

Strong decays of P -wave mixing heavy-light 1^+ states^{*}

Zhi-Hui Wang(王志会)^{1,2,1)} Yi Zhang(张怡)¹ Tian-Hong Wang(王天鸿)² Yue Jiang(姜越)²

Qiang Li(李强)³ Guo-Li Wang(王国利)²

¹ School of Electrical & Information Engineering, North Minzu University, Yinchuan 750021, China

² Department of Physics, Harbin Institute of Technology, Harbin 150001, China

³Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

Abstract: Many P -wave mixing heavy-light 1^+ states have not yet been discovered by experiment, while others have been discovered but without width information, or with large uncertainties on the widths. In this paper, the strong decays of the P -wave mixing heavy-light 1^+ states D^0 , D^\pm , D_s^\pm , B^0 , B^\pm and B_s are studied by the improved Bethe-Salpeter (B-S) method with two conditions of mixing angle θ : one is $\theta=35.3^\circ$; the other is considering a correction to the mixing angle $\theta=35.3^\circ+\theta_1$. Valuable predictions for the strong decay widths are obtained: $\Gamma(D_1^0)=232$ MeV, $\Gamma(D_1^\pm)=21.5$ MeV, $\Gamma(D_1^\pm)=232$ MeV, $\Gamma(D_1^\pm)=21.5$ MeV, $\Gamma(D_{s1}^\pm)=0.0101$ MeV, $\Gamma(D_{s1}^\pm)=0.950$ MeV, $\Gamma(B_1^\pm)=263$ MeV, $\Gamma(B_1^\pm)=16.8$ MeV, $\Gamma(B_{s1}^\pm)=0.01987$ MeV and $\Gamma(B_{s1}^\pm)=0.412$ MeV. It is found that the decay widths of D_{s1}^\pm and B_{s1}^\pm are very sensitive to the mixing angle. The results will provide theoretical assistance to future experiments.

Keywords: P -wave mixing states, strong decay, mixing angle, improved B-S method

PACS: 12.38.Lg, 12.39.Ki, 12.39.Pn **DOI:** 10.1088/1674-1137/42/12/123101

1 Introduction

Heavy-light mesons are very important in hadron physics. During the past few years, a lot of interesting processes have been obtained for the heavy-light mesons. Nowadays, many more excited heavy-light mesons have been discovered by experiments. For the P -wave D_J^* mesons, $D_0^*(2400)^0$, $D_1(2430)^0$, $D_1(2420)^0$, $D_2^*(2460)^0$ and their charged isospin partners $D_0^*(2400)^\pm$, $D_1(2420)^\pm$, $D_2^*(2460)^\pm$ have been listed in Particle Data Group (PDG) 2016 edition [1]. Among these, we have little knowledge about $D_1(2430)^0$, which has large errors for the decay width, and its charged isospin partner has not been observed. Four P -wave D_{sJ}^* mesons, $D_{s0}^*(2317)^\pm$, $D_{s1}(2460)^\pm$, $D_{s1}(2536)^\pm$ and $D_{s2}^*(2573)$, have been observed in experiments [1]. The upper bounds on the total decay width of the $D_{s0}^*(2317)^\pm$ and $D_{s1}(2460)^\pm$ meson are 3.8 MeV at 95% confidence level and 3.5 MeV at 95% confidence level [1], respectively. The full width of $D_{s1}(2536)^\pm$ is rather narrow: $\Gamma = 0.92 \pm 0.05$ MeV. In 2007, the D0 Collaboration reported two separate excited B mesons, $B_1(5721)^0$ and $B_2^*(5747)^0$, in fully re-

constructed decays to $B^{(*)}\pi^-$ [2]. The CDF Collaboration observed two orbitally excited narrow B^0 mesons in 2009 [3]. They then updated the measurement of the properties of orbitally excited B^0 and B_s^0 mesons in 2014 [4]. The LHCb Collaboration gave precise measurements for the masses and widths of the $B_1(5721)^{0,+}$ and $B_2^*(5747)^{0,+}$ states in 2015 [5]. The CDF Collaboration reported their observations of $B_{s1}(5830)^0$ and $B_{s2}^*(5840)^0$ in 2008 [6]. Later the D0 Collaboration confirmed the existence of $B_{s2}^*(5840)^0$ and indicated that $B_{s1}(5830)^0$ was not observed with available data [7]. The LHCb Collaboration confirmed the existence of $B_{s1}(5830)^0$ and $B_{s2}^*(5840)^0$ in the $B^{(*)}K^-$ [8] channel. The discovery of these excited states not only enriched the spectroscopy of P -wave heavy-light mesons but also provided us an opportunity to research the properties of P -wave heavy-light mesons.

Besides the experimental progress, there have been a lot of theoretical efforts to investigate the properties of the P -wave heavy-light mesons, especially for the newly observed states. In heavy quark effective theory (HQET) [9, 10], the angular momentum of the light

Received 8 July 2018, Published online 19 October 2018

^{*} Supported by the National Natural Science Foundation of China (NSFC) (11405004, 11405037, 11505039, 11575048), the Key Scientific Research Projects in 2017 at North Minzu University (2017KJ11), Ningxia First-Class Discipline and Scientific Research Projects (Electronic Science and Technology) (NXYLXK2017A07) and the Fundamental Research Funds for the Central Universities, North Minzu University (2018XYZDX08)

1) E-mail: 2013086@nun.edu.cn



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP³ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

quark $j_q = s_q + L$ (s_q and L are the spin and the orbital angular momentum, respectively, of the light quark) is a good quantum number when the heavy quark is in the $m_Q \rightarrow \infty$ limit, and can be used to label the states, so the physical heavy-light states can be described by HQET. As well as HQET, the mass spectroscopy [11–14] and strong decays of P -wave heavy-light mesons have also been studied by other methods [15–29]. The strong decay of P -wave heavy-light mesons can help us understand the properties of these mesons and establish the heavy-light meson spectroscopy.

The mesons can be described by the Bethe-Salpeter (B-S) equation. Reference [30] took the B-S equation to describe the light mesons π and K , then they calculated the mass and decay constant of π by the B-S amplitudes [31]. The weak decays [32] and the strong decays [33] have also been studied, by combining with the Dyson-Schwinger equation. In this paper, however, we describe the properties of heavy mesons and the matrix elements of strong decays by an improved B-S method, which includes two improvements [34]. One improvement is that relativistic wavefunctions are used which describe bound states with definite quantum numbers, and the relativistic forms of wavefunctions are solutions of the full Salpeter equations. The other is that the matrix elements of strong decays are obtained with relativistic wavefunctions as input. So the improved B-S method is good to describe the properties and decays of heavy mesons with relativistic corrections.

