Corrections to R_D and R_{D^*} in the BLMSSM *

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Abstract: The deviation of the measurement of R_D (R_{D^*}) from the Standard Model (SM) expectation is 2.3σ (3.1σ). R_D (R_{D^*}) is the ratio of the branching fraction of $\overline{B}\to D\tau\overline{\nu}_\tau$ ($\overline{B}\to D^*\tau\overline{\nu}_\tau$) to that of $\overline{B}\to Dl\overline{\nu}_l$ ($\overline{B}\to D^*l\overline{\nu}_l$), where l=e or μ. This anomaly may imply the existence of new physics (NP). In this paper, we restudy this problem in the supersymmetric extension of the Standard Model with local gauged baryon and lepton numbers (BLMSSM), and give one-loop corrections to R_D (R_{D^*}).

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1 Introduction

The Standard Model (SM) is the most successful particle physics model to date. It gives accurate predictions for a significant number of experiments. However, for some experiments, it cannot give a good explanation. In the last few years, the experimental measurements of $R_{D^{(*)}}$ (the ratio of the branching fraction of $\overline{B}\to D\tau\overline{\nu}_\tau$ ($\overline{B}\to D^*\tau\overline{\nu}_\tau$) to that of $\overline{B}\to Dl\overline{\nu}_l$ ($\overline{B}\to D^*l\overline{\nu}_l$), where l=e or μ) show deviations from the SM theoretical predictions – these measurements are larger than SM expectations. Therefore, in order to explain these anomalies, it is necessary for us to try some new physics (NP) models.

The SM expectations for $R_{D^{(*)}}$ are: $R_{D_{\rm SM}}=0.299\pm 0.011$ in Ref. [1], $R_{D_{\rm SM}}=0.299\pm0.003$ in Ref. [2], $R_{D_{\rm SM}}=0.300\pm0.008$ in Ref. [3], $R_{D_{\rm SM}}=0.300\pm0.011$ in Ref. [4], $R_{D_{\rm SM}}=0.299\pm0.003$ in Ref. [5], $R_{D^*{}_{\rm SM}}=0.254\pm0.004$ in Ref. [4], $R_{D^*{}_{\rm SM}}=0.257\pm0.003$ in Ref. [5] and $R_{D^*{}_{\rm SM}}=0.252\pm0.003$ in Ref. [6]. The relevant experimental results for $R_{D^{(*)}}$ are listed in Table 1.

R_D=0.407±0.039±0.024 and R_{D^*}=0.304±0.013±0.007 exceed the SM predictions by 2.3σ and 3.1σ respectively. These anomalies have caused physicists to seek a variety of ways to explain the experimental data [15–35]. Most physicists tend to seek the solutions in NP models. So, various NP models have been used, such as charged Higgs [30–32] and lepton flavor violation [33–35]. The supersymmetric extension of the SM is a popular choice in various NP models. In fact, theorists have been fond of the minimal supersymmetric model (MSSM) for a long time. However, baryon number (B) should be broken because of the matter-antimatter asymmetry in the Universe. The neutrino oscillation experiments imply that neutrinos have tiny masses, therefore lepton number (L) also needs to be broken. A minimal supersymmetric extension of the SM with local gauged B and L (BLMSSM) [36, 37] is more promising. Thus, we try to deal with the anomalies of $R_{D^{(*)}}$ in the BLMSSM.

In our work, we use effective field theory to do the theoretical calculation. The effective Lagrangian is described by the four fermion operators and the corresponding Wilson coefficients (WCs). NP contributions with non-zero WCs are possible solutions to the $R_{D^{(*)}}$ anomalies [38]. After considering all the 10 independent 6-dimensional operators and calculating the values of the corresponding WCs at one-loop level, we obtain the theoretical values of $R_{D^{(*)}}$ in the BLMSSM.

This paper is organised as follows. In Section 2, we introduce some content of the BLMSSM. In Section 3, we give the mass matrices of the BLMSSM particles that we use. In Section 4, we write down the needed couplings. In Section 5, we provide the relevant formulae, including

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observables $R_{D^{(*)}}$ and the effective Lagrangian with all the four fermion operators. In Section 6, we show the one-loop Feynman diagrams that can correct $R_{D^{(*)}}$. At the same time, NP contributions of some diagrams are given by WCs. In Section 7, we present our numerical results. Finally, we summarise our findings in Section 8. Some integral formulae are shown in the Appendix.

Table 1. The measurements of $R_{D^{(*)}}$.

observable	experiment	measured value
R_D	2012 BaBar	0.440±0.058±0.042 [7, 8]
	2015 Belle	0.375±0.064±0.026 [9]
	2017 HFAG average	0.407±0.039±0.024 [10]
R_{D^*}	2012 BaBar	0.332±0.024±0.018 [7, 8]
	2015 Belle	0.293±0.038±0.015 [9]
	2015 LHCb	0.336±0.027±0.030 [11]
	2016 Belle	0.302±0.030±0.011 [12]
	2017 Belle	$0.270\pm0.035^{+0.028}_{-0.025}$ [13]
	2017 LHCb	0.291±0.019±0.026±0.013 [14]
	2017 HFAG average	0.304±0.013±0.007 [10]

2 Some content of the BLMSSM

As an extension of the MSSM, the BLMSSM includes many new fields [39, 40]. The exotic quarks $(\hat{Q}_4,\hat{U}_4^c,\hat{D}_4^c,\hat{Q}_5^c,\hat{U}_5,\hat{D}_5)$ are used to deal with the B anomaly. The exotic leptons $(\hat{L}_4,\hat{E}_4^c,\hat{N}_4^c,\hat{L}_5^c,\hat{E}_5,\hat{N}_5)$ are used to cancel the L anomaly. The exotic Higgs superfields $\hat{\Phi}_B,\hat{\varphi}_B$ are introduced to break baryon number spontaneously with non-zero vacuum expectation values (VEVs). The exotic Higgs superfields $\hat{\Phi}_L,\hat{\varphi}_L$ are introduced to break lepton number spontaneously with non-zero VEVs. The model introduces the right-handed neutrinos N_R^c, so we can obtain tiny masses of neutrinos through the see-saw mechanism. The model also includes the superfields $\hat{X}$ to make the exotic quarks unstable.

The superpotential of the BLMSSM is [41]:

$$\begin{aligned}
\mathcal{W}_{\rm BLMSSM} &= \mathcal{W}_{\rm MSSM}+\mathcal{W}_B+\mathcal{W}_L+\mathcal{W}_X,\\
\mathcal{W}_B &= \lambda_Q\hat{Q}_4\hat{Q}_5^c\hat{\Phi}_B+\lambda_U\hat{U}_4^c\hat{U}_5\hat{\varphi}_B+\lambda_D\hat{D}_4^c\hat{D}_5\hat{\varphi}_B\\
&\quad+\mu_B\hat{\Phi}_B\hat{\varphi}_B+Y_{u_4}\hat{Q}_4\hat{H}_u\hat{U}_4^c+Y_{d_4}\hat{Q}_4\hat{H}_d\hat{D}_4^c\\
&\quad+Y_{u_5}\hat{Q}_5^c\hat{H}_d\hat{U}_5+Y_{d_5}\hat{Q}_5^c\hat{H}_u\hat{D}_5,\\
\mathcal{W}_L &= Y_{e_4}\hat{L}_4\hat{H}_d\hat{E}_4^c+Y_{\nu_4}\hat{L}_4\hat{H}_u\hat{N}_4^c+Y_{e_5}\hat{L}_5^c\hat{H}_u\hat{E}_5\\
&\quad+Y_{\nu_5}\hat{L}_5^c\hat{H}_d\hat{N}_5+Y_\nu\hat{L}\hat{H}_u\hat{N}^c+\lambda_{N^c}\hat{N}^c\hat{N}^c\hat{\varphi}_L\\
&\quad+\mu_L\hat{\Phi}_L\hat{\varphi}_L,\\
\mathcal{W}_X &= \lambda_1\hat{Q}\hat{Q}_5^c\hat{X}+\lambda_2\hat{U}^c\hat{U}_5\hat{X}'+\lambda_3\hat{D}^c\hat{D}_5\hat{X}'+\mu_X\hat{X}\hat{X}',
\end{aligned}\tag{1}$$

where $\mathcal{W}_{\rm MSSM}$ is the superpotential of the MSSM.

The soft breaking terms $\mathcal{L}_{\rm soft}$ of the BLMSSM can be written in the following form [36, 37, 41]:

$$\begin{aligned}
\mathcal{L}_{\rm soft} &= \mathcal{L}_{\rm soft}^{\rm MSSM}-(m^2_{\tilde{\nu}^c})_{IJ}\tilde{N}_I^{c*}\tilde{N}_J^c-m^2_{\tilde{Q}_4}\tilde{Q}_4^\dagger\tilde{Q}_4-m^2_{\tilde{U}_4}\tilde{U}_4^{c*}\tilde{U}_4^c-m^2_{\tilde{D}_4}\tilde{D}_4^{c*}\tilde{D}_4^c\\
&\quad-m^2_{\tilde{Q}_5}\tilde{Q}_5^{c\dagger}\tilde{Q}_5^c-m^2_{\tilde{U}_5}\tilde{U}_5^*\tilde{U}_5-m^2_{\tilde{D}_5}\tilde{D}_5^*\tilde{D}_5-m^2_{\tilde{L}_4}\tilde{L}_4^\dagger\tilde{L}_4-m^2_{\tilde{\nu}_4}\tilde{N}_4^{c*}\tilde{N}_4^c\\
&\quad-m^2_{\tilde{e}_4}\tilde{E}_4^{c*}\tilde{E}_4^c-m^2_{\tilde{L}_5}\tilde{L}_5^{c\dagger}\tilde{L}_5^c-m^2_{\tilde{\nu}_5}\tilde{N}_5^*\tilde{N}_5-m^2_{\tilde{e}_5}\tilde{E}_5^*\tilde{E}_5-m^2_{\Phi_B}\Phi_B^*\Phi_B\\
&\quad-m^2_{\varphi_B}\varphi_B^*\varphi_B-m^2_{\Phi_L}\Phi_L^*\Phi_L-m^2_{\varphi_L}\varphi_L^*\varphi_L-\Big(M_B\lambda_B\lambda_B+M_L\lambda_L\lambda_L+h.c.\Big)\\
&\quad+\Big\{A_{u_4}Y_{u_4}\tilde{Q}_4H_u\tilde{U}_4^c+A_{d_4}Y_{d_4}\tilde{Q}_4H_d\tilde{D}_4^c+A_{u_5}Y_{u_5}\tilde{Q}_5^cH_d\tilde{U}_5+A_{d_5}Y_{d_5}\tilde{Q}_5^cH_u\tilde{D}_5\\
&\quad+A_{BQ}\lambda_Q\tilde{Q}_4\tilde{Q}_5^c\Phi_B+A_{BU}\lambda_U\tilde{U}_4^c\tilde{U}_5\varphi_B+A_{BD}\lambda_D\tilde{D}_4^c\tilde{D}_5\varphi_B+B_B\mu_B\Phi_B\varphi_B+h.c.\Big\}\\
&\quad+\Big\{A_{e_4}Y_{e_4}\tilde{L}_4H_d\tilde{E}_4^c+A_{\nu_4}Y_{\nu_4}\tilde{L}_4H_u\tilde{N}_4^c+A_{e_5}Y_{e_5}\tilde{L}_5^cH_u\tilde{E}_5+A_{\nu_5}Y_{\nu_5}\tilde{L}_5^cH_d\tilde{N}_5\\
&\quad+A_\nu Y_\nu\tilde{L}H_u\tilde{N}^c+A_{\nu^c}\lambda_{\nu^c}\tilde{N}^c\tilde{N}^c\varphi_L+B_L\mu_L\Phi_L\varphi_L+h.c.\Big\}\\
&\quad+\Big\{A_1\lambda_1\tilde{Q}\tilde{Q}_5^cX+A_2\lambda_2\tilde{U}^c\tilde{U}_5X'+A_3\lambda_3\tilde{D}^c\tilde{D}_5X'+B_X\mu_XXX'+h.c.\Big\}.
\end{aligned}\tag{2}$$

