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Simulation of longitudinal dynamics of laser-cooled and RF-bunched C^{3+} ion beams at heavy ion storage ring CSRe*

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Abstract: Laser cooling of Li-like C^{3+} and O^{4+} relativistic heavy ion beams is planned at the experimental Cooler Storage Ring (CSRe). Recently, a preparatory experiment to test important prerequisites for laser cooling of relativistic $^{12}C^{3+}$ ion beams using a pulsed laser system has been performed at the CSRe. Unfortunately, the interaction between the ions and the pulsed laser cannot be detected. In order to study the laser cooling process and find the optimized parameters for future laser cooling experiments, a multi-particle tracking method has been developed to simulate the detailed longitudinal dynamics of laser-cooled ion beams at the CSRe. Simulations of laser cooling of the $^{12}C^{3+}$ion beams by scanning the frequency of the RF-buncher or continuous wave (CW) laser wavelength have been performed. The simulation results indicate that ion beams with a large momentum spread could be laser-cooled by the combination of only one CW laser and the RF-buncher, and show the requirements of a successful laser cooling experiment. The optimized parameters for scanning the RF-buncher frequency or laser frequency have been obtained. Furthermore, the heating effects have been estimated for laser cooling at the CSRe. The Schottky noise spectra of longitudinally modulated and laser-cooled ion beams have been simulated to fully explain and anticipate the experimental results. The combination of Schottky spectra from the highly sensitive resonant Schottky pick-up and the simulation methods developed in this paper will be helpful to investigate the longitudinal dynamics of RF-bunched and ultra-cold ion beams in the upcoming laser cooling experiments at the CSRe.

Keywords: storage ring, laser cooling, longitudinal dynamics, Schottky noise spectrum, multi-particle simulation method

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1 Introduction

Laser cooling is one of the most powerful techniques to reach high phase space densities for relativistic heavy ion beams at storage rings [1, 2]. Compared with stochastic cooling [3, 4] and electron cooling [5, 6], laser cooling is expected to obtain a much higher cooling rate, resulting in higher phase space densities to achieve phase transition for ordered beams or crystallized beams [7–9]. So far, beams of several ion species have been successfully cooled at heavy ion storage rings, including $^{7}Li^{+}$ ion beams at the TSR in Heidelberg and ASTRID in Aarhus, $^{24}Mg^{+}$ ion beams at the S-LSR in Kyoto, and $^{12}C^{3+}$ ion beams at the ESR in Darmstadt [10–16]. However, none of the laser cooling experiments has achieved phase-transition to obtain crystallized highly charged heavy ion beams, and there are still many challenges to reaching this state. Systematic simulations and experimental studies are urgently needed to make progress in laser cooling at storage rings. Currently, most laser cooling simulations and experiments are concentrated on the longitudinal phase space. The transverse laser cooling of stored ion beams can be realized through the intra-beam scattering (IBS) effect and dispersion coupling effect, which were investigated at the TSR [17, 18] at MPIK and the S-LSR at Kyoto University [19]. However, it will be very difficult to investigate transverse laser cooling of relativistic ion beams at the experimental Cooler Storage Ring (CSRe) in Lanzhou.

Progress is being made in laser cooling experiments for relativistic $^{12}C^{3+}$ and $^{16}O^{4+}$ ion beams at the CSRe [20–22]. Recently, a test experiment towards laser cool-

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ing of $^{12}C^{3+}$ ion beams at an energy of 122 MeV/u was performed with a pulsed laser system. The aims of the experiment were to test the experimental method of building laser systems in an accelerator machine and overlapping the laser with ion beams. For laser cooling schemes, a CW laser is used for bunched ion beams. To investigate the dynamics of laser-cooled and RF-bunched heavy ion beams and the properties of ultra-cold ion beams, a molecular dynamics simulation was carried out at the storage ring S-LSR, which is a compact ion storage ring in Kyoto University [2].

In this paper, a multi-particle tracking simulation method is developed. The longitudinal dynamics of RF-bunched and laser-cooled $^{12}C^{3+}$ ion beams with a kinetic energy of 122 MeV/u are simulated to study the laser cooling process and find the optimized parameters of the cooling process at the CSRe. The heating effects are simulated. Moreover, a simulation to produce the Schottky noise spectra of the RF-bunched and laser-cooled ion beams is performed to explain and anticipate the experimental results, especially for ultra-cold ion beams. The combination of the simulation and the experiments will be very helpful to investigate the properties of ultra-cold ion beams at the CSRe in the future.