We have previously studied the strong decay of P -wave B_s^* mesons by the improved B-S method [35], and we have also calculated the production of P -wave mesons in B , B_s and B_c mesons [36–38]. The wavefunctions of mesons are obtained by considering the quantum number J^P or J^{PC} for different states. P -wave 1^+ states are labelled as 3P_1 and 1P_1 in our model. For unequal mass systems, the 3P_1 and 1P_1 states are not physical states, while the two physical states $P_1^{1/2}$ and $P_1^{3/2}$ are mixtures of 3P_1 and 1P_1 [9, 10]. In Refs. [35–38], the mixing angle is taken as a definite value $\theta \approx 35.3^\circ$ for the P -wave 1^+ heavy-light mesons. However, in reality the heavy quark is not infinitely heavy in P -wave 1^+ states. The mixing angle between 3P_1 and 1P_1 is not a fixed value when considering the correction to the heavy quark limit; there is a shift from the value $\theta \approx 35.3^\circ$, and the shift is different for the different P -wave mixing 1^+ heavy-light states [15, 19, 39]. In this paper, the strong decays of P -wave mixing 1^+ heavy-light states (1^{++} , 1^+) (just like D_1^0 and D_1^0) are studied with two conditions of mixing angle θ . One condition is $\theta = 35.3^\circ$; the other considers the correction to the mixing angle $\theta = 35.3^\circ + \theta_1$. The influence of the shift in the mixing angle on the strong decay of P -wave mixing 1^+ heavy-light states is discussed afterwards.

The paper is organized as follows. In Section 2, we give the formulation and hadronic matrix elements of strong decays. We show the relativistic wavefunctions of initial mesons and final mesons in Section 3; We talk about the mixing of 3P_1 and 1P_1 states in Section 4, and in Section 5, we present the corresponding results and conclusions.

2 Formulation and hadronic matrix elements of strong decays

In this section, we will show the formulations and the transition matrix elements of P -wave mixing states. The quantum numbers of P -wave mixing states (1^{++} , 1^+) are both 1^+ . Considering the limitations of the phase spaces, the ground P -wave 1^+ states only have an OZI-allowed dominant strong decay channel: $1^+ \rightarrow 1^- 0^-$. The pseudoscalar 0^- state must be the light meson K , π , and the 1^- state is a heavy-light meson in the final states.

2.1 Strong decays of 1^+ states

In order to calculate the strong decays of the two P -wave mixing states, we take the channel $D_1^0 \rightarrow D^{*+} \pi^-$ as an example in Fig. 1. According to the reduction formula, PCAC relation and low energy theorem, the corresponding amplitude can be written as [40, 41]:

$$T(D_1^0 \rightarrow D^{*+} \pi^-) = \frac{P_{f_2}^\mu}{f_\pi} \langle D^{*+}(P_{f_1}) | \bar{d} \gamma_\mu \gamma_5 u | D_1^0(P) \rangle, \quad (1)$$

where P , P_{f_1} , P_{f_2} are the momenta of D_1^0 and the final states D^{*+} and π^- , respectively. f_π is the decay constant of π^- . $\langle D^{*+}(P_{f_1}) | \bar{d} \gamma_\mu \gamma_5 u | D_1^0(P) \rangle$ is the hadronic matrix element.

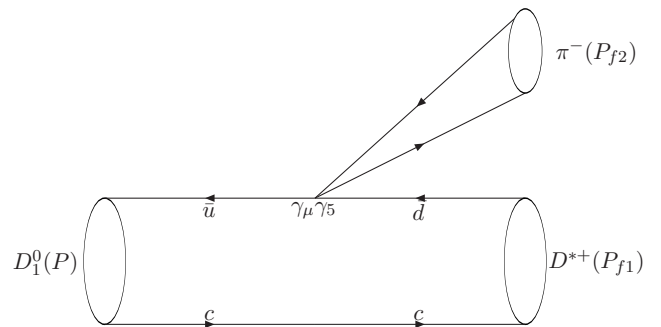


Fig. 1. Strong decay of $D_1^0 \rightarrow D^{*+} \pi^-$.

With Eq. (1), we obtain the strong decay width formula,

$$\Gamma = \frac{|\vec{P}_{f_1}|}{24\pi M^2} \Sigma |T(D_1^0 \rightarrow D^{*+} \pi^-)|^2. \quad (2)$$

M is the mass of the initial meson D_1^0 , and \vec{P}_{f_1} is the

three-momentum of the final heavy meson D^{*+} .

With the hadronic matrix element

$$\langle D^{*+}(P_{f1}) | \bar{d} \gamma_\mu \gamma_5 u | D_1^0(P) \rangle,$$

we will obtain the result of Eq. (1) and Eq. (2). The hadronic matrix element will be discussed in the next subsection.

2.2 Hadronic matrix elements of strong decays

In this subsection, we will give the calculation of the hadronic matrix element by the improved B-S method. The improved B-S method, which considers relativistic effects, is a good way to describe bound states. Using the improved B-S method, the instantaneous approach and the Mandelstam formalism [42], we can get the hadronic matrix elements. As an example, the hadronic matrix element of $D_1^0 \rightarrow D^{*+} \pi^-$ can be written as [34],

$$\begin{aligned} & \langle D^{*+}(P_{f1}) | \bar{d} \gamma_\mu \gamma_5 u | D_1^0(P) \rangle \\ &= \int \frac{d\vec{q}_f}{(2\pi)^3} Tr \left[\bar{\varphi}_{P_{f1}}^{++}(\vec{q}_f) \gamma_\mu \gamma_5 \varphi_P^{++}(\vec{q}) \frac{\not{P}}{M} \right], \end{aligned} \quad (3)$$

where P and M are the momentum and mass of the initial state D_1^0 , and \vec{q} and $\vec{q}_f = \vec{q} - \frac{m_c}{m_c + m_d} \vec{P}_{f1}$ are the relative three-momentum of the quark and antiquark in the initial state D_1^0 and final state D^{*+} , respectively. $\varphi_P^{++}(\vec{q})$ and $\varphi_{P_{f1}}^{++}(\vec{q}_f)$ are the positive energy wavefunctions of D_1^0 and D^{*+} , which are given in the next section.

3 Relativistic wavefunctions

In the improved B-S method, which is based on the constituent quark model, the forms of wave functions are obtained by considering the quantum numbers J^P or J^{PC} for different bound states, which are labelled as $^3S_1(1^-)$, $^3P_1(1^+)$ and $^1P_1(1^+)$ and so on. In this paper, we consider the strong decays of the P -wave mixing states ($1'^+$, 1^+), which are mixtures of $^3P_1(1^+)$ and $^1P_1(1^+)$. So we only discuss the relativistic wavefunctions of the $^1P_1(1^+)$, $^3P_1(1^+)$ and $^3S_1(1^-)$ states.