The $SU(2)_L$ singlets $\Phi_L,\varphi_L,\Phi_B,\varphi_B$ and the $SU(2)_L$ doublets H_u,H_d are:

$$\begin{aligned}
&\Phi_L=\frac{1}{\sqrt{2}}\Big(v_L+\Phi_L^0+iP_L^0\Big),\quad \varphi_L=\frac{1}{\sqrt{2}}\Big(\overline{v}_L+\varphi_L^0+i\overline{P}_L^0\Big),\\
&\Phi_B=\frac{1}{\sqrt{2}}\Big(v_B+\Phi_B^0+iP_B^0\Big),\quad \varphi_B=\frac{1}{\sqrt{2}}\Big(\overline{v}_B+\varphi_B^0+i\overline{P}_B^0\Big),\\
&H_u=\begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}}\Big(v_u+H_u^0+iP_u^0\Big)\end{pmatrix},\\
&H_d=\begin{pmatrix} \frac{1}{\sqrt{2}}\Big(v_d+H_d^0+iP_d^0\Big) \\ H_d^-\end{pmatrix}.
\end{aligned}\tag{3}$$

The $SU(2)_L$ singlets Φ_L, φ_L, Φ_B, φ_B and the $SU(2)_L$ doublets H_u, H_d should obtain non-zero VEVs $v_L,\overline{v}_L,v_B,\overline{v}_B$ and v_u,v_d respectively. Therefore, the local gauge symmetry $SU(2)_L\otimes U(1)_Y\otimes U(1)_B\otimes U(1)_L$ breaks down to the electromagnetic symmetry $U(1)_e$.

3 Mass matrices for some BLMSSM particles

Lepneutralinos are made up of λ_L (the superpartner of the new lepton boson), and ψ_{Φ_L} and ψ_{φ_L} (the superpartners of the $SU(2)_L$ singlets Φ_L and φ_L). The mass

mixing matrix of lepneutralinos M_{LN} is shown in the basis $(i\lambda_L,\psi_{\Phi_L},\psi_{\varphi_L})$ [42–45]. $\chi^0_{L_i}$ $(i=1,2,3)$ are mass eigenstates of lepneutralinos. The masses of the three lepneutralinos are obtained from diagonalizing M_{LN} by Z_{N_L}:

$$\begin{aligned}&M_{LN}=\begin{pmatrix}2M_L & 2v_Lg_L & -2\bar{v}_Lg_L\\ 2v_Lg_L & 0 & -\mu_L\\ -2\bar{v}_Lg_L & -\mu_L & 0\end{pmatrix},\\&i\lambda_L=Z^{1i}_{N_L}k^0_{L_i},\ \ \psi_{\Phi_L}=Z^{2i}_{N_L}k^0_{L_i},\\&\psi_{\varphi_L}=Z^{3i}_{N_L}k^0_{L_i},\ \ \chi^0_{L_i}=\begin{pmatrix}k^0_{L_i}\\ \bar{k}^0_{L_i}\end{pmatrix}.\end{aligned}\tag{4}$$

The slepton mass squared matrix becomes

$$\begin{pmatrix}(\mathcal{M}^2_{\tilde{L}})_{LL} & (\mathcal{M}^2_{\tilde{L}})_{LR}\\ (\mathcal{M}^2_{\tilde{L}})^\dagger_{LR} & (\mathcal{M}^2_{\tilde{L}})_{RR}\end{pmatrix},\tag{5}$$

which is diagonalized by the matrix $Z_{\tilde{L}}$. $(\mathcal{M}^2_{\tilde{L}})_{LL}$, $(\mathcal{M}^2_{\tilde{L}})_{LR}$ and $(\mathcal{M}^2_{\tilde{L}})_{RR}$ are:

$$\begin{aligned}(\mathcal{M}^2_{\tilde{L}})_{LL} =\ & \frac{(g_1^2-g_2^2)(v_d^2-v_u^2)}{8}\delta_{IJ}+g_L^2(\bar{v}_L^2-v_L^2)\delta_{IJ}\\&+m^2_{l^I}\delta_{IJ}+(m^2_{\tilde{L}})_{IJ},\\(\mathcal{M}^2_{\tilde{L}})_{LR} =\ & \frac{\mu^* v_u}{\sqrt{2}}(Y_l)_{IJ}-\frac{v_u}{\sqrt{2}}(A'_l)_{IJ}+\frac{v_d}{\sqrt{2}}(A_l)_{IJ},\\(\mathcal{M}^2_{\tilde{L}})_{RR} =\ & \frac{g_1^2(v_u^2-v_d^2)}{4}\delta_{IJ}-g_L^2(\bar{v}_L^2-v_L^2)\delta_{IJ}\\&+m^2_{l^I}\delta_{IJ}+(m^2_{\tilde{R}})_{IJ}.\end{aligned}\tag{6}$$

The mass squared matrix of sneutrino $\mathcal{M}_{\tilde{n}}$ with $\tilde{n}^T=(\tilde{\nu},\tilde{N}^c)$ reads [46]

$$\begin{pmatrix}\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}^*_I\tilde{\nu}_J) & \mathcal{M}^2_{\tilde{n}}(\tilde{\nu}_I\tilde{N}^c_J)\\ (\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}_I\tilde{N}^c_J))^\dagger & \mathcal{M}^2_{\tilde{n}}(\tilde{N}^{c*}_I\tilde{N}^c_J)\end{pmatrix}.\tag{7}$$

$\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}^*_I\tilde{\nu}_J)$, $\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}_I\tilde{N}^c_J)$ and $\mathcal{M}^2_{\tilde{n}}(\tilde{N}^{c*}_I\tilde{N}^c_J)$ are:

$$\begin{aligned}\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}^*_I\tilde{\nu}_J) =\ & \frac{g_1^2+g_2^2}{8}(v_d^2-v_u^2)\delta_{IJ}+g_L^2(\overline{v}_L^2-v_L^2)\delta_{IJ}\\&+\frac{v_u^2}{2}(Y^\dagger_\nu Y_\nu)_{IJ}+(m^2_{\tilde{L}})_{IJ},\\\mathcal{M}^2_{\tilde{n}}(\tilde{\nu}_I\tilde{N}^c_J) =\ & \mu^*\frac{v_d}{\sqrt{2}}(Y_\nu)_{IJ}-v_u\overline{v}_L(Y^\dagger_\nu\lambda_{N^c})_{IJ}\\&+\frac{v_u}{\sqrt{2}}(A_N)_{IJ}(Y_\nu)_{IJ},\\\mathcal{M}^2_{\tilde{n}}(\tilde{N}^{c*}_I\tilde{N}^c_J) =\ & -g_L^2(\overline{v}_L^2-v_L^2)\delta_{IJ}+\frac{v_u^2}{2}(Y^\dagger_\nu Y_\nu)_{IJ}\\&+2\overline{v}_L^2(\lambda^\dagger_{N^c}\lambda_{N^c})_{IJ}+(m^2_{\tilde{N}^c})_{IJ}\\&+\mu_L\frac{v_L}{\sqrt{2}}(\lambda_{N^c})_{IJ}-\frac{\overline{v}_L}{\sqrt{2}}(A_{N^c})_{IJ}(\lambda_{N^c})_{IJ}.\end{aligned}\tag{8}$$

Then the masses of the sneutrinos are obtained by using the formula $Z^\dagger_{\tilde{\nu}}\mathcal{M}^2_{\tilde{n}}Z_{\tilde{\nu}}=\mathrm{diag}(m^2_{\tilde{\nu}^1},\ m^2_{\tilde{\nu}^2},\ m^2_{\tilde{\nu}^3},\ m^2_{\tilde{\nu}^4},\ m^2_{\tilde{\nu}^5},\ m^2_{\tilde{\nu}^6})$.

