Weak decays of doubly heavy baryons: "decay constants" *

Xiao-Hui Hu(胡晓会)[1] Yue-Long Shen(沈月龙)[2] Wei Wang(王伟)[1;1)] Zhen-Xing Zhao(赵振兴)[1;2)]

[1] INPAC, Shanghai Key Laboratory for Particle Physics and Cosmology,
MOE Key Laboratory for Particle Physics, Astrophysics and Cosmology,
School of Physics and Astronomy, Shanghai Jiao-Tong University, Shanghai 200240, China
[2] College of Information Science and Engineering, Ocean University of China, Qingdao 266100, China

Abstract: Inspired by the recent observation of the Ξ_{cc}^{++} by the LHCb Collaboration, we explore the "decay constants" of doubly heavy baryons in the framework of QCD sum rules. With the Ξ_{cc},Ξ_{bc},Ξ_{bb}, and Ω_{cc},Ω_{bc},Ω_{bb} baryons interpolated by three-quark operators, we calculate the correlation functions using the operator product expansion and include the contributions from operators up to dimension six. On the hadron side, we consider both contributions from the lowest-lying states with $J^P=1/2^+$ and from negative parity baryons with $J^P=1/2^-$. We find that the results are stable and the contaminations from negative parity baryons are not severe. These results are ingredients for the QCD study of weak decays and other properties of doubly-heavy baryons.

Keywords: doubly heavy baryons, decay constant, QCD sum rules

PACS: 12.39.Hg **DOI:** 10.1088/1674-1137/42/12/123102

1 Introduction

For many years, it was widely believed that doubly heavy baryons with two charm and/or bottom quarks exist in reality, but experimental searches for such baryons took some time. The SELEX Collaboration first reported the discovery of Ξ_{cc}^{+} in the $\Lambda_c^+K^-\pi^+$ final state sixteen years ago [1, 2], with the mass measured as $m_{\Xi_{cc}^+}=(3519\pm1)$ MeV [1, 2]. However, the SELEX-like Ξ_{cc}^+ signal was not confirmed by later experiments [3–7]. In 2017, in the $\Lambda_c^+K^-\pi^+\pi^+$ final state, the LHCb Collaboration observed the doubly charmed baryon Ξ_{cc}^{++} with the mass [8]:

$$m_{\Xi_{cc}^{++}}=(3621.40\pm0.72\pm0.27\pm0.14)\text{ MeV}. \quad (1)$$

In order to decipher the internal nature of doubly heavy baryons and uncover the underlying dynamics in the transition, more experimental investigations of their production and decays are urgently needed. Meanwhile, further theoretical studies on weak decays of doubly heavy baryons will be of great importance [9–30], and in particular, there is a need for solid QCD analyses of weak decays and production.

In this work, we present an analysis of the "decay constant" of doubly heavy baryons in the framework of QCD sum rules (QCDSR). QCDSR have been extensively applied to study hadron masses, decay constants and transition form factors, the mixing matrix elements of K-meson and B-meson systems, etc. [31–40]. In this approach, hadrons are interpolated by the corresponding quark operators. The correlation functions of these operators can be handled using the operator product expansion (OPE), where the short-distance coefficients and long-distance quark-gluon interactions are separated. The former are calculable in QCD perturbation theory, whereas the latter can be parameterized in terms of vacuum condensates. The QCD result is then matched, via a dispersion relation, onto the observable characteristics of hadronic states. Due to various advantages, the QCDSR have been used to calculate the masses of doubly heavy baryons in Refs. [9, 25, 41–46]. The main motive of this work is to study "decay constants" using the QCDSR. The "decay constants" defined by the interpolating current are necessary inputs for studies of other properties of doubly heavy baryons in QCDSR, for example the heavy-to-light transition form factors.

The rest of the paper is arranged as follows. In Sec-

Received 28 June 2018, Published online 29 October 2018

∗ Supported by National Natural Science Foundation of China (11575110, 11655002, 11735010), Natural Science Foundation of Shanghai (15DZ2272100, 15ZR1423100), Shanghai Key Laboratory for Particle Physics and Cosmology and Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education

1) E-mail: wei.wang@sjtu.edu.cn

2) E-mail: star_0027@sjtu.edu.cn

Table 1. Quantum numbers and quark content for the ground state of doubly heavy baryons. The s_h denotes the spin of the heavy quark system.

baryon	quark content	s_h^π	J^P	baryon	quark content	s_h^π	J^P
Ξ_{cc}	$\{cc\}q$	1^+	$1/2^+$	Ξ_{bb}	$\{bb\}q$	1^+	$1/2^+$
Ξ_{cc}^*	$\{cc\}q$	1^+	$3/2^+$	Ξ_{bb}^*	$\{bb\}q$	1^+	$3/2^+$
Ω_{cc}	$\{cc\}s$	1^+	$1/2^+$	Ω_{bb}	$\{bb\}s$	1^+	$1/2^+$
Ω_{cc}^*	$\{cc\}s$	1^+	$3/2^+$	Ω_{bb}^*	$\{bb\}s$	1^+	$3/2^+$
Ξ_{bc}'	$\{bc\}q$	0^+	$1/2^+$	Ω_{bc}'	$\{bc\}s$	0^+	$1/2^+$
Ξ_{bc}	$\{bc\}q$	1^+	$1/2^+$	Ω_{bc}	$\{bc\}s$	1^+	$1/2^+$
Ξ_{bc}^*	$\{bc\}q$	1^+	$3/2^+$	Ω_{bc}^*	$\{bc\}s$	1^+	$3/2^+$

tion 2, we will present the calculation of correlation function in QCD sum rules, including the explicit expressions of the spectral functions. We include both the contributions from the $J^P=1/2^+$ baryons and the contamination from the $J^P=1/2^-$ baryons. Section 3 is devoted to the numerical results. A summary is presented in the last section.

2 QCD sum rules study

A doubly heavy baryon is made of two heavy quarks and one light quark. The quantum numbers and quark content of the ground states are given in Table 1. In this work we will study the $J^P=1/2^+$ baryons, which can only weakly decay.

2.1 QCD sum rules with only positive parity baryons

The interpolating current for the Ξ_{QQ} and Ω_{QQ} is chosen as

$$J_{\Xi_{QQ}} = \epsilon_{abc}\left(Q_a^T C\gamma^\mu Q_b\right)\gamma_\mu\gamma_5 q_c, \tag{2}$$

$$J_{\Omega_{QQ}} = \epsilon_{abc}\left(Q_a^T C\gamma^\mu Q_b\right)\gamma_\mu\gamma_5 s_c, \tag{3}$$

where $Q=c$ or $Q=b$. For the Ξ_{bc} and Ω_{bc}, we choose

$$J_{\Xi_{bc}} = \frac{1}{\sqrt{2}}\epsilon_{abc}\left(b_a^T C\gamma^\mu c_b+c_a^T C\gamma^\mu b_b\right)\gamma_\mu\gamma_5 q_c, \tag{4}$$

$$J_{\Omega_{bc}} = \frac{1}{\sqrt{2}}\epsilon_{abc}\left(b_a^T C\gamma^\mu c_b+c_a^T C\gamma^\mu b_b\right)\gamma_\mu\gamma_5 s_c. \tag{5}$$

In the above equations, we have considered the $s_h^\pi=1^+$ baryons only.

