The $^{4}_{\Lambda\Lambda}$n system *

H. Garcilazo[1;1)] A. Valcarce[2;2)] J. Vijande[3;3)]

[1] Escuela Superior de Física y Matemáticas, Instituto Politécnico Nacional, Edificio 9, 07738 México D.F., Mexico
[2] Departamento de Física Fundamental and IUFFyM, Universidad de Salamanca, E-37008 Salamanca, Spain
[3] Unidad Mixta de Investigación en Radiofísica e Instrumentación Nuclear en Medicina (IRIMED),
Instituto de Investigación Sanitaria La Fe (IIS-La Fe)-Universitat de Valencia (UV) and IFIC (UV-CSIC), Valencia, Spain

Abstract: Using local central Yukawa-type Malfliet-Tjon interactions reproducing the low-energy parameters and phase shifts of the nn system, and the latest updates of the nΛ and $\Lambda\Lambda$ Nijmegen ESC08c potentials, we study the possible existence of a $^{4}_{\Lambda\Lambda}$n bound state. Our results indicate that the $^{4}_{\Lambda\Lambda}$n is unbound, being just above threshold. We discuss the role played by the $^1S_0$ nn repulsive term of the Yukawa-type Malfliet-Tjon interaction.

Keywords: baryon-baryon interactions, few-body systems

PACS: 21.45.-v, 25.10.+s, 11.80.Jy **DOI:** 10.1088/1674-1137/41/7/074102

1 Introduction

It is well-established that two-body bound states made of neutrons and/or Λ's, the lightest hyperon, do not exist. The situation is much more cumbersome for three-, four- and in general few-body systems made of nucleons and hyperons [1]. For example, it has been proposed that dineutrons could become bound in the presence of additional nucleons [2]. This is the mechanism responsible for the properties of some bound nuclei that have a neutron excess, like ^{11}Li, where a pair of external neutrons form a remote halo around the core of ^{9}Li [3]. Such a possibility has recently been raised in a lighter system by the experimental HypHI Collaboration [4], suggesting the existence of a neutral bound state of two neutrons and a Λ hyperon, $^{3}_{\Lambda}$n. They analyze the experimental data obtained from the reaction ^{6}Li $+^{12}$C at 2 A GeV to study the invariant mass distribution of d$+\pi^-$ and t$+\pi^-$. The signal observed in the invariant mass distributions of d$+\pi^-$ and t$+\pi^-$ final states was attributed to a strangeness-changing weak process corresponding to the two- and three-body decays of an unknown bound state of two neutrons associated with a Λ, $^{3}_{\Lambda}$n, via $^{3}_{\Lambda}\mathrm{n}\to \mathrm{t}+\pi^-$ and $^{3}_{\Lambda}\mathrm{n}\to \mathrm{t}^*+\pi^-\to \mathrm{d}+\mathrm{n}+\pi^-$. This is an intriguing conclusion, since one would naively expect the nnΛ system to be unbound. In the nnΛ system the two nucleons interact in the $^1S_0$ partial wave, while in the npΛ system they interact in the $^3S_1$ partial wave. Thus, since the nucleon-nucleon (NN) interaction in the $^1S_0$ channel is weaker than in the $^3S_1$ channel, and the npΛ system is bound by only 0.13 MeV, one may have anticipated that the nnΛ system should be unbound. The absence of binding of the nnΛ system was first demonstrated by Dalitz and Downs [5] using a variational approach, and later from the solution of the Faddeev equations with separable interactions [6]. The theoretical debate on the possible existence of a neutral bound state of two neutrons and a Λ hyperon, $^{3}_{\Lambda}$n, is still open and has lately deserved an important theoretical effort [1, 7–11].

In the four-body case, the analysis of the missing-mass spectrum in the double-charge-exchange reaction ^{4}He(^{8}He,^{8}Be) at 186 MeV/u has unveiled the possible existence of a tetraneutron resonance 0.83 ± 0.65(stat)$\pm$ 1.25(syst) MeV above the threshold of four-neutron decay with a significance level of 4.9σ [12]. In 2002, one collaboration claimed to have found a bound tetraneutron in a ^{14}Be breakup reaction [13]. This result remains unconfirmed, and theorists quickly showed that, based on the best knowledge of the NN interaction, the existence of a bound tetraneutron was nearly impossible, although they could not rule out the existence of a short-lived resonant state on the basis of a dineutron-dineutron structure [14–17]. The stability of a tetraneutron state cannot be established even with potentials made artificially deeper to produce a dineutron bound state (the