3.1 Wavefunctions of the 1P_1 state

The general expression for the Salpeter wave function of the 1P_1 state, which has $J^P = 1^+$ (or $J^{PC} = 1^{+-}$ for quarkonium), can be written as [35, 43]:

$$\varphi_{1^{+-}}(\vec{q}) = q_\perp \cdot \epsilon \left[f_1(\vec{q}) + \frac{\not{P}}{M} f_2(\vec{q}) + \frac{q_\perp}{M} f_3(\vec{q}) + \frac{\not{P} q_\perp}{M^2} f_4(\vec{q}) \right] \gamma_5, \quad (4)$$

where $q_\perp = q - \frac{q \cdot P}{M}$ is the relative momentum between the quark and anti-quark in the state, P and M are the momentum and mass of the 1^+ meson, and ϵ is the polarization vector. In the center-of-mass system of the meson, $q_\perp = (0, \vec{q})$. The wave functions, f_1 , f_2 , f_3 and f_4 , which are functions of q_\perp^2 , are not independent, but

are related because of the constraint equations of the Salpeter equation [35, 43],

$$f_3 = -\frac{M(w_1 - w_2)}{m_1 w_2 + m_2 w_1} f_1, \quad f_4 = -\frac{M(w_1 + w_2)}{m_1 w_2 + m_2 w_1} f_2, \quad (5)$$

where m_1, m_2, w_1, w_2 are the masses and momenta of the quark and anti-quark in the 1^+ state, respectively, and $w_1 = \sqrt{m_1^2 + \vec{q}^2}$ and $w_2 = \sqrt{m_2^2 + \vec{q}^2}$. With this wavefunction we can obtain the corresponding positive wavefunction of the 1P_1 state,

$$\varphi_{1^{++}}(\vec{q}) = q_\perp \cdot \epsilon \left[a_1 + a_2 \frac{\not{P}}{M} + a_3 \frac{q_\perp}{M} + a_4 \frac{\not{P} q_\perp}{M^2} \right] \gamma_5, \quad (6)$$

where the coefficients $a_1 \dots a_4$ have been defined in Ref. [35],

$$\begin{aligned} a_1 &= \frac{1}{2} (f_1(\vec{q}) + \frac{w_1 + w_2}{m_1 + m_2} f_2(\vec{q})), & a_2 &= \frac{m_1 + m_2}{w_1 + w_2} a_1, \\ a_3 &= -\frac{M(w_1 - w_2)}{m_1 w_2 + w_1 m_2} a_1, & a_4 &= -\frac{M(m_1 + m_2)}{m_1 w_2 + w_1 m_2} a_1. \end{aligned}$$

3.2 Wavefunctions of the 3P_1 state

The general expression for the Salpeter wave function of the 3P_1 state, for which $J^P = 1^+$ (or $J^{PC} = 1^{++}$ for quarkonium), can be written as [35, 43]:

$$\begin{aligned} \varphi_{3P_1}(\vec{q}) &= i \varepsilon_{\mu\nu\alpha\beta} \frac{P^\nu}{M} q_\perp^\alpha \epsilon^\beta \left[g_1 \gamma^\mu + g_2 \frac{\not{P}}{M} \gamma^\mu \right. \\ &\quad \left. + g_3 \frac{q_\perp}{M} \gamma^\mu + g_4 \frac{\not{P} \gamma^\mu q_\perp}{M^2} \right]. \end{aligned} \quad (7)$$

According to the relations of the constraint equations of the Salpeter equation [35, 43], we have:

$$g_3 = -\frac{M(w_1 - w_2)}{m_1 w_2 + m_2 w_1} g_1, \quad g_4 = -\frac{M(w_1 + w_2)}{m_1 w_2 + m_2 w_1} g_2. \quad (8)$$

Then, we can get the positive energy wavefunction of the 3P_1 state [35],

$$\varphi_{3P_1}^{++}(\vec{q}) = i \varepsilon_{\mu\nu\alpha\beta} \frac{P^\nu}{M} q_\perp^\alpha \epsilon^\beta \gamma^\mu \left[b_1 + b_2 \frac{\not{P}}{M} + b_3 \frac{q_\perp}{M} + b_4 \frac{\not{P} q_\perp}{M^2} \right]. \quad (9)$$

The coefficients $b_1 \dots b_4$ have been defined in Ref. [35],

$$\begin{aligned} b_1 &= \frac{1}{2} (g_1(\vec{q}) + \frac{w_1 + w_2}{m_1 + m_2} g_2(\vec{q})), & b_2 &= -\frac{m_1 + m_2}{w_1 + w_2} b_1, \\ b_3 &= \frac{M(w_1 - w_2)}{m_1 w_2 + w_1 m_2} b_1, & b_4 &= -\frac{M(m_1 + m_2)}{m_1 w_2 + w_1 m_2} b_1. \end{aligned}$$

3.3 Wavefunctions of the 3S_1 state

The general form for the relativistic wavefunction of the vector state $J^P = 1^-$ (or $J^{PC} = 1^{--}$ for quarkonium) can be written as eight terms, which are constructed by

P_{f_1} , q_{f_\perp} , ϵ_1 and gamma matrices [44],

$$\begin{aligned} \varphi_{1^-}(\vec{q}_f) = & q_{f_\perp} \cdot \epsilon_1 \left[f'_1 + \frac{\mathcal{P}_{f_1}}{M_{f_1}} f'_2 + \frac{q'_{f_\perp}}{M_{f_1}} f'_3 + \frac{\mathcal{P}_{f_1} q'_{f_\perp}}{M_{f_1}^2} f'_4 \right] \\ & + M_{f_1} \not{\epsilon}_1 f'_5 + \not{\epsilon}_1 \mathcal{P}_{f_1} f'_6 + (q'_{f_\perp} \not{\epsilon}_1 - q_{f_\perp} \cdot \epsilon_1) f'_7 \\ & + \frac{1}{M_{f_1}} (\mathcal{P}_{f_1} \not{\epsilon}_1 q'_{f_\perp} - \mathcal{P}_{f_1} q_{f_\perp} \cdot \epsilon_1) f'_8, \end{aligned} \quad (10)$$

where ϵ_1 is the polarization vector of the vector meson in the final state.

