Signatures of shell evolution in alpha decay across the $N = 126$ shell closure*

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Abstract: Within the alpha-cluster model, we particularly investigate the alpha decay of exotic nuclei in the vicinity of the $N = 126$ neutron shell plus the $Z = 82$ proton shell. The systematics of alpha-preformation probability (P_α), as an indicator of the shell effect, is deduced from the ratio of the experimental decay width to the calculated one. Through the comparative analysis of the P_α trend in the $N = 124 - 130$ isotonic chain, the $N = 126$ and $Z = 82$ shell closures are believed to strongly affect the formation of the alpha particle before its penetration. Additionally, the P_α variety in Po and Rn isotopes is presented as another proof for such an influence. More importantly, it may be concluded that the expected neutron (or proton) shell effect gradually fades away along with the increasing valence proton (or neutron) number. The odd-even staggering presented in the P_α value is also discussed.

Keywords: alpha-preformation factor, shell effect, cluster model

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1 Introduction

Since the discovery of radioactivity, alpha decay has played an important role in the research of nuclear structural properties [1], especially in the recent fascinating studies of synthesis and decay of superheavy elements, shape coexistence of exotic nuclei and so on [2–4]. Furthermore, experimental alpha decay schemes are not only convenient to execute, but also provide us with a large amount of spectroscopic information. This is valuable for further understanding of the structural characteristics in unstable nuclei. A couple of decades ago, the problem of whether the $Z = 82$ shell closure is stable in neutron-deficient nuclei was tested by the detection of fine structure in alpha decay [5]. Very recently, due to the extensive accumulation of experimental alpha decay data, the shell effect on the alpha decay process when crossing a magic shell has attracted special attention [6]. Meanwhile, the evolution of the shell closure towards higher proton numbers has been examined via the analysis of the alpha decay spectrum, which is a significant topic, in particular when extending to the extremes of proton-to-neutron ratio and mass number [7].

Usually, the systematic analysis of the corresponding experimental data, such as decay energies Q_α and half-lives $T_{1/2}$, can give hints of the shell effect. To gain a deep and reliable insight, the alpha-preformation probability at the surface of the parent nucleus, as a critical resource of nuclear structure, should be considered. In 1928, the Geiger-Nuttall law was interpreted as the quantum tunneling of a preformed alpha particle through the potential barrier by Gamow [8] and independently by Gurney and Condon [9]. From then on, the alpha decay has commonly been taken as a two-step process, namely the formation of the emitted alpha particle and its subsequential penetration. In this sense, the alpha preformation factor, or other equivalent quantities (like the reduced width), can be extracted from the measured decay half-lives accompanied by Q_α values [6, 7, 10–12]. Obviously, the systematics of deduced P_α values can be employed to detect the aforementioned structural properties, especially for nuclei around the shell closure, which is the objective of the present study. Based on various effective models, extensive studies have been devoted to exotic alpha decay in the closed shell region, including a particular focus on the alpha preformation

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factor or equivalent variables [13–22]. It should be noted that the alpha-daughter interaction potential is a fundamental input during the above extraction procedure. However, according to our previous analysis, the varying trends of P_α from different models are actually consistent with each other, even though the magnitudes can be very different [23]. Therefore, regardless of the choice of potential, the P_α trend can reveal information on structural evolution. In this paper, we propose to make use of the effective alpha-core potential to deduce the vital preformation factor. We mainly concentrate on the variation of this quantity for nearly closed-shell nuclei, seeking knowledge of the shell influence and shell evolution. Moreover, the phenomenon of odd-even staggering in the P_α trend will be discussed.

2 Theoretical approach

Given that the alpha emitters in which we are interested are generally in the vicinity of the shell closure, these nuclei are usually spherical or nearly-spherical. Consequently, the parent nucleus in alpha decay can be considered as a two-body system comprising a preformed alpha particle interacting with the spherical daughter nucleus. In this cluster model picture, the alpha-core interaction potential, as the sum of the nuclear and Coulomb parts plus the Langer modified centrifugal term, is written as [13, 24]

$$V(r)=V_N(r)+V_C(r)+\frac{\hbar^2}{2\mu}\frac{(L+1/2)^2}{r^2}, \tag{1}$$

where r is the separation between the center of the alpha particle and that of the daughter nucleus. μ is the reduced mass of the alpha-core system, and L is the angular momentum carried by the emitted alpha particle. Here the nuclear potential $V_N(r)$ is described as a "cosh" geometry of depth V_0, diffuseness a, and radius R,