The QCDSR analysis starts with the two-point correlator:

$$\Pi(q) = \mathrm{i}\int \mathrm{d}^4x e^{\mathrm{i}q\cdot x}\langle 0|T[J(x),\bar{J}(0)]|0\rangle, \tag{6}$$

where the interpolating current has been given in the above, and $\bar{J}$ is defined as

$$\bar{J}=J^\dagger\gamma^0. \tag{7}$$

A Lorentz structure analysis implies that the two-point correlation function has the form:

$$\Pi(q)=\not{q}\Pi_1(q^2)+\Pi_2(q^2). \tag{8}$$

On the hadronic side, one can insert the complete set of hadronic states into the correlator and then the correlator can be expressed as a dispersion integral over a physical spectral function:

$$\Pi(q) = \lambda_H^2\frac{\not{q}+m_H}{m_H^2-q^2}+\frac{1}{\pi}\int_{s_0}^\infty \mathrm{d}s\frac{\mathrm{Im}\Pi(s)}{s-q^2}, \tag{9}$$

where H can be a ground-state doubly heavy baryon and m_H denotes its mass. In obtaining the above expression, the polarization summation for spinors has been used:

$$\sum_s u(q,s)\bar{u}(q,s)=\not{q}+m_H. \tag{10}$$

The pole residue λ_H is defined as

$$\langle 0|J_H|H(q,s)\rangle=\lambda_H u(q,s). \tag{11}$$

The mass dimension for λ_H is 3, while in analogy with the meson case, it is convenient to use the "decay constant" with the definition

$$\langle 0|J_H|H(q,s)\rangle=f_H m_H^2 u(q,s). \tag{12}$$

On the OPE side, we will work at leading order in α_s in this work and include the condensate contributions up to dimension six. The full propagator for the heavy quark is given as

$$S_{ij}^Q(x) = \mathrm{i}\int\frac{\mathrm{d}^4k}{(2\pi)^4}e^{-\mathrm{i}k\cdot x}\left[\frac{\delta_{ij}}{\not{k}-m_Q}-\frac{g_s G_{\alpha\beta}^a t_{ij}^a}{4}\frac{\sigma^{\alpha\beta}(\not{k}+m_Q)+(\not{k}+m_Q)\sigma^{\alpha\beta}}{(k^2-m_Q^2)^2}\right.$$
$$\left.+\frac{g_s D_\alpha G_{\beta\lambda}^n t_{ij}^n(f^{\lambda\beta\alpha}+f^{\lambda\alpha\beta})}{3(k^2-m_Q^2)^4}-\frac{g_s^2(t^at^b)_{ij}G_{\alpha\beta}^a G_{\mu\nu}^b(f^{\alpha\beta\mu\nu}+f^{\alpha\mu\beta\nu}+f^{\alpha\mu\nu\beta})}{4(k^2-m_Q^2)^5}\right], \tag{13}$$

with

$$f^{\lambda\alpha\beta}=(\not{k}+m_Q)\gamma^\lambda(\not{k}+m_Q)\gamma^\alpha(\not{k}+m_Q)\gamma^\beta(\not{k}+m_Q), \quad (14)$$

$$f^{\alpha\beta\mu\nu} = (\not{k}+m_Q)\gamma^\alpha(\not{k}+m_Q)\gamma^\beta(\not{k}+m_Q)\gamma^\mu(\not{k}+m_Q) \times\gamma^\nu(\not{k}+m_Q), \quad (15)$$

where $t^n=\lambda^n/2$ and λ^n is the Gell-Mann matrix, and the i,j are the color indices. The full propagator for light quarks is given as

$$\begin{aligned} S_{ij}(x) = {} & \frac{i\delta_{ij}\not{x}}{2\pi^2x^4}-\frac{\delta_{ij}}{12}\langle\bar{q}q\rangle-\frac{\delta_{ij}x^2\langle\bar{q}g_s\sigma Gq\rangle}{192} \\ & +\frac{i\delta_{ij}x^2\not{x}\langle\bar{s}g_s\sigma Gs\rangle m_q}{1152} \\ & -\frac{ig_sG_{\alpha\beta}t^a_{ij}(\not{x}\sigma^{\alpha\beta}+\sigma^{\alpha\beta}\not{x})}{32\pi^2x^2}. \end{aligned} \quad (16)$$

With the quark propagators one can express the correlation function in terms of a dispersion relation as:

$$\Pi_i(q^2) = \int_{(m_Q+m_{Q'})^2}^{\infty} \mathrm{d}s\frac{\rho_i(s)}{s-q^2}, \quad i=1,2, \quad (17)$$

where the spectral density is given by the imaginary part of the correlation function:

$$\rho_i(s) = \frac{1}{\pi}\mathrm{Im}\Pi_i^{\mathrm{OPE}}(s). \quad (18)$$

After equating the two expressions for $\Pi(q^2)$ based on the quark-hadron duality, and making a Borel transformation, we can write the sum rules as

$$\lambda_H^2\mathrm{e}^{-m_H^2/M^2} = \int_{(m_Q+m_{Q'})^2}^{s_0}\mathrm{d}s\rho_1(s)\mathrm{e}^{-s/M^2}, \quad (19)$$

$$\lambda_H^2 m_H\mathrm{e}^{-m_H^2/M^2} = \int_{(m_Q+m_{Q'})^2}^{s_0}\mathrm{d}s\rho_2(s)\mathrm{e}^{-s/M^2}. \quad (20)$$

The spectral functions ρ_1 and ρ_2 are given as follows:

$$\rho_1^{\mathrm{pert}}(s) = \frac{6}{(2\pi)^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{\mathrm{d}\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{\mathrm{d}\beta}{\beta}\Big([\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]^2+(1-\alpha-\beta)m_Qm_{Q'}[\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]\Big), \quad (21)$$

$$\begin{aligned} \rho_1(s) = {} & \rho_1^{\mathrm{pert}}(s)+\frac{\langle g_s^2G^2\rangle}{72}\left(m_Q^2\frac{\partial^3}{(\partial m_Q^2)^3}+m_{Q'}^2\frac{\partial^3}{(\partial m_{Q'}^2)^3}\right)\rho_1^{\mathrm{pert}}(s) \\ & +\frac{4m_Qm_{Q'}\langle g_s^2G^2\rangle}{(4\pi)^4}\left(\frac{\partial^2}{(\partial m_Q^2)^2}+\frac{\partial^2}{(\partial m_{Q'}^2)^2}\right)\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{\mathrm{d}\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{\mathrm{d}\beta}{\beta}(1-\alpha-\beta)(\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2) \\ & +\frac{2\langle g_s^2G^2\rangle}{(4\pi)^4}\left(\frac{\partial}{\partial m_Q^2}+\frac{\partial}{\partial m_{Q'}^2}\right)\int_{\alpha_{\min}}^{\alpha_{\max}}\mathrm{d}\alpha\int_{\beta_{\min}}^{1-\alpha}\mathrm{d}\beta(3\alpha m_Q^2+3\beta m_{Q'}^2-m_Qm_{Q'}-4\alpha\beta s), \end{aligned} \quad (22)$$

$$\begin{aligned} \rho_2(s) = {} & -\frac{\langle\bar{q}q\rangle}{2\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}}\mathrm{d}\alpha(3\alpha(1-\alpha)s-2\alpha m_Q^2-2(1-\alpha)m_{Q'}^2+2m_Qm_{Q'})-\frac{\langle\bar{q}g_s\sigma Gq\rangle}{8\pi^2}\left(1+\frac{s}{M^2}\right)A(s) \\ & -\frac{2\langle\bar{q}g_s\sigma Gq\rangle}{(4\pi)^2}\left((\alpha_{\max}-\alpha_{\min})+\frac{1}{2s(\alpha_{\max}-\alpha_{\min})}[\alpha_{\max}(1-\alpha_{\max})s+\alpha_{\min}(1-\alpha_{\min})s+4m_Qm_{Q'}]\right), \end{aligned} \quad (23)$$

with

$$A(s)=\frac{-s^3+(m_Q^2+m_{Q'}^2)s^2+(m_Q^2-4m_Qm_{Q'}+m_{Q'}^2)[s(m_Q^2+m_{Q'}^2)-(m_Q^2-m_{Q'}^2)^2]}{2s^2\sqrt{(s+m_Q^2-m_{Q'}^2)^2-4m_Q^2s}}. \quad (24)$$

