

On the constituent counting rule for hard exclusive processes involving multi-quark states^{*}

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Abstract: At high energy, the cross section at finite scattering angle of a hard exclusive process falls off as a power of the Manderstam variable s . If all involved quark-gluon compositions undergo hard momentum transfers, the fall-off scaling is determined by the underlying valence structures of the initial and final hadrons, known as the constituent counting rule. In spite of the complication due to helicity conservation, it has been argued that when applied to exclusive process with exotic multiquark states, the counting rule is a powerful way to determine the valence degrees of freedom inside hadron exotics. In this work, we demonstrate that for hadrons with hidden flavors, the naive application of the constituent counting rule is problematic, since it is not mandatory for all components to participate in hard scattering at the scale \sqrt{s} . We illustrate the problems in the viewpoint based on effective field theory. We clarify the misleading results that may be obtained from the constituent counting rule in exclusive processes with exotic candidates such as Z_c^\pm ($\bar{c}c\bar{u}d/\bar{c}c\bar{d}u$), Z_b^\pm ($\bar{b}b\bar{u}d/\bar{b}b\bar{d}u$), $X(3872)$, etc.

Keywords: hadron exotics, constituent scaling rule, effective field theory

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1 Introduction

The concept of valence quarks has played a very important role in the classification of hadronic states. Hundreds of hadrons have been discovered in experiments, and most of them are accommodated in the valence quark model: mesons and baryons are composed of a quark-antiquark pair and three quarks, respectively. Here, “quarks” refers to the valence degrees of freedom. Hadrons beyond such configurations are dubbed as exotic, and searching for them, in particular those with exotic quantum numbers which cannot be formed by the

above-mentioned simple configurations, to show their existence or nonexistence is of utmost importance in understanding low-energy nonperturbative quantum chromodynamics (QCD), because color confinement allows such color-singlet states¹⁾. Thanks to worldwide experiments during the last decade or so at the e^+e^- and hadron colliders, lots of new structures as candidates of various hadron resonances, narrow peaks or broad bumps in invariant mass distributions, have been reported with properties different from quark model expectations. It is probable that some of these could be interpreted as exotic multiquark states. Most of these

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1) Classical large N_c arguments state that $q\bar{q}q\bar{q}$ tetraquark states are absent in the large N_c limit (see Refs. [1, 2]). However, this conclusion was challenged in Ref. [3] where it was argued that tetraquark states can exist in the large N_c limit with narrow widths.

new discoveries are in the heavy quarkonium mass region (for reviews, see, e.g., Refs. [4–7]). Among them, a milestone is the discovery of the X(3872) by the Belle Collaboration in 2003 and confirmed by several other experiments later on [8–11]. Since then, the study of these observed structures has been a key topic in hadron physics.

Taking the XYZ states in the charmonium mass region as an example, a number of interpretations have been proposed, including normal quarkonia, hybrid states, compact multiquark states, hadro-charmonia, hadron molecules and effects due to kinematical singularities. Most of these interpretations are based on quark model notations, assuming explicitly or implicitly that the number of (valence) quarks is well defined, even when discussing the production of multiquarks at very high momentum transfer. Then, a central question discussing the observed candidates of exotic hadrons (including proposing new measurements of the properties of these states and searching for new structures) is: how can one determine the valence quark-gluon compositions of a hadron? In a special case of a hadron located very close to an S -wave threshold of two other hadrons, one can in fact measure the valence hadron component since hadrons, being asymptotic states, can go on-shell. However, it becomes very complicated when one tries to determine the quark-gluon components. This is because of confinement, which tells us that quarks and gluons are not asymptotic states and cannot be measured directly in experiments. In a system with hidden flavor, and defining the numbers of quarks and antiquarks as n_q and $n_{\bar{q}}$, respectively, then $n_q - n_{\bar{q}}$ is well defined because of baryon number conservation, while $n_q + n_{\bar{q}}$ is not. In particular, one does not expect the latter to take a definite value for a given hadron in different processes happening at different energy scales. However, the latter is the key quantity discussed in some papers in the literature, the conclusions of which are thus model-dependent and sometimes problematic. This argument can be made clearer by showing why the constituent “counting rule” fails for multiquark states in hard exclusive processes.