Received 8 March 2017

∗ Supported by COFAA-IPN (México), Ministerio de Economía, Industria y Competitividad and EU FEDER (FPA2013-47443, FPA2015-69714-REDT, FPA2016-77177), Junta de Castilla y León (SA041U16) and Generalitat Valenciana PrometeoII/2014/066

1) E-mail: humberto@esfm.ipn.mx

2) E-mail: valcarce@usal.es

3) E-mail: javier.vijande@uv.es

dineutron is a virtual state 66 keV above the two-neutron threshold), due to the Pauli principle which forbids two identical fermions from occupying the same quantum state. For four-neutrons, only one pair can be in the lowest-energy state, forcing the second pair into a state of higher energy, thereby making the tetraneutron unstable. Thus, one could think of the stability of a modified tetraneutron with Bose statistics, where a pair of neutrons is replaced by a pair of neutral light baryons, in this way enforcing antisymmetrization with all particles in the lowest-energy state. This is the case for the ${}^{4}_{\Lambda\Lambda}$n $=(\mathrm{n},\mathrm{n},\Lambda,\Lambda)$, recently discussed in Ref. [1] and suggested as a possible Borromean state.

The relevance of the addition of further baryons to an almost bound two-body system has also been discussed recently by some of us looking for stable bound states of N's and Ξ's. In Ref. [18] we pointed out that when a two-baryon interaction is attractive, if the system is merged with nuclear matter and the Pauli principle does not impose severe restrictions, the attraction may be reinforced. Simple examples of the effect of a third or a fourth baryon in two-baryon systems could be given. The deuteron, $(I)J^P=(0)1^+$, is bound by 2.225 MeV, while the triton, $(I)J^P=(1/2)1/2^+$, is bound by 8.480 MeV, and the α particle, $(I)J^P=(0)0^+$, is bound by 28.295 MeV. The binding per nucleon B/A increases from $1:3:7$. A similar argument could be employed for strangeness -1 systems. While there is no evidence for dibaryon states, the hypertriton ${}^{3}_{\Lambda}$H, $(I)J^P=(0)1/2^+$, is bound with a separation energy of 130±50 keV, and the ${}^{4}_{\Lambda}$H, $(I)J^P=(0)0^+$, is bound with a separation energy of 2.12±0.01(stat)±0.09(syst) MeV [19]. This cooperative effect of the attraction in two-body subsystems when merged in few-baryon states was also made evident in the prediction of a ΣNN quasibound state in the $(I)J^P=(1)1/2^+$ channel very near threshold [20]. Such a ΣNN quasibound state has been recently suggested in ^{3}He$(\mathrm{K}^-,\pi^{\mp})$ reactions at 600 MeV/c [21].

Thus, if a second Λ were added to the uncertain nnΛ state, the weakly attractive ΛΛ interaction [22] and the reinforcement of the NΛ potential, without paying a price for antisymmetry requirements, may give rise to a stable bound state. Our goal in this paper is to study the ${}^{4}_{\Lambda\Lambda}$n state, making use of potentials compatible with the low-energy data and phase-shifts of the nn, nΛ, and ΛΛ systems. A first examination of this problem has been presented in Ref. [1] based on potentials with a single Yukawa attractive term or a Morse parametrization.

2 Two-body interactions

For the identical pairs nn and ΛΛ, the S wave interaction is in the ${}^1S_0$ channel due to the Pauli principle, while for the NΛ pair both ${}^1S_0$ and ${}^3S_1$ channels contribute. It is well-known that the NN ${}^1S_0$ channel is almost bound, the virtual state lying slightly below the nn threshold in the unphysical sheet. In the case of the NN ${}^1S_0$ channel we use the Malfliet-Tjon I model [23] with the parameters given in Ref. [24]. For the two-body interactions containing Λ's, NΛ and ΛΛ, we use the most recent update of the ESC08c Nijmegen potentials [25–27]. Regarding the two-body interactions containing a single Λ, they are constrained by a simultaneous fit to the combined NN and YN scattering data, supplied with constraints on the YN and YY interaction originating from the G-matrix information on hypernuclei [25]. The ΛΛ strangeness -2 interaction is mainly determined by the NN and YN data, and $SU(3)$ symmetry [26, 27]. It takes account of the pivotal results of strangeness -2 physics, the NAGARA [22] and the KISO [28] events. Although other double-Λ hypernuclei events, like the DEMACHIYANAGI and HIDA events [29], are not explicitly taken into account, the G-matrix nuclear matter study of Ξ^- capture both in ^{12}C and ^{14}N (see section VII of Ref. [26]), concludes that the ΞN attraction in the ESC08c potential is consistent with the Ξ-nucleus binding energies given by the emulsion data of the twin Λ-hypernuclei.