The integration limits are given by $\alpha_{\min}=[s-m_Q^2+m_{Q'}^2-\sqrt{(s-m_Q^2+m_{Q'}^2)^2-4m_{Q'}^2s}]/(2s)$, $\alpha_{\max}=[s-m_Q^2+m_{Q'}^2+\sqrt{(s-m_Q^2+m_{Q'}^2)^2-4m_{Q'}^2s}]/(2s)$, and $\beta_{\min}=\alpha m_Q^2/(s\alpha-m_{Q'}^2)$. For the $\Omega_{QQ'}$, one needs to replace the condensates correspondingly. The integration lower bound $(m_Q+m_{Q'})^2$ is replaced by $(m_Q+m_{Q'}+m_s)^2$.

In Ref. [41], the authors obtained a similar expression to our Eq. (19):

$$\begin{aligned} \rho_1(s) = {} & -\frac{3}{2^4\pi^4}\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{\mathrm{d}\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{\mathrm{d}\beta}{\beta}[\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]^2\frac{3}{2^2\pi^4}m_Qm_{Q'} \\ & \times\int_{\alpha_{\min}}^{\alpha_{\max}}\frac{\mathrm{d}\alpha}{\alpha}\int_{\beta_{\min}}^{1-\alpha}\frac{\mathrm{d}\beta}{\beta}(1-\alpha-\beta)[\alpha\beta s-\alpha m_Q^2-\beta m_{Q'}^2]-\frac{5m_q\langle\bar{q}q\rangle}{2^3\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}}\mathrm{d}\alpha\alpha(1-\alpha). \end{aligned} \quad (25)$$

A few remarks are in order.

- We did not include the mass corrected quark condensate. This might have some impact in the case of $\Omega_{cc,bc,bb}$.

- However, the gluon condensate contribution, which was anticipated to be more important, is missing in Eq. (25).

- In the massless limit, we have the spectral function:

$$\rho_1(s)=\frac{s^2}{64\pi^4}+\frac{2\langle g_s^2G^2\rangle}{(4\pi)^4}. \tag{26}$$

  Our result is fully consistent with Ref. [47]:

$$\begin{aligned}&\lambda_H^2 \mathrm{e}^{-m_H^2/M^2}\\ =&\frac{1}{2(2\pi)^4}\left[M^6\left(1-\mathrm{e}^{-s_0/M^2}\left(1+\frac{s_0}{M^2}+\frac{1}{2}\frac{s_0^2}{M^4}\right)\right)\right.\\ &\left.+\frac{\langle g_s^2G^2\rangle}{4}M^2\left(1-\mathrm{e}^{-s_0/M^2}\right)\right].\end{aligned} \tag{27}$$

- In Ref. [41], the predicted mass $m_{\Xi_{cc}}=(4.26\pm 0.19)$ GeV is much larger than the experimental result $m^{\rm exp}_{\Xi_{cc}^{++}}=3.621\,\mathrm{GeV}$.

2.2 QCD sum rules with both positive and negative parity baryons

In the above analysis, only the $1/2^+$ baryons were considered. An interpolating current for the negative parity $1/2^-$ baryon can be defined as

$$J_-\equiv i\gamma_5 J_+, \tag{28}$$

where J_+ is given in Eqs. (2-5). When the complete set of hadron states is inserted into the correlation function in Eq. (6), both the positive and the negative parity single-particle states can contribute [48, 49].

When taking into account the $1/2^-$ single-particle states, Eq. (9) is rewritten as

$$\Pi(q)=\lambda_+^2\frac{\not q+m_+}{m_+^2-q^2}+\lambda_-^2\frac{\not q-m_-}{m_-^2-q^2}+\frac{1}{\pi}\int_{s_0}^{\infty}\mathrm{d}s\frac{\mathrm{Im}\Pi(s)}{s-q^2}, \tag{29}$$

where $\lambda_\pm$ ($m_\pm$) stands for the "decay constant" (mass) of positive or negative parity baryons. The λ_+ is the "decay constant" λ_H we have defined in Eq. (11). The λ_- is defined as

$$\langle 0|J_H^+|H(1/2^-,q,s)\rangle=i\gamma_5\lambda_- u(q,s). \tag{30}$$

At the hadronic level, one can take the imaginary part of the correlation function as follows:

$$\begin{aligned}\frac{1}{\pi}\mathrm{Im}\Pi(s)=&\lambda_+^2(\not q+m_+)\delta(s-m_+^2)+\lambda_-^2(\not q-m_-)\delta(s-m_-^2)+\cdots\\ =&\not q\rho_1^{\rm had}(s)+\rho_2^{\rm had}(s),\end{aligned} \tag{31}$$

with

$$\begin{aligned}\rho_1^{\rm had}(s)&=\lambda_+^2\delta(s-m_+^2)+\lambda_-^2\delta(s-m_-^2)]+\cdots,\\ \rho_2^{\rm had}(s)&=m_+\lambda_+^2\delta(s-m_+^2)-m_-\lambda_-^2\delta(s-m_-^2)]+\cdots.\end{aligned} \tag{32}$$

Here the ellipsis stand for the contributions from higher resonances and the continuum spectra. Considering the combination $\sqrt{s}\rho_1^{\rm had}+\rho_2^{\rm had}$, and introducing the exponential function $\exp(-s/M^2)$ to suppress these contributions, one can separate the λ_+ contributions:

$$\begin{aligned}&\int_\Delta^{s_0}\mathrm{d}s[\sqrt{s}\rho_1^{\rm had}(s)+\rho_2^{\rm had}(s)]\exp\left(-\frac{s}{M^2}\right)\\ =&2m_+\lambda_+^2\exp\left(-\frac{m_+^2}{M^2}\right),\end{aligned} \tag{33}$$

where s_0 is the threshold of the continuum states and M^2 is the Borel parameter.

On the OPE side, we compute the correlation function $\Pi(q)$ to obtain the QCD spectral densities

$$\frac{1}{\pi}\mathrm{Im}\Pi(s)=\not q\rho_1^{\rm OPE}(s)+\rho_2^{\rm OPE}(s). \tag{34}$$

Taking the quark-hadron duality below the continuum threshold s_0, we arrive at the following QCD sum rule:

$$\begin{aligned}&2m_+\lambda_+^2\exp\left(-\frac{m_+^2}{M^2}\right)\\ =&\int_\Delta^{s_0}\mathrm{d}s[\sqrt{s}\rho_1^{\rm OPE}(s)+\rho_2^{\rm OPE}(s)]\exp\left(-\frac{s}{M^2}\right).\end{aligned} \tag{35}$$

Here Δ is the threshold parameter, $\Delta=(m_Q+m_{Q'})^2$ for $\Xi_{QQ'}$, and $\Delta=(m_Q+m_{Q'}+m_s)^2$ for $\Omega_{QQ'}$.

