

Acceleration of particles in Einstein-Maxwell-dilaton black holes^{*}

Pu-Jian Mao(毛普健)^{1,3} Ran Li(李然)^{2;1)} Lin-Yu Jia(贾林玉)³ Ji-Rong Ren(任继荣)^{3;2)}

¹Institute of High Energy Physics and Theoretical Physics Center for Science Facilities, Chinese Academy of Sciences, 19B Yuquan Road, Beijing 100049, China

²Department of Physics, Henan Normal University, Xinxiang 453007, China

³Institute of Theoretical Physics, Lanzhou University, Lanzhou 730000, China

Abstract: It has recently been pointed out that, under certain conditions, the energy of particles accelerated by black holes in the center-of-mass frame can become arbitrarily high. In this paper, we study the collision of two particles in the case of four-dimensional charged nonrotating, extremal charged rotating and near-extremal charged rotating Kaluza-Klein black holes as well as the naked singularity case in Einstein-Maxwell-dilaton theory. We find that the center-of-mass energy for a pair of colliding particles is unlimited at the horizon of charged nonrotating Kaluza-Klein black holes, extremal charged rotating Kaluza-Klein black holes and in the naked singularity case.

Keywords: black hole, particles accelerator, center-of-mass energy

PACS: 97.60.Lf, 04.70.-s, 04.50.Cd **DOI:** 10.1088/1674-1137/41/6/065101

1 Introduction

The Planck scale defines the meeting point of gravity and quantum mechanics. The probe of the Planck-scale physics also contributes to the discovery of extra dimensions of space-time and the Grand Unification Theory. However, compared with the Planck energy of 10^{16} TeV, the largest terrestrial accelerator, the Large Hadron Collider, which can detect physics at collision energies of order 10^1 TeV, is too low to probe Planck-scale physics. There is a very, very large gap between the Planck scale and our current experimental techniques. So some other new physics mechanisms should be proposed for probing Planck-scale physics. The collision of particles around a black hole may provide a possible detection.

Bañados, Silk and West (BSW) [1] recently investigated the maximum center-of-mass energy of particles colliding around a Kerr black hole. Their result showed that the maximum energy grows with a , which is the unit angular momentum of the black hole. Remarkably, as the black hole becomes extremal, they found a fascinating and important property of the extremal Kerr black hole, that two particles freely falling from rest at spatial infinity can collide at the horizon with arbitrarily high center-of-mass (CM) energy. Such a black hole could serve as a particle accelerator and may provide a visible probe of Planck-scale physics. To achieve that,

the black hole must have a maximized angular momentum and one of the colliding particles should have orbital angular momentum per unit rest mass $l = 2$. Subsequently, in Refs. [2, 3], the authors argued that the CM energy is in fact limited, because there is always a small deviation of the spin of an astrophysical black hole from its maximal value. According to the work of Thorne [23], the dimensionless spin of astrophysical black holes should not exceed $a = 0.998$. In terms of the small parameter $\epsilon = 1 - a$, Jacobson and Sotiriou got the maximal CM energy $\frac{E_{CM}^{\max}}{m_0} \sim 4.06\epsilon^{-1/4} + \mathcal{O}(\epsilon^{1/4})$ in Ref. [3]. Tak-

ing $a = 0.998$ as a limit, one can obtain the maximal CM energy per unit mass 19.20, which is a finite value. Meanwhile, Lake showed that the CM energy of the collision at the inner horizon of the black hole is generically divergent [4] and the colliding particles can have arbitrary angular momentum per unit rest mass that could fall into the black hole. But he claimed soon after [4] that the collision at the inner horizon actually could not take place [5], which leads to no divergence of the CM energy. On the other hand, Grib and Pavlov suggested that the CM energy can be unlimited in the case of multiple scattering [6–9]. The universal property of acceleration of particles for black holes was investigated in Refs. [10–12]. In Refs. [13, 14], the property of the CM energy for two colliding particles in the background of a

Received 11 January 2017, Revised 19 February 2017

^{*} Supported by NSFC (11575202, 11205048), Foundation for Young Key Teacher of Henan Normal University and Cuiying Programme of Lanzhou University (225000-582404) and Fundamental Research Fund for Physics and Mathematic of Lanzhou University (LZULL200911)

1) E-mail: liran@htu.edu.cn

2) E-mail: renjr@lzu.edu.cn

©2017 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

charged spinning black hole was discussed. One of the important results of [13, 14] is that the CM energy can still be unlimited despite the deviation of the spin from its maximal value. The BSW approach was then applied to the collision of particles plunging from the innermost stable circular orbit and last stable orbit near the horizon in Refs. [15, 16]. There are also some investigations on naked singularities [17–20] and (anti-) de Sitter backgrounds [21].

