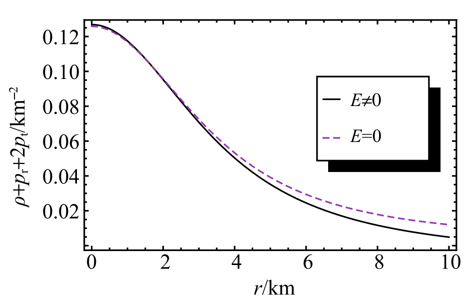

Fig. 9. Energy condition.

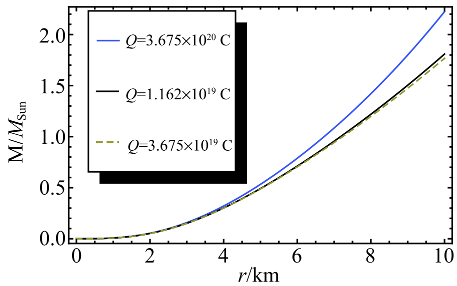

Fig. 10. Total mass.

As motivated in Section 1, very dense baryonic matter is speculated to exist in SQS [67, 68]. If such SQS should exist, they should be made of chemically equilibrated SQM, which requires the presence of electrons inside SQS that play a crucial role in producing electric field. The SQM at the core of the SQS consists of roughly equal numbers of up, down and strange quarks and a relatively small number of electrons. However, as this composition changes toward the surface of the SQS, a higher number of electrons is required to maintain electric charge neutrality [13]. The actual value of the electric field may depend on electrostatic effects, including Debye screening, surface tension at the interface, whether or not the matter is in a superconducting state, etc. It is possible that some color superconducting phase could exist in the core of the SQS where the densities are above nuclear matter densities and temperatures are of the order of tens of keV. At asymptotic densities, the attractive QCD interaction between the quarks at the Fermi surface causes them to condense in Cooper pairs [69–71]. Such an SQS made of color superconducting SQM, which could be either in the color-flavor-locked phase or in the 2-flavor color superconducting phase, is speculated to enhance the electric field.

6 Conclusion

We develop a general approach to solve an Einstein-Maxwell system to describe a static spherically symmetric anisotropic strange matter distribution with linear equation of state in terms of two generating functions. This is examined by choosing Tolmann IV type potential for the gravitational potential g_{11} and a physically reasonable choice for electromagnetic field that generalizes the uncharged model reported recently [55]. Through a physical analysis, the generated models are shown to satisfy all major physical properties of a realistic star such as regularity of the gravitational potentials at the origin; positive definiteness of the energy density and the radial pressure at the origin; vanishing of the radial pressure at some finite radius; monotonic decrease of the energy density and the radial pressure with increasing radius; non-singularity of charge density; and the matching of interior metric smoothly with the Reissnar-Nordstrom exterior metric at the boundary of the stellar object. Moreover, the model satisfies the energy conditions required for normal matter. We clarify that the inclusion of net electric charge distribution inside the SQS can significantly enhance the star mass and thus support recent observations of very heavy two-solar mass pulsars. The extracted numerical values for charge and electric fields corroborate with previous studies and the graphical representations of the bag model illustrate the physical viability.

References

1 P. C. C. Freire, C. G. Bassa, N. Wex et al, Mon. Not. R. Astron. Soc., **412**: 2763 (2011)
2 J. A. Pons, F. M. Walter, J. M. Lattimer et al, Astrophys. J., **564**: 981 (2002)
3 M. Dey, I. Bombaci, J. Dey et al, Phys. Lett. B, **438**: 123(1998)
4 X. -D. Li, I. Bombaci, M. Dey et al, Phys. Rev. Lett., **83**: 3776 (1999)
5 R. X. Xu, G. J. Qiao, and B. Zhang, Astrophys. J., **522**: L109 (1999)
6 P. B. Demorest, T. Pennucci, S. M. Ransom et al, Nature, **467**: 1081 (2010)
7 J. Antoniadis, P. C. C. Freire, N. Wex et al, Science, **340**: 1233232 (2013)
8 M. K. Mak, P. N. Dobson Jr, and T. Harko, Europhys. Lett., **55**: 310 (2001)
9 F. Weber, M. Meixner, R. P. Negreiros et al, Int. J. Mod. Phys. E , **16**: 1165 (2007)
10 F. Weber, R. Negreiros, and P. Rosenfield, Astrophysics and Space Science Library, **357**: 213 (2009)
11 F. Weber, O. Hamil, K. Minura et al, Int. J. Mod. Phys. D, **19**: 1427 (2010)
12 R. P. Negreiros, F. Weber, M. Malheiro et al, Phys. Rev. D, **80**: 083006 (2009)
13 C. Alcock, E. Farhi, and A. Olinto, Astrophys. J., **310**: 261 (1986)
14 V. V. Usov, Phys. Rev. D, **70**: 067301 (2004)
15 V. V. Usov, T. Harko, and K. S. Cheng, ApJ, **620**, 915 (2005)
16 R. P. Negreiros, Igor N. Mishustin, S. Schramm et al Phys. Rev. D, **82**: 103010 (2010)
17 W. B. Bonnor, Mon. Not. R. Astron. Soc., **29**: 443 (1965)
18 J. D. Bekenstein, Phys. Rev. D, **4**, 2185 (1971)
19 F. de Felice, Y. Yu, and J. Fang, Mon. Not. R. Astron. Soc., **277**: L17 (1995)
20 E. Witten, Phys. Rev. D, **30**: 272 (1984)
21 A. R. Bodmer, Phys. Rev. D, **4**: 1601 (1971)
22 A. Chodos, R. L. Jaff, K. Johnson et al, Phys. Rev. D, **9**: 3471 (1974)
23 E. Farhi, and R. L. Jaffe, Phys. Rev. D, **30**: 2379 (1984)
24 H. Sotani, K. Kohri, and T. Harada, Phys. Rev. D, **69**: 084008 (2004)
25 P. Hansel, J. L. Zdunik, and R. Schaeffer, Astron. Astrophys., **160**: 121 (1986)
26 Kettnr Ch, F. Weber, M. K. Weigel et al, Phys. Rev. D, **51**: 1440 (1995)
27 H. Müller, Nucl. Phys. A, **618**: 349 (1997)
28 G. F. Burgio, M. Baldo, P. K. Sahu et al, Phys. Rev. C, **66**:

025802 (2002)
29 M. K. Mak, and T. Harko, Int. J. Mod. Phys. D, **13**: 149 (2004)
30 M. Di Toro, A. Drago, T. Gaitanos et al, Nucl. Phys. A, **775**: 102 (2006)
31 O. E. Nicotra, M. Baldo, G. F. Burgio et al Phys. Rev. D, **74**: 123001 (2006)
32 S. Thirukkanesh, and F. C. Ragel, Pramana-J. Phys., **81**: 275 (2013)
33 D. Gondek-Rosinska, T. Bulik, L. Zdunik et al, Astron. Astrophys., **363**: 1005 (2000)
34 G. Lugones, and J. E. Horvath, Phys. Rev. D, **66**: 074017 (2002)
35 M. G. Alford, S. Han, and M. Prakash, Phys. Rev. D, **88**: 083013 (2013)
36 X. Y. Lai, and R. X. Xu, Mon. Not. R. Astron. Soc. **398**: L31L35 (2009)
37 S. Thirukkanesh, and F. C. Ragel, Astrophys. Space Sci., **354**: 415 (2014)
38 M. Ruderman, Annu. Rev. Astron. Astrophys., **10**: 27 (1972)
39 R. L. Bowers, and E. P. T. Liang, Astrophys. J., **188**: 657 (1974)
40 R. Chan, L. Herrera, and N. O. Santos, Mon. Not. R. Astron. Soc., **265**: 533 (1993)
41 L. Herrera, and N. O. Santos, Phys. Rep., **286**: 53 (1997)
42 F. Weber, *Pulsars as astrophysical observatories for nuclear and particle physics* (Institute of Physics, Bristol, 1999).
43 A. Pérez Martlnez, H. Pérez Rojas, and H. J. Mosquera Cuesta, Eur. Phys. J. C, **29**: 111 (2003)
44 N. Anderson, G. Comer, and K. Glampedakis, Nucl. Phys. A, **763**: 212 (2005)
45 P. B. Jones, Phys. Rev. D, **64**: 084003 (2001)
46 E. N. E. van Dalen, and A. E. L. Dieperink, Phys. Rev. C, **69**: 025802 (2004)
47 M. K. Mak, and T. Harko, Chin. J. Astron. Astrophysics, **2**: 248 (2002)
48 R Sharma and S D Maharaj, Mon. Not. R. Astron. Soc., **375**: 1265 (2007)
49 S Thirukkanesh and S D Maharaj, Class. Quantum Grav., **25**: 235001 (2008)
50 M. Esculpi, and E. Alom, Eur. Phys. J. C, **67**: 521 (2010)
51 S. D. Maharaj, J. M. Sunzu, and S. Ray, Eur. Phys. J Plus, **129**: 3 (2014)
52 P. M. Takisa, S. D. Maharaj, and S. Ray, Astrophys. Space Sci., **354**: 463 (2014)
53 P. M. Takisa, S. Ray, and S. D. Maharaj, Astrophys. Space Sci., **350**: 733 (2014)
54 S. K. Maurya, Y. K. Gupta, and S. Ray, arXiv preprint, arXiv:1502.01915 (2015)
55 S. Thirukkanesh, and F. C. Ragel, Astrophys. Space Sci., **352**: 743 (2014)
56 R. C. Tolman, Phys. Rev., **55**: 364 (1939)
57 M. S. R. Delgaty, and K. Lake, Comput. Phys. Commun., **115**: 395 (1998)
58 F. S. N. Lobo, Class. Quantum Grav., **23**: 1525 (2006)
59 P. S. Florides, J. Phys. A, Math. Gen., **16**: 1419 (1983)
60 M. Kalam, A. A. Usmani, F. Rahaman et al Int. J. Theor. Phys., **52**: 3319 (2013)
61 A. Schmitt, *Dense Matter in Compact Stars: A Pedagogical Introduction*, Springer-Verlag Berlin Heidelberg, (2010) p. 15-23.
62 J. E. Madsen, Phys. Rev. Let., **100**: 151102 (2008)
63 K. Lake, Phys. Rev. D, **19**: 2847 (1979)
64 K. Maeda, and H. Sato, Prog. Theor. Phys., **70**: 772 (1983)
65 H. -J Schmidt, Gen. Relativ. Gravit., **16**, 1053 (1984)
66 H. Abreu, H. Hernòndez and L. A. Nez, Class. Quantum Grav. **24**, 4631 (2007)
67 P. Slane, D. J. Helfand, and S. S. Murray, Astrophys. J., **571**: L45 (2002)
68 J. J. Drake, H. L. Marshall, S. Dreizler et al Astrophys. J., **572**: 996 (2002)
69 M. G. Alford, Ann. Rev. Nucl. Part. Sci., **51**, 131 (2001)
70 G. Nardulli, Riv. Nuovo Cim., **25N3**: 1 (2002)
71 S. Reddy, Phys. Polon. B, **33**: 4101 (2002)