The structure of this paper is as follows. First, a brief description of the setup for laser cooling experiments at the CSRe is presented, followed by the details of the simulation method, and the simulation results of laser cooling. Last but not least, analysis of the Schottky noise spectra of RF-bunched and laser-cooled ion beams is introduced and discussed.

2 Experimental setup for laser cooling of relativistic C^{3+} ion beams at the CSRe

The layout of the laser cooling setup at the CSRe is shown in Fig. 1. The laser cooling experiment of relativistic heavy ion beams at the CSRe has already been described in another paper [23]. For laser cooling experiments, $^{12}C^{3+}$ ions are produced, accelerated, and finally injected into the CSRe at an energy of 122 MeV/u (at a velocity of 47% of the speed of light). A suitable laser overlaps the ion beams all along the straight section (~ 20 m) where the new Schottky-noise pick-up and optical diagnostic of UV-PMT and CPM are located. The UV-PMT and CPM are the ultraviolet-sensitive photomultiplier tube and Channeltron photomultiplier to detect the fluorescence. A specially designed RF-buncher system has been installed to bunch the beam [21]. The relevant parameters for laser cooling of C^{3+} ion beams at the CSRe are listed in Table 1.

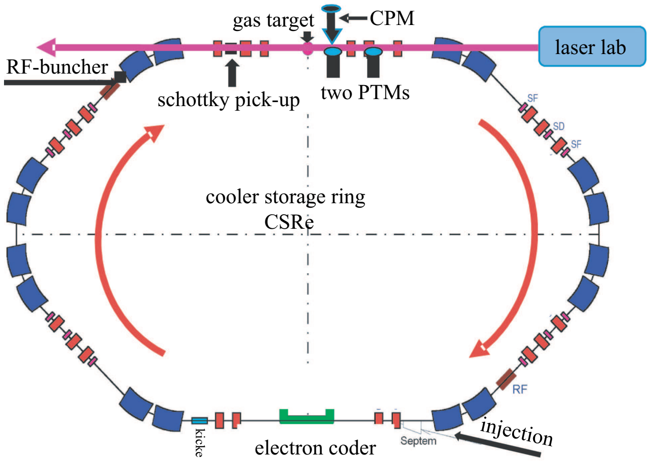


Fig. 1. (color online) Schematic of the CSRe and experimental setup for laser cooling.

Table 1. Main parameters for laser cooling experiments at the CSRe.

CSRe parameters	
circumference/m	128.80
ion species	$^{12}C^{3+}$
beam energy/(MeV/u)	122
relativistic factor $\beta\gamma$	0.47,1.13
revolution frequency/MHz	1.087
transition energy γ_t	2.629
slip factor η	0.64
laser system	
laser type	CW laser
laser wavelength/nm	257.5
laser power/mW	40
scan range/(GHz/10ms)	26
cooling optical transition	
optical transition $2S_{1/2}\rightarrow 2P_{1/2}$/nm	155.07
natural line width Γ/MHz	42
RF-buncher system	
voltage/V	1
harmonic number	15th
scan range/Hz	~500

3 Multi-particle tracking simulation of RF-bunched and laser-cooled ion beams

Laser cooling of relativistic heavy ion beams at the CSRe storage ring will be achieved by the combination of a CW laser and a RF-buncher, as described in Ref. [21]. The purposes of the RF-buncher include bunching the ion beams, restricting the ions in the bucket and providing a counter-force for the laser scattering force. Ion beams with large momentum spread could be laser-cooled by scanning the frequency of the RF-buncher to bring the ions to resonant interaction with the fixed laser. In the laser cooling simulation, the potential of the RF-buncher is sinusoidal wave, as:

$$V = V_0 \sin 2\pi h\phi. \tag{1}$$

Here V_0 is the voltage amplitude, h is the harmonic number, and ϕ is the phase value, which is equivalent to time. The RF-buncher can be operated at various harmonic numbers of the revolution frequency. As a result, the ion beams will be modulated longitudinally by the RF-buncher.