We have constructed the two-body amplitudes for all subsystems entering the four-body problem studied by solving the Lippmann–Schwinger equation of each (i,j) channel,

$$t^{ij}(p,p';e) = V^{ij}(p,p')+\int_0^\infty p''^2\mathrm{d}p''V^{ij}(p,p'') \times\frac{1}{e-p''^2/2\mu}t^{ij}(p'',p';e), \tag{1}$$

where

$$V^{ij}(p,p')=\frac{2}{\pi}\int_0^\infty r^2\mathrm{d}r\, j_0(pr)V^{ij}(r)j_0(p'r), \tag{2}$$

and the two-body potentials consist of an attractive and a repulsive Yukawa term, i.e.,

$$V^{ij}(r)=-A\frac{\mathrm{e}^{-\mu_A r}}{r}+B\frac{\mathrm{e}^{-\mu_B r}}{r}. \tag{3}$$

The parameters of the ΛN and ΛΛ channels were obtained by fitting the low-energy data and the phase-shifts of each channel as given in the most recent update of the strangeness -1 [25] and -2 [26] ESC08c Nijmegen potential. The low-energy data and the parameters of these models, together with those of the NN interaction from Ref. [24], are given in Table 1. It is worth noting that the scattering length and effective range of the most recent updates of the ΛΛ interaction derived from chiral effective field theories are very much like those of the ESC08c Nijmegen potential (see Table 2 of Ref. [30]), unlike the earlier version used in Ref. [1] (see Table 4 of Ref. [31]), reporting remarkably small effective ranges.

Table 1. Low-energy parameters and parameters of the local central Yukawa-type potentials given by Eq. (3) for the NN [24], ΛN [25], and ΛΛ [26] systems contributing to the $(I)J^P=(1)0^+$ ${}^{4}_{\Lambda\Lambda}$n state. See text for details.

	(i,j)	a/fm	r_0/fm	A/(MeV fm)	μ_A/fm^{-1}	B/(MeV fm)	μ_B/fm^{-1})
NN	(1,0)	−23.56	2.88	513.968	1.55	1438.72	3.11
ΛN	(1/2,0)	−2.62	3.17	416	1.77	1098	3.33
	(1/2,1)	−1.72	3.50	339	1.87	968	3.73
ΛΛ	(0,0)	−0.853	5.126	121	1.74	926	6.04

If it is assumed that only singlet and triplet S wave contribute in the two-particle channel, the parametrization of the NN interaction used in this work, set III for the triplet partial wave and set I for the singlet partial wave, gives a triton binding energy of 8.3 MeV [23]. The effect of the repulsive core on the singlet two-body channel is crucial to get this result, while the repulsion on the triplet two-body channel has almost no effect on the binding. In fact, if the repulsive core in the singlet partial wave is not considered, the triton gains around 2 MeV of binding (see Table II of Ref. [32]). Based on predictions for separable potentials, in Ref. [23] it is suggested that the inclusion of the tensor force in the triplet interaction changes the binding energy by 0.3 MeV. Indeed, this is the result obtained in Ref. [33], where, as can be seen in Table III of that reference, a five-channel calculation (S and D partial waves) differs from a two-channel calculation (only S partial waves) by about 0.3 MeV. The influence of local tensor forces in Malfliet-Tjon Yukawa type interactions has also been studied in Ref. [34], showing that the inclusion of tensor forces reduces the binding energy of the three-body problem by 1 to 1.5 MeV, depending on the D wave percentage. Thus, the local Yukawa-type potentials with tensor interaction would lack binding in the three-body problem at a difference of separable potentials that would drive to overbinding [35]. Note that in the ${}^{4}_{\Lambda\Lambda}$n the NN ${}^3S_1$ partial wave does not contribute, so although this system is free of any uncertainty related to the triplet partial wave, the repulsive core in the singlet NN channel might play some role.