3 Numerical results

In the numerical analysis, the quark masses used are [50]: $m_c=1.35\pm0.10\,\mathrm{GeV}, m_b=4.60\pm0.10$ GeV, $m_s=0.12\pm0.01\,\mathrm{GeV}$, while the u and d quarks are taken as massless. Similar values have been taken in Ref. [42].

The vacuum condensates used are [31, 41, 51–54]: $\langle\bar qq\rangle=-(0.24\pm0.01\,\mathrm{GeV})^3$, $\langle\bar ss\rangle=(0.8\pm0.1)\langle\bar qq\rangle$, $\langle g_s^2G^2\rangle=0.88\pm0.25$ GeV^4, $\langle\bar qg_s\sigma Gq\rangle=m_0^2\langle\bar qq\rangle$, $\langle\bar sg_s\sigma Gs\rangle=m_0^2\langle\bar ss\rangle$ and $m_0^2=0.8\pm0.1\,\mathrm{GeV}^2$ at the energy scale $\mu=1$ GeV.

The baryon masses used in the analysis of the decay constants are given in Table 2. For the Ξ_{cc}^{++} mass, we adopt the experimental value [8], and we use the isospin symmetry for the Ξ_{cc}^+. For other baryons, we use the lattice QCD results from Ref. [55].

The continuum threshold $\sqrt{s_0}$ used is $0.4\sim0.6$ GeV higher than the corresponding baryon mass, where we have assumed that the energy gap between the ground state and the first radial excited state is approximately 0.5 GeV [56].

Complying with the standard procedure of QCD sum rule analysis, the Borel parameter M^2 is varied to find

Table 2. Masses (in units of GeV) of doubly heavy baryons. We adopt the experimental value for the mass of Ξ_{cc} [8] and the lattice QCD results from Ref. [55].

baryons	Ξ_{cc}	Ω_{cc}	Ξ_{bb}	Ω_{bb}	Ξ_{bc}	Ω_{bc}
masses	3.621 [8]	3.738 [55]	10.143 [55]	10.273[55]	6.943 [55]	6.998 [55]

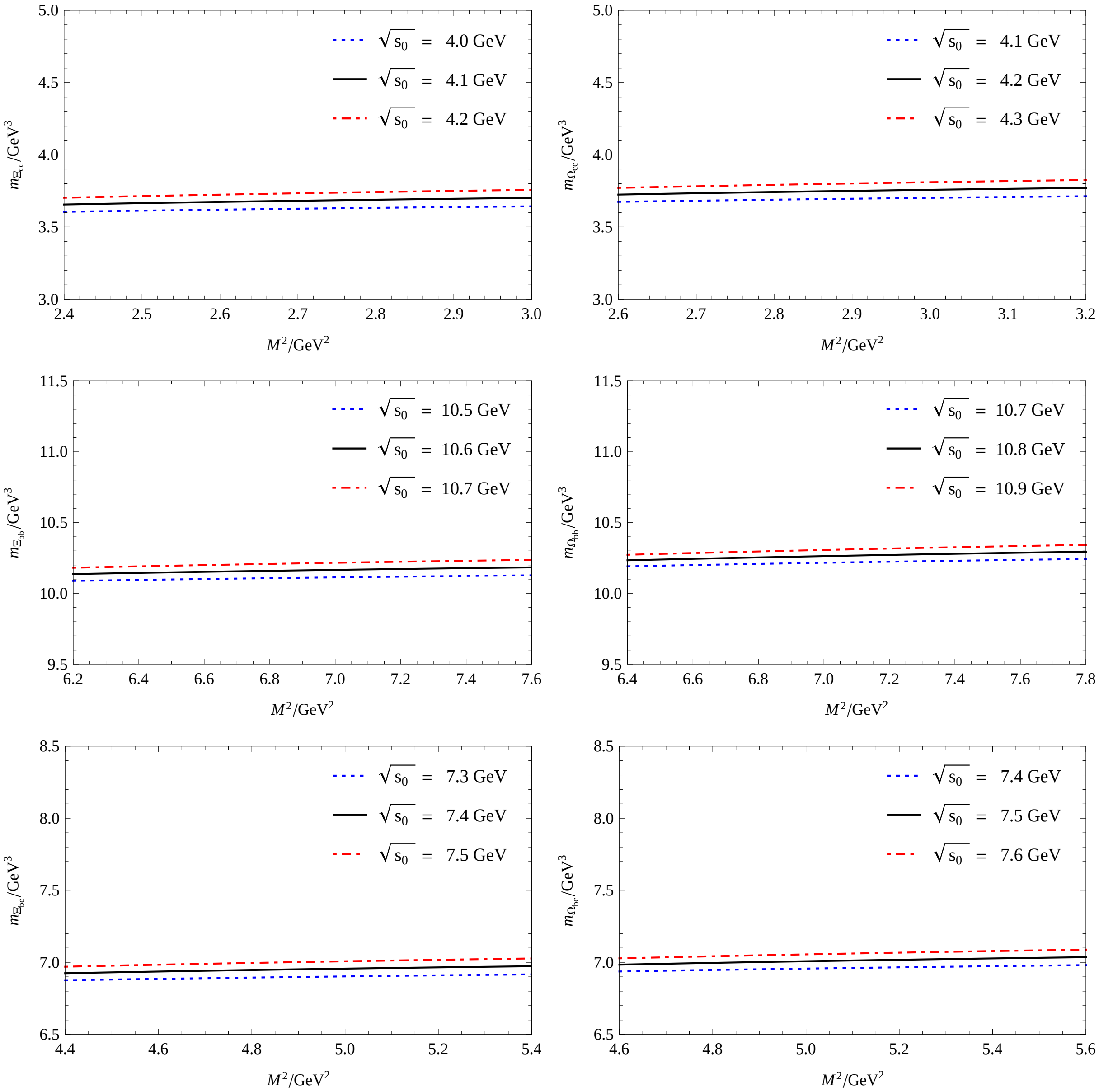

Fig. 1. (color online) The M^2-dependence of the masses of Ξ_{cc}, Ω_{cc} (top two figures), Ξ_{bb}, Ω_{bb} (middle two figures), Ξ_{bc} and Ω_{bc} (bottom two figures). The sum rule in Eq. (19) is considered. The inputs are taken as m_c=1.35GeV, m_b=4.60GeV, and m_s=0.12GeV, and the condensate parameters are taken at μ=1GeV.