$$V_N(r)=-V_0\frac{1+\cosh(R/a)}{\cosh(r/a)+\cosh(R/a)}. \tag{2}$$

The Coulomb potential is given by a form corresponding to a point alpha cluster interacting with a uniformly charged spherical core of radius R,

$$V_C(r)=\begin{cases}\dfrac{Z_\alpha Z_d e^2}{r} & r\geqslant R\\ \dfrac{Z_\alpha Z_d e^2}{2R}\left[3-\left(\dfrac{r}{R}\right)^2\right] & r\leqslant R,\end{cases} \tag{3}$$

where Z_α and Z_d are respectively the charge numbers of the alpha particle and the daughter nucleus. Notice that the modified centrifugal barrier $(L+1/2)^2$ is used instead of $L(L+1)$ to guarantee the sequential integrals are well defined for all possible angular momentums [13, 24].

Through solving the equation $V(r)=Q$ (Q is the experimental decay energy), the three classical turning points r_1, r_2, and r_3 (in order of increasing distance from the origin) are obtained. The value of r_1 is actually close to zero for small L. Meanwhile, the nuclear potential V_N sharply vanishes in the asymptotic region, so r_3 can be determined by solving the corresponding quadratic equation. Considering the appropriate global quantum number $G=2n+L$ according to the Wildermuth and Tang condition [25], the depth V_0 for nuclear potential can be determined for each decay by adjusting the Bohr-Sommerfeld quantization condition,

$$\int_{r_1}^{r_2}\mathrm{d}r\sqrt{\frac{2\mu}{\hbar^2}[Q-V(r)]}=(G-L+1)\frac{\pi}{2}=(2n+1)\frac{\pi}{2}. \tag{4}$$

Here this requirement corresponds to the quasibound state of relative motion with n inner nodes plus the decay energy Q. Once the V_0 value is obtained, the decay width in the semi-classical approximation can be calculated by

$$\varGamma=P_\alpha F\frac{\hbar^2}{4\mu}\exp\left(-2\int_{r_2}^{r_3}\mathrm{d}r k(r)\right), \tag{5}$$

where P_α is the alpha particle preformation probability, and $k(r)$ is the wave number $k(r)=\sqrt{2\mu/\hbar^2|Q-V(r)|}$. The normalization factor F is given by

$$F\int_{r_1}^{r_2}\mathrm{d}r\frac{1}{k(r)}\cos^2\left(\int_{r_1}^{r}\mathrm{d}r' k(r')-\frac{\pi}{4}\right)=1. \tag{6}$$

It should be noted that the squared cosine term can be substituted by its average value of $\frac{1}{2}$ without clear detriment to the accuracy. The focus in the present study is on the alpha preformation factor, namely P_α. The alpha decay half-life is related to the width by $T_{1/2}=\hbar\ln 2/\varGamma$. Hence the experimental decay width can in turn be obtained from the experimental half-life, and the P_α value can then be deduced via Eq. (5).

Considering that the radius parameter R in the Coulomb potential is widely used as $R=1.2A^{1/3}$ fm [26, 27], this choice is adopted here and the parameter a is fixed as $a=0.6$ fm. As mentioned before, the specific choice of the fixed parameter does not in fact affect the trend of variation of P_α. In other words, the different parameters will not change our final conclusion at all. To be more specific about the global quantum number G, details of its value are introduced as follows. In the case of a single alpha cluster, the G value is given by [25]

$$G=2n+L=\sum_{i=1}^{4}g_i, \tag{7}$$

where g_i is the corresponding oscillator number of the nucleon forming the cluster, which is restricted by the Pauli principle. Here we take $g_i=6$ for nucleons beyond the $N=126$ shell closure, $g_i=5$ for nucleons in the $82 < Z,N \leqslant 126$ shell, and $g_i = 4$ for nucleons in the $50 < Z,N \leqslant 82$ shell. Subsequently, we focus on the behavior of P_α in nuclei near the magic shell via the above procedure.