3 The four-body problem

The four-body problem has been addressed by means of a generalized variational method. The nonrelativistic Hamiltonian is given by,

$$H=\sum_{i=1}^{4}\frac{\vec{p}_i^{\,2}}{2m_i}+\sum_{i<j=1}^{4}V(\vec{r}_{ij}), \tag{4}$$

where the potentials $V(\vec{r}_{ij})$ have been discussed in the previous section. For each channel s, the variational wave function is the tensor product of a spin ($|S_{s_1}\rangle$), isospin ($|I_{s_2}\rangle$), and radial ($|R_{s_3}\rangle$) component,

$$|\phi_s\rangle=|S_{s_1}\rangle\otimes|I_{s_2}\rangle\otimes|R_{s_3}\rangle\,, \tag{5}$$

where $s\equiv\{s_1,s_2,s_3\}$. Once the spin and isospin parts are integrated out, the coefficients of the radial wave function are obtained by solving the system of linear equations,

$$\sum_{s'\,s}\sum_{i}\beta_{s_3}^{(i)}[\langle R_{s'_3}^{(j)}|H|R_{s_3}^{(i)}\rangle-E\langle R_{s'_3}^{(j)}|R_{s_3}^{(i)}\rangle\delta_{s,s'}]=0 \qquad \forall j, \tag{6}$$

where the eigenvalues are obtained by a minimization procedure.

For the description of the four-body wave function we consider the Jacobi coordinates:

$$\begin{aligned}
&\vec{r}_{NN}=\vec{x}=\vec{r}_1-\vec{r}_2\,,\\
&\vec{r}_{\Lambda\Lambda}=\vec{y}=\vec{r}_3-\vec{r}_4\,,\\
&\vec{r}_{NN-\Lambda\Lambda}=\vec{z}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)-\frac{1}{2}(\vec{r}_3+\vec{r}_4)\,,\\
&\vec{R}_{\rm CM}=\vec{R}=\frac{\sum m_i\vec{r}_i}{\sum m_i}.
\end{aligned} \tag{7}$$

The total wave function should have well-defined permutation properties under the exchange of identical particles. The most general S wave radial wave function may depend on the six scalar quantities that can be constructed with the Jacobi coordinates of the system. They are: $\vec{x}^2$, $\vec{y}^2$, $\vec{z}^2$, $\vec{x}\cdot\vec{y}$, $\vec{x}\cdot\vec{z}$ and $\vec{y}\cdot\vec{z}$. We define the variational spatial wave function as a linear combination of generalized Gaussians,

$$|R_{s_3}\rangle=\sum_{i=1}^{n}\beta_{s_3}^{(i)}R_{s_3}^{i}(\vec{x},\vec{y},\vec{z})=\sum_{i=1}^{n}\beta_{s_3}^{(i)}R_{s_3}^{i}\,, \tag{8}$$

where n is the number of Gaussians used for each spin-isospin component. $R_{s_3}^i$ depends on six variational parameters: a_s^i, b_s^i, c_s^i, d_s^i, e_s^i, and f_s^i, one for each scalar quantity. Therefore, the four-body system will depend on $6{\times}n{\times}n_s$ variational parameters, where n_s is the number of different channels allowed by the Pauli principle. Equation (8) should have well defined permutation symmetry under the exchange of both N's and Λ's,

$$\begin{aligned}
P_{12}(\vec{x}\rightarrow-\vec{x})R_{s_3}^i &= P_xR_{s_3}^i\\
P_{34}(\vec{y}\rightarrow-\vec{y})R_{s_3}^i &= P_yR_{s_3}^i,
\end{aligned} \tag{9}$$

where P_x and P_y are −1 for antisymmetric states, (A), and +1 for symmetric ones, (S).