Recently, it has been argued in Refs. [12–16] that the differential cross section for high energy production of multiquark states should scale to a certain power of s , the center-of-mass energy squared, predicted based on their expected valence quark structures. More explicitly, for a generic process $a + b \rightarrow c + d$, the cross-section is argued to obey the behavior [12–16]:

$$\frac{d\sigma}{dt} \sim s^{2-n} f(\theta_{\text{cm}}), \quad (1)$$

with $n = n_a + n_b + n_c + n_d$. Here, s and t are the Mandelstam variables, θ_{cm} is the scattering angle in the center-of-mass frame, and n_h is the number of constituents in

the particle h . Here a, b, c, d denotes a generic lepton or hadron including the four-quark structure tetraquark and five-quark structure pentaquark. An ordinary meson has $n_i = 2$, a meson-meson molecule or a tetraquark has $n_i = 4$, and a pentaquark has $n_i = 5$, all of which amounts to $n_q + n_{\bar{q}}$ defined before. The investigated processes include $\pi^- + p \rightarrow K^0 + \Lambda(1405)$, $\gamma + p \rightarrow K^+ + \Lambda(1405)$ [12, 13, 16], the exclusive electron-positron annihilations [14, 15] and so on. Moreover, within the tetraquark framework, the authors of Ref. [15] have argued that based on distinctive falloffs of the cross sections in center-of-mass energy, it is possible to distinguish whether the tetraquarks are segregated into di-meson molecules, diquark-antidiquark pairs, or more democratically arranged four-quark states.

Were the constituent counting rule right, it would provide a very powerful and straightforward tool to access the valence quark structures of the exotic hadrons. But unfortunately as we have argued above and will show below in more detailed, for hadrons with hidden-flavor quarks it is problematic to apply such a naive constituent counting rule. To be explicit, we will first consider a simpler example involving only ordinary mesons, $e^+e^- \rightarrow VP$ with V and P denoting ordinary light flavor vector and pseudoscalar mesons, respectively. This reaction does not follow the naive scaling rule shown in Eq. (1). Then we will adopt the effective field theory and point out the problems in the derivation of the misleading scaling behavior in Eq. (1).

2 Constituent scaling rule?

At very high energy with $\sqrt{s} \gg \Lambda_{\text{QCD}}$, exclusive processes can be understood in the perturbation theory of QCD [17]. The scaling behavior exists in the factorization limit and can be formally derived by matching the full theory QCD to the low-energy effective field theory. When factorization is applicable, one can formally separate the interactions according to the involved scales:

$$\begin{aligned} & T \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right] \\ &= T \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right]_{>\mu} \\ & \quad \times T \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right]_{<\mu} \end{aligned} \quad (2)$$

where T stands for time ordering, μ is the factorization scale and for high energy processes we can have $\mu \sim \sqrt{s}$. The perturbation theory at high energy allows one to express the matrix elements of Heisenberg operators in terms of free local operators and the interaction terms. By including the interaction, we have:

$$\begin{aligned}
 \langle f | \mathcal{O}_H(0) | i \rangle &= \langle f | T \left[\mathcal{O} \times \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right]_{>\mu} \right. \\
 &\quad \left. \times T \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right]_{<\mu} \right] | i \rangle \\
 &\sim \langle f | T \left[\mathcal{O}' \times \exp \left[i \int d^4x \mathcal{L}_{\text{int}}(x) \right]_{<\mu} \right] | i \rangle \\
 &\equiv \langle f | \mathcal{O}'_{H,\mu} | i \rangle,
 \end{aligned} \quad (3)$$