The up scalar quark mass squared matrix in the BLMSSM is given by

$$\begin{pmatrix}(\mathcal{M}^2_{\tilde{U}})_{LL} & (\mathcal{M}^2_{\tilde{U}})_{LR}\\ (\mathcal{M}^2_{\tilde{U}})^\dagger_{LR} & (\mathcal{M}^2_{\tilde{U}})_{RR}\end{pmatrix},\tag{9}$$

which is diagonalized by the matrix $Z_{\tilde{U}}$. $(\mathcal{M}^2_{\tilde{U}})_{LL}$, $(\mathcal{M}^2_{\tilde{U}})_{LR}$ and $(\mathcal{M}^2_{\tilde{U}})_{RR}$ are:

$$\begin{aligned}(M^2_{\tilde{U}})_{LL} =\ & -\frac{e^2(v_d^2-v_u^2)(1-4c_W^2)}{24s_W^2c_W^2}+\frac{v_u^2Y_u^2}{2}+(Km^2_{\tilde{Q}}K^\dagger)^T\\&+\frac{g_B^2}{6}(v_B^2-\bar{v}_B^2),\\(M^2_{\tilde{U}})_{RR} =\ & \frac{e^2(v_d^2-v_u^2)}{6c_W^2}+\frac{v_u^2Y_u^2}{2}+m^2_{\tilde{U}}-\frac{g_B^2}{6}(v_B^2-\bar{v}_B^2),\\(M^2_{\tilde{U}})_{LR} =\ & -\frac{1}{\sqrt{2}}\Big(v_d(A'_u+Y_u\mu^*)+v_uA_u\Big).\end{aligned}\tag{10}$$

The down scalar quark mass squared matrix in the BLMSSM is given by

$$\begin{pmatrix}(\mathcal{M}^2_{\tilde{D}})_{LL} & (\mathcal{M}^2_{\tilde{D}})_{LR}\\ (\mathcal{M}^2_{\tilde{D}})^\dagger_{LR} & (\mathcal{M}^2_{\tilde{D}})_{RR}\end{pmatrix},\tag{11}$$

which is diagonalized by the matrix $Z_{\tilde{D}}$. $(\mathcal{M}^2_{\tilde{U}})_{LL}$, $(\mathcal{M}^2_{\tilde{U}})_{LR}$ and $(\mathcal{M}^2_{\tilde{U}})_{RR}$ are:

$$\begin{aligned}(M^2_{\tilde{D}})_{LL} =\ & -\frac{e^2(v_d^2-v_u^2)(1+2c_W^2)}{24s_W^2c_W^2}+\frac{v_d^2Y_d^2}{2}+(m^2_{\tilde{Q}})^T\\&+\frac{g_B^2}{6}(v_B^2-\bar{v}_B^2),\\(M^2_{\tilde{D}})_{RR} =\ & -\frac{e^2(v_d^2-v_u^2)}{12c_W^2}+\frac{v_d^2Y_d^2}{2}+m^2_{\tilde{D}}-\frac{g_B^2}{6}(v_B^2-\bar{v}_B^2),\\(M^2_{\tilde{D}})_{LR} =\ & \frac{1}{\sqrt{2}}\Big(v_u(-A'_d+Y_d\mu^*)+v_dA_d\Big).\end{aligned}\tag{12}$$

In the basis $(\psi_{\nu^I_L},\psi_{N^{cI}_R})$, the neutrino mass mixing matrix is diagonalized by Z_ν [46]:

$$\begin{aligned}&Z^T_\nu\begin{pmatrix}0 & \frac{v_u}{\sqrt{2}}(Y_\nu)^{IJ}\\ \frac{v_u}{\sqrt{2}}(Y^T_\nu)^{IJ} & \frac{v_L}{\sqrt{2}}(\lambda_{N^c})^{IJ}\end{pmatrix}Z_\nu\\&=\mathrm{diag}(m_{\nu^\alpha}),\quad \alpha=1\ldots6.\\&\psi_{\nu^I_L}=Z^{I\alpha}_\nu k^0_{N_\alpha},\quad \psi_{N^{cI}_R}=Z^{(I+3)\alpha}_\nu k^0_{N_\alpha},\\&\nu^\alpha=\begin{pmatrix}k^0_{N_\alpha}\\ \bar{k}^0_{N_\alpha}\end{pmatrix}.\end{aligned}\tag{13}$$

ν^α denotes the mass eigenstates of the neutrino fields mixed by the left-handed and right-handed neutrinos. In this paper, we deal with the neutrinos by an approximation, $Z_\nu\approx1$, so the theoretical values at tree level are consistent with those in the SM.

4 Necessary couplings

In the BLMSSM, due to the superfields $\tilde{N}^c$, we deduce the corrections to the couplings in the MSSM. The couplings for W-l-ν and W-$\tilde{L}$-$\tilde{\nu}$ read

$$\mathcal{L}_{Wl\nu}=-\frac{e}{\sqrt{2}s_W}W_\mu^+\sum_{I=1}^{3}\sum_{\alpha=1}^{6}Z_\nu^{I\alpha*}\bar{\nu}^\alpha\gamma^\mu P_L l^I, \tag{14}$$

$$\mathcal{L}_{W\tilde{L}\tilde{\nu}}=-\frac{ie}{\sqrt{2}s_W}W_\mu^-\sum_{I=1}^{3}\sum_{i,\alpha=1}^{6}(Z_{\tilde{L}}^{Ii}Z_{\tilde{\nu}}^{I\alpha})(\tilde{L}_i^+(\overrightarrow{\partial^\mu}-\overleftarrow{\partial^\mu})\tilde{\nu}^\alpha). \tag{15}$$

From the interactions of gauge and matter multiplets $ig\sqrt{2}T_{ij}^a(\lambda^a\psi_jA_i^*-\bar{\lambda}^a\bar{\psi}_iA_j)$, the l-χ_L^0-$\tilde{L}$ coupling is deduced here:

$$\mathcal{L}_{l\chi_L^0\tilde{L}}=\sqrt{2}g_L\bar{\chi}_{L_j}^0\left(Z_{N_L}^{1j}Z_{\tilde{L}}^{Ii}P_L-Z_{N_L}^{1j*}Z_{\tilde{L}}^{(I+3)i}P_R\right)l^I\tilde{L}_i^+ +\text{h.c.} \tag{16}$$

The ν-χ_L^0-$\tilde{\nu}$ coupling is

$$\begin{aligned}\mathcal{L}_{\nu\chi_L^0\tilde{\nu}} =& \Big[\sqrt{2}g_LZ_{N_L}^{1i}Z_\nu^{I\alpha}Z_{\tilde{\nu}}^{Jj*}\delta^{IJ}\\ &-(Z_{N_L}^{3i}(\lambda_{N^c}^{IJ}+\lambda_{N^c}^{JI})+\sqrt{2}g_LZ_{N_L}^{1i}\delta^{IJ})\\ &\times Z_\nu^{(I+3)\alpha}Z_{\tilde{\nu}}^{(J+3)j*}\Big]\bar{\chi}_{L_i}^0P_L\nu^\alpha\tilde{\nu}^{j*}+\text{h.c.}\end{aligned} \tag{17}$$

We also obtain the $\chi^\pm$-l-$\tilde{\nu}$ coupling and the $\chi^\pm$-$\tilde{L}$-ν coupling:

$$\begin{aligned}\mathcal{L}_{\chi^\pm l\tilde{\nu}} =& -\sum_{I,J=1}^{3}\sum_{\alpha=1}^{6}\bar{\chi}_j^-\Bigg(Y_l^{IJ}Z_-^{2j*}(Z_{\tilde{\nu}}^{I\alpha})^*P_R\\ &+\left[\frac{e}{s_W}Z_+^{1j}(Z_{\tilde{\nu}}^{I\alpha})^*+Y_\nu^{IJ}Z_+^{2j}(Z_{\tilde{\nu}}^{(I+3)\alpha})^*\right]P_L\Bigg)l^J\tilde{\nu}^{\alpha*}\\ &+\text{h.c.}\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\chi^\pm\tilde{L}\nu} =& -\sum_{I,J=1}^{3}\sum_{i=1}^{2}\sum_{j,\alpha=1}^{6}\bar{\chi}_i^+\Bigg(Y_\nu^{IJ}Z_+^{2i*}Z_{\tilde{L}}^{Ij}Z_\nu^{(J+3)\alpha*}P_R\\ &+\left[\frac{e}{s_W}Z_-^{1i}Z_{\tilde{L}}^{Ij}+Y_l^{IJ}Z_-^{2i}Z_{\tilde{L}}^{(I+3)j}\right]Z_\nu^{J\alpha}P_L\Bigg)\nu^\alpha\tilde{L}_j^+\\ &+\text{h.c.}\end{aligned} \tag{18}$$

The χ^0-$\tilde{\nu}$-ν coupling in the BLMSSM becomes

$$\begin{aligned}\mathcal{L}_{\chi^0\tilde{\nu}\nu} =& \Bigg[Z_\nu^{I\alpha}Z_{\tilde{\nu}}^{Jj*}\frac{e}{\sqrt{2}s_Wc_W}(Z_N^{1i}s_W-Z_N^{2i}c_W)\\ &+\frac{Y_\nu^{IJ}}{\sqrt{2}}Z_N^{4i}(Z_\nu^{I\alpha}Z_{\tilde{\nu}}^{(J+3)j*}+Z_\nu^{(I+3)\alpha}Z_{\tilde{\nu}}^{Jj*})\Bigg]\\ &\times\bar{\chi}_i^0P_L\nu^\alpha\tilde{\nu}^{j*}+\text{h.c.}\end{aligned} \tag{19}$$

All the other couplings used are consistent with the MSSM.

5 Formulae

5.1 Observables

The observable $R_{D^{(*)}}$ is defined as

$$R_{D^{(*)}}=\frac{\mathcal{B}_\tau^{D^{(*)}}}{\mathcal{B}_l^{D^{(*)}}}=\frac{\mathcal{B}(\overline{B}\to D^{(*)}\tau\bar{\nu}_\tau)}{\mathcal{B}(\overline{B}\to D^{(*)}l\bar{\nu}_l)}. \tag{20}$$

$\mathcal{B}_\ell^{D^{(*)}}$, the branching fraction, is given by [4]:

$$\mathcal{B}_\ell^{D^{(*)}}=\int\mathcal{N}|p_{D^{(*)}}|(2a_\ell^{D^{(*)}}+\frac{2}{3}c_\ell^{D^{(*)}})\mathrm{d}q^2, \tag{21}$$

where $l=e$ or μ, and ℓ denotes any lepton (e,μ or τ). q^2 is the invariant mass squared of the lepton-neutrino system, whose integral interval is $[m_\ell^2,(M_B-M_{D^{(*)}})^2]$. $\mathcal{N}$, the normalisation factor, is given by

$$\mathcal{N}=\frac{\tau_BG_F^2|V_{cb}|^2q^2}{256\pi^3M_B^2}(1-\frac{m_\ell^2}{q^2})^2. \tag{22}$$

Here τ_B is the lifetime of the $B-$meson. $G_F=\sqrt{2}e^2/8m_W^2s_W^2$ is the Fermi coupling constant. $|p_{D^{(*)}}|$, the absolute value of the $D^{(*)}-$meson momentum, is given by

$$|p_{D^{(*)}}|=\frac{\sqrt{(M_B^2)^2+(M_{D^{(*)}}^2)^2+(q^2)^2-2(M_B^2M_{D^{(*)}}^2+M_{D^{(*)}}^2q^2+q^2M_B^2)}}{2M_B}. \tag{23}$$