In this paper, we will investigate the property of the collision of particles in the background of Einstein-Maxwell-dilaton gravity. Firstly we calculate geodesic equations in Kaluza-Klein spacetime in Einstein-Maxwell-dilaton theory. Then, we study the CM energy for collisions taking place at the horizon of three cases of Kaluza-Klein black holes. We find the CM energy is unlimited for a pair of point particles colliding at the horizon, with some fine tuning for the charged nonrotating case and the extremal rotating case. The result of the near-extremal case shows that the CM energy is in fact limited because the critical angular momentum is too large for the geodesics of particles to reach the horizon. Then we obtain the numerical result of the maximal CM energy per unit mass for some different value of $\epsilon = 1 - a$ in the case of a near-extremal Kaluza-Klein black hole, due to the difficulty in finding the exact result. Lastly, the Kaluza-Klein naked singularity is also considered in the present work. We find that the CM energy of collision between two particles can be arbitrarily high in this case.

This paper is organized as follows. In Section 2, we give a brief review of four-dimensional Kaluza-Klein black holes and obtain the geodesic equations in the background of Kaluza-Klein spacetime. In Section 3, we study the CM energy for collisions taking place at the horizon for different cases of Kaluza-Klein black holes and in the naked singularity case. The last section is devoted to discussion and conclusions.

2 Geodesic equations in Kaluza-Klein spacetime

We begin with a brief review of the Kaluza-Klein black hole. It is derived by a dimensional reduction of the boosted five-dimensional Kerr solution to four dimensions. It is an exact solution of Einstein-Maxwell-Dilaton theory. The metric is explicitly given by [22]

$$ds^2 = -\frac{1-Z}{B}dt^2 - \frac{2aZ\sin^2\theta}{B\sqrt{1-\nu^2}}dtd\varphi + \frac{B\Sigma}{\Delta}dr^2 + B\Sigma d\theta^2 + \left[B(r^2+a^2) + a^2\sin^2\theta\frac{Z}{B} \right] \sin^2\theta d\varphi^2, \quad (1)$$

where

$$Z = \frac{2\mu r}{\Sigma},$$

$$\begin{aligned} B &= \sqrt{1 + \frac{\nu^2 Z}{1-\nu^2}}, \\ \Sigma &= r^2 + a^2 \cos^2\theta, \\ \Delta &= r^2 - 2\mu r + a^2. \end{aligned} \quad (2)$$

The gauge potential is

$$A = \frac{\nu}{2(1-\nu^2)} \frac{Z}{B^2} dt - \frac{a\nu\sin^2\theta}{2\sqrt{1-\nu^2}} \frac{Z}{B^2} d\varphi. \quad (3)$$

The physical mass M , the charge Q , and the angular momentum J are expressed in terms of the boost parameter ν , the mass parameter μ , and the specific angular momentum a as

$$\begin{aligned} M &= \mu \left[1 + \frac{\nu^2}{2(1-\nu^2)} \right], \\ Q &= \frac{\mu\nu}{1-\nu^2}, \\ J &= \frac{\mu a}{\sqrt{1-\nu^2}}. \end{aligned} \quad (4)$$

The outer and inner horizons are respectively defined at

$$r_{\pm} = \mu \pm \sqrt{\mu^2 - a^2}. \quad (5)$$

Thus, $\mu = a$ corresponds to the extremal black hole with one degenerate horizon. The components of the inverse metric are

$$\begin{aligned} g^{tt} &= -\frac{B(r^2+a^2) + a^2\sin^2\theta\frac{Z}{B}}{\Delta}, \\ g^{rr} &= \frac{\Delta}{B\Sigma}, \quad g^{\theta\theta} = \frac{1}{B\Sigma}, \\ g^{\varphi\varphi} &= \frac{1-Z}{B\Delta\sin^2\theta}, \quad g^{t\varphi} = -\frac{aZ}{B\Delta\sqrt{1-\nu^2}}. \end{aligned} \quad (6)$$

Because there are two Killing vectors $\left(\frac{\partial}{\partial t}\right)^\mu$ and $\left(\frac{\partial}{\partial\varphi}\right)^\mu$, we have two conserved quantities along a geodesic motion for a test particle with charge e as follows

$$E = -g_{\mu\sigma} \left(\frac{\partial}{\partial t}\right)^\mu [u^\sigma + eA^\sigma] = -(g_{tt}\dot{t} + g_{t\varphi}\dot{\varphi} - eA_t), \quad (7)$$

$$L = g_{\mu\sigma} \left(\frac{\partial}{\partial\varphi}\right)^\mu [u^\sigma + eA^\sigma] = g_{t\varphi}\dot{t} + g_{\varphi\varphi}\dot{\varphi} - eA_\varphi, \quad (8)$$

where E and L correspond to the constant energy and angular momentum along a geodesic motion, respectively. It is easy to solve the above equations for \dot{t} and