where in the last step we have formally integrated out the interactions above the factorization scale μ using operator product expansion, and obtained a new set of generic low energy effective operators \mathcal{O}' . The interaction below μ contains no information on the $1/\sqrt{s}$ scaling and thus the fall-off scaling can be obtained by counting the power scaling in the operator \mathcal{O}' . It is possible to include the running effects due to the renormalization group and resummation of double logarithms known as the Sudakov logarithms. For simplicity, we do not consider these effects here since the leading power behavior will be unaltered. An implication of the above analysis is that the s -scaling of a process at high energies is given by that contained in the operator \mathcal{O}' , and one gets the scaling easily by identifying the number of lines attached to the vertex described by that operator to the n in Eq. (1).

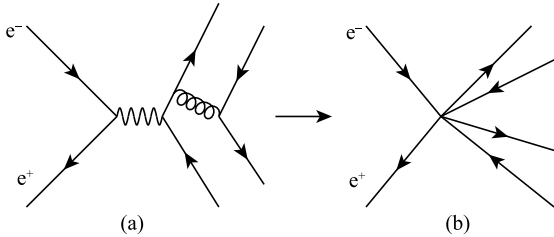


Fig. 1. Feynman diagrams for $e^+e^- \rightarrow VP$ with the quark-antiquark pair produced by a gluon. (a) and (b) stand for the diagram in full theory and EFT, where hard propagators are shrunk to a point, respectively.

A simplest example to apply the constituent counting rule is the e^+e^- annihilation into two light mesons, whose typical Feynman diagram is given in Fig. 1(a). The s power dependence is normally determined by the constituent scaling rule as given in Eq. (1) [17], with $n = 6$ since a meson is made of two quarks. For $e^+e^- \rightarrow VP$ where V is a vector meson and P is a light pseudoscalar meson, there exists a helicity flip since the vector meson can only be transversely polarized. Thus the cross section receives an additional suppression in $1/s$:

$$\sigma(e^+e^- \rightarrow VP) \propto \frac{1}{s^4}. \quad (4)$$

This differs from Eq. (1) which would give $1/s^3$ (notice that here we have the total cross section while Eq. (1)

refers to the differential cross section). Recent measurements of $e^+e^- \rightarrow KK^*$ by the Belle Collaboration [18] at 10.58 GeV and CLEO Collaboration [19] show consistency with the above scaling in Eq. (4) (see also measurements by BES [20] and BaBar [21]).

However, if the vector meson is composed of a pair of quarks with the same flavor, like the ρ^0, ω, ϕ and J/ψ mesons, the fall-off scaling behavior will be different at high energies. We show a production mechanism in Fig. 2(a), which leads to the scaling behavior:

$$\sigma(e^+e^- \rightarrow VP) \propto \frac{1}{s^2}, \quad (5)$$

as can be read off from Fig. 2(b) where the effective interaction vertex has $n = 5$. This production mechanism is suppressed by the fine structure constant $\alpha_{\text{em}} \sim 1/137$ and thus is less important at low energies. At very high energies, however, this new diagram will provide the dominant contribution and it gives a scaling rule different from that obtained by naively counting the number of valence quarks in the mesons [22].

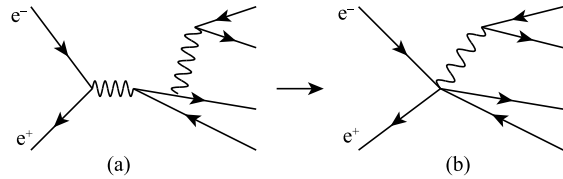


Fig. 2. Feynman diagrams for $e^+e^- \rightarrow VP$ with both V and P being neutral. The neutral vector meson is produced through a photon. Integrating out high-off-shell propagators, one obtains (b) from (a).