The expressions for a_ℓ^D and c_ℓ^D are [4]:

$$\begin{aligned}a_\ell^D =& 8\Bigg\{\frac{M_B^2|p_D|^2}{q^2}(|\mathcal{C}_{VL}^\ell|^2+|\mathcal{C}_{VR}^\ell|^2)\mathbf{F}_+^2+\frac{(M_B^2-M_D^2)^2}{4(m_b-m_c)^2}(|\mathcal{C}_{SL}^\ell|^2+|\mathcal{C}_{SR}^\ell|^2)\mathbf{F}_0^2\\ &+m_\ell\left[\frac{(M_B^2-M_D^2)^2}{2q^2(m_b-m_c)}(\mathcal{R}(\mathcal{C}_{SL}^\ell\mathcal{C}_{VL}^{\ell*})+\mathcal{R}(\mathcal{C}_{SR}^\ell\mathcal{C}_{VR}^{\ell*}))\mathbf{F}_0^2+\frac{4M_B^2|p_D|^2}{q^2(M_B+M_D)}(\mathcal{R}(\mathcal{C}_{TL}^\ell\mathcal{C}_{VL}^{\ell*})+\mathcal{R}(\mathcal{C}_{TR}^\ell\mathcal{C}_{VR}^{\ell*}))\mathbf{F}_+\mathbf{F}_T\right]\\ &+m_\ell^2\left[\frac{(M_B^2-M_D^2)^2}{4q^4}(|\mathcal{C}_{VL}^\ell|^2+|\mathcal{C}_{VR}^\ell|^2)\mathbf{F}_0^2+\frac{4|p_D|^2M_B^2}{q^2(M_B+M_D)^2}(|\mathcal{C}_{TL}^\ell|^2+|\mathcal{C}_{TR}^\ell|^2)\mathbf{F}_T^2\right]\Bigg\},\end{aligned} \tag{24}$$

$$c_\ell^D = 8\Big\{\frac{4M_B^2|p_D|^2}{(M_B+M_D)^2}(|\mathcal{C}_{TL}^\ell|^2+|\mathcal{C}_{TR}^\ell|^2)\mathbf{F}_T^2-\frac{M_B^2|p_D|^2}{q^2}(|\mathcal{C}_{VL}^\ell|^2+|\mathcal{C}_{VR}^\ell|^2)\mathbf{F}_+^2 +m_\ell^2\Big[\frac{|p_D|^2M_B^2}{q^4}(|\mathcal{C}_{VL}^\ell|^2+|\mathcal{C}_{VR}^\ell|^2)\mathbf{F}_+^2-\frac{4|p_D|^2M_B^2}{(M_B+M_D)^2q^2}(|\mathcal{C}_{TL}^\ell|^2+|\mathcal{C}_{TR}^\ell|^2)\mathbf{F}_T^2\Big]\Big\}. \quad (25)$$

The full expressions for $a_\ell^{D^*}$, $c_\ell^{D^*}$ and all form factors ($\mathbf{F}_T(q^2)$, $\mathbf{F}_+(q^2)$ and $\mathbf{F}_0(q^2)$, etc) are given in Refs. [4, 47].

5.2 Effective Lagrangian

We use effective field theory to calculate the theoretical values. The effective Lagrangian for the $b\to c\ell\bar{\nu}^\ell$ process is

$$\mathcal{L}_{\rm eff}^{b\to c\ell\bar{\nu}\ell} = \sqrt{2}G_FV_{cb}(\mathcal{C}_{VL}^\ell\mathcal{O}_{VL}^\ell+\mathcal{C}_{VR}^\ell\mathcal{O}_{VR}^\ell+\mathcal{C}_{AL}^\ell\mathcal{O}_{AL}^\ell +\mathcal{C}_{AR}^\ell\mathcal{O}_{AR}^\ell+\mathcal{C}_{SL}^\ell\mathcal{O}_{SL}^\ell+\mathcal{C}_{SR}^\ell\mathcal{O}_{SR}^\ell+\mathcal{C}_{PL}^\ell\mathcal{O}_{PL}^\ell +\mathcal{C}_{PR}^\ell\mathcal{O}_{PR}^\ell+\mathcal{C}_{TL}^\ell\mathcal{O}_{TL}^\ell+\mathcal{C}_{TR}^\ell\mathcal{O}_{TR}^\ell), \quad (26)$$

where V_{cb}=0.04, and the full set of operators is [48]:

$$\begin{aligned}
&\mathcal{O}_{VL}^\ell=[\bar{c}\gamma^\mu b][\bar{\ell}\gamma_\mu P_L\nu^\ell], && \mathcal{O}_{VR}^\ell=[\bar{c}\gamma^\mu b][\bar{\ell}\gamma_\mu P_R\nu^\ell],\\
&\mathcal{O}_{AL}^\ell=[\bar{c}\gamma^\mu\gamma_5 b][\bar{\ell}\gamma_\mu P_L\nu^\ell], && \mathcal{O}_{AR}^\ell=[\bar{c}\gamma^\mu\gamma_5 b][\bar{\ell}\gamma_\mu P_R\nu^\ell],\\
&\mathcal{O}_{SL}^\ell=[\bar{c}b][\bar{\ell}P_L\nu^\ell], && \mathcal{O}_{SR}^\ell=[\bar{c}b][\bar{\ell}P_R\nu^\ell],\\
&\mathcal{O}_{PL}^\ell=[\bar{c}\gamma_5 b][\bar{\ell}P_L\nu^\ell], && \mathcal{O}_{PR}^\ell=[\bar{c}\gamma_5 b][\bar{\ell}P_R\nu^\ell],\\
&\mathcal{O}_{TL}^\ell=[\bar{c}\sigma^{\mu\nu} b][\bar{\ell}\sigma_{\mu\nu}P_L\nu^\ell], && \mathcal{O}_{TR}^\ell=[\bar{c}\sigma^{\mu\nu} b][\bar{\ell}\sigma_{\mu\nu}P_R\nu^\ell].
\end{aligned} \quad (27)$$

In the SM, $\mathcal{C}_{VL}^\ell=-\mathcal{C}_{AL}^\ell=1$ and all the other WCs vanish. In the BLMSSM, we calculate all the WCs at one-loop level to obtain the theoretical values.

6 Feynman diagrams

In the BLMSSM, the one-loop Feynman diagrams for the lepton sector that can correct the anomalies are shown in Fig. 1 and Fig. 2.

6.1 Penguin-type Feynman diagrams

6.1.1 The WCs

The one-loop Feynman diagrams in Fig. 1(a),(b),(c) and (d) are all UV divergent. Focusing on Fig. 1(a), the three lepneutralinos χ_L^0 are new particles in the BLMSSM, and they play very important roles in this decay process. So taking Fig. 1(a) as an example, the non-zero WCs in Eq. (26) are given as follows:

$$\begin{aligned}
\mathcal{C}_{VL(a)}^\ell &= \Big[\sum_{\beta,s=1}^{6}\sum_{j=1}^{3}\frac{\mathcal{B}_1^{\ell sj}\mathcal{A}_2^{\beta\ell j}\mathcal{A}_3^{\beta s}\mathcal{A}_4}{m_W^2}\frac{1}{64\pi^2}(\Delta_{UV}+1 -F_{21}(x_{\chi^0_{L_j}},x_{\tilde{L}_s},x_{\tilde{\nu}^\beta}))\Big]/(\sqrt{2}G_FV_{cb})\\
&=\frac{g_L^2}{16\pi^2}\Delta_{UV}+\text{finite terms},\\
\mathcal{C}_{AL(a)}^\ell &= -\mathcal{C}_{VL(a)}^\ell.
\end{aligned} \quad (28)$$

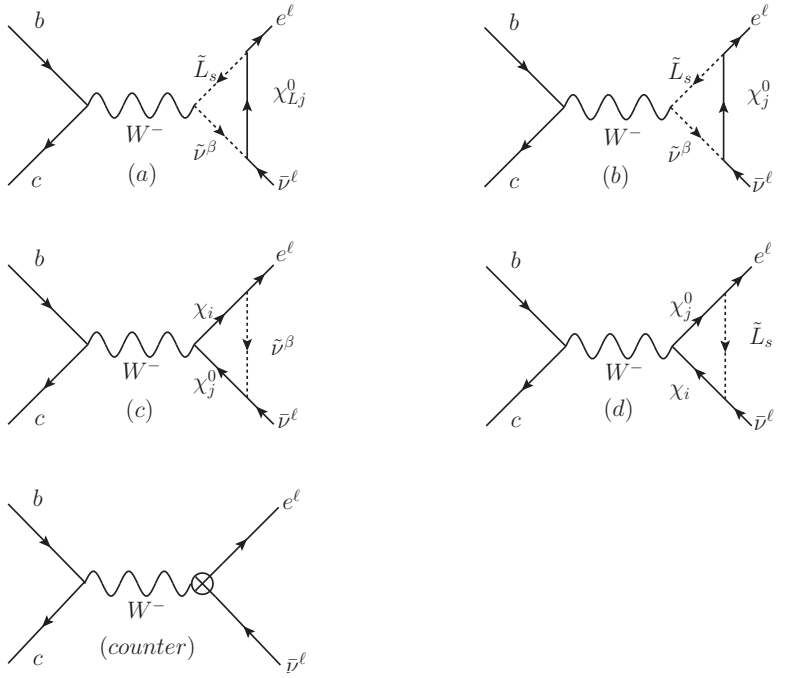

Fig. 1. The penguin-type Feynman diagrams that can correct $R_{D^{(*)}}$ in the BLMSSM.