$\dot{\varphi}$ as

$$\begin{aligned} \dot{t} &= \frac{B(r^2 + a^2) + a^2 \frac{Z}{B} \sin^2 \theta}{\Delta} (E - eA_t) \\ &\quad - \frac{aZ}{B\Delta\sqrt{1-\nu^2}} (L + eA_\varphi) , \\ \dot{\varphi} &= \frac{aZ}{B\Delta\sqrt{1-\nu^2}} (E - eA_t) + \frac{1-Z}{B\Delta\sin^2\theta} (L + eA_\varphi) . \end{aligned} \quad (9)$$

Substituting these solutions into the normalization condition $g_{\mu\nu}u^\mu u^\nu = -1$ on the equatorial plane, $\theta = \frac{\pi}{2}$ and $\dot{\theta} = 0$, one will arrive at

$$\dot{r} = \frac{\Delta}{B\Sigma} R(r) , \quad (10)$$

where

$$\begin{aligned} \frac{\Delta}{B\Sigma} R^2(r) &= \frac{B(r^2 + a^2) + a^2 \frac{Z}{B}}{\Delta} (E - eA_t)^2 \\ &\quad - 2 \frac{aZ}{B\Delta\sqrt{1-\nu^2}} (E - eA_t)(L + eA_\varphi) \\ &\quad - \frac{1-Z}{B\Delta} (L + eA_\varphi)^2 - 1 . \end{aligned} \quad (11)$$

Now we have solved the geodesic equations on the equatorial plane in Kaluza-Klein spacetime. In the next section, we will turn to the CM energy for particles colliding in this background.

3 Center-of-mass energy for collisions in Kaluza-Klein spacetime

The energy in the center-of-mass frame for a pair of point particles colliding is computed by the formula [1]

$$E_{\text{CM}} = \sqrt{2}m_0\sqrt{1 - g_{\mu\nu}u_1^\mu u_2^\nu} , \quad (12)$$

where u_1^μ and u_2^ν are the 4-velocities of the two particles. For the case that the particles begin at rest at infinity and the collision energy comes solely from gravitational acceleration, the particles follow geodesics with energy $E \geq 1$. Consider two particles coming from infinity with $E_1 = E_2 = 1$ and approaching the black hole with different angular momenta L_1 and L_2 . Taking into account the metric of the Kaluza-Klein black hole (1) on the equatorial plane, we obtain the CM energy for collision with the help of (9) (10) and (12)

$$\frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{K}{B\Delta} , \quad (13)$$

where

$$\begin{aligned} K &= [B^2(r^2 + a^2) + a^2 Z](1 - e_1 A_t)(1 - e_2 A_t) \\ &\quad + (Z - 1)(L_1 + e_1 A_\varphi)(L_2 + e_2 A_\varphi) \\ &\quad - \frac{aZ}{\sqrt{1-\nu^2}} [(1 - e_1 A_t)(L_2 + e_2 A_\varphi) \\ &\quad + (1 - e_2 A_t)(L_1 + e_1 A_\varphi)] \\ &\quad - \left\{ [B^2(r^2 + a^2) + a^2 Z](1 - e_1 A_t)^2 \right. \\ &\quad + (Z - 1)(L_1 + e_1 A_\varphi)^2 \\ &\quad \left. - \frac{2aZ}{\sqrt{1-\nu^2}} (1 - e_1 A_t)(L_1 + e_1 A_\varphi) - B\Delta \right\}^{\frac{1}{2}} \\ &\quad \times \left\{ [B^2(r^2 + a^2) + a^2 Z](1 - e_2 A_t)^2 \right. \\ &\quad + (Z - 1)(L_2 + e_2 A_\varphi)^2 \\ &\quad \left. - \frac{2aZ}{\sqrt{1-\nu^2}} (1 - e_2 A_t)(L_2 + e_2 A_\varphi) - B\Delta \right\}^{\frac{1}{2}} . \end{aligned} \quad (14)$$

We have obtained the CM energy of two colliding particles in Kaluza-Klein spacetime. Now we are ready to investigate the CM energy for different cases of Kaluza-Klein black holes and for the naked singularity case.