3 Numerical results and discussion

Our attention is mainly given to those alpha emitters around the $N = 126$ shell, which is supposed to provide a large amount of attractive structural information. Initially, we systematically investigate the alpha decay in the isotonic chains of $N = 124$, 126, 128, 130. As mentioned before, the P_α value is the main quantity in the present study. The detailed results are presented in Table 1. The first column gives the corresponding parent nucleus, and the following two columns present the spin-parity of the parent and daughter [28, 29]. The experimental decay energies and half-lives [28, 29] are listed in the fourth and fifth columns, while the calculated half-lives, without the consideration of the alpha preformation factor (namely $P_\alpha = 1$), are shown in the sixth column. The interesting alpha preformation factors are located in the last column. As one can see from the table, the P_α values in the $N = 126$ isotonic chain are obviously smaller than those in other isotones. Besides, the present alpha preformation factors in the $N = 126$ isotones and those based on the Generalized Liquid Drop Model (GLDM) [16] and the Generalized Density-Dependent Cluster Model (GDDCM) [18], are displayed in Fig. 1 as an example of the comparison between different theoretical results. Indeed, although the P_α values from different models correspond to different magnitudes, they show similar trends to each other, which is consistent with our previous conjecture [23]. More interestingly, it is found that the present result of ^{212}Po, i.e. $P_\alpha = 0.188$, is quite compatible with that extracted from the experimental double differential energy spectrum [30]. The adopted Woods-Saxon potential in Ref. [30], in particular along with similar radius (R) and diffuseness (a) parameters, is actually quite close to the present nuclear potential. One may infer that the extracted P_α value mainly depends on the chosen effective potential, despite the specific deduction procedure, which is worth studying deeply. Besides, as mentioned in Ref. [30], the formation process of the emitted alpha cluster before its penetration is pictured as two protons and two neutrons, at the surface of the parent nucleus, being combined into the alpha particle.

In order to obtain a better and more straightforward insight, the P_α value is plotted versus the proton number of the parent nucleus in Fig. 2, for different isotonic chains. Clearly, the P_α line of $N = 126$ is lowest, further implying that the $N = 126$ shell strongly affects the formation process of the emitted alpha particle. The P_α values of parent nuclei close to the $Z = 82$ shell are the smallest for each isotonic chain, which shows the effect of the proton shell closure on the alpha decay. More significantly, when the proton number of the parent nucleus increases and approaches $Z = 92$ (uranium), the preformation factors P_α in different isotones gradually become comparable with each other. This can be considered as positive evidence that the neutron shell effect on the alpha decay process appears to be weak for nuclei with high atomic numbers, which is consistent with the conclusion in Ref. [7]. On the other hand, there is

Table 1. Detailed calculation results for the isotopic chains with $N = 124$, 126, 128, 130, where the alpha-preformation factor is listed in the last column.