According to the relations of the constraint equations of the Salpeter equation [35, 43], we have:

$$\begin{aligned} f'_1 &= \frac{[q'^2_{f_\perp} f'_3 + M_{f_1}^2 f'_5] (m'_1 m'_2 - w'_1 w'_2 + q'^2_{f_\perp})}{M_{f_1} (m'_1 + m'_2) q'^2_{f_\perp}}, \\ f'_7 &= \frac{f'_5 (q_{f_\perp}) M_{f_1} (-w'_1 + w'_2)}{(m'_1 w'_2 + m'_2 w'_1)}, \\ f'_2 &= \frac{[-q'^2_{f_\perp} f'_4 + M_{f_1}^2 f'_6 (q_{f_\perp})] (m'_1 w'_2 - m'_2 w'_1)}{M_{f_1} (w'_1 + w'_2) q'^2_{f_\perp}}, \\ f'_8 &= \frac{f'_6 M_{f_1} (w'_1 w'_2 - m'_1 m'_2 - q'^2_{f_\perp})}{(m'_1 + m'_2) q'^2_{f_\perp}}. \end{aligned}$$

The relativistic positive wavefunction of the 3S_1 state can be written as:

$$\begin{aligned} \varphi_{1^{++}}(\vec{q}_f) = & b_1 \not{\epsilon}_1 + b_2 \not{\epsilon}_1 \mathcal{P}_{f_1} + b_3 (q'_{f_\perp} \not{\epsilon}_1 - q_{f_\perp} \cdot \epsilon_1) \\ & + b_4 (\mathcal{P}_{f_1} \not{\epsilon}_1 q'_{f_\perp} - \mathcal{P}_{f_1} q_{f_\perp} \cdot \epsilon_1) \\ & + q_{f_\perp} \cdot \epsilon_1 (b_5 + b_6 \mathcal{P}_{f_1} + b_7 q'_{f_\perp} + b_8 q_{f_\perp} \mathcal{P}_{f_1}), \end{aligned} \quad (11)$$

where we first define the parameters n_i , which are functions of f'_i (3S_1 wave functions):

$$\begin{aligned} n_1 &= f'_5 - f'_6 \frac{(w'_1 + w'_2)}{(m'_1 + m'_2)}, \\ n_2 &= f'_5 - f'_6 \frac{(m'_1 + m'_2)}{(w'_1 + w'_2)}, \\ n_3 &= f'_3 + f'_4 \frac{(m'_1 + m'_2)}{(w'_1 + w'_2)}, \end{aligned}$$

then we define the parameters b_i , which are functions of f'_i and n_i :

$$\begin{aligned} b_1 &= \frac{M_{f_1}}{2} n_1, \\ b_2 &= -\frac{(m'_1 + m'_2)}{2(w'_1 + w'_2)} n_1, \\ b_3 &= \frac{M_{f_1} (w'_2 - w'_1)}{2(m'_1 w'_2 + m'_2 w'_1)} n_1, \\ b_4 &= \frac{(w'_1 + w'_2)}{2(w'_1 w'_2 + m'_1 m'_2 - q'^2_{f_\perp})} n_1, \\ b_5 &= \frac{1}{2M_{f_1}} \frac{(m'_1 + m'_2) (M_{f_1}^2 n_2 + q'^2_{f_\perp} n_3)}{(w'_1 w'_2 + m'_1 m'_2 + q'^2_{f_\perp})}, \end{aligned}$$

$$\begin{aligned} b_6 &= \frac{1}{2M_{f_1}^2} \frac{(w'_1 - w'_2) (M_{f_1}^2 n_2 + q'^2_{f_\perp} n_3)}{(w'_1 w'_2 + m'_1 m'_2 + q'^2_{f_\perp})}, \\ b_7 &= \frac{n_3}{2M_{f_1}} - \frac{f'_6 M_{f_1}}{(m'_1 w'_2 + m'_2 w'_1)}, \\ b_8 &= \frac{1}{2M_{f_1}^2} \frac{w'_1 + w'_2}{m'_1 + m'_2} n_3 - f'_5 \frac{w'_1 + w'_2}{(m'_1 + m'_2) (w'_1 w'_2 + m'_1 m'_2 - q'^2_{f_\perp})}. \end{aligned}$$

4 Mixing of 3P_1 and 1P_1 states

The heavy quark is in the $m_Q \rightarrow \infty$ limit in the heavy-light mesons, so the properties of heavy-light mesons are similar to those of the hydrogen atom (which are determined by the orbital electron). The spin of the heavy quark decouples and the properties of heavy-light mesons are determined by light degrees of freedom alone. For the P -wave heavy-light mesons (orbital angular momentum $L=1$), the total angular momentum $j_q = s_q + L$ of the light quark has two conditions ($j_q = \frac{1}{2}$ and $j_q = \frac{3}{2}$) with $s_q = \frac{1}{2}$. The whole angular momentum $J = s_Q + j_q$ and the spin of the heavy quark $S_Q = \frac{1}{2}$, so there are two degenerate doublets for P -wave heavy-light mesons: $j_q = \frac{1}{2}$ S doublet ($J^P = 0^+, 1^+$) states and $j_q = \frac{3}{2}$ T doublet ($J^P = 1^+, 2^+$) states. In this paper, we only talk about the two physical P -wave heavy-light mixing $J^P = 1^+$ states. In order to distinguish the two 1^+ states, we use the notation $P_1^{1/2}$ (which is from the $j_q = \frac{1}{2}$ S doublet) and $P_1^{3/2}$ (which is from the $j_q = \frac{3}{2}$ T doublet) to describe the two physical P -wave heavy-light mixing states.

In our model, we give expressions for the wavefunctions in term of the quantum numbers J^P (or J^{PC}), which describe the equal mass systems in heavy mesons very well. There are two physical P -wave states $^3P_1(1^{++})$ and $^1P_1(1^{+-})$ for equal mass systems, but they are not physical states when there is no charge conjugation parity for an unequal mass system. In the heavy quark limit, the physical states $P_1^{1/2}$ and $P_1^{3/2}$ of P -wave heavy-light mesons can be written as [9, 10, 45]:

$$\begin{aligned} |P_1^{3/2}\rangle &= \sqrt{\frac{2}{3}} |^1P_1\rangle + \sqrt{\frac{1}{3}} |^3P_1\rangle \\ |P_1^{1/2}\rangle &= -\sqrt{\frac{1}{3}} |^1P_1\rangle + \sqrt{\frac{2}{3}} |^3P_1\rangle, \end{aligned} \quad (12)$$

where Eq. (12) is the same as the result of Ref. [35, 38] if we take $\theta = 35.3^\circ$. So the wavefunctions for the physical $P_1^{1/2}$ and $P_1^{3/2}$ states can be obtained by the mixing relations of the 3P_1 and 1P_1 wavefunctions which were shown in Section 3. However, when we solve the B-S equation, the mass of the heavy quark is a fixed value which is not infinite. So, considering the correction to the heavy quark limit, the physical states 1^{++} and 1^+ are mixtures of $P_1^{1/2}$ and $P_1^{3/2}$ [15, 19, 39],

$$|1^+\rangle = \cos\theta_1 |P_1^{3/2}\rangle + \sin\theta_1 |P_1^{1/2}\rangle$$