3.1 Charged nonrotating Kaluza-Klein black hole

The acceleration of particles by a Reissner-Nordström black hole has been discussed in Ref. [11]. Charged nonrotating Kaluza-Klein spacetime is very different from the Reissner-Nordström case because there is only one event horizon, which leads to the absence of an extremal nonrotating Kaluza-Klein black hole. The metric of the charged nonrotating Kaluza-Klein spacetime is

$$ds^2 = -\frac{\Delta}{r^2 B} dt^2 + \frac{r^2 B}{\Delta} dr^2 + Br^2 d\theta^2 + Br^2 \sin^2 \theta d\varphi^2 , \quad (15)$$

where

$$\begin{aligned} B &= \sqrt{1 + \frac{\nu^2 Z}{1 - \nu^2}} , \\ Z &= \frac{2\mu}{r} , \\ \Delta &= r^2 - 2\mu r . \end{aligned} \quad (16)$$

The gauge potential is given by

$$A = \frac{\nu}{2(1-\nu^2)} \frac{Z}{B^2} dt . \quad (17)$$

The horizon lies at $r_h = 2\mu$. According to Eq. (13) and Eq. (14), we can obtain the CM energy of two radial motion particles colliding in charged nonrotating Kaluza-Klein spacetime as

$$\frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{K_1}{B\Delta}, \quad (18)$$

where

$$K_1 = B^2 r^2 (1 - e_1 A_t) (1 - e_2 A_t) - \sqrt{[B^2 r^2 (1 - e_1 A_t)^2 - B\Delta][B^2 r^2 (1 - e_2 A_t)^2 - B\Delta]}. \quad (19)$$

It appears that E_{CM}^2 diverges at $r = r_h$, but this is not true because, although not totally obvious, the numerator vanishes at that point as well. After some calculations, the CM energy is

$$\frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{1}{2} \left(\frac{1 - e_2 \frac{\nu}{2}}{1 - e_1 \frac{\nu}{2}} + \frac{1 - e_1 \frac{\nu}{2}}{1 - e_2 \frac{\nu}{2}} \right). \quad (20)$$

If one of the particles participating in the collision has the critical charge $e = \frac{2}{\nu}$, the CM energy will blow up at the horizon. Thus we have shown that non-extremal black holes could also serve as particle accelerators and provide a visible probe of Planck-scale physics.

3.2 Extremal charged rotating Kaluza-Klein black hole

In the case $a = \mu$, which corresponds to the extremal Kaluza-Klein black hole, we obtain the form of the CM energy of two uncharged particles colliding at the degenerate horizon, after some tedious calculations, as:

$$E_{\text{CM}}^{\text{KK}}(r \rightarrow r_+)$$

$$= \sqrt{2} m_0 \left[1 + \frac{(1 + \nu^2)(L_1 - L_2)^2}{2\sqrt{1 - \nu^4} \left(L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}} \right) \left(L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}} \right)} + \frac{1}{2} \left(\frac{L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}}}{L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}}} + \frac{L_2 - \frac{2\mu}{\sqrt{1 - \nu^2}}}{L_1 - \frac{2\mu}{\sqrt{1 - \nu^2}}} \right) \right]^{\frac{1}{2}}, \quad (21)$$

Clearly, when L_1 or L_2 takes the critical angular momentum $L_c = \frac{2\mu}{\sqrt{1 - \nu^2}}$, the CM energy $E_{\text{CM}}^{\text{KK}}$ will be unlimited, which means that the particles can collide with arbitrarily high CM energy at the horizon. We expect that, in the case $\nu = 0$, the CM energy (21) in the background of a Kaluza-Klein black hole should reduce to the one in the background of a Kerr black hole. After some calculations, we find that the CM energy is exactly consistent with that of Ref. [1] in the case $\nu = 0$.

We plot \dot{r}^2 and $E_{\text{CM}}^{\text{KK}}$ in Fig. 1 and Fig. 2, from which we can see that there exists a critical angular momentum $L_c = \frac{2\mu}{\sqrt{1 - \nu^2}}$ for the geodesics of particle to reach the horizon. If $L > L_c$, the geodesics never reach the horizon. On the other hand, if the angular momentum is too small, the particle will fall into the black hole and the CM energy for the collision is limited. However, when L_1 or L_2 takes the angular momentum $L = \frac{2\mu}{\sqrt{1 - \nu^2}}$, the CM energy is unlimited. As a result, it may provide a unique probe for Planck-scale physics.