If we now define the function,

$$\begin{aligned}
g(s_1,s_2,s_3) \;=\; &\mathrm{Exp}\big(-a_s^i\vec{x}^2-b_s^i\vec{y}^2-c_s^i\vec{z}^2-s_1d_s^i\vec{x}\cdot\vec{y}\\
&-s_2e_s^i\vec{x}\cdot\vec{z}-s_3f_s^i\vec{y}\cdot\vec{z}\big),
\end{aligned} \tag{10}$$

and the vectors,

$$\vec{G}_s^i = \begin{pmatrix} g(+,+,+) \\ g(-,+,-) \\ g(-,-,+) \\ g(+,-,-) \end{pmatrix}, \tag{11}$$

and

$$\begin{aligned} \vec{\alpha}_{SS} &= (+,+,+,+) \\ \vec{\alpha}_{SA} &= (+,-,+,-) \\ \vec{\alpha}_{AS} &= (+,+,-,-) \\ \vec{\alpha}_{AA} &= (+,-,-,+), \end{aligned} \tag{12}$$

we can build any symmetry for the radial wave function, $(P_xP_y)=(SS)$, (SA), (AS) and (AA),

$$\begin{aligned} (SS) &\Rightarrow R_1^i=\vec{\alpha}_{SS}\cdot\vec{G}_s^i \\ (SA) &\Rightarrow R_2^i=\vec{\alpha}_{SA}\cdot\vec{G}_s^i \\ (AS) &\Rightarrow R_3^i=\vec{\alpha}_{AS}\cdot\vec{G}_s^i \\ (AA) &\Rightarrow R_4^i=\vec{\alpha}_{AA}\cdot\vec{G}_s^i, \end{aligned} \tag{13}$$

including all possible relative orbital angular momenta coupled to an S wave. The radial wave function described in this section is adequate to describe not only bound states, but is also flexible enough to describe states of the continuum with a reasonable accuracy [36–38].

The numerical method described in this section has been successfully tested in different few-body calculations in comparison with the hyperspherical harmonic formalism, see for example Refs. [38, 39], or the stochastic variational approach of Ref. [37] for some of the results presented in Ref. [40].

4 Results and discussion

Let us first of all show the reliability of the input potentials. We compare in Fig. 1 the NΛ and ΛΛ phase shifts reported by the ESC08c Nijmegen potential and those obtained by our fits with the two-body potential of Eq. (3) and the parameters given in Table 1. There is good agreement. Once we have described the phase shifts, the NΛ and ΛΛ potentials include in an effective manner the coupling to other two-body channels, such as the NΣ or NΞ two-body systems.