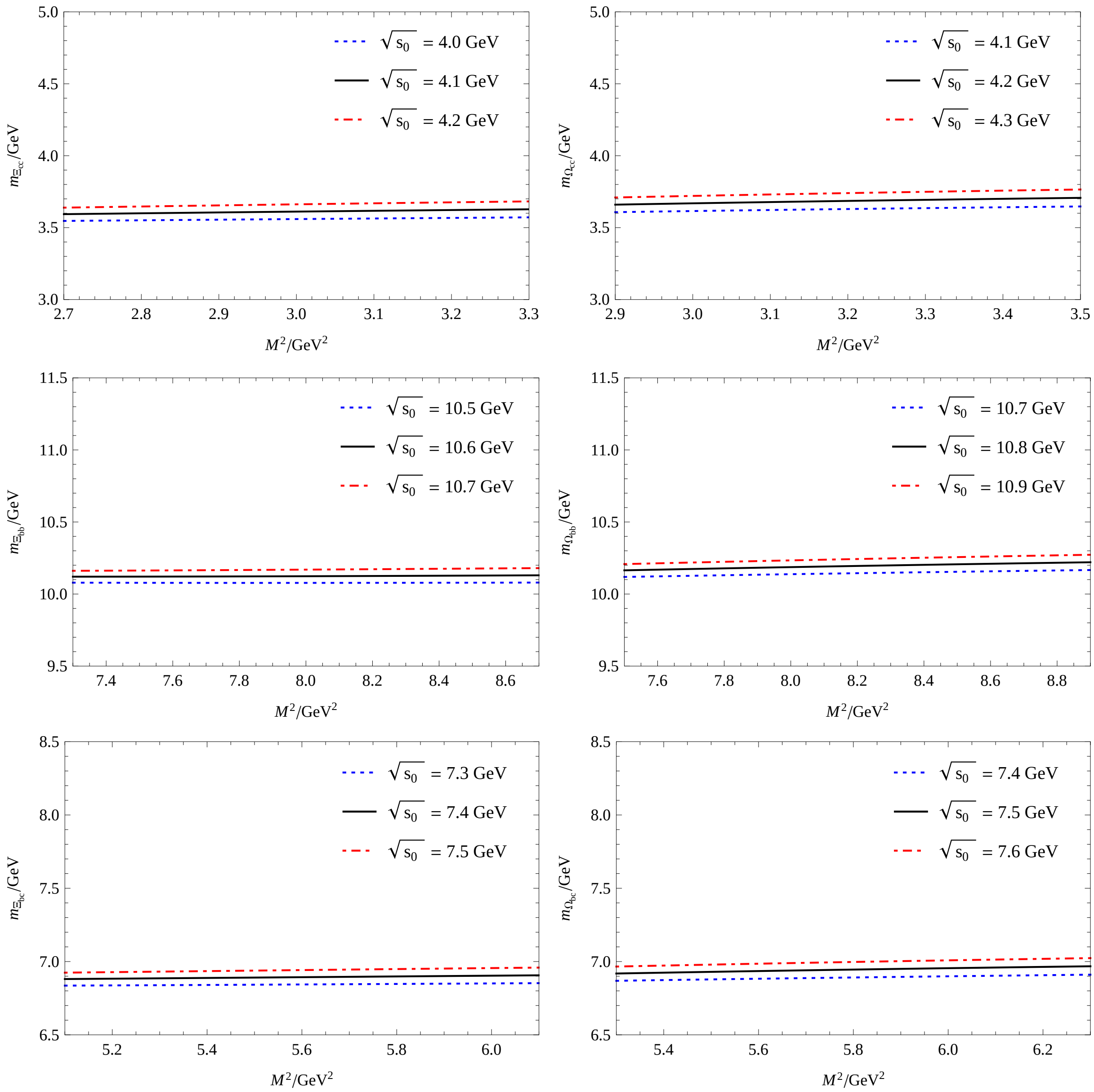

Fig. 2. (color online) The M^2-dependence of the masses of Ξ_{cc}, Ω_{cc} (top two figures), Ξ_{bb}, Ω_{bb} (middle two figures), Ξ_{bc} and Ω_{bc} (bottom two figures). The sum rule in Eq. (35) is considered. The inputs are taken as m_c=1.35GeV, m_b=4.60GeV, and m_s=0.12GeV, and the condensate parameters are taken at μ=1GeV.

Table 3. Theoretical predictions for the masses (in units of GeV) of the doubly heavy baryons. The results listed under "this work #1" are predicted using Eq. (19) while those under "this work #2" use Eq. (35). The uncertainties of the relavant parameters, including M^2, s_0, the quark masses and the condensates, have been taken into account. For purposes of comparison, some other QCDSR results from Ref. [41] and Ref. [42] and the lattice QCD results from Ref. [55] are listed. Our results are consistent with Ref. [42] and Ref. [55] but somewhat different from Ref. [41].

baryon	this work #1	this work #2	Ref. [41]	Ref. [42]	Ref. [55]	experiment
Ξ_{cc}	3.68±0.08	3.61±0.09	4.26±0.19	3.57±0.14	3.610±0.023±0.022	3.6214±0.0008
Ω_{cc}	3.75±0.08	3.69±0.09	4.25±0.20	3.71±0.14	3.738±0.020±0.020	–
Ξ_{bb}	10.16±0.09	10.12±0.10	9.78±0.07	10.17±0.14	10.143±0.030±0.023	–
Ω_{bb}	10.27±0.09	10.19±0.10	9.85±0.07	10.32±0.14	10.273±0.027±0.020	–
Ξ_{bc}	6.95±0.09	6.89±0.10	6.75±0.05	–	6.943±0.033±0.028	–
Ω_{bc}	7.01±0.09	6.95±0.09	7.02±0.08	–	6.998±0.027±0.020	–

the optimal stability window, in which the perturbative contribution should be larger than the condensate contributions and the pole contribution larger than the continuum contribution.

The sum rule in Eq. (19) will be numerically analyzed since it is expected to have better convergence than the sum rule in Eq. (20).

3.1 Masses

Differentiating Eq. (19) or Eq. (35) with respect to $-1/M^2$, one can extract the mass of the doubly heavy baryon as

$$m_H^2 = \frac{\int_\Delta^{s_0} \mathrm{d}s \rho_1(s) s \mathrm{e}^{-s/M^2}}{\int_\Delta^{s_0} \mathrm{d}s \rho_1(s) \mathrm{e}^{-s/M^2}} \tag{36}$$

or

$$m_H^2 = \frac{\frac{\mathrm{d}}{\mathrm{d}(-1/M^2)} \int_\Delta^{s_0} \mathrm{d}s \left[\sqrt{s}\rho_1(s) + \rho_2(s)\right] \mathrm{e}^{-s/M^2}}{\int_\Delta^{s_0} \mathrm{d}s \left[\sqrt{s}\rho_1(s) + \rho_2(s)\right] \mathrm{e}^{-s/M^2}}. \tag{37}$$

The optimal stability window for M^2 can be determined as follows. The upper bounds of the Borel parameters M^2 can be determined by the requirement that the pole contribution should be larger than the continuum contribution, while the lower bound can be determined by the requirement that the perturbative contribution should be larger than the quark condensate contribution. For the sum rule in Eq. (35), $M^2_{\rm max}$ = 3.3, 3.5, 8.7, 9.5, 6.1, 6.3 GeV^2 and $M^2_{\rm min}$ = 2.7, 2.0, 7.1, 5.2, 4.7, 3.6 GeV^2 for Ξ_{cc}, Ω_{cc}, Ξ_{bb}, Ω_{bb}, Ξ_{bc} and Ω_{bc} respectively. The optimal windows for M^2 can be chosen as [2.7,3.3], [2.9,3.5], [7.3,8.7], [7.5,8.9], [5.1,6.1] and [5.3,6.3] respectively. For the sum rule in Eq. (19), the optimal windows for M^2 can be chosen as [2.4,3.0], [2.6,3.2], [6.2,7.6], [6.4,7.8], [4.4,5.4] and [4.6,5.6] respectively.

The dependence of the predicted mass m_H on the Borel parameter M^2 is shown in Figs. 1 and 2, where the sum rules in Eq. (19) and Eq. (35) are adopted, respectively. Using Eq. (19), we obtain

$$m_{\Xi_{cc}} = (3.68 \pm 0.08)\ \mathrm{GeV},$$

where only the positive parity baryons are taken into account. When the contamination from the $1/2^-$ baryon is considered, we find the mass is slightly changed:

$$m_{\Xi_{cc}} = (3.61 \pm 0.09)\ \mathrm{GeV}.$$

Here the uncertainties of the relevant parameters, including M^2, s_0, the quark masses and the condensates, have been taken into account. It can be seen that our values are consistent with the experimental value when the errors are taken into account. Our results are also consistent with other estimates, for instance Ref. [42]. A collection of the results can be found in Table 3.