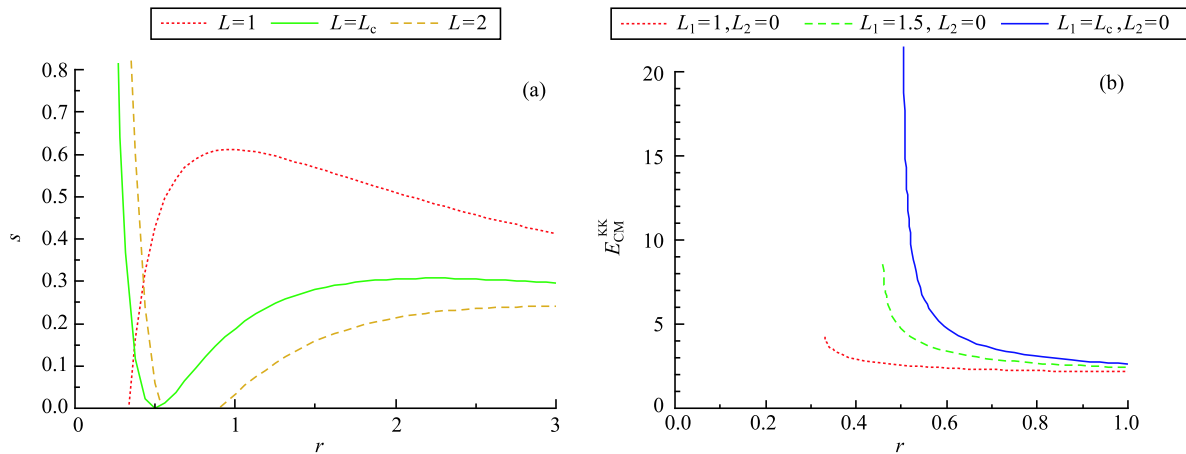


Fig. 1. (color online) For an extremal Kaluza-Klein Black hole with $J = \frac{\sqrt{3}}{4}$ and $M = 1$, (a) the variation of $s = r^2$ with radius for three different values of angular momentum, and (b) the variation of $E_{\text{CM}}^{\text{KK}}$ with radius for three combinations of L_1 and L_2 . For $L_1 = L_c$, $E_{\text{CM}}^{\text{KK}}$ blows up at the horizon.

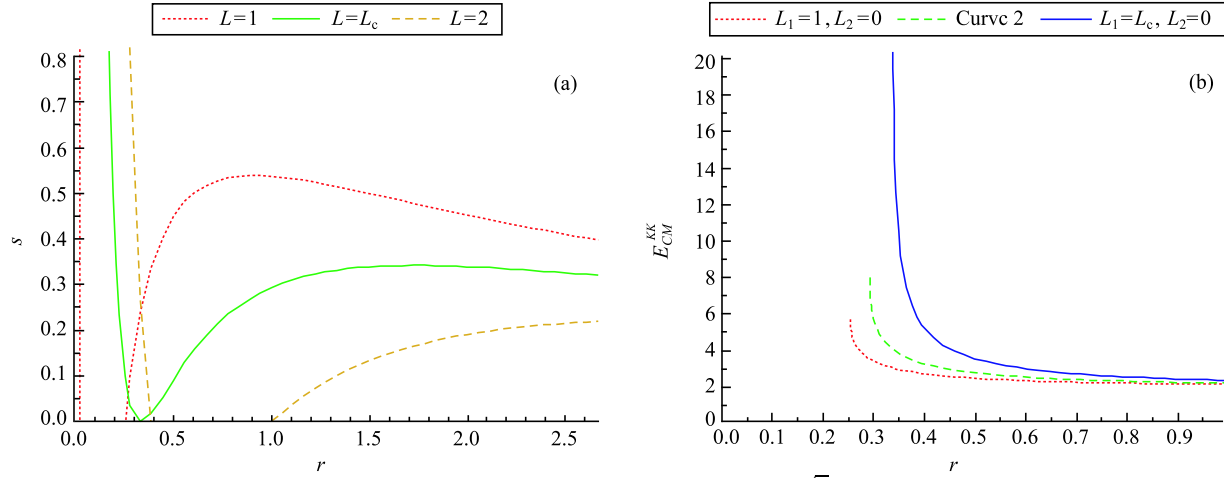


Fig. 2. (color online) For an extremal Kaluza-Klein Black hole with $J = \frac{\sqrt{5}}{9}$ and $M = 1$, (a) the variation of $s = r^2$ with radius for three different values of angular momentum, and (b) the variation of $E = E_{CM}^{KK}$ with radius for three combinations of L_1 and L_2 . For $L_1 = L_c$, E_{CM}^{KK} blows up at the horizon.