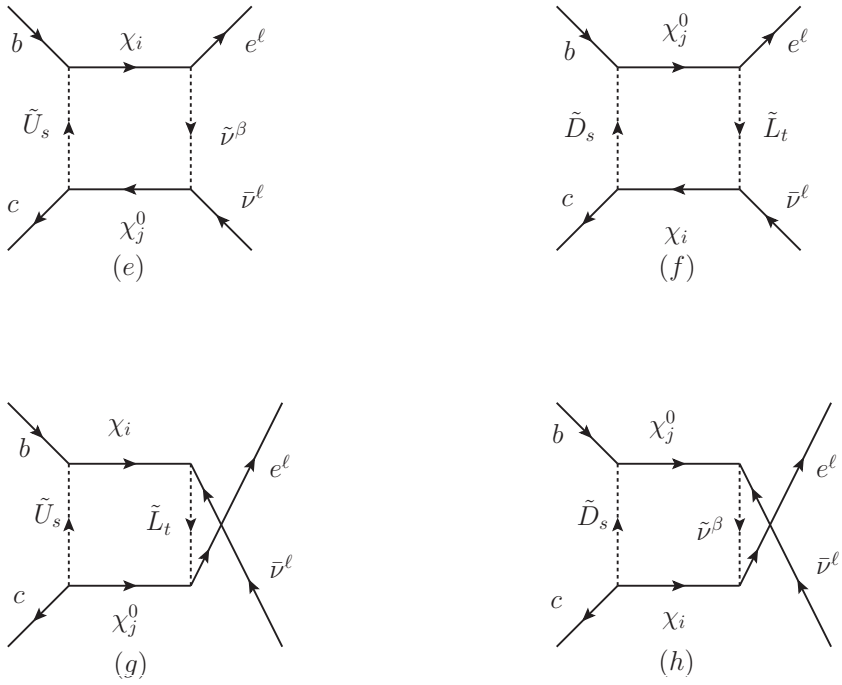

Fig. 2. The box-type Feynman diagrams that can correct $R_{D^{(*)}}$ in the BLMSSM.

Here, we use the unitary characteristics of the rotation matrices. In Eq. (28),

$$\begin{aligned}\mathcal{B}_1^{\ell sj} &= \sqrt{2}g_L Z_{N_L}^{1j*} Z_{\tilde{L}}^{\ell s*},\\ \mathcal{A}_2^{\beta\ell j} &= \sum_{I=1}^{3}\Big(\sqrt{2}g_L Z_{N_L}^{1j} Z_{\nu}^{I\ell} Z_{\tilde{\nu}}^{I\beta} - (2\lambda_{N^c}^{II} Z_{N_L}^{3j} + \sqrt{2}g_L Z_{N_L}^{1j})\\ &\quad \times Z_{\nu}^{(I+3)\ell} Z_{\tilde{\nu}}^{(I+3)\beta *}\Big),\\ \mathcal{A}_3^{\beta s} &= -\sum_{I=1}^{3}\Big(\frac{e}{\sqrt{2}s_W} Z_{\tilde{\nu}}^{I\beta} Z_{\tilde{L}}^{Is}\Big), \quad \mathcal{A}_4 = -\frac{e}{\sqrt{2}s_W} V_{cb}. \end{aligned} \tag{29}$$

$\Delta_{UV}=1/\epsilon+\ln(4\pi\kappa^2/\Lambda_{\rm NP}^2)-\gamma_E$, $\frac{1}{\varepsilon}$ is an infinite term, the mass scale κ is introduced in the dimensional regularization, $\Lambda_{\rm NP}$ is the NP scale, and γ_E is the Euler-Mascheroni constant. x_i represents $\frac{m_i^2}{\Lambda_{\rm NP}^2}$, and the concrete form of formula $F_{21}(x_1,x_2,x_3)$ is given in the Appendix.

We can see that the infinite terms of the WCs of Fig. 1(a) are $\mathcal{C}_{VL(a)}^{\ell(IF)} = \frac{g_L^2}{16\pi^2}\Delta_{UV}$ and $\mathcal{C}_{AL(a)}^{\ell(IF)} = -\mathcal{C}_{VL(a)}^{\ell(IF)}$. Similarly, the infinite terms of the WCs of the following three diagrams (Fig. 1(b), (c) and (d)) are given as follows:

$$\begin{aligned}&\mathcal{C}_{VL(b)}^{\ell(IF)} = \frac{e^2}{64\pi^2 s_W^2 c_W^2}(1-2c_W^2)\Delta_{UV}, \quad && \mathcal{C}_{AL(b)}^{\ell(IF)} = -\mathcal{C}_{VL(b)}^{\ell},\\ &\mathcal{C}_{VL(c)}^{\ell(IF)} = \frac{e^2}{32\pi^2 s_W^2}\Delta_{UV}, && \mathcal{C}_{AL(c)}^{\ell(IF)} = -\mathcal{C}_{VL(c)}^{\ell},\\ &\mathcal{C}_{VL(d)}^{\ell(IF)} = \Big(\frac{e^2}{32\pi^2 s_W^2} + \frac{(Y_l^\ell)^2}{32\pi^2}\Big)\Delta_{UV}, && \mathcal{C}_{AL(d)}^{\ell(IF)} = -\mathcal{C}_{VL(d)}^{\ell}. \end{aligned} \tag{30}$$

Now we should deal with the UV divergences by renormalization procedures.

6.1.2 The counter term in the on-shell scheme

Considering the final state lepton and neutrino are both real particles, we use the on-shell scheme to eliminate the infinite terms. To obtain finite results, the contributions from the counter terms for the vertex $\overline{l^I}\nu^I W^-$ are necessary. The counter term formula for the vertex $\overline{l^I}\nu^I W^-$ is:

$$\begin{aligned}\delta V_{\overline{l^I}\nu^I W^-}^{\mu\ (\rm OS)} &= \frac{-ie}{2\sqrt{2}s_W}\Big(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_Z^2 - \delta m_W^2}{m_Z^2 - m_W^2} + 2\delta e + \delta Z_L^{l\ I}\\ &\quad + \delta Z_L^{\nu I} + \delta Z_{WW}\Big)\gamma^\mu P_L, \end{aligned} \tag{31}$$

Following the method in Refs. [49–51], we obtain the needed renormalization constants in the BLMSSM.

We calculate the Z boson self-energy diagram (loop particles are sneutrinos or sleptons) and get the renormalization constant δm_Z^2:

$$\begin{aligned}\frac{\delta m_Z^2}{m_Z^2} &= \Big[\frac{e^2}{32\pi^2 s_W^2 c_W^2}(1-2s_W^2)^2 + \frac{e^2}{16\pi^2 c_W^2}\Big]\Delta_{UV}\\ &\quad - \frac{e^2}{s_W^2 c_W^2}\Big\{\frac{1}{4}\sum_{j=1}^{6} F_1(x_{\tilde{\nu}_j}, x_{\tilde{\nu}_j})\\ &\quad + \sum_{\alpha,\beta=1}^{6} |(\mathcal{G})_{\alpha\beta}|^2 F_1(x_{\tilde{L}_\alpha}, x_{\tilde{L}_\beta})\Big\}. \end{aligned} \tag{32}$$

Through calculating the W boson self-energy diagram (with sneutrinos and sleptons in the loop), we can obtain:

$$\frac{\delta m_W^2}{m_Z^2}=\frac{e^2c_W^2}{32\pi^2s_W^2}\Delta_{UV}-\frac{e^2c_W^2}{2s_W^2}\sum_{i=1}^{6}\sum_{\alpha=1}^{6}|(\eta)_{i\alpha}|^2F_1(x_{\tilde{\nu}_\alpha},x_{\tilde{L}_i}),$$
$$\delta Z_{WW}=-\frac{e^2}{32\pi^2s_W^2}\Delta_{UV}+\frac{e^2}{2s_W^2}\sum_{i=1}^{6}\sum_{\alpha=1}^{6}|(\eta)_{i\alpha}|^2F_1(x_{\tilde{\nu}_\alpha},x_{\tilde{L}_i}). \tag{33}$$

The renormalization constant of charge is obtained from virtual sleptons:

$$\delta e=\frac{e^2}{16\pi^2}\Delta_{UV}-\frac{1}{2}e^2\sum_{i=1}^{6}F_1(x_{\tilde{L}_i},x_{\tilde{L}_i}). \tag{34}$$

In the same way, we give the renormalization constants $\delta Z_L^{\nu\,I}$ and $\delta Z_L^{l\;I}$ for neutrinos and leptons respectively:

$$\begin{aligned}
\delta Z_L^{\nu\,I} &= \frac{-e^2}{32\pi^2s_W^2}\Big(\frac{1}{2c_W^2}+1+(\frac{s_WY_l^I}{e})^2+(\frac{\sqrt{2}s_Wg_L}{e})^2\Big)\Delta_{UV}-\frac{e^2}{2s_W^2c_W^2}\sum_{i=1}^{4}\sum_{\alpha=1}^{6}|(\zeta^I)_{\alpha i}|^2F_2(x_{\tilde{\nu}_\alpha},x_{\chi_i^0})\\
&\quad-\frac{e^2}{s_W^2}\sum_{i=1}^{2}\sum_{\alpha=1}^{6}|(\mathcal{P}^I)_{\alpha i}|^2F_2(x_{\tilde{L}_\alpha},x_{\chi_i^-})-\frac{e^2}{2s_W^2c_W^2}\sum_{i=1}^{3}\sum_{\alpha=1}^{6}|(\zeta'^I)_{\alpha i}|^2F_2(x_{\tilde{\nu}_\alpha},x_{\chi_{Li}^0}),\\
\delta Z_L^{l\;I} &= \frac{-e^2}{32\pi^2s_W^2}\Big(\frac{1}{2c_W^2}+1+(\frac{s_WY_l^I}{e})^2+(\frac{\sqrt{2}s_Wg_L}{e})^2\Big)\Delta_{UV}\\
&\quad-\frac{e^2}{s_W^2}\sum_{\alpha=1}^{6}\sum_{i=1}^{2}\Big\{|(\mathcal{B}_i)^{I\alpha}|^2F_2+x_{e^I}\Big[|(\mathcal{B}_i)^{I\alpha}|^2+|(\mathcal{A}_i)^{I\alpha}|^2+2\mathbf{Re}[(\mathcal{A}_i^\dagger)^{I\alpha}(\mathcal{B}_i)^{I\alpha}]\Big]F_3\Big\}(x_{\tilde{\nu}_\alpha},x_{\chi_i^-})\\
&\quad-e^2\sum_{j=1}^{4}\sum_{i=1}^{6}\Big\{x_{e^I}\Big[\frac{|(\mathcal{D}^I)_{ij}|^2}{2s_W^2}+\frac{\sqrt{2}}{s_W}\mathbf{Re}[(\mathcal{C}^I)_{ij}^\dagger(\mathcal{D}^I)_{ij}]+|(\mathcal{C}^I)_{ij}|^2\Big]F_3+\frac{1}{2s_W^2}|(\mathcal{D}^I)_{ij}|^2F_2\Big\}(x_{\tilde{L}_i},x_{\chi_j^0})\\
&\quad-e^2\sum_{j=1}^{3}\sum_{i=1}^{6}\Big\{x_{e^I}\Big[\frac{|(\mathcal{D}'^I)_{ij}|^2}{2s_W^2}+\frac{\sqrt{2}}{s_W}\mathbf{Re}[(\mathcal{C}'^I)_{ij}^\dagger(\mathcal{D}'^I)_{ij}]+|(\mathcal{C}'^I)_{ij}|^2\Big]F_3+\frac{1}{2s_W^2}|(\mathcal{D}'^I)_{ij}|^2F_2\Big\}(x_{\tilde{L}_i},x_{\chi_{Lj}^0}),
\end{aligned} \tag{35}$$