To study the movement of the RF-bunched C^{3+} ions in longitudinal phase space, a multi-particle tracking code is developed. The discrete longitudinal dynamics equation is used:

$$\begin{cases} \delta_{n+1}=\delta_n+\dfrac{qeV}{\beta^2 E}(\sin\phi_n-\sin\phi_s) \\ \phi_{n+1}=\phi_n+2\pi h\eta\delta_{n+1} \end{cases}, \qquad (2)$$

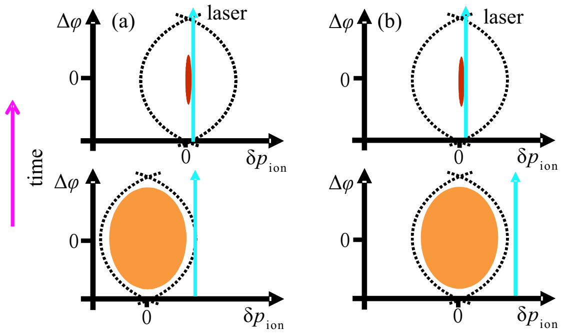
where β is the relativistic factor, qe is the ion charge, η is the slip factor, h is the harmonic number, δ_n and ϕ_n are the momentum spread and the phase value at the nth turn respectively, and δ_{n+1} and ϕ_{n+1} are the momentum spread and the phase value at the $(n+1)$th turn respectively. The phase ϕ_{n+1} depends on the new off-momentum coordinate δ_{n+1}. The ϕ_s is the phase of a synchrotron particle [24].

In laser cooling experiments at the CSRe, a CW laser will be employed to resonantly interact with the ion beams in the counter-propagating direction. The optical transition $2s_{1/2}\rightarrow 2p_{1/2}$ of C^{3+} ions with energy of 122 MeV/u, where the transition energy is equivalent to a laser wavelength $\lambda_0 = 155.07$ nm, could be excited by a laser with wavelength of 257.5 nm, as a result of the relativistic Doppler effect [25, 26]. However, the longitudinal modulation of the ions will make the ions oscillate inside the bucket of the RF-buncher. In the simplified model, the simulation contains only longitudinal dynamics, assuming that the transverse motion is unaffected by the laser. We assume that the laser is always switched on while the ions circulate at the CSRe.

For laser cooling with the CW laser, the spectral linewidth of the laser is very narrow (< 10 MHz). As a result, the momentum acceptance of the cooling force is also correspondingly narrow ($\sim 5\times10^{-8}$). To increase the momentum acceptance of the cooling force, it is necessary to scan the frequency of the RF-buncher or laser [14, 27].

4 Simulation results and discussion

Two possible schemes for laser cooling of RF-bunched ion beams are shown in Fig. 2. In the first scheme, the RF-buncher frequency is scanned while the CW laser frequency is fixed, resulting in the ions inside the bucket being accelerated to resonantly interact with the laser and then cooled down. In the second scheme, the laser frequency is scanned while the RF-buncher frequency is fixed, which means the ions will be continually pushed into the center of the bucket and then cooled down. All the main simulation parameters are the same as the experimental parameters in the multi-particle tracking simulation. Usually, the laser cooling is carried out after electron cooling. For all of the simulation results presented below, the initial momentum spread of the ion beams was set to $\sim1.6\times10^{-5}$ for 3 sigma, which was achieved by the beam parameters after the electron cooling experiment, and 10000 particles with Gaussian distributions were stored in every bunch.



scheme 1:
fixed laser frequency
scanned RF-buncher frequency

scheme 2:
scanned laser frequency
fixed RF-buncher frequency

Fig. 2. (color online) Two schemes of laser cooling of relativistic ion beams by a CW laser and a RF-buncher.

4.1 Laser cooling with scanned RF-buncher and fixed laser frequency

Figure 3 schematically illustrates the principle of laser cooling of RF-bunched ion beams for the first scheme as shown in Fig. 2(a), scanning the RF-buncher frequency while the laser frequency is fixed. The phase space trajectory of a single C^{3+}particle is shown. Each point represents the phase and energy deviations of the ion in the bucket of the RF-buncher. Figure 4 shows the simulated longitudinal momentum spread as a function of turn numbers for various scanning speeds of the RF-buncher frequency. The best scanning speed of the RF-buncher frequency is 20 Hz/s, and a minimum momentum spread of approximately 5×10^{-9} can be reached if heating effects such as the intra-beam scattering effect and the space charge effect are not taken into account. However, the ion beams could be heated by the laser scattering force when the RF-buncher frequency is scanned excessively. Figure 5 shows the particle distribution in longitudinal phase space at 0, 2×10^6 and 4.4×10^6 turns. By detuning the bunching frequency relative to the laser, the momentum acceptance range of the laser could be increased to cover the initial momentum spread of the ion beams.