3.3 Near-extremal charged rotating Kaluza-Klein black hole

For the near-extremal case, we also obtain the CM

$$E_{CM}^{KK}(r \rightarrow r_+) = \sqrt{2}m_0 \left[1 + \frac{\left(\frac{r_+^2}{a^2} + \nu^2\right)(L_1 - L_2)^2}{2B(r_+)(1 - \nu^2) \left(L_1 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}\right) \left(L_2 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}\right)} + \frac{1}{2} \left(\frac{L_1 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}}{L_2 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}} + \frac{L_2 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}}{L_1 - \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}} \right) \right]^{\frac{1}{2}}. \quad (22)$$

Naively, the CM energy E_{CM}^{KK} will be divergent when L_1 or L_2 takes the critical angular momentum

$$L_c = \frac{r_+^2 + a^2}{a\sqrt{1 - \nu^2}}. \quad (23)$$

However, a careful analysis shows that the critical angular momentum L_c is too large for the geodesics of the particle to reach the horizon. That is, a freely falling particle with this critical angular momentum will be reflected before it reaches the horizon. The turning point of an initially ingoing particle is located at the larger root of the equation

$$V_{\text{eff}} = 0, \quad (24)$$

where V_{eff} is the effective potential for the radial motion. For an uncharged particle coming from infinity with $E = 1$, it is given by

$$V_{\text{eff}} = \frac{1}{B^2 r^4} \left[\left(aL - \frac{r^2 + a^2}{\sqrt{1 - \nu^2}} \right)^2 - \Delta \left(L - \frac{a}{\sqrt{1 - \nu^2}} \right)^2 \right]$$

energy of two uncharged particles colliding at the outer horizon:

$$-\Delta \frac{r^2 \nu^2}{1 - \nu^2} - \Delta B r^2, \quad (25)$$

where $\Delta = r^2 - 2\mu r + a^2$ and $B = \sqrt{1 + \frac{2\mu\nu^2}{r(1 - \nu^2)}}$. We find that $V_{\text{eff}} = 0$ at the horizon when the freely falling particle has the critical angular momentum L_c . However, one can derive at the horizon that

$$\frac{dV_{\text{eff}}}{dr} \Big|_{r=r_+} = -\frac{\Delta'}{B^2 r_+^4} \times \left[\left(L_c - \frac{a}{\sqrt{1 - \nu^2}} \right)^2 + \frac{r_+^2 \nu^2}{1 - \nu^2} + B r_+^2 \right], \quad (26)$$

where $\Delta' = \frac{d\Delta}{dr} \Big|_{r=r_+}$ and $\Delta \Big|_{r=r_+} = 0$ has been used. Since r_+ is the larger root of $\Delta = 0$, $\Delta' = r_+ - r_- > 0$.

Given the fact that the boost parameter $\nu^2 < 1$, hence $\frac{dV_{\text{eff}}}{dr}|_{r=r_+} < 0$. That means the effective potential will decrease when r grows from r_+ . Thus there will be at least one root located at $r_+ < r < \infty$. Consequently, the freely falling particle with critical angular momentum L_c will be reflected before reaching the horizon. Hence the arbitrarily high CM energy will not be achieved in the case of collision of two freely falling particles from infinity. For particles which can reach the horizon, their angular momentum should be smaller than the angular momentum of a circular orbit particle. We will refer to the angular momentum of a circular orbit particle as the maximal angular momentum L_{max} . The angular momentum of a circular orbit particle and the radius of the circular orbit are defined implicitly from the solution of

the following equations

$$V_{\text{eff}} = \frac{dV_{\text{eff}}}{dr} = 0. \tag{27}$$

Though an analytical definition of L_{max} is very hard to get from Eq. (27) due to the presence of r in the expression of B , one can always solve it numerically when other parameters μ, ν, a are specified. Taking the unit $\mu = 1$ and one of the particles falling with L_{max} , while the other particle falls without orbital angular momentum, we list the numerical results of CM energy per unit mass in Table 1 for different small parameter $\epsilon = 1 - a$ and different values of ν . The results show that the CM energy is in fact limited and grows slowly as the black hole spin approaches its maximal value.

Table 1. The CM energy per unit rest mass $\frac{E_{\text{cm}}}{m_0}$ for a KK black hole with spin $a = 1 - \epsilon$ and $L_1 = L_{\text{max}}, L_2 = 0$.