where the vertex couplings are given by

$$\begin{aligned}
&(\mathcal{A}_i)^{I\alpha}=\frac{Y_l^Is_W}{e}Z_-^{2i*}Z_{\tilde{\nu}}^{I\alpha*},\qquad(\mathcal{B}_i)^{I\alpha}=Z_+^{1i}Z_{\tilde{\nu}}^{I\alpha*},\qquad(\eta)_{i\alpha}=Z_{\tilde{\nu}}^{J\alpha}Z_{\tilde{L}}^{Ji},\\
&(\mathcal{P}^I)_{\alpha i}=\frac{-Y_l^Js_W}{e}Z_\nu^{JI}Z_{\tilde{L}}^{(J+3)\alpha*}Z_-^{2i*}-Z_\nu^{JI}Z_{\tilde{L}}^{J\alpha*}Z_-^{1i*},\\
&(\zeta^I)_{\alpha i}=Z_{\tilde{\nu}}^{J\alpha*}Z_\nu^{JI}(Z_N^{1i}s_W-Z_N^{2i}c_W),\qquad(\mathcal{C}^I)_{ij}=\frac{-\sqrt{2}}{c_W}Z_{\tilde{L}}^{(I+3)i}Z_N^{1j*}+\frac{Y_l^I}{e}Z_{\tilde{L}}^{Ii}Z_N^{3j*},\\
&(\mathcal{D}^I)_{ij}=\frac{Z_{\tilde{L}}^{Ii}}{c_W}(Z_N^{1j}s_W+Z_N^{2j}c_W)+\frac{\sqrt{2}s_WY_l^I}{e}Z_{\tilde{L}}^{(I+3)i}Z_N^{3j},\\
&(\mathcal{G})_{\alpha\beta}=\frac{1}{2}Z_{\tilde{L}}^{J\alpha}Z_{\tilde{L}}^{J\beta*}-s_W^2\delta^{\alpha\beta},\qquad(\mathcal{C}'^I)_{ij}=\frac{-\sqrt{2}g_L}{e}Z_{NL}^{1j*}Z_L^{(I+3)i},\qquad(\mathcal{D}'^I)_{ij}=\frac{2g_Ls_W}{e}Z_{NL}^{1j}Z_L^{Ii},\\
&(\zeta'^I)_{\alpha i}=\frac{\sqrt{2}s_Wc_W}{e}(\sqrt{2}g_LZ_{NL}^{1i}Z_\nu^{JI}Z_{\tilde{\nu}}^{J\alpha*}-[2\lambda_{N^c}^{JJ}Z_{NL}^{3i}+\sqrt{2}g_LZ_{NL}^{1i}]Z_\nu^{(J+3)I}Z_{\tilde{\nu}}^{(J+3)\alpha*}).
\end{aligned} \tag{36}$$

The functions F_1,F_2, and F_3 are as follows:

$$\begin{aligned}
F_1(x_1,x_2) &= \frac{1}{288\pi^2(x_1-x_2)^3}[6(x_1-3x_2)x_1^2\ln x_1+6(3x_1-x_2)x_2^2\ln x_2-(x_1-x_2)(5x_1^2-22x_1x_2+5x_2^2)],\\
F_2(x_1,x_2) &= \frac{(2x_2-x_1)(x_2-x_1+x_1\ln x_1)-x_2^2\ln x_2}{32\pi^2(x_1-x_2)^2},\\
F_3(x_1,x_2) &= \frac{x_1^2+2x_1x_2(\ln x_2-\ln x_1)-x_2^2}{32\pi^2(x_1-x_2)^3}.
\end{aligned} \tag{37}$$

If $x_1=x_2$, they simplify to

$$F_1(x_1,x_2)=\frac{\ln x_1}{48\pi^2},\quad F_2(x_1,x_2)=-\frac{\ln x_1}{32\pi^2}+\frac{1}{64\pi^2},\quad F_3(x_1,x_2)=\frac{1}{96\pi^2 x_1}. \tag{38}$$

Now, the WCs of Fig. 1(counter) read:

$$\begin{aligned}\mathcal{C}^{\ell}_{VL(counter)} &= \frac{1}{2}\Big(\frac{\delta m_Z^2}{m_Z^2}-\frac{\delta m_Z^2-\delta m_W^2}{m_Z^2-m_W^2}+2\delta e+\delta Z_L^{l\,\ell}+\delta Z_L^{\nu\,\ell}+\delta Z_{WW}\Big),\\ \mathcal{C}^{\ell}_{AL(counter)} &= -\mathcal{C}^{\ell}_{VL(counter)}.\end{aligned} \tag{39}$$

The corresponding $\mathcal{C}^{\ell(IF)}_{VL(counter)}$ and $\mathcal{C}^{\ell(IF)}_{AL(counter)}$ are:

$$\begin{aligned}\mathcal{C}^{\ell(IF)}_{VL(counter)} &= \frac{1}{2}\Big\{\Big[\frac{e^2}{32\pi^2 s_W^2 c_W^2}(1-2s_W^2)^2+\frac{e^2}{16\pi^2 c_W^2}\Big]\Delta_{UV}\\ &\quad-\Big[\Big(\frac{e^2}{32\pi^2 s_W^2 c_W^2}(1-2s_W^2)^2+\frac{e^2}{16\pi^2 c_W^2}\Big)\Delta_{UV}-\frac{e^2 c_W^2}{32\pi^2 s_W^2}\Delta_{UV}\Big]\frac{m_Z^2}{m_Z^2-m_W^2}\\ &\quad+\frac{e^2}{8\pi^2}\Delta_{UV}-\frac{e^2}{32\pi^2 s_W^2}\Delta_{UV}+\frac{-e^2}{16\pi^2 s_W^2}\Big[\frac{1}{2c_W^2}+1+\Big(\frac{s_W Y_l^{\ell}}{e}\Big)^2+\Big(\frac{\sqrt{2}s_W g_L}{e}\Big)^2\Big]\Delta_{UV}\Big\},\\ \mathcal{C}^{\ell(IF)}_{AL(counter)} &= -\mathcal{C}^{\ell(IF)}_{VL(counter)}.\end{aligned} \tag{40}$$

It is easy to test that the infinite terms in the sum of Fig. 1(a),(b),(c),(d) and (counter) vanish: $\mathcal{C}^{\ell(IF)}_{VL}=\mathcal{C}^{\ell(IF)}_{VL(a)}+\mathcal{C}^{\ell(IF)}_{VL(b)}+\mathcal{C}^{\ell(IF)}_{VL(c)}+\mathcal{C}^{\ell(IF)}_{VL(d)}+\mathcal{C}^{\ell(IF)}_{VL(counter)}=0$, similarly, $\mathcal{C}^{\ell(IF)}_{AL}=0$. Therefore, the divergences are completely eliminated. Note that the infinite terms in the sum of Fig. 1(a), (b), (c) and (d) can be eliminated by the counter terms. However, a single diagram in Fig. 1(b), (c), (d), such as Fig. 1(b), cannot be counteracted individually in the on-shell scheme.

6.2 Box-type Feynman diagrams

Taking Fig. 2(e) as an example, the corresponding WCs are given as follows:

$$\begin{aligned}\mathcal{C}^{\ell}_{VL(e)} &= \sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4}\Big[\frac{\mathcal{B}_1^{\beta\ell i}\mathcal{B}_2^{is}\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j}m_{\chi_j^0}m_{\chi_i}}{\Lambda^4_{\rm NP}}\frac{1}{64\pi^2}F_{11}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\\ &\quad-\frac{\mathcal{B}_1^{\beta\ell i}\mathcal{A}_2^{is}\mathcal{B}_3^{sj}\mathcal{A}_4^{\beta\ell j}}{\Lambda^2_{\rm NP}}\frac{1}{128\pi^2}F_{21}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\Big]/(\sqrt{2}G_F V_{cb}),\end{aligned} \tag{41}$$

$$\begin{aligned}\mathcal{C}^{\ell}_{AL(e)} &= \sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4}\Big[\frac{\mathcal{B}_1^{\beta\ell i}\mathcal{B}_2^{is}\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j}m_{\chi_j^0}m_{\chi_i}}{\Lambda^4_{\rm NP}}\frac{1}{64\pi^2}F_{11}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\\ &\quad+\frac{\mathcal{B}_1^{\beta\ell i}\mathcal{A}_2^{is}\mathcal{B}_3^{sj}\mathcal{A}_4^{\beta\ell j}}{\Lambda^2_{\rm NP}}\frac{1}{128\pi^2}F_{21}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\Big]/(\sqrt{2}G_F V_{cb}),\end{aligned} \tag{42}$$

$$\begin{aligned}\mathcal{C}^{\ell}_{SL(e)} &= \sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4}\Big[\frac{\mathcal{A}_1^{\beta\ell i}\mathcal{A}_2^{is}\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j}m_{\chi_j^0}m_{\chi_i}}{\Lambda^4_{\rm NP}}\frac{1}{64\pi^2}F_{11}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\\ &\quad+\frac{\mathcal{A}_1^{\beta\ell i}\mathcal{B}_2^{is}\mathcal{B}_3^{sj}\mathcal{A}_4^{\beta\ell j}}{\Lambda^2_{\rm NP}}\frac{1}{64\pi^2}F_{21}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\Big]/(\sqrt{2}G_F V_{cb}),\end{aligned} \tag{43}$$

$$\begin{aligned}\mathcal{C}^{\ell}_{TL(e)} &= \Big[\sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4}\frac{\mathcal{A}_1^{\beta\ell i}\mathcal{A}_2^{is}\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j}m_{\chi_j^0}m_{\chi_i}}{\Lambda^4_{\rm NP}}\frac{1}{128\pi^2}F_{11}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\Big]\\ &\quad/(\sqrt{2}G_F V_{cb}),\end{aligned} \tag{44}$$