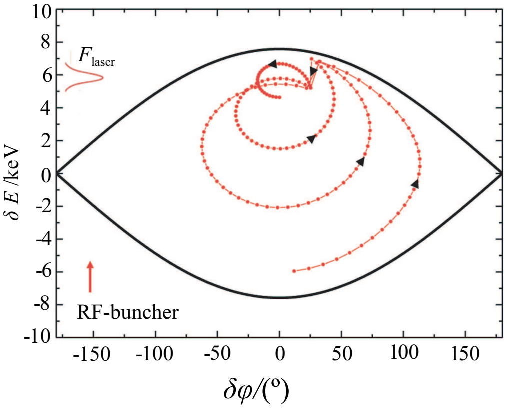


Fig. 3. (color online) Simulation of longitudinal dynamics of a laser-cooled single C^{3+} particle in a bunch. Every 2000th turn is indicated by a dot. For the purposes of illustration, the actual cooling rate has been exaggerated by a factor of 1000.

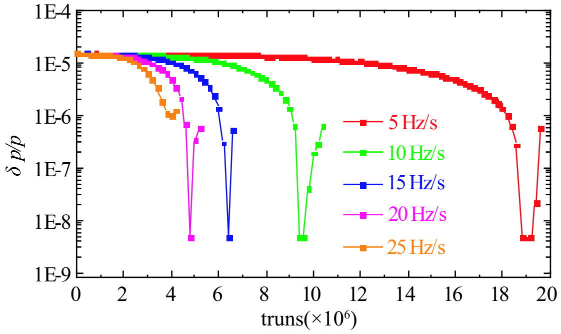


Fig. 4. (color online) Momentum spread reduction versus the turn number during laser cooling. The laser frequency is fixed. Five scanning speeds of the RF-buncher frequency are simulated.

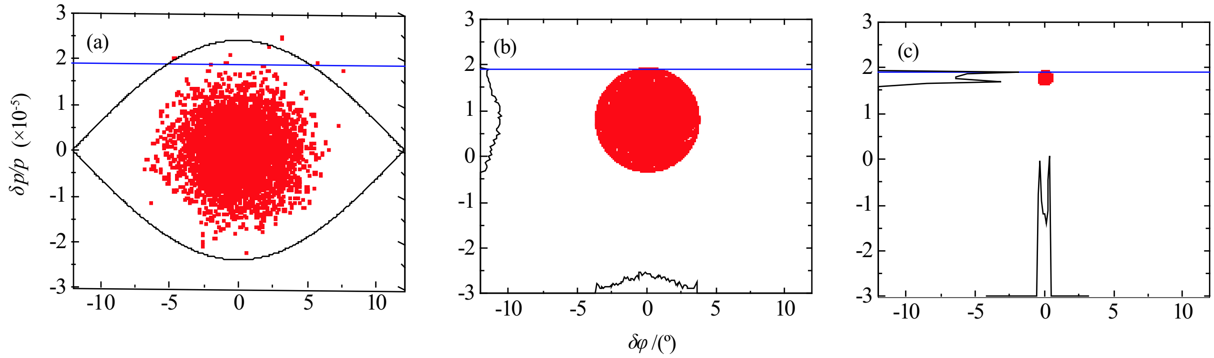


Fig. 5. (color online) Longitudinal phase space distribution of C^{3+} during laser cooling at different turns: (a) initial distribution, (b) 2×10^6 turns, and (c) 4.4×10^6 turns.

4.2 Laser cooling with scanned laser and fixed RF-buncher frequency

For the laser cooling scheme shown in Fig. 2 (b), the simulated cooling process of a single C^{3+} particle is shown schematically in Fig. 6. The laser frequency is scanned from the separatrix to the center of the bucket. As a result, ion beams with a large momentum spread can be cooled down. Various scanning speeds of the laser frequency were simulated and the results are shown in Fig. 7. A scanning speed of 0.25 GHz/s for the laser is the optimum condition. However, the ion beams will be heated if the laser frequency scans over the center of the bucket. Figure 8 shows the particle distribution in longitudinal phase space at 0, 1×10^7 and 1.9×10^7turns.