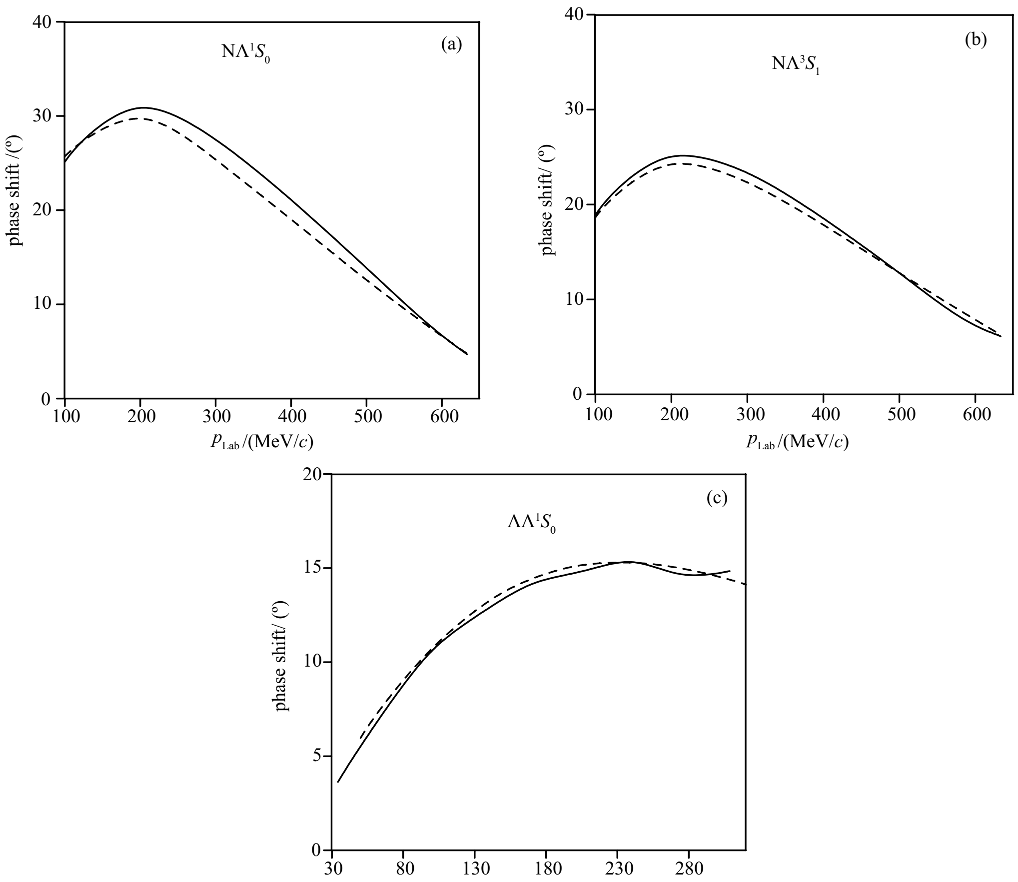

Fig. 1. (a) NΛ $^1S_0$ phase shifts. The solid line stands for the results of the ESC08c Nijmegen potential and the dashed line for the results of the two-body potential of Eq. (3) with the parameters given in Table 1. (b) Same as (a) for the NΛ $^3S_1$ phase shifts. (c) Same as (a) for the ΛΛ $^1S_0$ phase shifts.

We have also tested the two-body interactions in the three-body problem of systems made of N's and Λ's. The hypertriton is bound by 144 keV, and the nnΛ system is unbound. The reasonable description of the two- and three-body problem gives confidence to address the study of the nnΛΛ state.

Using the variational method described in the last section, we have evaluated the binding energy of the nnΛΛ system with quantum numbers $(I)J^P=(1)0^+$. The system is unbound, appearing just above threshold, and thus it does not seem to be Borromean, a four-body bound state without two- or three-body stable subsystems. An unbound result was also reported in Ref. [41], although in this case the authors made use of repulsive Gaussian-type potentials for any of the two-body subsystems (see the figure on p. 475) that do not allow for the existence of any bound state.

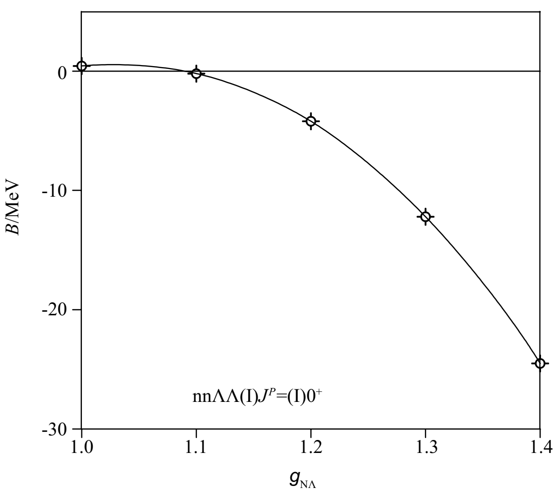

Fig. 2. Binding energy of the $(I)J^P=(1)0^+$ nnΛΛ state as a function of the multiplicative factor, $g_{\rm N\Lambda}$, in the attractive part of $V^{\rm N\Lambda}(r)$ interaction for $g_{\rm NN}=g_{\Lambda\Lambda}=1$.

We have studied the dependence of the binding on the strength of the attractive part of the different two-body interactions entering the four-body problem. For this purpose we have used the following interactions,

$$V^{B_1B_2}(r)=-g_{B_1B_2}A\frac{{\rm e}^{-\mu_A r}}{r}+B\frac{{\rm e}^{-\mu_B r}}{r} \qquad (14)$$

with the same parameters given in Table 1. The system hardly gets bound for a reasonable increase of the strength of the attractive part of the ΛΛ interaction, $g_{\Lambda\Lambda}$. Although one cannot exclude that the genuine ΛΛ interaction in dilute states such as the one studied here could be slightly stronger that the one reported in Ref. [26], one needs $g_{\Lambda\Lambda}\geqslant 1.8$ to get a bound state, which would destroy the agreement with the ESC08c Nijmegen ΛΛ phase shifts. Note that this is also a very sensitive parameter for the study of double-Λ hypernuclei [42]. Taking a factor 1.2 in the attractive part of the ${}^1S_0$ NN interaction, that would make the ${}^1S_0$ NN potential as strong as the ${}^3S_1$ [23] and thus the singlet S wave would develop a dineutron bound state, the four-body system would start to be bound. The situation is slightly different when dealing with the NΛ interaction. We have used a common factor $g_{\rm N\Lambda}$ for the attractive part of the two NΛ partial waves, ${}^1S_0$ and ${}^3S_1$. We show in Fig. 2 the binding energy of the $(I)J^P=(1)0^+$ nnΛΛ state as a function of the multiplicative factor $g_{\rm N\Lambda}$, for $g_{\rm NN}=g_{\Lambda\Lambda}=1$. As one can see, the four-body system develops a bound state for $g_{\rm N\Lambda}=1.1$.