Integrating out the interactions at $\sqrt{s} \gtrsim \mu$ as shown in Fig. 2(b), we obtain the relevant matrix element via the photon

$$\langle \rho^0 | A_\perp^\mu | 0 \rangle, \quad (6)$$

with the A_μ being a photon field. This matrix element is nonzero, and it should be compared to the matrix element of the quark fields ψ :

$$\langle \rho^0 | \bar{\psi} \gamma_\perp^\mu \psi | 0 \rangle. \quad (7)$$

The above matrix element is obtained by integrating out the off-shell quark and gluon in Fig. 1(a), and it receives a suppression factor Λ^2/s compared to the photon matrix element in Eq. (6). It is straightforward to understand this behavior through the diagrams in Figs. 1 and 2. In Fig. 1(a), all internal propagators have typically large off-shellness: $p^2 \sim s$. In Fig. 2(a), the virtuality of the second photon, equal to the mass square of the vector

meson, is much smaller. Thus, to accommodate with the constituent scaling rule, one can technically count the valence degrees of freedom of the neutral vector meson as $n_i = 1$ since it is produced by a photon, which amounts to counting the number of lines attached to the effective vertex. The lesson one can learn from the above example is: not all of the ingredients undergo the hard momentum transfer at scale \sqrt{s} . The fall-off power scaling is determined by the leading-power operator at the scale $\mu = \sqrt{s}$, which has a nonzero matrix element with the hadron. Actually, the original constituent counting rule is applicable at finite scattering angles. If the scattering angle is small, at least two of the involved particles are collinear which can then be produced via soft momentum transfers.

Let us switch to the exclusive production of multi-quark states, and take $e^+e^- \rightarrow Z_c^\pm \pi^\mp$ as an example. In Ref. [15], it has been argued that its cross section obeys the fall-off scaling as

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}c\bar{d}u)\pi^-(\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \stackrel{?}{\propto} \frac{1}{s^4}, \quad (8)$$

where the $Z_c^+(\bar{c}c\bar{d}u)$ is a tetraquark state composed of two quarks and two antiquarks. We have put a question mark to the above scaling since we believe the above scaling behavior is problematic at very high energies in the $s \rightarrow \infty$ limit. We show a production mechanism in Fig. 3(a). In this diagram, the heavy quark pair $\bar{c}c$ is generated from the QCD vacuum, and thus such a contribution is suppressed by $\mathcal{O}(1/m_c^2)$. But since the main focus of this work is the fall-off scaling in terms of the collision energy, we are less interested in the $1/m_c^2$ suppression. Integrating out the off-shell intermediate propagators at the scale \sqrt{s} we find that the Z_c behaves as an ordinary $\bar{q}q$ meson and the s dependence scaling of the cross-section is determined by the light quarks of Z_c :

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+\pi^-)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \propto \frac{1}{s^2}, \quad (9)$$

which can again be obtained by counting the number of lines attached to the effective vertex. Apparently, this production mechanism will become dominant at very high energy with $\sqrt{s} \gg m_c$.

A further example is $e^+e^- \rightarrow Z_c^\pm Z_c^\mp$, argued to follow the fall-off scaling [15]

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}c\bar{d}u)Z_c^-(\bar{c}c\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \stackrel{?}{\propto} \frac{1}{s^6}, \quad (10)$$

which should be corrected to

$$\frac{\sigma(e^+e^- \rightarrow Z_c^+(\bar{c}c\bar{d}u)Z_c^-(\bar{c}c\bar{u}d))}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \propto \frac{1}{s^2}. \quad (11)$$

The above results are applicable for longitudinal polarization, while the transverse polarized case should be further suppressed by $1/s$.

The above discussions on Z_c^\pm production are valid in both the tetraquark (diquark-anti-diquark or a democratically arranged four-quark state), and hadronic molecular pictures. The fall-off scaling behavior in a negative power of s is the same in both scenarios, and therefore one can hardly distinguish a compact tetraquark from a meson-meson molecule.