4.3 Heating effects for laser cooling at the CSRe

In the above simulation results of laser cooling, it is also shown that the laser cooling can obtain much higher cooling rates and very cold ion beams. Therefore, the

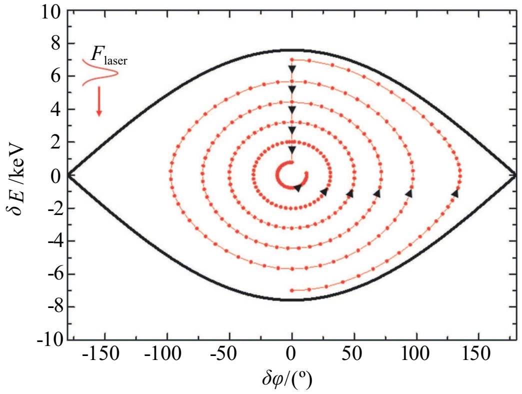


Fig. 6. (color online) Simulation of the cooling process for a single C^{3+} particle while the laser frequency is scanned from the separatrix to the center of the bucket. Every 2000th turn is indicated by a dot. For the purposes of illustration, the actual cooling rate has been exaggerated by a factor of 1000.

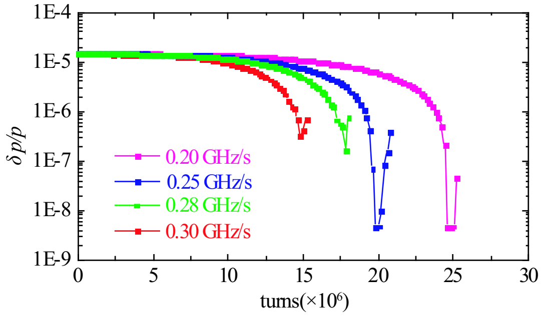


Fig. 7. (color online) Momentum spread reduction versus turn number. Four speeds for scanning the laser frequency were simulated.

heating effects will be obvious compared with stochastic cooling and electron cooling. For laser-cooled ion beams, the space charge effect and the IBS effect become important. The space charge effect is complicated in the storage ring. It can locally change the distribution of particles in phase space, but it will only become very important for laser cooling experiments when the density of the ions becomes high. In the simulation, we do not have large numbers of ions stored inside the CSRe, and the density of ions inside the bucket is not so high. In addition, C^{3+} ions have low charge state and the space charge effect can be carefully ignored in this simulation. The IBS is also complicated in the storage ring. It is the key factor which determines the final momentum spread of the laser-cooled ion beams. In the paper, a standard and simplified IBS formula can be used to calculate the IBS growth rate [28]. It has confirmed in the electron cooling experiment and can be written as:

$$\tau^{-1}=\frac{1}{\sigma_p^2}\frac{\mathrm{d}\sigma_p^2}{\mathrm{d}t}=\frac{r^2cN\Lambda}{8\beta^3\gamma^3\varepsilon_x^{3/2}\left\langle\beta_\perp^{1/2}\right\rangle\sqrt{\pi/2}\sigma_s\sigma_p^2}, \qquad (3)$$

where r is the radius of the carbon atom, c is the speed of light in vacuum, N is the number of particles, Λ is the Coulomb logarithm for IBS ($\Lambda\approx 10-20$), β and γ are the relativistic factors, ε_x is the transverse emittance, which is a constant in the simulation, $\beta_\perp$ is the average envelope, σ_s is the bunch length, and σ_p is the momentum deviation. For the first laser cooling scheme, the IBS effect is simulated while the ion beams are cooled by the CW laser, and the results are shown in Fig. 9. It shows that the IBS growth increases obviously when the momentum spread is less than 5×10^{-6}. The final momentum spread is 2.35×10^{-6}, which is a balanced state between the laser cooling and the IBS heating effect.