3.2 Decay constants

The dependence of the "decay constants" λ_H on the Borel parameter M^2 is shown in Figs. 3 and 4, where the sum rules in Eq. (19) and Eq. (35) are adopted, respectively. The numerical results for the "decay constants" can be found in Table 4.

Table 4. Decay constants λ_H (in units of GeV^3) for the doubly heavy baryons. The results listed under "this work #1" are predicted using Eq. (19) while those under "this work #2" use Eq. (35). The uncertainties of the relevant parameters, including M^2, s_0, the quark masses, the condensates and the baryon masses, have been taken into account. For purposes of comparison, the QCDSR results from Ref. [42] are listed.

baryon	this work #1	this work #2	Ref. [42]
Ξ_{cc}	0.113±0.029	0.109±0.021	0.115±0.027
Ω_{cc}	0.140±0.033	0.123±0.024	0.138±0.030
Ξ_{bb}	0.303±0.094	0.281±0.071	0.252±0.064
Ω_{bb}	0.404±0.112	0.347±0.083	0.311±0.077
Ξ_{bc}	0.191±0.053	0.176±0.040	–
Ω_{bc}	0.217±0.056	0.188±0.041	–

A few remarks are in order.

- When including the contributions from the $1/2^-$ baryons, the threshold parameter might be somewhat higher. In this analysis, we have used approximately the same values.
- Comparing the two sets of results in Table 4, one can see that the negative parity baryons do not give significant modifications.
- We can see from Table 4 that the decay constants of $\Omega_{QQ'}$ are slightly larger than those of $\Xi_{QQ'}$.

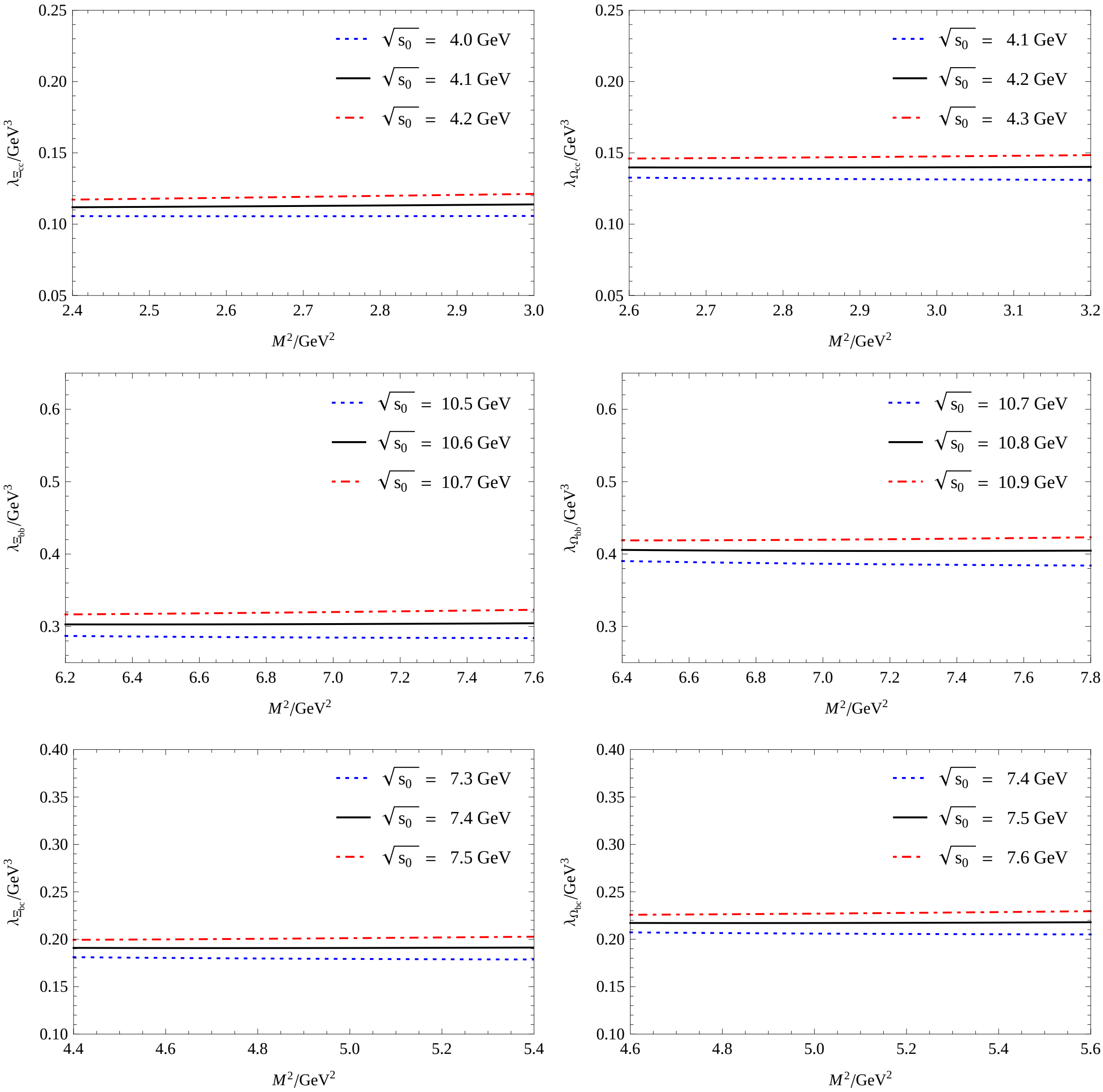

Fig. 3. (color online) The M^2-dependence of the decay constants of Ξ_{cc}, Ω_{cc} (top two figures), Ξ_{bb}, Ω_{bb} (middle two figures), Ξ_{bc} and Ω_{bc} (bottom two figures). The continuum thresholds are taken as $\sqrt{s_0}=4.0\sim4.2$ GeV, $\sqrt{s_0}=4.1\sim4.3$ GeV, $\sqrt{s_0}=10.5\sim10.7$ GeV, $\sqrt{s_0}=10.7\sim10.9$ GeV, $\sqrt{s_0}=7.3\sim7.5$ GeV and $\sqrt{s_0}=7.4\sim7.6$ GeV, respectively. The sum rule in Eq. (19) is considered.