parent	I_i^π	I_f^π	Q/MeV	$T_{1/2}^{\rm expt}$/s	$T_{1/2}^{\rm calc}$/s	P_α
			$N = 124$			
^{208}Po	0^+	0^+	5.215	9.14×10^{7}	6.23×10^{6}	0.068
^{209}At	$9/2^-$	$9/2^-$	5.757	4.70×10^{5}	2.69×10^{4}	0.057
^{210}Rn	0^+	0^+	6.158	9.00×10^{3}	1.04×10^{3}	0.116
^{211}Fr	$9/2^-$	$9/2^-$	6.663	2.13×10^{2}	1.98×10^{1}	0.093
^{212}Ra	0^+	0^+	7.031	1.53×10^{1}	2.11×10^{0}	0.138
^{213}Ac	$9/2^-$	$9/2^-$	7.501	7.38×10^{-1}	1.07×10^{-1}	0.145
^{214}Th	0^+	0^+	7.827	8.71×10^{-2}	2.33×10^{-2}	0.268
^{215}Pa	$9/2^-$	$9/2^-$	8.240	1.40×10^{-2}	2.72×10^{-3}	0.194
^{216}U	0^+	0^+	8.530	4.50×10^{-3}	9.42×10^{-4}	0.209
			$N = 126$			
^{210}Po	0^+	0^+	5.407	1.20×10^{7}	4.93×10^{5}	0.041
^{211}At	$9/2^-$	$9/2^-$	5.982	6.20×10^{4}	1.96×10^{3}	0.032
^{212}Rn	0^+	0^+	6.385	1.43×10^{3}	9.67×10^{1}	0.067
^{213}Fr	$9/2^-$	$9/2^-$	6.904	3.46×10^{1}	2.2×10^{0}	0.064
^{214}Ra	0^+	0^+	7.273	2.46×10^{0}	2.55×10^{-1}	0.104
^{215}Ac	$9/2^-$	$9/2^-$	7.746	1.70×10^{-1}	1.57×10^{-2}	0.092
^{216}Th	0^+	0^+	8.071	2.60×10^{-2}	3.44×10^{-3}	0.132
^{217}Pa	$9/2^-$	$9/2^-$	8.493	3.60×10^{-3}	4.76×10^{-4}	0.132
^{218}U	0^+	0^+	8.773	5.10×10^{-4}	1.78×10^{-4}	0.349
			$N = 128$			
^{212}Po	0^+	0^+	8.954	2.79×10^{-7}	5.24×10^{-8}	0.188
^{213}At	$9/2^-$	$9/2^-$	9.254	1.25×10^{-7}	2.33×10^{-8}	0.186
^{214}Rn	0^+	0^+	9.208	2.70×10^{-7}	6.24×10^{-8}	0.231
^{215}Fr	$9/2^-$	$9/2^-$	9.540	8.60×10^{-8}	2.29×10^{-8}	0.266
^{216}Ra	0^+	0^+	9.526	1.82×10^{-7}	5.59×10^{-8}	0.307
^{217}Ac	$9/2^-$	$9/2^-$	9.832	6.90×10^{-8}	2.39×10^{-8}	0.346
^{218}Th	0^+	0^+	9.849	1.17×10^{-7}	4.69×10^{-8}	0.401
^{219}Pa	$9/2^-$	$9/2^-$	10.08	5.30×10^{-8}	2.99×10^{-8}	0.564
			$N = 130$			
^{214}Po	0^+	0^+	7.833	1.64×10^{-4}	5.07×10^{-5}	0.310
^{215}At	$9/2^-$	$9/2^-$	8.178	1.00×10^{-4}	1.27×10^{-5}	0.127
^{216}Rn	0^+	0^+	8.200	4.50×10^{-5}	2.46×10^{-5}	0.546
^{217}Fr	$9/2^-$	$9/2^-$	8.469	1.90×10^{-5}	1.01×10^{-5}	0.533
^{218}Ra	0^+	0^+	8.546	2.52×10^{-5}	1.52×10^{-5}	0.604
^{219}Ac	$9/2^-$	$9/2^-$	8.830	1.18×10^{-5}	5.89×10^{-6}	0.499
^{220}Th	0^+	0^+	8.953	9.70×10^{-6}	6.09×10^{-6}	0.628
^{221}Pa	$9/2^-$	$9/2^-$	9.248	5.90×10^{-6}	2.51×10^{-6}	0.425
^{222}U	0^+	0^+	9.480	4.70×10^{-6}	1.45×10^{-6}	0.309

still a rather large uncertainty in the alpha decay half-life of ^{218}U, which is directly related to the extraction of its P_α value. Meanwhile, nuclear pairing may also play an significant role in inducing alpha clustering, particularly when nucleon numbers increase [6, 12]. Therefore, the specific reason causing the exotic evolution of the alpha preformation factor deserves further investigation. Moreover, the odd-even nuclei are involved in our present calculations, and they provide the same conjecture. In detail, one may conclude that valence protons away from the proton shell $Z=82$ somewhat compensate for the influence of the neutron shell $N=126$. Besides, one can see that in the $N=128$, 130 isotonic chains, the preformation factor varies smoothly and gradually maintains similar values, except for ^{215}At (worth further investigation). This situation indicates the fading of the $Z=82$ shell closure when going to high neutron numbers. Meanwhile, it is actually compatible with the common strategy that the P_α values are fixed as a constant for any given kind of nucleus in the open shell region [13, 24, 27].