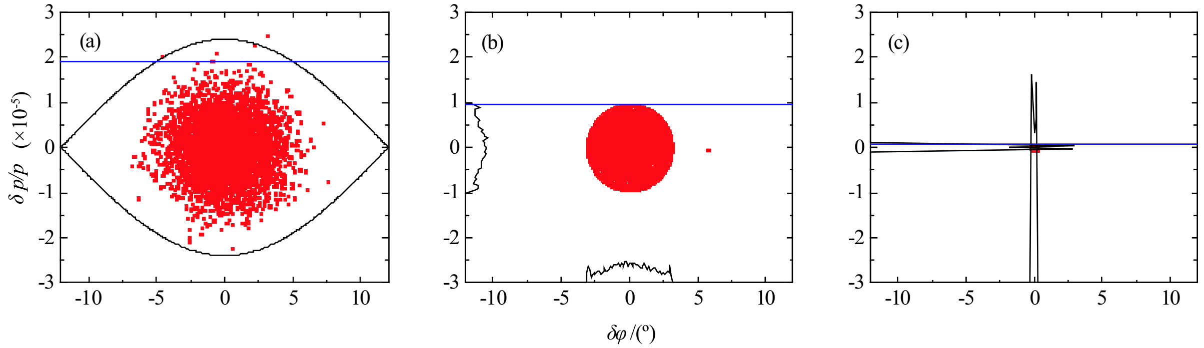


Fig. 8. (color online) Longitudinal phase space distribution of C^{3+} during laser cooling at: (a) the initial distribution, (b) 1×10^7 turns, and (c) 1.9×10^7 turns.

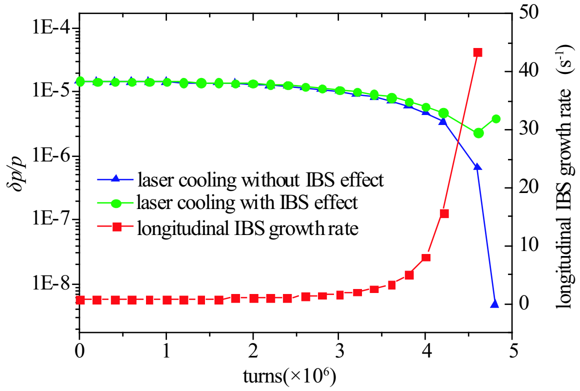


Fig. 9. (color online) Laser cooling with IBS effect. The red curve is the longitudinal IBS growth versus the turn numbers. The blue curve is the momentum spread reduction with the turn numbers without the IBS effect. The green curve is the momentum spread reduction with the turn numbers with the IBS effect.

4.4 Schottky noise signal of the RF-bunched and laser-cooled ion beams

Longitudinal Schottky signal measurements are a widely used tool for the determination of longitudinal dynamical properties of ion beams at storage rings, including momentum spread and the synchrotron frequency. In order to compare with the experimental results, a simulation to produce the Schottky noise spectra of the longitudinally modulated and laser-cooled ion beams was performed. For the RF-bunched ion beams, every individual particle executes oscillations at the synchrotron frequency $f_s=\Omega_s/2\pi$. It can be written as

$$f_s=\frac{f_r}{\beta}\sqrt{\frac{qeh\eta U_b}{2\pi\gamma mc^2}}, \tag{4}$$

which depends on the revolution frequency f_r, the beam velocity βc, the ion charge qe, the momentum slip factor η, the harmonic number h, the effective RF-bunching voltage U_b, and the relativistic Lorentz factor γ, while m is the mass of the ion and c is the speed of light in vacuum. The time of passage of the particle in front of the Schottky pick-up is modulated according to [29]:

$$\tau_i(t)=\hat{\tau}_i\sin(\Omega_s t+\varphi_i), \tag{5}$$

where $\tau_i(t)$ is the time difference with respect to the synchronous particle (frequency f_0) and $\hat{\tau}_i$ is the amplitude of the synchrotron oscillation. In the time domain, the beam current is obtained from the modulated time of passage. It can be written as:

$$i_i(t)=eqf_r+2eqf_r\mathrm{Re}\left\{\sum_{n=1}^{+\infty}\exp jn\omega_r(t+\hat{\tau}_i\sin(\Omega_s t+\varphi_i))\right\}. \tag{6}$$

Using the relation:

$$\exp[j(z\sin\theta)]=\sum_{p=-\infty}^{+\infty}J_p(z)\mathrm{e}^{jp\theta},$$

where J_p is the Bessel function of order p, one can expand the n^{th} harmonic in Eq. (6) and obtain:

$$i_n=2eqf_r\mathrm{Re}\left\{\sum_{p=-\infty}^{+\infty}J_p n\omega_r\hat{\tau}_i\mathrm{e}^{j(n\omega_r t+p\Omega_s t+p\varphi_i)}\right\}. \tag{7}$$