Table 1. Mass spectra of the P -wave mixing states $1'^+$ and 1^+ , in units of MeV. ‘ex.’ means the experimental data from the PDG [1], and ‘th.’ means our prediction.

states	th.	ex.	states	th.	ex.	states	th.	ex.
$D_1^{0'}$	2427.0	$2427 \pm 26 \pm 25$	$D_{s1}^{\prime\pm}$	2460.0	2459.6 ± 0.9	$B_1^{0'}$	5710.0	–
D_1^0	2422.0	2421.4 ± 0.6	D_{s1}^{\pm}	2536.0	2535.18 ± 0.24	B_1^0	5726.0	5726.0 ± 1.3
$D_1^{\prime\pm}$	2427.0	–	B'_{s1}	5820.0	–	$B_1^{\prime\pm}$	5710.0	–
D_1^{\pm}	2422.0	2423.2 ± 2.4	B_{s1}	5829.0	5828.78 ± 0.35	B_1^{\pm}	5726.0	$5726.8^{+3.2}_{-4.0}$

 Table 2. Decay widths of two-body strong decays of D^0 mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.1 \pm 0.05) \text{ rad} = -(5.7 \pm 2.9)^\circ$ [15, 19].

mode	$\theta = 35.3^\circ$	$\theta = 35.3^\circ + \theta_1$	[17]	[21]	[22]	[24]	[1]
$D_1^{0'} \rightarrow D^* \pi$	232	$228 \sim 232$	–	244	272	220	$384^{+107}_{-75} \pm 74$
$D_1^0 \rightarrow D^* \pi$	17.3	$17.6 \sim 21.5$	11	25	22	21.6	27.4 ± 2.5

$$|1'^+\rangle = -\sin\theta_1 |P_1^{3/2}\rangle + \cos\theta_1 |P_1^{1/2}\rangle, \quad (13)$$

when in the heavy quark limit, e.g: $\theta_1 = 0^\circ$, $|1'^+\rangle = |P_1^{1/2}\rangle$ and $|1^+\rangle = |P_1^{3/2}\rangle$. For the 1^+ states D and D_s , $\theta_1 = -(0.1 \pm 0.05) \text{ rad} = -(5.7 \pm 2.9)^\circ$ [15, 19]. For the 1^+ states B and B_s , $\theta_1 = -(0.03 \pm 0.015) \text{ rad} = -(1.72 \pm 0.86)^\circ$ [19]. Taking Eq. (12) into Eq. (13), we can get the relation of $1'^+$, 1^+ and 3P_1 , 1P_1 ,

$$\begin{aligned} |1^+\rangle &= \cos\theta |{}^1P_1\rangle + \sin\theta |{}^3P_1\rangle \\ |1'^+\rangle &= -\sin\theta |{}^1P_1\rangle + \cos\theta |{}^3P_1\rangle, \end{aligned} \quad (14)$$

where $\theta = 35.3^\circ + \theta_1$. According to Eq. (14), we have obtained the mass spectra of the P -wave mixing states $1'^+$ and 1^+ by solving the full Salpeter equation in Table 1.

5 Numerical results and discussion

In order to fix the Cornell potentials and masses of quarks, we take these parameters: $a=e=2.7183$, $\lambda=0.21 \text{ GeV}^2$, $\Lambda_{\text{QCD}} = 0.27 \text{ GeV}$, $\alpha = 0.06 \text{ GeV}$, $m_u = 0.305 \text{ GeV}$, $m_d = 0.311 \text{ GeV}$, $m_s = 0.500 \text{ GeV}$, $m_b = 4.96 \text{ GeV}$, $m_c = 1.62 \text{ GeV}$, etc [46], which are best to fit the mass spectra of ground state B and D mesons and other heavy mesons. We get the following masses of the ground states: $M_{D^{*0}} = 2.007 \text{ GeV}$, $M_{D^{*\pm}} = 2.010 \text{ GeV}$, $M_{D_s^{*\pm}} = 2.112 \text{ GeV}$, $M_{B^*} = 5.325 \text{ GeV}$, and $M_{B_s^*} = 5.415 \text{ GeV}$. For the light mesons, the masses and decay constants are: $M_\pi = 0.140 \text{ GeV}$, $f_\pi = 0.130 \text{ GeV}$, $M_K = 0.494 \text{ GeV}$, and $f_K = 0.156 \text{ GeV}$ [1].

5.1 $D_1^{0'}$, D_1^0 and $D_1^{\prime\pm}$, D_1^\pm

The $D_1^{0'}$, D_1^0 and D_1^\pm states have been listed in the PDG [1]. $D_1^{0'}$ is a broad state with large uncertainty, and D_1^0 and D_1^\pm are narrow states with small uncertainty, but there is no evidence of another state $D_1^{\prime\pm}$ in experiment. We therefore predict the mass of $D_1^{\prime\pm}$ to be the same as $D_1(2430)^0$ by the improved B-S method in Table 1. The two-body strong decays of these states only happens in the $D^* \pi$ channel, which is OZI-allowed.

We calculate the transition matrix elements by the wavefunctions numerically, and get the strong decay

widths of $D_1^{0'}$ and D_1^0 . Our predicted results and those of other authors are given in Table 2. The results are shown for the two conditions of the mixing angle θ : $\theta = 35.3^\circ$, and $\theta = 35.3^\circ + \theta_1$. We find that the results of $D_1^{0'} \rightarrow D^* \pi$ are very close for the two conditions. Both of them are smaller than the central experiment value, but if we consider the large uncertainty of the experiment value for $D_1^{0'}$, our results seem reasonable. We also find that our results are consistent with the results of Ref. [21] and Ref. [24], but smaller than the result of Ref. [22]. Though the predicted masses of the P -wave mixing states are similar for different models, the predicted decay widths are much different. The situation is similar in other channels. For example, in our prediction of $D_1^0 \rightarrow D^* \pi$, when we consider the correction to the heavy quark limit, the decay width is increased, which is consistent with the results of other models and close to the lower limit of the experimental value. In Fig. 2, we plot the relation of mixing angle θ to the strong decay widths of $D_1^{0'}$ and D_1^0 . The $D_1^{0'}$ meson is a broad state and the influence of the mixing angle is very small. The width of the D_1^0 meson is at the bottom of the curve, and is sensitive to the mixing angle. We determine the mixing angle $\theta = 35.3^\circ + \theta_{1\text{min}} = 26.7^\circ$ to be the best description of the experimental value. But for the $D_1^{0'}$ meson, since there