	$\epsilon=0.1$	$\epsilon=0.01$	$\epsilon=0.001$	$\epsilon=0.0001$	$\epsilon=0.00001$
$\nu=0$	4.21767	7.10481	12.43999	22.01962	39.10101
$\nu=0.1$	4.21931	7.10426	12.43576	22.00991	39.08243
$\nu=0.2$	4.22482	7.10430	12.42667	21.98763	39.03917
$\nu=0.3$	4.23623	7.11010	12.42368	21.97348	39.00878
$\nu=0.4$	4.25740	7.13106	12.44599	22.00338	39.05618
$\nu=0.5$	4.29529	7.18270	12.52408	22.13337	39.28214
$\nu=0.6$	4.36278	7.29163	12.70810	22.45441	39.84939
$\nu=0.7$	4.48641	7.50928	13.09206	23.13507	41.05854
$\nu=0.8$	4.73260	7.95838	13.89599	24.56740	43.60709
$\nu=0.9$	5.34624	9.07637	15.89588	28.12961	49.94483

3.4 Naked singularity case

Lastly, we will consider the naked singularity case. If we take a near-extremal Kaluza-Klein naked singularity, we expect that it is possible for an ingoing and an outgoing particle to collide, just like the case of the Kerr naked singularity [19]. Then the CM energy for collision in the Kaluza-Klein naked singularity is

$$\frac{E_{\text{CM}}^2}{2m_0^2} = 1 + \frac{\mathcal{K}}{B\Delta}, \tag{28}$$

where

$$\begin{aligned} \mathcal{K} = & [B^2(r^2 + a^2) + a^2Z](1 - e_1A_t)(1 - e_2A_t) \\ & + (Z - 1)(L_1 + e_1A_\varphi)(L_2 + e_2A_\varphi) \\ & - \frac{aZ}{\sqrt{1 - \nu^2}}[(1 - e_1A_t)(L_2 + e_2A_\varphi) \\ & + (1 - e_2A_t)(L_1 + e_1A_\varphi)] \\ & + \left\{ [B^2(r^2 + a^2) + a^2Z](1 - e_1A_t)^2 \right. \\ & + (Z - 1)(L_1 + e_1A_\varphi)^2 \\ & \left. - \frac{2aZ}{\sqrt{1 - \nu^2}}(1 - e_1A_t)(L_1 + e_1A_\varphi) - B\Delta \right\}^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned} & \times \left\{ [B^2(r^2 + a^2) + a^2Z](1 - e_2A_t)^2 \right. \\ & + (Z - 1)(L_2 + e_2A_\varphi)^2 \\ & \left. - \frac{2aZ}{\sqrt{1 - \nu^2}}(1 - e_2A_t)(L_2 + e_2A_\varphi) - B\Delta \right\}^{\frac{1}{2}}. \tag{29} \end{aligned}$$

Let us work in the unit $\mu = 1$. If the collision happens to take place at $r = 1$, the CM energy of collision between two particles can be very high in the limit that the deviation of an extremal Kaluza-Klein naked singularity is very small ($\Delta \rightarrow 0$). Essentially, the divergence of the CM energy comes from the extremality of the naked singularity. Nonetheless, the range of allowed angular momentum of a freely falling particle is not arbitrary, to guarantee that it can turn back at some point to become an outgoing particle. The turning point of an initially ingoing particle is located at the larger root of the equation

$$V_{\text{eff}} = 0, \tag{30}$$

where V_{eff} is the effective potential for the radial motion. For an uncharged particle coming from infinity with $E = 1$, it is given by

$$V_{\text{eff}} = - \left[1 + \frac{a^2}{r^2} + \frac{2a^2}{r^3 B^2} - \frac{4aL}{r^3 B^2 \sqrt{1-\nu^2}} - \frac{(r-2)L^2}{r^3 B^2} - \frac{r^2 - 2r + a^2}{Br^2} \right], \quad (31)$$

where $B = \sqrt{1 + \frac{2\nu^2}{r(1-\nu^2)}}$. For the case without a real

root of Eq. (30), the particle will hit the singularity eventually. To have a collision of the two particles at $r = 1$, one just needs to solve the range of allowed angular momentum from Eq. (30) to guarantee that the following condition is satisfied: one of the particles should have a turning point at $r_t < 1$ and the other particle should not turn back before reaching $r = 1$. Though the analytical solution to Eq. (30) is extremely hard, if not impossible, to get, one can always check the existence of a solution numerically when all the parameters are specified.

4 Discussion and conclusions

In this paper, we have investigated the CM energy for two colliding particles in Kaluza-Klein spacetime. The Kaluza-Klein black hole is an exact solution in Einstein-Maxwell-dilaton theory in four-dimensional spacetime. When the charge Q vanishes, it just describes the Kerr black hole. Hence there is a restriction that our result should not be in contradiction with that of the Kerr black hole when $Q = 0$, which has been proved by the calculations. Our results show that an extremal Kaluza-Klein black hole can serve as a particle accelerator with arbitrarily high CM energy when one of the colliding particles has the fine-tuned angular momentum $L = \frac{2\mu}{\sqrt{1-\nu^2}}$.