Each revolution frequency line (nf_r) now splits into an infinity of synchrotron satellites, spaced by f_s, the amplitudes of which are proportional to the Bessel function of argument $n\omega_r\hat{\tau}_i$ [29]

For the RF-bunched C^{3+}ion beams with the kinetic energy of 122 MeV/u, every individual particle executes synchrotron oscillations at the CSRe. The phase value of the individual ions is φ_i randomly distributed synchrotron phases from 0 to the total bunch length τ_m. They are collected and are equivalent to the time of passage of the ions at the position of the Schottky-noise pick-up. It can be written as

$$\varphi_i(N)=2\pi N+\varphi_i, \tag{8}$$

where i is the particle number, N is the turn number, $\varphi_i(N)$ is the phase of the ith particle at the Nth turn, and φ_i is the phase of the ith particle at every turn. The number of particles at each sampling is counted. The Schottky noise spectrum of the RF-bunched and laser-cooled ion beams in the frequency domain can be obtained from the time domain by a fast Fourier transform (FFT). The simulation result is shown in Fig. 10. It shows sharp and pronounced peaks at a spacing determined by the synchrotron oscillation frequency of the ions in the bucket. The synchrotron frequency of the ions inside the bucket can be extracted from the frequency between every two adjacent peaks in the Schottky spectrum. In Fig. 10, the abscissa is the frequency deviation relative to the revolution frequency, the central peak represents the revolution frequency, and the frequency between every two adjacent peaks represents the synchrotron frequency, which is about 326 Hz. This is in perfect agreement with the experimental result [22]. The momentum spread of the RF-bunched ion beams is then given by the synchrotron frequency spread as

$$\frac{\delta p}{p}=\frac{1}{\eta}\frac{\delta f}{hf_r}, \tag{9}$$

which is related to the revolution frequency f_r.

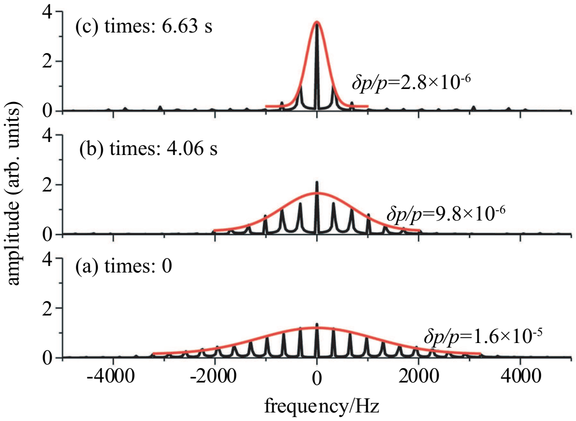


Fig. 10. (color online) Simulation results of the Schottky noise spectrum of RF-bunched and laser-cooled $^{12}C^{3+}$ion beams at 0 s, 4.06 s and 6.63 s after injection. The scanning speed of laser frequency is 0.25 GHz/s. The momentum spread is 1.6×10^{-5}, 9.8×10^{-6} and 2.8×10^{-6} respectively.

5 Conclusions

The longitudinal dynamics of laser cooling of RF-bunched ion beams has been simulated by a multi-particle tracking method. The simulation results show that ion beams with a large momentum spread could be cooled by the combination of a CW laser and a RF-buncher. The optimized parameters for scanning the RF-buncher frequency or laser frequency have been obtained. The IBS effect was estimated for the CSRe, finding that its growth rate increases greatly when the ion beams are highly cooled, and that it determines the final momentum spread of the laser-cooled ion beams. In addition, the Schottky noise spectra of the longitudinal modulated ion beams was simulated, and the momentum spread of laser-cooled ion beams and the synchrotron frequency of the ions inside the bucket could be deduced. For successful laser cooling experiments at the CSRe, the frequency of the laser must be appropriate to resonantly interact with the ion beams. The scanning speeds of the RF-buncher frequency and the laser frequency must be suitable. The laser must overlap the ion beams. The combination of simulation and experimental results of laser cooling of relativistic ion beams at the CSRe will provide a novel method to investigate the longitudinal dynamics of ultra-cold ion beams. It should be noted that the investigations of laser cooling of heavy ion beams at the CSRe are directly relevant to laser cooling and precision laser spectroscopy of highly charged and relativistic heavy ions at future large facilities such as the High Intensity heavy ion Accelerator Facility (HIAF) in China. At this facility, such experiments could even open up a new field for atomic physics and nuclear physics.

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