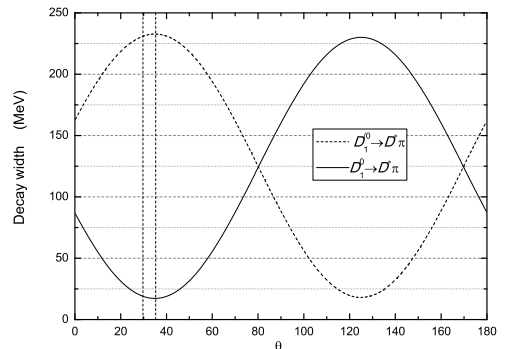
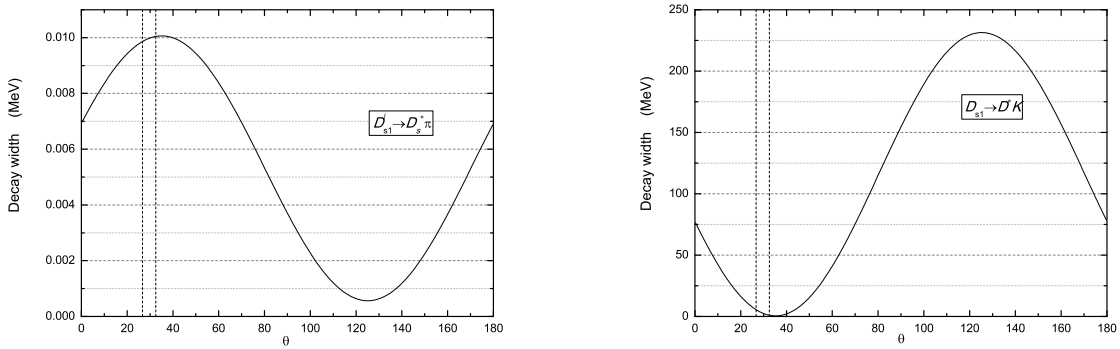

 Fig. 2. Decay widths of $D_1^{0'}$ and D_1^0 .

Table 3. Decay widths of two-body strong decays of D^\pm mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.1 \pm 0.05) \text{ rad} = -(5.7 \pm 2.9)^\circ$ [15, 19].

mode	$\theta=35.3^\circ$	$\theta=35.3^\circ+\theta_1$	[17]	[21]	[22]	[24]	[1]
$D_1^{\prime\pm} \rightarrow D^* \pi$	232	228~232	–	244	272	220	–
$D_1^\pm \rightarrow D^* \pi$	17.2	18.1~22.0	11	25	22	21.6	25 ± 6

 Table 4. Decay widths of two-body strong decays of D_s mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.1 \pm 0.05) \text{ rad} = -(5.7 \pm 2.9)^\circ$ [15, 19].

mode	$\theta=35.3^\circ$	$\theta=35.3^\circ+\theta_1$	[16]	[17]	[21]	[22]	[24]	[26]	[1]
$D_{s1}^{\prime\pm} \rightarrow D_s^* \pi$	0.0101	0.00984~0.0100	0.0215	–	~ 0.010	–	–	0.01141	< 3.5
$D_{s1}^\pm \rightarrow D^* K$	0.449	0.950~5.46	–	< 1	0.340	0.800	0.350	–	0.92 ± 0.05


 Fig. 3. Decay widths of $D_{s1}^{\prime\pm}$ and D_{s1}^\pm .

are large uncertainties on the experimental value, much stronger experimental confirmation is needed in the future. We give the strong decay widths of $D_1^{\prime\pm}$ and D_1^\pm in Table 3. The strong decay widths of $D_1^{\prime\pm}$ and D_1^\pm are very similar to the results for $D_1^{\prime 0}$ and D_1^0 , because of the masses of the light quarks in the P -wave mixing states are very close: $m_u \approx m_d$. The strong decays of $D_1^{\prime\pm}$ also provide a good way to observe this meson in experiment.

5.2 $D_{s1}^{\prime\pm}$ and D_{s1}^\pm

Both $D_{s1}^{\prime\pm}$ and D_{s1}^\pm have small widths. The $D_{s1}^{\prime\pm}$ meson is below the threshold of D^* and K , so the OZI-allowed strong decay is forbidden, and the dominant strong decay channel of the $D_{s1}^{\prime\pm}$ meson is the isospin symmetry violating decay via π^0 - η mixing, which is $D_{s1}^{\prime\pm} \rightarrow D_s^* \pi \eta \rightarrow D_s^{\prime\pm} \pi^0$ [47]. Because of the small value of the mixing parameter $t_{\pi\eta} = \langle \pi^0 | \mathcal{H} | \eta \rangle = -0.003 \text{ GeV}^2$ [47], the decay is heavily suppressed, and the decay width is very narrow. The mass of D_{s1}^\pm is larger than the threshold of $D^* K$, but the width is also very narrow because of the small kinematic range.

We have given two results of our method in Table 4: (1) $\theta=35.3^\circ$; (2) considering the correction $\theta=35.3^\circ+\theta_1$. For the strong decay $D_{s1}^{\prime\pm} \rightarrow D_s^* \pi$, the results are very similar for the two conditions and close to the results of Ref. [21] and Ref. [26]. The results for $D_{s1}^{\prime\pm} \rightarrow D_s^* \pi$ are smaller than the result from Ref. [16], but they are reli-

able compared with the experimental result. We find that the results for $D_{s1}^\pm \rightarrow D^* K$ are sensitive to the mixing angle θ . When the mixing angle $\theta = 35.3^\circ$, the decay width of D_{s1}^\pm is close to the results of Ref. [21] and Ref. [24], but smaller than the result of Ref. [22] and the experimental value in Ref. [1]. With the correction to the mixing angle, we get the decay width of D_{s1}^\pm as: $\Gamma(D_{s1}^\pm) = 0.950 \sim 5.46 \text{ MeV}$. In order to compare with the experimental data, we plot the relation of strong decay width $D_{s1}^{\prime\pm}$ and D_{s1}^\pm against mixing angle θ in Fig. 3. The results for $D_{s1}^{\prime\pm}$ are at the bottom of the curve and close to zero with both conditions, so the result for D_{s1}^\pm is sensitive to mixing angle. It shows that with $\theta = 32.5^\circ$, the corresponding result $\Gamma(D_{s1}^\pm) = 0.950 \text{ MeV}$, which is consistent with the experimental data of $D_{s1}(2536)^\pm$: $\Gamma = 0.92 \pm 0.05 \text{ MeV}$ [1].

5.3 B_1^0 , B_1^0 and $B_1^{\prime\pm}$, B_1^\pm

Experiment has only observed B_1^0 and B_1^\pm , which are considered to be $B_1(5721)^0$ and $B_1(5721)^+$ [2–4], and there is no evidence for the $B_1^{\prime 0}$ and $B_1^{\prime\pm}$. So we first predict the masses of the $B_1^{\prime 0}$ and $B_1^{\prime\pm}$, then calculate the strong decays of these mesons. Because of the large kinematic range, all of these four states have OZI-allowed strong decay channels.