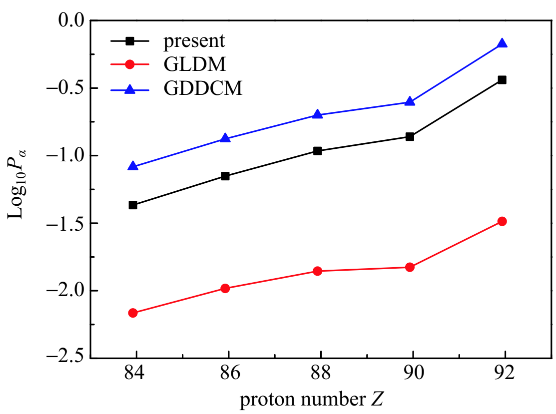

Fig. 1. (color online) The present P_α (in logarithm scale) values are compared with those from the GLDM and GDDCM calculations for even-even nuclei in the $N=126$ isotonic chain.

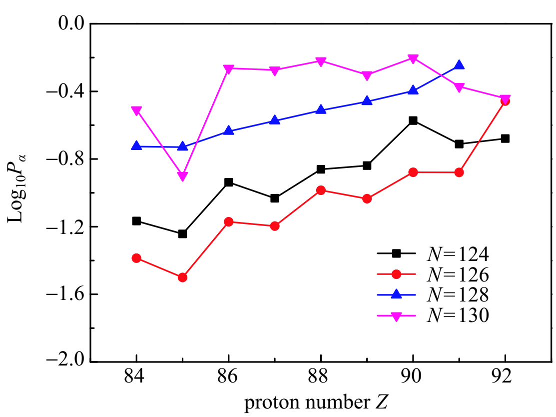

Fig. 2. (color online) Variety of P_α (in logarithm scale) with the proton number of alpha emitter for isotones with $N=124$, 126, 128 and 130, including both even-even and odd-even nuclei.

In addition, we have paid extra attention to the isotopic chains with $Z=84$ and 86, to pursue more knowledge about the structural information via the alpha decay process. In Fig. 3, the preformation factor P_α is plotted with the neutron number of the parent nucleus, for the Po and Rn isotopes separately. The P_α value comes down sharply when the neutron number approaches the $N=126$ neutron shell, which again reveals the drastic effect of shell closure. It is also found that the alpha preformation factor stabilizes as long as the neutron number is far away from the closed shell. More interestingly, by combining Figs. 2 and 3, one can clearly see that there is an odd-even staggering effect in the variation of P_α. However, this phenomenon seems to fade away after the neutron number exceeds 126 (i.e., $N=128$ and 130) for the isotonic chain in Fig. 2. For the $N=128$ and 130 isotones, the two neutrons forming the emitted alpha cluster are supposed to come from the region above the $N=126$ major shell, which may slim the coupling interaction between them and the protons below the shell closure. Consequently, this may attenuate the blocking effect of the unpaired proton, affecting the formation of the emitted alpha particle, leading to the weakening of odd-even staggering. Furthermore, as reported in Refs. [18–21] the preformation factor in alpha decay can be connected with the valence nucleon number. This phenomenon deserves further investigation.

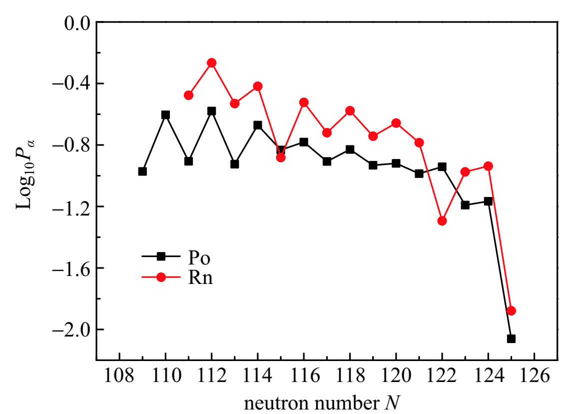

Fig. 3. (color online) Same as Fig. 2 but for the Po and Rn isotopes.

4 Summary

In summary, on the basis of experimental alpha decay data, we have extracted the alpha preformation factor P_α for those nuclei near the shell closure, within the alpha cluster model. By analyzing the P_α trend in the isotonic chains with $N=124$, 126, 128 and 130 plus the Po and Rn isotopes, the drastic effect of the $N=126$ and $Z=82$ shell on the alpha decay process has been clearly demonstrated. A possible explanation is that the weakening evolution of shell effect, along with the increasing of the valence nucleon number away from the nearest shell closure, leads to the observed P_α behavior. The odd-even staggering involved in the alpha preformation process has also been detected.

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