

R^4 corrections to holographic Schwinger effect^{*}

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Abstract: We consider R^4 corrections to the holographic Schwinger effect in an AdS black hole background and a confining D3-brane background. The potential between a test particle pair are performed for both backgrounds. We find there is no potential barrier in the critical electric field, which means that the system becomes catastrophically unstable. It is shown that for both backgrounds, increasing the inverse 't Hooft coupling parameter $1/\lambda$ enhances the Schwinger effect. We also discuss the possible relation between the Schwinger effect and the viscosity-entropy ratio η/s in strong coupling.

Keywords: holographic Schwinger effect, AdS/CFT, potential analysis

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1 Introduction

The Schwinger effect is a non-perturbative phenomenon in quantum electrodynamics(QED), which describes how the virtual electron-positron pairs can become real particles in a strong electric-field. The pair-production rate Γ has been calculated in the condition of the weak-coupling and weak-field by Schwinger [1]. Later, it was generalized to arbitrary-coupling and weak-field by Affleck-Alvarez-Manton (AAM) [2], via the relation

$$\Gamma \sim \exp\left(-\frac{\pi m^2}{eE} + \frac{e^2}{4}\right), \quad (1)$$

where m is the electron mass, e is the elementary electric charge, and E is the external electric field. Today, we know that this non-perturbative effect is explained as a tunneling process and is not restricted to QED, but ubiquitously in quantum field theories coupled to a U(1) gauge field. Thus, the Schwinger effect may be an important tool to understand the vacuum structure and non-perturbative aspects of string theory, and also quantum field theories.

Since AdS/CFT correspondence calls the duality between the type IIB superstring theory formulated on $\text{AdS}_5 \times S^5$ and $\mathcal{N}=4$ SYM in four dimensions (which realize a construction that is coupled with the U(1) gauge field [3–5]), it is interesting to consider the Schwinger effect in the context of the AdS/CFT correspondence.

One of the problems in the formula of AAM is that the critical value E does not satisfy the weak-field con-

dition $eE \ll m^2$, where the E is a certain value of the electric field that makes the Schwinger effect occur without a tunneling process. In 2011, Semenoff and Zarembo computed the production rate of the W bosons in the Coulomb branch of $\mathcal{N}=4$ SYM theory [6]

$$\Gamma \sim \exp\left[-\frac{\sqrt{\lambda}}{2}\left(\sqrt{\frac{E_c}{E}} - \sqrt{\frac{E}{E_c}}\right)^2\right], \quad E_c = \frac{2\pi m^2}{\sqrt{\lambda}}, \quad (2)$$

where the value of critical field E_c exactly coincides with the critical value of the Dirac-Born-Infeld (DBI) action of the probe D3-brane with an electric field turning on. λ is the 't Hooft coupling constant. From the expression of production rate, one can anticipate the law of particle production to λ :

- 1) $E = E_c$, the production rate of particles $\Gamma \sim \exp(0)=1$ irrelevant to λ .
- 2) $E < E_c$ or $E > E_c$, the production rate of particles $\Gamma \sim \exp(-\frac{\sqrt{\lambda}}{2}\beta)$ decreases by increasing the λ ($\beta = [\sqrt{E_c/E} - \sqrt{E/E_c}]^2 > 0$).

There are many attempts to address the Schwinger effect in this direction after Semenoff and Zarembo's work. For instance, the potential in the holographic Schwinger effect has been analyzed in [7]. The holographic Schwinger effect in a confining D3-brane background with chemical potential was studied in [8]. One review about this topic was shown in [9]. Ordinarily, due to the existence of the stringy effect, there are many higher derivative corrections to the Schwinger effect. Although there is little knowledge about the forms of higher derivative corrections, generic corrections can

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be expected to exist when considering the vastness of the string landscape [10].

It is known that calculation of the holographic Schwinger effect is highly related to string theory, so it is natural to consider various stringy corrections. The R^2 corrections to Schwinger effect have been studied in [11]. In this research, we would like to study how R^4 corrections affect the Schwinger effect. Besides, it was shown in [12] that $\eta/s \geq 1/4\pi$ could be violated in theories with R^4 corrections, which makes it very interesting to study the connection between the shear viscosity and the Schwinger effect in these R^4 theories.

In this paper, we will study R^4 corrections to the Schwinger effect. The organization of this paper is as follows. In section 2, the background with R^4 corrections is briefly reviewed. In section 3, the potential analysis for the AdS black hole background with R^4 corrections and the evaluation of the critical electric field from the DBI action is performed. In section 4, the Schwinger effect in a confining D3-brane background with R^4 corrections is studied as well. Section 5 is devoted to conclusion and discussion.