We show the strong decay widths for the P -wave mixing B mesons, under the same two conditions as the P -

Table 5. Decay widths of two-body strong decays of B^0 mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.03 \pm 0.015)$ rad $= -(1.72 \pm 0.86)^\circ$ [19].

mode	$\theta = 35.3^\circ$	$\theta = 35.3^\circ + \theta_1$	[20]	[24]	[27]	[1]
$B_1^{\prime 0} \rightarrow B^* \pi$	262.4	262.1~262.4	250	219	139	–
$B_1^0 \rightarrow B^* \pi$	15.6	15.6~16.0	–	30	20	$23 \pm 3 \pm 4$

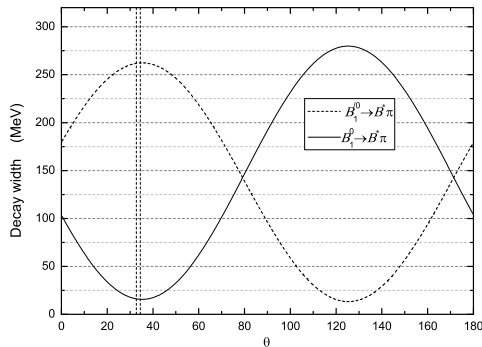
 Table 6. Decay widths of two-body strong decays of B^\pm mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.03 \pm 0.015)$ rad $= -(1.72 \pm 0.86)^\circ$ [19].

mode	$\theta = 35.3^\circ$	$\theta = 35.3^\circ + \theta_1$	[20]	[24]	[27]	[1]
$B_1^{\prime \pm} \rightarrow B^* \pi$	263	262.4~262.9	250	219	139	–
$B_1^\pm \rightarrow B^* \pi$	16.0	16.1~16.8	–	30	20	31 ± 6

 Table 7. Decay widths of two-body strong decays of B_s mixing states $1'^+$ and 1^+ , in units of MeV, with $\theta_1 = -(0.03 \pm 0.015)$ rad $= -(1.72 \pm 0.86)^\circ$ [19].

mode	$\theta = 35.3^\circ$	$\theta = 35.3^\circ + \theta_1$	[16]	[17]	[24]	[26]	[28]	[29]	[1]
$B_{s1}^{\prime} \rightarrow B_s^* \pi$	0.01987	0.01986~0.01987	0.0215	–	–	0.01036	–	–	–
$B_{s1} \rightarrow B^* K$	0.0396	0.0834~0.412	–	< 1	0.4~1	–	$0.7 \pm 0.3 \pm 0.3$	0.098	$0.5 \pm 0.3 \pm 0.3$

wave mixing D mesons, in Table 5 and Table 6. For comparison, we list the experimental data and some results from other model predictions, and plot the relations of the decay widths of B_1^0 and B_1^0 against the mixing angle θ in Fig. 4. For B_1^0 and B_1^{\pm} , the decay widths with the two conditions are very close, which is consistent with the result of the QCD sum rules: $\Gamma(B_1^0) \simeq 250$ MeV [20]. The decay widths of B_1^0 and B_1^\pm are close to the result of Ref. [20] and smaller than the results of Ref. [24] and Ref. [27]. The results for B_1^\pm are also much smaller than the experimental value, but the results for B_1^0 are very close to the upper limit of the experimental value. Because the experimental results for B_1^\pm have large uncertainties, they need to be confirmed experimentally in the future, and our results will provide theoretical assistance.


 Fig. 4. Decay widths of B_1^0 and B_1^0 .

5.4 B_{s1}^{\prime} and B_{s1}

In order to compare the results with the two conditions of mixing angle θ for the P -wave B_s^* mixing states

B_{s1}^{\prime} and B_{s1} , we also show the corresponding strong decay widths of B_{s1}^{\prime} and B_{s1} in Table 7. For the B_{s1}^{\prime} meson, the OZI-allowed strong decay is forbidden, and the dominant strong decay of the B_{s1}^{\prime} meson is the isospin symmetry violating decay via π^0 - η mixing [47]. So the decay widths of B_{s1}^{\prime} are very close for both conditions, and the influence of the mixing angle is very small. For the decay width of the B_{s1} meson, however, there is a big difference between the two conditions, and the influence of the angle is very large. In our calculation, we plot the decay widths of the mixed states B_{s1}^{\prime} and B_{s1} as functions of θ in Fig. 5. The results for the two conditions are at the bottom of the curve and close to zero, which is the same as for D_{s1}^{\pm} . If we take the mixing angle $\theta = 32.7^\circ$, we get the decay width $\Gamma(B_{s1}) = 0.412$ MeV, which is close to the experimental data, within the large uncertainties.

In conclusion, we have studied the strong decays of the P -wave mixing heavy-light 1^+ states by the improved B-S method with two conditions of mixing angle θ : $\theta = 35.3^\circ$, and considering a correction to the mixing angle, $\theta = 35.3^\circ + \theta_1$. We find that, for the P -wave $1'^+$ mesons (D_1^0 , D_1^{\pm} , D_{s1}^{\pm} , B_1^0 , B_1^{\pm} and B_{s1}^{\prime}) and some 1^+ states (B_1^0 and B_1^{\pm}), the influence of mixing angle θ between 3P_1 and 1P_1 is very small, and the results with the two conditions are very close. However, for some of the P -wave 1^+ mesons (D_1^0 , D_1^{\pm} , D_{s1}^{\pm} and B_{s1}), the influence of the mixing angle θ between 3P_1 and 1P_1 is large; especially for the D_{s1}^{\pm} and B_{s1} mesons, there is a large discrepancy between the two conditions. For the D_1^0 and D_1^{\pm} states, we take the mixing angle $\theta = 26.7^\circ$, which is the best description of the experimental value. For the D_{s1}^{\pm} state, the mixing angle $\theta = 32.5^\circ$ is the best description of the experimental value. For the B_{s1} state, the result for the mixing angle $\theta = 32.7^\circ$ is close to the exper-