In Ref. [1], the ${}^{\;4}_{\Lambda\Lambda}$n system was studied based on the fit of Nijmegen-RIKEN [43, 44] or chiral effective field theory [31] low-energy parameters by means of a single Yukawa attractive term or a Morse parametrization. The method used to solve the four-body problem is similar to the one we have used in our calculation, so the results might be directly comparable. Our improved description of the two- and three-body subsystems and the introduction of the repulsive barrier for the ${}^1S_0$ NN partial wave, relevant for the study of the triton binding energy (see Table II of Ref. [32]), leads to a four-body state just above threshold, that cannot get bound by a reliable modification in the two-body subsystems. As clearly explained in Ref. [1], the window of Borromean binding is more and more reduced for potentials with harder inner cores.

As already discussed in Ref. [1], many effects are still to be taken into account before arriving at any definitive conclusion. Among the refinements that would eliminate uncertainties, it would be a future challenge to consider three-body forces that may have an attractive component, as suggested when studying the triton and ^{4}He [45]. Although by fitting the NΛ phase shifts, the coupling to the NΣ system has been included in an effective manner, it would also be interesting to unfold the effective ΛN interaction, separating the contribution from ΛN↔ΣN. As has been discussed in the literature [7, 8, 20, 46–48], the hypertriton does not get bound by considering only NNΛ channels, but it is necessary to include also NNΣ channels. Similar considerations hold for the ΛΛ↔NΞ coupling, that is expected to play a minor role in this case, because the nucleon generated in the transition must occupy an excited p-shell, the lowest s-shell being forbidden by the Pauli principle [42, 49].

5 Summary

In brief, based on a reasonable approach to the interactions of two-body subsystems contributing to the $(I)J^P=(1)0^+$ nnΛΛ state, it does not present a bound state. We have fitted not only the low-energy parameters of the two-body subsystems, but also the phase-shifts. We have considered the repulsive barrier in the two-body interactions, which is relevant for a correct description of the triton binding energy. We have also studied the strange three-body subsystems involved in the problem, the hypertriton bound by 144 keV, and the nnΛ system that it is unbound. Thus, the $^{4}_{\Lambda\Lambda}$n four-body system does not seem to be Borromean. Finally, although our arguments on the unbound nature of the $^{4}_{\Lambda\Lambda}$n are strong, one should bear in mind how delicate the few-body problem in the regime of weak binding is, as demonstrated in Ref. [42].