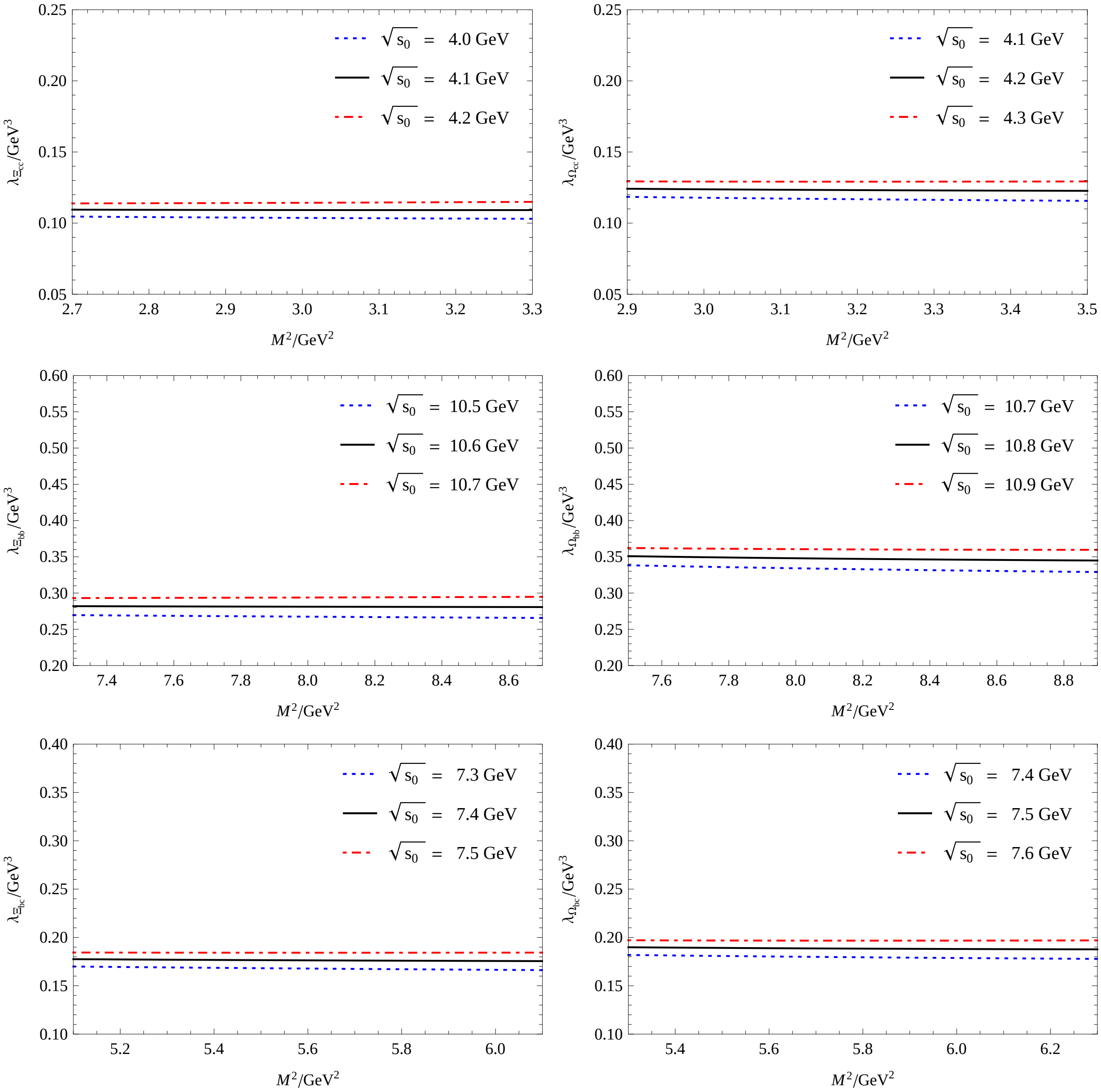

Fig. 4. (color online) The M^2-dependence of the decay constants of Ξ_{cc}, Ω_{cc} (top two figures), Ξ_{bb}, Ω_{bb} (middle two figures), Ξ_{bc} and Ω_{bc} (bottom two figures). The continuum thresholds are taken as $\sqrt{s_0}=4.0\sim4.2$ GeV, $\sqrt{s_0}=4.1\sim4.3$ GeV, $\sqrt{s_0}=10.5\sim10.7$ GeV, $\sqrt{s_0}=10.7\sim10.9$ GeV, $\sqrt{s_0}=7.3\sim7.5$ GeV and $\sqrt{s_0}=7.4\sim7.6$ GeV, respectively. The sum rule in Eq. (35) is considered.

4 Conclusion

In this work we have calculated the "decay constants" for the doubly heavy baryons Ξ_{cc}, Ω_{cc}, Ξ_{bb}, Ω_{bb}, Ξ_{bc} and Ω_{bc} using QCD sum rules. In the calculation we have included both the positive and negative parity baryons, and found that the $1/2^-$ contamination is not severe. The extracted results for the decay constants are ingredients for the study of weak decays and other properties of doubly heavy baryons, including their lifetimes [57–59].

The authors are grateful to Pietro Colangelo, Fulvia De Fazio, Zhi-Gang Wang, Yu-Ming Wang, and Fu-Sheng Yu for useful discussions.