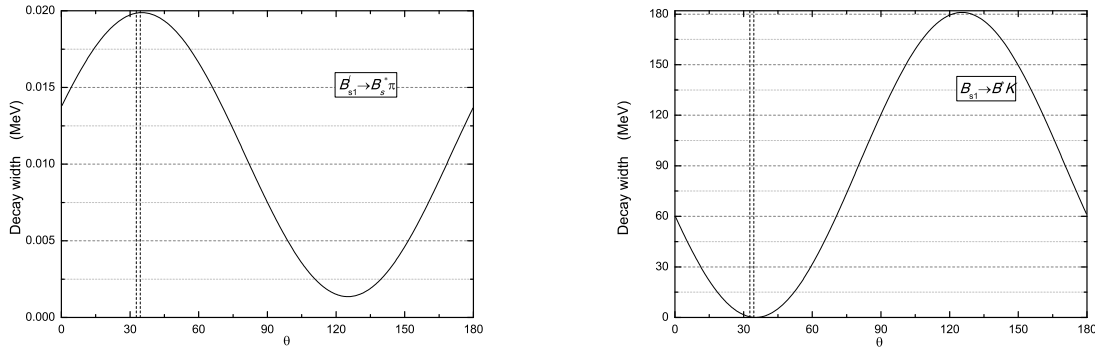


Fig. 5. Decay widths of B'_{s1} and B_{s1} .

imental data, within large uncertainties. In this paper, we have studied the strong decays of some special states which have not yet been discovered in experiment, such as $D'_1{}^\pm$, $B'_1{}^0$, $B'_1{}^\pm$ and B'_{s1} . This will provide theoretical assistance to future experiments. We have also investi-

gated the strong decays of $D'_{s1}{}^\pm$, $B_1{}^0$, $B_1{}^\pm$ and B_{s1} with large uncertainties for the experimental data, and given predicted results, which need to be confirmed by future experiments.

References

- 1 C. Patrignani et al (Particle Data Group), Chin. Phys. C, **40**: 100001 (2016)
- 2 V. M. Abazov et al (D0 Collaboration), Phys. Rev. Lett., **99**: 172001 (2007)
- 3 T. Aaltonen et al (CDF Collaboration), Phys. Rev. Lett., **102**: 102003 (2009)
- 4 T. Aaltonen et al (CDF Collaboration), Phys. Rev. D, **90**: 012013 (2014)
- 5 R. Aaij et al (LHCb Collaboration), JHEP, **04**: 024 (2015)
- 6 T. Aaltonen et al (CDF Collaboration), Phys. Rev. Lett., **100**: 082001 (2008)
- 7 V. M. Abazov et al (D0 Collaboration), Phys. Rev. Lett., **100**: 082002 (2008)
- 8 R. Aaij et al (LHCb Collaboration), Phys. Rev. Lett., **110**: 151803 (2013)
- 9 N. Isgur and M. B. Wise, Phys. Lett. B, **232**: 113 (1989)
- 10 N. Isgur and M. B. Wise, Phys. Lett. B, **237**: 527 (1990)
- 11 D. Mohler and R. M. Woloshyn, Phys. Rev. D, **84**: 054505 (2011)
- 12 Y. Sun, Q. T. Song, D. Y. Chen et al. Phys. Rev. D, **89**: 054026 (2014)
- 13 L. Y. Xiao and X. H. Zhong, Phys. Rev. D, **90**: 074029 (2014)
- 14 H. X. Chen, W. Chen, X. Liu et al, Rept. Prog. Phys., **80**(7): 076201 (2017)
- 15 H. Y. Cheng, Phys. Rev. D, **68**: 094005 (2003)
- 16 W. A. Bardeen, E. J. Eichten, and C. T. Hill, Phys. Rev. D, **68**: 054024 (2003)
- 17 E. J. Eichten, C. T. Hill, and C. Quigg, Phys. Rev. Lett., **71**: 4116 (1993)
- 18 M. Cleven et al, Eur. Phys. J. A, **50**: 149 (2014)
- 19 Y. B. Dai and S. L. Zhu, Phys. Rev. D, **58**: 074009 (1998)
- 20 S. L. Zhu and Y. B. Dai, Mod. Phys. Lett. A, **14**: 2367 (1999)
- 21 S. Godfrey, Phys. Rev. D, **72**: 054029 (2005)
- 22 F. E. Close and E. S. Swanson, Phys. Rev. D, **72**: 094004 (2005)
- 23 W. Wei, P. Z. Huang, and S. L. Zhu, Phys. Rev. D, **73**: 034004 (2006)
- 24 X. H. Zhong and Q. Zhao, Phys. Rev. D, **78**: 014029 (2008)
- 25 X. G. Wu and Q. Zhao, Phys. Rev. D, **85**: 034040 (2012)
- 26 F. K. Guo, P. N. Shen, and H. C. Chiang, Phys. Lett. B, **647**: 133 (2007)
- 27 A. H. Orsland and H. Hogaasen, Eur. Phys. J. C, **9**: 503 (1999)
- 28 Z. G. Wang, Eur. Phys. J. C, **129**: 186 (2014)
- 29 Z. G. Luo, X. L. Chen, and X. Liu, Phys. Rev. D, **79**: 074020 (2009)
- 30 P. Maris and C. D. Roberts, Phys. Rev. C, **56**: 3369 (1997)
- 31 P. Maris, C. D. Roberts, and P. C. Tandy, Phys. Lett. B, **420**: 267 (1998)
- 32 M. A. Ivanov, Yu. L. Kalinovsky, P. Maris et al, Phys. Rev. C, **57**: 1991 (1998)
- 33 B. El-Bennich, M. A. Ivanov, and C. D. Roberts, Phys. Rev. C, **83**: 025205 (2011)
- 34 C. H. Chang, J. K. Chen, and G. L. Wang, Commun. Theor. Phys., **46**: 467 (2006)
- 35 Z. H. Wang, G. L. Wang, H. F. Fu et al, Phys. Lett. B, **706**: 389 (2012)
- 36 Y. Jiang, G. L. Wang, T. H. Wang et al, Chin. Phys. Lett., **30**: 101101 (2013)
- 37 Z. H. Wang, G. L. Wang, H. F. Fu et al, Int. J. Mod. Phys. A, **27**: 1250049 (2012)
- 38 Z. H. Wang, G. L. Wang, and C. H. Chang, J. Phys. G: Nucl. Part. Phys., **39**: 015009 (2012)
- 39 H. Y. Cheng and C. K. Chua, Phys. Rev. D, **74**: 034020 (2006)
- 40 C. H. Chang, C. S. Kim, and G. L. Wang, Phys. Lett. B, **623**: 218 (2005)
- 41 J. M. Zhang and G. L. Wang, Phys. Lett. B, **684**: 221 (2010)
- 42 S. Mandelstam, Proc. R. Soc. London, **233**: 248 (1955)
- 43 G. L. Wang, Phys. Lett. B, **650**: 15 (2007)
- 44 G. L. Wang, Phys. Lett. B, **633**: 492 (2006)
- 45 J. Rosner, Commun. Nucl. Part. Phys., **16**: 109 (1986)
- 46 C. H. Chang and G. L. Wang, Science in China Series G, **53**: 2005 (2010)
- 47 R. Dashen, Phys. Rev., **183**: 1245 (1969)