References

1 J. -M. Richard, Q. Wang, and Q. Zhao, Phys. Rev. C, **91**: 014003 (2015)
2 A. B. Migdal, Sov. J. Nucl. Phys., **16**: 238 (1973)
3 C. A. Bertulani and V. Zelevinsky, Nature, **532**: 448 (2016)
4 C. Rappold et al (HypHI Collaboration), Phys. Rev. C, **88**: 041001R (2013)
5 R. H. Dalitz and B. W. Downs, Phys. Rev., **110**: 958 (1958); Phys. Rev., **111**: 967 (1958); Phys. Rev., **114**: 593 (1959)
6 H. Garcilazo, J. Phys. G, **13**: L63 (1987)
7 H. Garcilazo and A. Valcarce, Phys. Rev. C, **89**: 057001 (2014)
8 E. Hiyama, S. Ohnishi, B. F. Gibson, and Th. A. Rijken, Phys. Rev. C, **89**: 061302(R) (2014)
9 A. Gal and H. Garcilazo, Phys. Lett. B, **736**: 93 (2014)
10 S. -I. Ando, U. Raha, and Y. Oh, Phys. Rev. C, **92**: 024325 (2015)
11 I. R. Afnan and B. F. Gibson, Phys. Rev. C, **92**: 054608 (2015)
12 K. Kisamori et al, Phys. Rev. Lett., **116**: 052501 (2016)
13 F. M. Marqués et al, Phys. Rev. C, **65**: 044006 (2002)
14 C. A. Bertulani and V. Zelevinsky, J. Phys. G, **29**: 2431 (2003)
15 S. C. Pieper, Phys. Rev. Lett., **90**: 252501 (2003)
16 N. K. Timofeyuk, J. Phys. G, **29**: L9 (2003)
17 R. Lazauskas and J. Carbonell, Phys. Rev. C, **72**: 034003 (2005)
18 H. Garcilazo and A. Valcarce, Phys. Rev. C, **92**: 014004 (2015)
19 A. Esser et al (A1 Collaboration), Phys. Rev. Lett., **114**: 232501 (2015)
20 H. Garcilazo, A. Valcarce, and T. Fernández-Caramés, Phys. Rev. C, **76**: 034001 (2007); Phys. Rev. C, **75**: 034002 (2007)
21 T. Harada and Y. Hirabayashi, Phys. Rev. C, **89**: 054603 (2014)
22 H. Takahashi et al, Phys. Rev. Lett., **87**: 212502 (2001)
23 R. A. Malfliet and J. A. Tjon, Nucl. Phys. A, **127**: 161 (1969)
24 J. L. Friar, B. F. Gibson, G. Berthold, W. Glöckle, Th. Cornelius, H. Witala, J. Haidenbauer, Y. Koike, G. L. Payne, J. A. Tjon, and W. M. Kloet, Phys. Rev. C, **42**: 1838 (1990)
25 M. M. Nagels, Th. A. Rijken, and Y. Yamamoto, arXiv:1501.06636
26 M. M. Nagels, Th. A. Rijken, and Y. Yamamoto, arXiv:1504.02634
27 Th. A. Rijken and H. -F. Schulze, Eur. Phys. J. A, **52**: 21 (2016)
28 K. Nakazawa et al, Prog. Theor. Exp. Phys., (2015) 033D02
29 K. Nakazawa (KEK-E176, E373 and J-PARC E07 Collaborations), Nucl. Phys. A, **835**: 207 (2010)
30 J. Haidenbauer, U. -G. Meissner, and S. Petschauer, Nucl. Phys. A, **954**: 273 (2016)
31 H. Polinder, J. Haidenbauer, and U. -G. Meissner, Phys. Lett. B ,**653**: 29 (2007)
32 R. A. Malfliet and J. A. Tjon, Ann. of Phys., **61**: 425 (1970)
33 Y. Fujiwara, K. Miyagawa, M. Kohno, Y. Suzuki, and H. Nemura, Phys. Rev. C, **66**: 021001(R) (2002)
34 R. A. Malfliet and J. A. Tjon, Phys. Lett., **30B**: 293 (1969)
35 A. C. Phillips, Nucl. Phys. A, **107**: 209 (1968)
36 J. Vijande and A. Valcarce, Symmetry, **1**: 155 (2009)
37 Y. Suzuki and K. Varga, Lect. Not. Phys., **M54**: 1 (1998)
38 J. Vijande and A. Valcarce, Phys. Rev. C, **80**: 035204 (2009)
39 J. Vijande, E. Weissman, A. Valcarce, and N. Barnea, Phys. Rev. D, **76**: 094027 (2007)
40 J. Vijande, A. Valcarce, J. -M. Richard, and P. Sorba, Phys. Rev. D, **94**: 034038 (2016)
41 M. L. Lekala, G. J. Rampho, R. M. Adam, S. A. Sofianos, and V. B. Belyaev, Phys. of Atom. Nucl., **77**: 472 (2014)
42 H. Nemura, Y. Akaishi, and K. S. Myint, Phys. Rev. C, **67**: 051001(R) (2003)
43 Th. A. Rijken, M. M. Nagels, and Y. Yamamoto, Prog. Theor. Phys. Supp., **185**: 14 (2010)
44 Th. A. Rijken, M. M. Nagels, and Y. Yamamoto, Few-Body Syst., **54**: 801 (2013)
45 J. J. Bevelacqua, Phys. Rev. C, **16**: 2420 (1977)
46 K. Miyagawa, H. Kamada, W. Glöckle, and V.Stoks, Phys. Rev. C, **51**: 2905 (1995)
47 B. F. Gibson and D. R. Lehman, Phys. Rev. C, **16**: 1679 (1977)
48 B. F. Gibson and D. R. Lehman, Nucl. Phys. A, **329**: 308 (1979)
49 H. Garcilazo and A. Valcarce, Phys. Rev. Lett., **110**: 012503 (2013); Phys. Rev. Lett., **110**: 179202 (2013)