2 Setup

Let us focus on the contribution to the free energy F coming from the $\alpha'^3 R^4$, string correction [13–16] and the supergravity action. In the Einstein frame, using the convention including $(F_5)^2$ in the action and imposing self-duality after the equations of motion are derived, the structure of the tree level type IIB string effective action will be as follows [17, 18]:

$$S_{\text{IIB}} = \frac{1}{16\pi G_{10}} \int d^{10}x \sqrt{-g} \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{4 \cdot 5!} (F_5)^2 + k e^{-\frac{3}{2}\phi} \mathcal{W} + \dots \right), \quad (3)$$

where G_{10} is the ten-dimensional Newton constant, and $k = \frac{1}{8}\zeta(3)(\alpha')^3$. The term \mathcal{W} depends only on the four copies of the Weyl tensor

$$\mathcal{W} = C^{\alpha\beta\gamma\delta} C_{\mu\beta\gamma\nu} C_{\alpha}^{\rho\sigma\mu} C_{\rho\sigma\delta}^{\nu} + \frac{1}{2} C^{\alpha\delta\beta\gamma} C_{\mu\nu\beta\gamma} C_{\alpha}^{\rho\sigma\mu} C_{\rho\sigma\delta}^{\nu}. \quad (4)$$

For the purpose of computing the corrected Schwinger effect, one can use the Kaluza-Klein reduced five-dimensional action [19]

$$S = \frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left(R + \frac{12}{R^2} + k\mathcal{W} \right), \quad (5)$$

where R is the radius of curvature of AdS₅, and \mathcal{W} is given by Eq. (4) in five-dimensions. k is the expansion parameter, which is connected with the 't Hooft coupling

constant λ in $\mathcal{N}=4$ SYM by

$$k = \frac{\zeta(3)}{8} \lambda^{-\frac{3}{2}} \sim 0.15 \lambda^{-\frac{3}{2}}. \quad (6)$$

Because the higher derivative terms are treated as perturbations in the equations of motion, the reliable results will be restricted to small (or large) values of parameter k (or λ). Therefore, the DBI analysis and the SUGRA background are not accurate enough when the value of λ extends to its smaller interval. The values for the 't Hooft coupling λ were assumed as follows [20]

$$\lambda = \{8, 12, 20, 100\}, \quad (7)$$

Since the correction of the inverse 't Hooft coupling parameter $1/\lambda$ corresponds to the α' correction in string theory [21], the dependence of the α' correction effect on parameter λ may be studied. Theoretically, the α' correction effect decreases gradually as the 't Hooft coupling parameter λ increases, and it approaches zero as λ approaches infinity.

The function of the α' -corrected metric is [17, 20]

$$ds^2 = -\frac{r^2(1-w^{-4})}{R^2} T(w) dt^2 + \frac{r^2}{R^2} X(w) (dx^i)^2 + \frac{R^2 R(w)}{r^2(1-w^{-4})} dr^2, \quad (8)$$

where R is the AdS₅ space radius and r denotes the radial coordinate of the black brane geometry. $x^i (i=1, 2, 3)$ is the boundary coordinate. Besides

$$\begin{aligned} T(w) &= 1 - k \left(75w^{-4} + \frac{1225}{16}w^{-8} - \frac{695}{16}w^{-12} \right) + \dots, \\ X(w) &= 1 - \frac{25k}{16}w^{-8}(1+w^{-4}) + \dots, \\ R(w) &= 1 + k \left(75w^{-4} + \frac{1175}{16}w^{-8} - \frac{4585}{16}w^{-12} \right) + \dots, \end{aligned} \quad (9)$$

and $w = \frac{r}{r_h}$ with $r=r_h$ being the event horizon and $r=\infty$ the boundary. Further, λ can be related to the ratio of the shear viscosity to the entropy density η/s by [22–24]

$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} \right). \quad (10)$$

The temperature of the black hole is

$$T = \frac{r_h}{\pi R^2(1-k)}. \quad (11)$$

3 Potential analysis

Following the calculations of [7] to study the Schwinger effect for the background metric of (8), the

Nambu-Goto string action is

$$S = T_F \int d\tau d\sigma \mathcal{L} = T_F \int d\tau d\sigma \sqrt{\det G_{ab}}, \quad (12)$$

$$G_{ab} \equiv \frac{\partial x^\mu}{\partial \sigma^a} \frac{\partial x^\nu}{\partial \sigma^b} g_{\mu\nu},$$

where $T_F = \frac{1}{2\pi\alpha'}$ is the fundamental string tension, and $G_{ab}(a, b = 0, 1)$ is the induced metric on the string world sheet, using the static gauge

$$x^0 = \tau, \quad x^1 = \sigma, \quad x^2 = x^3 = \text{const}. \quad (13)$$

Assuming the radial direction depends only on σ

$$r = r(\sigma). \quad (14)$$

The metric(8) is calculated as

$$ds^2 = -\frac{r^2(1-w^{-4})}{R^2} T(w) dt^2 + \left(\frac{r^2 X(w)}{R^2} + \frac{R^2}{r^2} \frac{R(w)}{(1-w^{-4})} \dot{r}^2 \right) d\sigma^2. \quad (15)$$