$$\begin{aligned}\mathcal{C}^{\ell}_{PL(e)} &= \sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4}\Big[-\frac{(\mathcal{A}_1^{\beta\ell i}\mathcal{A}_2^{is}-\mathcal{B}_1^{\beta\ell i}\mathcal{B}_2^{is})\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j}m_{\chi_j^0}m_{\chi_i}}{\Lambda^4_{\rm NP}}\frac{1}{128\pi^2}F_{11}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\\ &\quad+\frac{\mathcal{A}_1^{\beta\ell i}\mathcal{B}_2^{is}\mathcal{B}_3^{sj}\mathcal{A}_4^{\beta\ell j}}{\Lambda^2_{\rm NP}}\frac{1}{64\pi^2}F_{21}(x_{\tilde{\nu}^\beta},x_{\chi_j^0},x_{\tilde{U}_s},x_{\chi_i})\Big]/(\sqrt{2}G_F V_{cb}),\end{aligned} \tag{45}$$

$$\mathcal{C}^{\ell}_{PR(e)} = \Big[\sum_{\beta,s=1}^{6}\sum_{i=1}^{2}\sum_{j=1}^{4} -\frac{(\mathcal{A}_1^{\beta\ell i}\mathcal{A}_2^{is}+\mathcal{B}_1^{\beta\ell i}\mathcal{B}_2^{is})\mathcal{A}_3^{sj}\mathcal{A}_4^{\beta\ell j} m_{\chi_j^0} m_{\chi_i}}{\Lambda^4_{\rm NP}} \frac{1}{128\pi^2} F_{11}(x_{\tilde{\nu}^\beta}, x_{\chi_j^0}, x_{\tilde{U}_s}, x_{\chi_i})\Big] /(\sqrt{2}G_F V_{cb}), \tag{46}$$

and all the other WCs in Eq. (26) vanish. In Eqs. (41-46),

$$\begin{aligned}
&\mathcal{A}_1^{\beta\ell i} = -Y_l^{\ell} Z_-^{2i} Z_{\tilde{\nu}}^{\ell\beta}, \quad \mathcal{A}_2^{is} = \sum_{I=1}^{3}\Big(\frac{-e}{s_W} Z_{\tilde{U}}^{Is*} Z_+^{1i} + Y_u^I Z_{\tilde{U}}^{(I+3)s*} Z_+^{2i}\Big) V_{I3},\\
&\mathcal{A}_3^{sj} = \frac{2\sqrt{2}e}{3c_W} Z_{\tilde{U}}^{5s} Z_N^{1j} - Y_u^2 Z_{\tilde{U}}^{2s} Z_N^{4j},\\
&\mathcal{A}_4^{\beta\ell j} = \sum_{I=1}^{3}\Big\{ Z_{\tilde{\nu}}^{I\beta*} Z_\nu^{I\ell} \frac{e}{\sqrt{2}s_W c_W}(Z_N^{1j} s_W - Z_N^{2j} c_W) + \sum_{J=1}^{3}\Big(\frac{Y_\nu^{IJ}}{\sqrt{2}} Z_N^{4j}(Z_\nu^{I\ell} Z_{\tilde{\nu}}^{(J+3)\beta*} + Z_\nu^{(I+3)\ell} Z_{\tilde{\nu}}^{J\beta*})\Big)\Big\},\\
&\mathcal{B}_1^{\beta\ell i} = -\frac{e}{s_W} Z_+^{1i*} Z_{\tilde{\nu}}^{\ell\beta} - \sum_{I=1}^{3}(Y_\nu^{\ell I} Z_+^{2i*} Z_{\tilde{\nu}}^{(I+3)\beta}), \quad \mathcal{B}_2^{is} = -\sum_{I=1}^{3}(Y_d^3 Z_{\tilde{U}}^{Is*} Z_-^{2i*}) V_{I3},\\
&\mathcal{B}_3^{sj} = -\frac{e}{\sqrt{2}s_W c_W} Z_{\tilde{U}}^{2s}\Big(\frac{1}{3} Z_N^{1j*} s_W + Z_N^{2j*} c_W\Big) - Y_u^2 Z_{\tilde{U}}^{5s} Z_N^{4j*}.
\end{aligned} \tag{47}$$

The formulae $F_{11}(x_1,x_2,x_3,x_4)$ and $F_{21}(x_1,x_2,x_3,x_4)$ are given in the Appendix.

7 Numerical results

For the numerical discussion, the parameters used are:
m_1=400 GeV, M_L=2000 GeV, μ_L=1600 GeV, $\tan\beta_{\rm L}$=0.1, v_L=1260 GeV, λ_{N^c}=1, $\Lambda_{\rm NP}$=1000 GeV, $(m^2_{\tilde{Q}})_{ii}$=$(m^2_{\tilde{U}})_{ii}$=$(m^2_{\tilde{D}})_{ii}$=$(m^2_{\tilde{N}^c})_{ii}$=3×10^6 GeV^2, $(A_l)_{ii}$=$(A_l^{'})_{ii}$=300 GeV, $(A_N)_{ii}$=$(A_{N^c})_{ii}$=500 GeV, and $(A_u)_{ii}$=$(A_d)_{ii}$=$(A_u^{'})_{ii}$=$(A_d^{'})_{ii}$=500 GeV, where i=1…3. If not otherwise noted, the non-diagonal elements of the parameters used should be zero. The Yukawa couplings of neutrinos Y_ν^{IJ} are of the order of $10^{-8}\sim10^{-6}$; their effects are tiny and can be ignored.

At present, all supersymmetric mass bounds are model-dependent. Based on the PDG [52] data, we consider the limitations on masses of the charginos and neutralinos (the strongest limitations are 345 GeV). In our work, the masses of charginos $m_{\chi^\pm}\simeq(1000\sim2000)$ GeV and the masses of neutralinos $m_{\chi^0}\simeq(400\sim2000)$ GeV, all of which can satisfy the mass bounds. The limits for the sleptons are around 290 GeV$\sim$450 GeV [53], which can be satisfied easily. The masses of squarks in this paper are larger than 1000 GeV, so the limits for squarks are also satisfied. In other words, the parameters given above and the parameter space to be discussed below can all satisfy the mass bounds.

7.1 Effects of parameters $m^2_{\tilde{L}}$ (or $m^2_{\tilde{R}}$) on $R_{D^{(*)}}$

We now focus on the effects of parameters $m^2_{\tilde{L}}$ (or $m^2_{\tilde{R}}$) on $R_{D^{(*)}}$. First, we set the parameters as follows: $\tan\beta = 10$, $m_2 = \mu = 1200$ GeV, $g_L = 0.1$, and $(m^2_{\tilde{L}})_{33}$=$(m^2_{\tilde{R}})_{33}$=3×10^{8}GeV^2. To study the impacts of these parameters on $R_{D^{(*)}}$, we use the parameters $(m^2_{\tilde{L}})_{11}$=$(m^2_{\tilde{R}})_{11}$=$(m^2_{\tilde{L}})_{22}$=$(m^2_{\tilde{R}})_{22}$=$3\times10^{\xi}$ GeV^2, where ξ is a variable. After calculation we obtain Fig. 3. Here, we used the central value of the SM prediction in our calculation. The left-hand diagram shows R_D and the right-hand diagram shows R_{D^*}.

We know the measurement of $R_{D^{(*)}_e}$ (which implies $l=e$ in Eq. (20)) is approximately equal to that of $R_{D^{(*)}_\mu}$ ($l=\mu$ in Eq. (20)). This is the reason why we set $(m^2_{\tilde{L}})_{11} = (m^2_{\tilde{R}})_{11} = (m^2_{\tilde{L}})_{22} = (m^2_{\tilde{R}})_{22}$. To solve the problem of $R_{D^{(*)}}$, we should violate lepton flavour symmetry for generations 1(2) and generation 3. Therefore, we suppose $(m^2_{\tilde{L}})_{33}$=$(m^2_{\tilde{R}})_{33}\neq(m^2_{\tilde{L}})_{11}$.

It is easy to see from Fig. 3 that $R_{D^{(*)}}$ decreases as ξ increases. Obviously, our results satisfy the decoupling rule. When the sleptons are very heavy, the BLMSSM results are very near the SM predictions. In fact, the SM predictions of $R_{D^{(*)}}$ cannot explain the experimental values well, and our goal is to increase the theoretical values. From the numerical analysis, the following relational expression should be set up: $(m^2_{\tilde{L}})_{11} = (m^2_{\tilde{R}})_{11} = (m^2_{\tilde{L}})_{22} = (m^2_{\tilde{R}})_{22} < (m^2_{\tilde{L}})_{33} = (m^2_{\tilde{R}})_{33}$. We need to select a set of reasonable parameters, and finally choose: $(m^2_{\tilde{L}})_{11}$=$(m^2_{\tilde{R}})_{11}$=$(m^2_{\tilde{L}})_{22}$=$(m^2_{\tilde{R}})_{22}$=5.5×10^{5}GeV^2, and $(m^2_{\tilde{L}})_{33} = (m^2_{\tilde{R}})_{33} = 3\times10^8$GeV^2. Up to now, our theoretical values of $R_{D^{(*)}}$ are only a little bigger than those of the SM, so we also need to study the effects of other parameters on $R_{D^{(*)}}$.

7.2 Effect of parameter g_L on $R_{D^{(*)}}$

Based on the above analysis, we use the following parameters:
$\tan\beta$=10, m_2=μ=1200 GeV,
$(m^2_{\tilde{L}})_{11}$=$(m^2_{\tilde{R}})_{11}$=$(m^2_{\tilde{L}})_{22}$=$(m^2_{\tilde{R}})_{22}$=5.5×10^{5}GeV^2, and
$(m^2_{\tilde{L}})_{33}$=$(m^2_{\tilde{R}})_{33}$=3×10^{8}GeV^2.