References

1 M. Mattson et al (SELEX Collaboration), Phys. Rev. Lett., **89**: 112001 (2002) doi: 10.1103/PhysRevLett.89.112001 [hep-ex/0208014].
2 A. Ocherashvili et al (SELEX Collaboration), Phys. Lett. B, **628**: 18 (2005), doi: 10.1016/j.physletb.2005.09.043, [hep-ex/0406033]
3 S. P. Ratti, Nucl. Phys. Proc. Suppl., **115**: 33 (2003), doi: 10.1016/S0920-5632(02)01948-5
4 B. Aubert et al (BaBar Collaboration), Phys. Rev. D, **74**: 011103 (2006), doi: 10.1103/PhysRevD.74.011103, [hep-ex/0605075]
5 R. Chistov et al (Belle Collaboration), Phys. Rev. Lett., **97**: 162001 (2006), doi: 10.1103/PhysRevLett.97.162001, [hep-ex/0606051]
6 R. Aaij et al (LHCb Collaboration), JHEP, **1312**: 090 (2013), doi: 10.1007/JHEP12(2013)090, arXiv: 1310.2538 [hep-ex]
7 Y. Kato et al (Belle Collaboration), Phys. Rev. D, **89**(5): 052003 (2014), doi: 10.1103/PhysRevD.89.052003, arXiv: 1312.1026 [hep-ex]
8 R. Aaij et al (LHCb Collaboration), Phys. Rev. Lett., **119**(11): 112001 (2017), doi: 10.1103/PhysRevLett.119.112001, arXiv: 1707.01621 [hep-ex]
9 H. X. Chen, Q. Mao, W. Chen, X. Liu, and S. L. Zhu, Phys. Rev. D, **96**(3): 031501 (2017), doi: 10.1103/PhysRevD.96.031501, arXiv: 1707.01779 [hep-ph]
10 F. S. Yu, H. Y. Jiang, R. H. Li, C. D. L, W. Wang, and Z. X. Zhao, arXiv: 1703.09086 [hep-ph]
11 W. Wang, F. S. Yu, and Z. X. Zhao, Eur. Phys. J. C, **77**(11): 781 (2017), doi: 10.1140/epjc/s10052-017-5360-1, arXiv: 1707.02834 [hep-ph]
12 H. S. Li, L. Meng, Z. W. Liu, and S. L. Zhu, Phys. Rev. D, **96**(7): 076011 (2017), doi: 10.1103/PhysRevD.96.076011, arXiv: 1707.02765 [hep-ph]
13 L. Meng, N. Li, and S. l. Zhu, arXiv: 1707.03598 [hep-ph]
14 W. Wang, Z. P. Xing, and J. Xu, Eur. Phys. J. C, **77**(11): 800 (2017), doi: 10.1140/epjc/s10052-017-5363-y, arXiv: 1707.06570 [hep-ph]
15 M. Karliner and J. L. Rosner, Phys. Rev. Lett., **119**(20): 202001 (2017), doi: 10.1103/PhysRevLett.119.202001, arXiv: 1707.07666 [hep-ph]
16 T. Gutsche, M. A. Ivanov, J. G. Krner, and V. E. Lyubovitskij, Phys. Rev. D, **96**(5): 054013 (2017), doi: 10.1103/PhysRevD.96.054013, arXiv: 1708.00703 [hep-ph]
17 H. S. Li, L. Meng, Z. W. Liu, and S. L. Zhu, arXiv: 1708.03620 [hep-ph]
18 Z. H. Guo, Phys. Rev. D, **96**(7): 074004 (2017), doi: 10.1103/PhysRevD.96.074004, arXiv: 1708.04145 [hep-ph]
19 Q. F. L, K. L. Wang, L. Y. Xiao, and X. H. Zhong, arXiv: 1708.04468 [hep-ph]
20 L. Y. Xiao, K. L. Wang, Q. f. Lu, X. H. Zhong, and S. L. Zhu, Phys. Rev. D, **96**(9): 094005 (2017), doi: 10.1103/PhysRevD.96.094005, arXiv: 1708.04384 [hep-ph]
21 N. Sharma and R. Dhir, arXiv: 1709.08217 [hep-ph]
22 Y. L. Ma and M. Harada, arXiv: 1709.09746 [hep-ph]
23 L. Meng, H. S. Li, Z. W. Liu, and S. L. Zhu, arXiv: 1710.08283 [hep-ph]
24 R. H. Li, C. D. L, W. Wang, F. S. Yu, and Z. T. Zou, Phys. Lett. B, **767**: 232 (2017), doi: 10.1016/j.physletb.2017.02.003, arXiv: 1701.03284 [hep-ph]
25 C. Y. Wang, C. Meng, Y. Q. Ma, and K. T. Chao, arXiv: 1708.04563 [hep-ph]
26 Y. J. Shi, W. Wang, Y. Xing, and J. Xu, Eur. Phys. J. C, **78**(1): 56 (2018), doi: 10.1140/epjc/s10052-018-5532-7, arXiv: 1712.03830 [hep-ph]
27 L. Y. Xiao, Q. F. L, and S. L. Zhu, arXiv: 1712.07295 [hep-ph]
28 X. Yao and B. Müller, arXiv: 1801.02652 [hep-ph]
29 Z. X. Zhao, arXiv: 1805.10878 [hep-ph]
30 Z. P. Xing and Z. X. Zhao, arXiv: 1807.03101 [hep-ph].
31 P. Colangelo and A. Khodjamirian, In *Shifman, M. (ed.): At the frontier of particle physics*, vol. 3 1495-1576 [hep-ph/0010175]
32 L. J. Reinders and S. Yazaki, Nucl. Phys. B, **288**: 789 (1987), doi: 10.1016/0550-3213(87)90238-0
33 H. Castillo, C. A. Dominguez, and M. Loewe, JHEP, **0503**: 012 (2005), doi: 10.1088/1126-6708/2005/03/012, [hep-ph/0302271]
34 V. V. Kiselev and A. V. Tkabladze, Sov. J. Nucl. Phys., **50**: 1063 (1989), [Yad. Fiz., **50**: 1714 (1989)]
35 S. Narison, Phys. Lett. B, **198**: 104 (1987), doi: 10.1016/0370-2693(87)90168-7
36 N. Bilic, C. A. Dominguez, and B. Guberina, Z. Phys. C, **39**: 351 (1988)
37 L. J. Reinders and S. Yazaki, Phys. Lett. B, **212**: 245 (1988), doi: 10.1016/0370-2693(88)90533-3
38 A. A. Penin and M. Steinhauser, Phys. Rev. D, **65**: 054006 (2002), doi: 10.1103/PhysRevD.65.054006, [hep-ph/0108110]
39 M. Neubert, Phys. Rev. D, **45**: 2451 (1992), doi: 10.1103/PhysRevD.45.2451
40 P. Colangelo, G. Nardulli, and N. Paver, Z. Phys. C, **57**: 43 (1993), doi: 10.1007/BF01555737
41 J. R. Zhang and M. Q. Huang, Phys. Rev. D, **78**: 094007 (2008), doi: 10.1103/PhysRevD.78.094007, arXiv: 0810.5396 [hep-ph]
42 Z. G. Wang, Eur. Phys. J. A, **45**: 267 (2010), doi: 10.1140/epja/i2010-11004-3, arXiv: 1001.4693 [hep-ph]
43 Z. G. Wang, Eur. Phys. J. C, **68**: 459 (2010), doi: 10.1140/epjc/s10052-010-1357-8, arXiv: 1002.2471 [hep-ph]
44 Z. G. Wang, Eur. Phys. J. A, **47**: 81 (2011), doi: 10.1140/epja/i2011-11081-8, arXiv: 1003.2838 [hep-ph]
45 T. M. Aliev, K. Azizi, and M. Savci, Nucl. Phys. A, **895**: 59 (2012), doi: 10.1016/j.nuclphysa.2012.09.009, arXiv: 1205.2873 [hep-ph]
46 T. M. Aliev, K. Azizi, and M. Savci, J. Phys. G, **40**: 065003 (2013), doi: 10.1088/0954-3899/40/6/065003, arXiv: 1208.1976 [hep-ph]
47 V. Braun, R. J. Fries, N. Mahnke, and E. Stein, Nucl. Phys. B, **589**: 381 (2000), Erratum: [Nucl. Phys. B, **607**: 433 (2001)], doi: 10.1016/S0550-3213(00)00516-2, 10.1016/S0550-3213(01)00254-1, [hep-ph/0007279]
48 D. Jido, N. Kodama, and M. Oka, Phys. Rev. D, **54**: 4532 (1996), doi: 10.1103/PhysRevD.54.4532, [hep-ph/9604280]
49 A. Khodjamirian, C. Klein, T. Mannel, and Y.-M. Wang, JHEP, **1109**: 106 (2011), doi: 10.1007/JHEP09(2011)106, arXiv: 1108.2971 [hep-ph]
50 C. Patrignani et al (Particle Data Group), Chin. Phys. C, **40**(10): 100001 (2016), doi: 10.1088/1674-1137/40/10/100001
51 Z. G. Wang, Commun. Theor. Phys., **66**(3): 335 (2016), doi: 10.1088/0253-6102/66/3/335, arXiv: 1602.08711 [hep-ph]
52 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B, **147**: 385 (1979), doi: 10.1016/0550-3213(79)90022-1
53 L. J. Reinders, H. Rubinstein, and S. Yazaki, Phys. Rept., **127**: 1 (1985), doi: 10.1016/0370-1573(85)90065-1
54 P. Colangelo, C. A. Dominguez, G. Nardulli, and N. Paver, Phys. Rev. D, **54**: 4622 (1996), doi: 10.1103/PhysRevD.54.4622, [hep-ph/9512334]
55 Z. S. Brown, W. Detmold, S. Meinel, and K. Orginos, Phys. Rev. D, **90**(9): 094507 (2014), doi: 10.1103/PhysRevD.90.094507, arXiv: 1409.0497 [hep-lat]
56 Z. G. Wang, Eur. Phys. J. A, **49**: 131 (2013), doi: 10.1140/epja/i2013-13131-7, arXiv: 1203.6252 [hep-ph]
57 P. Colangelo and F. De Fazio, Phys. Lett. B, **387**: 371 (1996), doi: 10.1016/0370-2693(96)01049-0, [hep-ph/9604425]
58 A. Lenz, Int. J. Mod. Phys. A, **30**(10): 1543005 (2015), doi: 10.1142/S0217751X15430058, arXiv: 1405.3601 [hep-ph]
59 M. Kirk, A. Lenz, and T. Rauh, arXiv: 1711.02100 [hep-ph]