For the near-extremal case, in terms of the small parameter $\epsilon = 1 - a$, we also obtain the numerical result of the maximal CM energy per unit mass for some different val-

ues of ϵ and ν . Our near-extremal result shows that the CM energy will not be so high, even in the very near-extremal case. On the other hand, with the vanished angular momentum J , the Kaluza-Klein black hole does not reduce to the Reissner-Nordström black hole. Our result in the nonrotating case shows that the CM energy of two charged colliding particles could also blow up, thus a non-extremal black hole could also provide a visible probe of Planck-scale physics. In both cases where arbitrarily high CM energy can be reached, a very specific angular momentum of the colliding particle needs to be chosen. Hence one must fine-tune the angular momentum of the ingoing particle. To overcome this issue, we also studied the collision of particles in the Kaluza-Klein naked singularity case. Our results show that the range of allowed angular momenta of freely falling particles to reach arbitrarily high CM energy will be much larger than a single fine-tuned value.

However, our calculations were performed without considering the back reaction effect of the accelerated particle pair on the background geometry of the Kaluza-Klein black hole. It should be pointed out that particles can be accelerated to arbitrarily high CM energy. Hence the background geometry may be destroyed and the back reaction effect should not be ignored. On the other hand, high energy concentrated at small scale will lead to gravitational collapse. So the Planck-scale physics induced by the collision of a particle pair with arbitrarily high CM energy is protected by the event horizon formed due to gravitational collapse, and cannot be observed externally. Hence, it would definitely be interesting and meaningful to explore the field theory interpretation of this classical effect in the future.

P. J. M. is grateful to Xiaolei Sun and Shi-Xiong Song for valuable information and helpful discussions.

References

- 1 M. Banados, J. Silk, and S. M. West, Phys. Rev. Lett., **103**: 111102 (2009)
- 2 E. Berti, V. Cardoso, L. Gualtieri, F. Pretorius, and U. Sperhake, Phys. Rev. Lett., **103**: 239001 (2009)
- 3 T. Jacobson and T. P. Sotiriou, Phys. Rev. Lett., **104**: 021101 (2010)
- 4 K. Lake, Phys. Rev. Lett., **104**: 211102 (2010)
- 5 K. Lake, Phys. Rev. Lett., **104**: 259903(E) (2010)
- 6 A. A. Grib and Yu. V. Pavlov, JETP Letters, **92**: 125 (2010)
- 7 A. A. Grib and Yu. V. Pavlov, Astropart. Phys., **34**: 581 (2011)
- 8 A. A. Grib and Yu. V. Pavlov, arXiv:1007.3222
- 9 A. A. Grib and Yu. V. Pavlov, Grav. Cosmol., **17**: 42 (2011)
- 10 O. B. Zaslavskii, Phys. Rev. D, **82**: 083004 (2010)
- 11 O. B. Zaslavskii, JETP Lett., **92**: 571 (2010)
- 12 O. B. Zaslavskii, Class. Quant. Grav., **28**: 105010 (2011)
- 13 S. W. Wei, Y. X. Liu, H. Guo, and C. E. Fu, Phys. Rev. D, **82**: 103005 (2010)
- 14 S. W. Wei, Y. X. Liu, H. T. Li, and F. W. Chen, JHEP, **12**: 066 (2010)
- 15 T. Harada and M. Kimura, Phys. Rev. D, **83**: 024002 (2011)
- 16 T. Harada and M. Kimura, Phys. Rev. D, **83**: 084041 (2011)
- 17 M. Patil and P. S. Joshi, Phys. Rev. D **82**: 104049 (2010)
- 18 M. Patil, P. S. Joshi, and D. Malafarina, Phys. Rev. D, **83**: 064007 (2011)
- 19 M. Patil and P. S. Joshi, Class. Quant. Grav., **28**: 235012 (2011)
- 20 M. Patil and P. S. Joshi, Phys. Rev. D, **84**: 104001 (2011)
- 21 Y. Li, J. Yang, Y. L. Li, S. W. Wei, and Y. X. Liu, Class. Quant. Grav., **28**: 225006 (2011)
- 22 G. W. Gibbons and D. L. Wiltshire, Ann. Phys. (N. Y.), **167**: 201 (1986)
- 23 K. S. Thorne, Astrophys. J., **191**: 507 (1974)