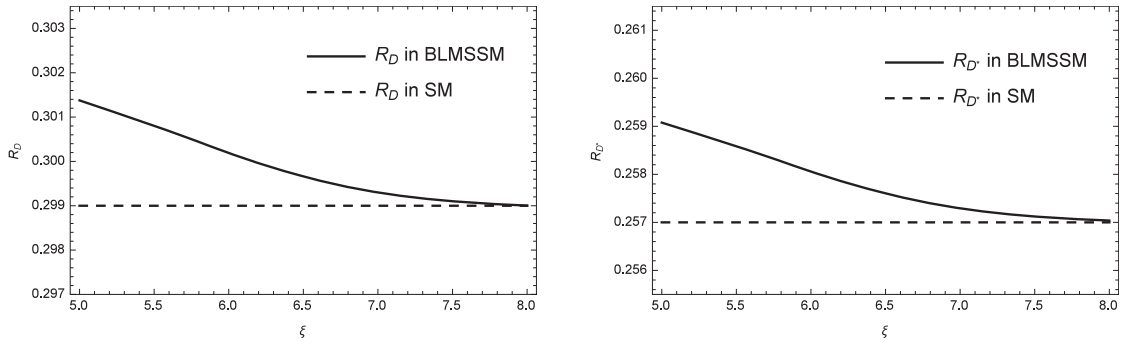

Fig. 3. R_D (left) and R_{D^*} (right) versus ξ with $(m^2_{\tilde{L}})_{11} = (m^2_{\tilde{R}})_{11} = (m^2_{\tilde{L}})_{22} = (m^2_{\tilde{R}})_{22} = 3\times10^{\xi}$ GeV^2, and $(m^2_{\tilde{L}})_{33}$=$(m^2_{\tilde{R}})_{33}$=3×10^{8}GeV^2.

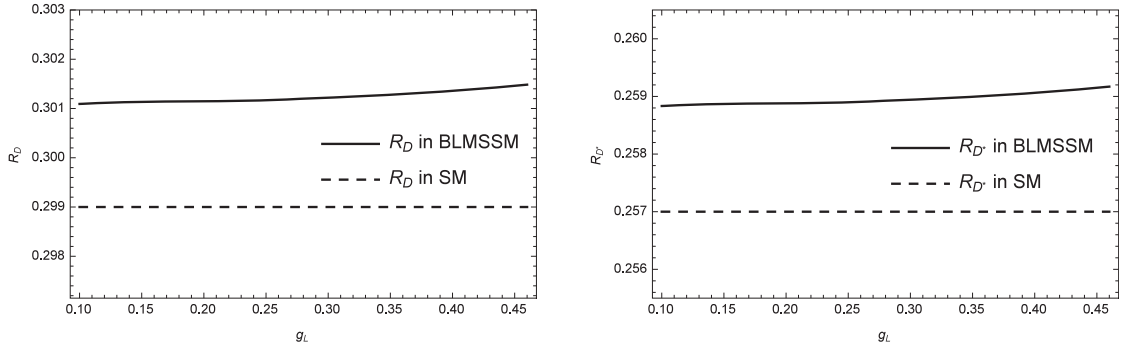

Fig. 4. R_D (left) and R_{D^*} (right) versus g_L.

g_L is the coupling constant of the vertexes $l\chi^0_L\tilde{L}$ and $\nu\chi^0_L\tilde{\nu}$. As a new parameter in the BLMSSM, g_L should affect $R_{D^{(*)}}$, which is of interest. The obtained numerical results are plotted in Fig. 4. The left-hand diagram shows R_D and the right-hand diagram shows R_{D^*}.

From Fig. 4, we can see that R_D and R_{D^*} both increase gently with increasing g_L. This is easy to understand: larger g_L improves the effects from NP. In order to get larger theoretical values of $R_{D^{(*)}}$, we need to choose a larger g_L. After considering the reasonableness of the range of parameter g_L, we use g_L=0.45. In this case, our numerical results are further improved.

7.3 Effects of parameters tanβ, m_2 and μ on $R_{D^{(*)}}$

We also research the effects of parameters tanβ, m_2 and μ on $R_{D^{(*)}}$. With the supposition $g_L = 0.45$, $(m^2_{\tilde{L}})_{11} = (m^2_{\tilde{R}})_{11} = (m^2_{\tilde{L}})_{22} = (m^2_{\tilde{R}})_{22} = 5.5\times10^5$GeV^2, $(m^2_{\tilde{L}})_{33} = (m^2_{\tilde{R}})_{33} = 3\times10^8$GeV^2, and $m_2 = \mu = M_\xi$, we scan the parameters of M_ξ versus tanβ in Fig. 5.

All the points in Fig. 5 can make R_D (R_{D^*}) reach 0.304 (0.261), and some particular points can bring R_D (R_{D^*}) to 0.305 (0.262). The theoretical values are improved, but they are not as big as we expected. However, our results are still better than those in the SM. All of the above discussions only consider the central values in the SM. If we consider the uncertainty of SM predictions $R_D = 0.299\pm0.003$ [5] and $R_{D^*} = 0.257\pm0.003$ [5], our theoretical value of R_D (R_{D^*}) can reach 0.308 (0.265), when we take the biggest value of the SM prediction.

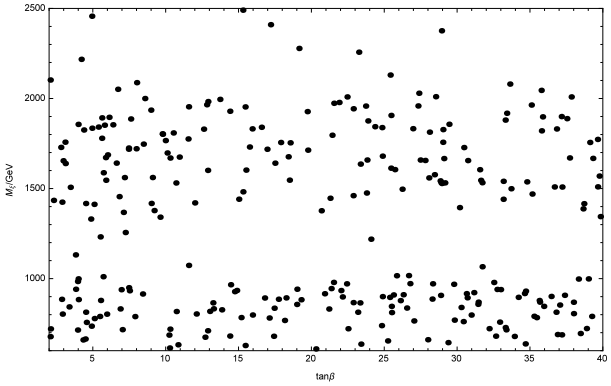

Fig. 5. The allowed parameters in the plane of M_ξ versus tanβ with $g_L = 0.45$, $(m^2_{\tilde{L}})_{11} = (m^2_{\tilde{R}})_{11} = (m^2_{\tilde{L}})_{22} = (m^2_{\tilde{R}})_{22} = 5.5\times10^5$GeV^2, $(m^2_{\tilde{L}})_{33} = (m^2_{\tilde{R}})_{33}$=3×10^{8}GeV^2.

8 Summary and future prospects

The SM cannot explain the experimental data for $R_{D^{(*)}}$ well, so we hold that SM should be the low energy effective theory of a large model. We think the BLMSSM is promising for testing in the future. Compared with the MSSM, there are new particles and new parameters in the BLMSSM, and the new contributions

from these are the keys to solve the anomalies in $R_{D^{(*)}}$. For instance, the three lepneutralinos χ_L^0 are new particles in the BLMSSM, and the Feynman diagram with χ_L^0 can give new contributions to $R_{D^{(*)}}$.

We find that the parameters $(m_{\tilde{L}}^2)_{ii}$ and $(m_{\tilde{R}}^2)_{ii}$ influence the theoretical results to some extent, and $R_{D^{(*)}e}$ is approximately equal to $R_{D^{(*)}\mu}$ only if there is a certain relationship between parameters $(m_{\tilde{L}}^2)_{ii}$ and $(m_{\tilde{R}}^2)_{ii}$. After that, the effect of parameter g_L is important, and we can further raise theoretical values when it takes some appropriate values. Finally, using the central value of the SM prediction we scan the parameter space, and bring the value of R_D (R_{D^*}) to 0.305 (0.262). Taking into account the SM uncertainty and adopting the biggest value in the SM, our result for R_D (R_{D^*}) can reach 0.308 (0.265).

In this paper, we use effective field theory to compute $R_{D^{(*)}}$ in the BLMSSM. The one-loop corrections to $R_{D^{(*)}}$ have an effect and the theoretical values can be increased (though they are not big improvements). We notice that the measurements of R_{D^*} (see Table 1) are not as large as the original measurements. This suggests that R_{D^*} perhaps is not so large. From the trend of experimental measurement, the experimental values of $R_{D^{(*)}}$ might be smaller in the future. In fact, without considering this case, the measurement of R_D (R_{D^*}) shows 2.3σ (3.1σ) deviation from its SM prediction, and our theoretical values are still better than the predictions given by SM. On the whole, the problem of R_D (R_{D^*}) should be further researched both experimentally and theoretically in the future.

Appendix

The formulae for the one-loop integral are:

$$\frac{(2\pi\kappa)^{4-D}}{i\pi^2}\int \mathrm{d}^D p \frac{1}{(p^2-m_1^2)(p^2-m_2^2)(p^2-m_3^2)} = -\frac{1}{\Lambda_{\rm NP}^2} F_{11}(x_1,x_2,x_3),$$
$$\frac{(2\pi\kappa)^{4-D}}{i\pi^2}\int \mathrm{d}^D p \frac{p^2}{(p^2-m_1^2)(p^2-m_2^2)(p^2-m_3^2)} = \frac{1}{\varepsilon} - \gamma_E + \ln(4\pi\kappa^2/\Lambda_{\rm NP}^2) + 1 - F_{21}(x_1,x_2,x_3),$$
$$\frac{(2\pi\kappa)^{4-D}}{i\pi^2}\int \mathrm{d}^D p \frac{1}{(p^2-m_1^2)(p^2-m_2^2)(p^2-m_3^2)(p^2-m_4^2)} = -\frac{1}{\Lambda_{\rm NP}^4} F_{11}(x_1,x_2,x_3,x_4),$$
$$\frac{(2\pi\kappa)^{4-D}}{i\pi^2}\int \mathrm{d}^D p \frac{p^2}{(p^2-m_1^2)(p^2-m_2^2)(p^2-m_3^2)(p^2-m_4^2)} = -\frac{1}{\Lambda_{\rm NP}^2} F_{21}(x_1,x_2,x_3,x_4),$$

$$\begin{aligned}
F_{11}(x_1,x_2,x_3) &= \frac{x_1\ln x_1}{(x_1-x_2)(x_1-x_3)} + \frac{x_2\ln x_2}{(x_2-x_1)(x_2-x_3)} + \frac{x_3\ln x_3}{(x_3-x_1)(x_3-x_2)},\\
F_{21}(x_1,x_2,x_3) &= \frac{x_1^2\ln x_1}{(x_1-x_2)(x_1-x_3)} + \frac{x_2^2\ln x_2}{(x_2-x_1)(x_2-x_3)} + \frac{x_3^2\ln x_3}{(x_3-x_1)(x_3-x_2)},\\
F_{11}(x_1,x_2,x_3,x_4) &= \frac{x_1\ln x_1}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{x_2\ln x_2}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}\\
&\quad + \frac{x_3\ln x_3}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} + \frac{x_4\ln x_4}{(x_4-x_1)(x_4-x_2)(x_4-x_3)},\\
F_{21}(x_1,x_2,x_3,x_4) &= \frac{x_1^2\ln x_1}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{x_2^2\ln x_2}{(x_2-x_1)(x_2-x_3)(x_2-x_4)}\\
&\quad + \frac{x_3^2\ln x_3}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} + \frac{x_4^2\ln x_4}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}.
\end{aligned}$$

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