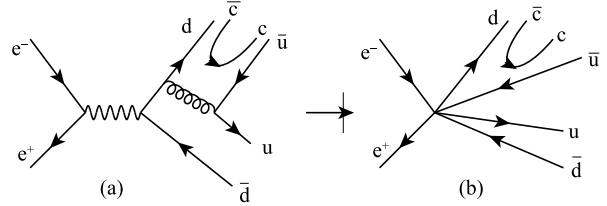


Fig. 3. Feynman diagrams at leading power in $1/s$ for $e^+e^- \rightarrow Z_c^-(\bar{c}c\bar{u}d)\pi^+$ at very high energies. The $\bar{c}c$ quark pair is generated by QCD interaction at the scale $\mu \sim m_c$ which is much lower than \sqrt{s} .

Reference [16] has applied the naive counting rule to the photoproduction of hyperon resonances and attempted to study the $\Lambda(1405)$. The $\Lambda(1405)$ has been expected to be a $\bar{K}N$ bound state [32–34]. The fitted constituent number is energy dependent, which can be understood since the constituent counting rule is an asymptotic behavior in the large energy limit, and will be distorted by finite energy corrections. At the largest collision energy, however the obtained constituent number is consistent with 3 for the $\Lambda(1405)$, despite the large errors. The fact that $n = 3$ does not imply that the $\Lambda(1405)$ is an ordinary uds baryon but instead it shows that $\Lambda(1405)$ is produced by producing three quarks at short distances.

Regarding the notable exotics candidate $X(3872)$, an important task in understanding its nature involves the discrimination of a two-quark configuration, a compact multi-quark configuration and a hadronic molecule [23–31]. Unlike the Z_c^\pm , the $X(3872)$ is neutral, and both the light quark–antiquark pair and charm–anticharm quark pair are hidden. So in hard exclusive processes the $X(3872)$ can be produced at short distances by two sets of operators:

$$\langle X | \bar{c} \Gamma c | 0 \rangle, \quad \langle X | \bar{q} \Gamma q | 0 \rangle, \quad (12)$$

with q being a light u/d quark field, and Γ denoting the Lorentz structure of the operator which can produce the X . The explicit form of the contributing operators depends on the process. For instance, Ref. [35] has explored the inclusive production of $X(3872)$ in B decays

and at hadron colliders, and pointed out that the most important term in the factorization formula should be the color-octet 3S_1 term. In exclusive B_c decays into the $X(3872)$, the $\langle X|\bar{c}\Gamma c|0\rangle$ contributes [36], and ratios of branching fractions can be predicted with a high precision under this mechanism, no matter whether the long-distance nature of X is a tetraquark or a hadronic molecule.

3 Conclusion

To understand the internal structure of hadron exotics, it is essential to work out the valence quark-gluon compositions. At low energy, since effective degrees of freedom are hadrons, and only integrated quantities can be observed, it is very hard to determine the valence components. It has been argued that the high energy process in which the quark-gluon degrees of freedom appear explicitly is helpful. At high energy, the cross section at large scattering angle of a hard exclusive process falls off as a power of the Manderstam variable s . If all involved quark-gluon compositions undergo hard momentum transfers, the fall-off scaling is determined by underlying valence structures of initial and final hadrons, which leads to a constituent counting rule. In this work, using $e^+e^- \rightarrow VP$ we have demonstrated that the naive application of the constituent counting rule is problem-

atic. We have illustrated the problem in the viewpoint based on effective field theory. To accommodate the constituent scaling rule, we have pointed out one should count the valence degree of freedom of the vector meson as $n_i = 1$, which violates the naive counting of a meson. The key to understand the paradox is that not all ingredients will undergo the hard momentum transfer at the scale \sqrt{s} . The fall-off power scaling is determined by the leading-power operator at the scale $\mu = \sqrt{s}$. It is unfortunate that the naive constituent counting rule does not work, but this is a consequence of quantum field theory. For exotic hadrons, we have discussed the productions of the $Z_c^\pm(\bar{c}c\bar{u}d/\bar{c}c\bar{d}u)$, $X(3872)$ and others in hard exclusive processes, where misleading results might be obtained from the naive constituent counting rule.

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