The induced metric G_{ab} is obtained

$$G_{ab} = \begin{pmatrix} \frac{r^2(1-w^{-4})}{R^2} T(w) & 0 \\ 0 & \frac{r^2}{R^2} X(w) + \frac{R^2}{r^2} \frac{R(w)}{(1-w^{-4})} \dot{r}^2 \end{pmatrix}, \quad (16)$$

where $\dot{r} = \frac{\partial r}{\partial \sigma}$. Then \mathcal{L} is expressed as

$$\mathcal{L} = \sqrt{\det G_{ab}} = \sqrt{\frac{r^4}{R^4} (1-w^{-4}) T(w) X(w) + \dot{r}^2 T(w) R(w)}. \quad (17)$$

Now, \mathcal{L} does not depend on σ explicitly; then, the corresponding Hamiltonian is a constant, which is

$$\mathcal{H} = \mathcal{L} - \frac{\partial \mathcal{L}}{\partial \dot{r}} \dot{r}. \quad (18)$$

Imposing the boundary condition at $\sigma = 0$,

$$\frac{dr}{d\sigma} = 0 \quad r = r_c \quad (r_h < r_c < r_0), \quad (19)$$

the conserved quantity becomes

$$\frac{\frac{r^4}{R^4} (1-w^{-4}) T(w) X(w)}{\sqrt{\frac{r^4}{R^4} (1-w^{-4}) T(w) X(w) + \dot{r}^2 T(w) R(w)}} = \text{const} \equiv \sqrt{\frac{r_c^4}{R^4} (1-w_c^{-4}) T(w_c) X(w_c)}, \quad (20)$$

with

$$T(w_c) = 1 - k \left(75 \left(\frac{r_h}{r_c} \right)^4 + \frac{1225}{16} \left(\frac{r_h}{r_c} \right)^8 - \frac{695}{16} \left(\frac{r_h}{r_c} \right)^{12} \right) + \dots,$$

$$X(w_c) = 1 - \frac{25k}{16} \left(\frac{r_h}{r_c} \right)^8 \left(1 + \left(\frac{r_h}{r_c} \right)^4 \right) + \dots \quad (21)$$

From the conserved quantity, we obtained the \dot{r}

$$\frac{dr}{d\sigma} = \frac{r^2}{R^2} \sqrt{\frac{(1-w^{-4})X(w)}{R(w)} \left[\frac{r^4(1-w^{-4})T(w)X(w)}{r_c^4(1-w_c^{-4})T(w_c)X(w_c)} - 1 \right]}. \quad (22)$$

By integrating the expression (22), the separation distance x of test particles on the probe brane is represented by

$$x = \frac{2R^2}{ar_0} \int_1^{1/a} \frac{dy}{y^2 \sqrt{1-w^{-4}}} \sqrt{\frac{R(w)}{X(w)} \frac{1}{\sqrt{\frac{y^4(1-w^{-4})T(w)X(w)}{(1-w_c^{-4})T(w_c)X(w_c)} - 1}}}, \quad (23)$$

with

$$T(w_c) = 1 - k \left(75 \left(\frac{b}{a} \right)^4 + \frac{1225}{16} \left(\frac{b}{a} \right)^8 - \frac{695}{16} \left(\frac{b}{a} \right)^{12} \right) + \dots,$$

$$X(w_c) = 1 - \frac{25k}{16} \left(\frac{b}{a} \right)^8 \left(1 + \left(\frac{b}{a} \right)^4 \right) + \dots, \quad (24)$$

where we have defined dimensionless quantities

$$y \equiv \frac{r}{r_c}, \quad a \equiv \frac{r_c}{r_0}, \quad b \equiv \frac{r_h}{r_0}. \quad (25)$$

The sum of the Coulomb potential (CP) and static energy (SE) in the R^4 correction is derived

$$V_{\text{CP+SE}} = 2T_F \int_0^{x/2} d\sigma \mathcal{L} = 2T_F r_0 a \int_1^{1/a} dy \sqrt{T(w)R(w)} \times \frac{1}{\sqrt{1 - \frac{(1-w_c^{-4})T(w_c)X(w_c)}{y^4(1-w^{-4})T(w)X(w)}}}. \quad (26)$$

Next, we calculate the critical electric field. The DBI

action is given by

$$S_{\text{DBI}} = -T_{D3} \int d^4x \sqrt{-\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu})}$$

$$T_{D3} = \frac{1}{g_s (2\pi)^3 \alpha'^2}, \quad (27)$$

where T_{D3} is the D3-brane tension.

As the virtue shows in Eq.(8), the induced metric $G_{\mu\nu}$ reads

$$G_{00} = -\frac{r^2}{R^2} (1-w^{-4}) T(w) \quad G_{11} = \frac{r^2}{R^2} X(w)$$

$$G_{22} = \frac{r^2}{R^2} X(w) \quad G_{33} = \frac{r^2}{R^2} X(w) \quad (28)$$

According to [25], $\mathcal{F}_{\mu\nu} = 2\pi\alpha' F_{\mu\nu}$, one obtains

$$G_{\mu\nu} + \mathcal{F}_{\mu\nu} = \begin{pmatrix} -\frac{r^2(1-w^{-4})}{R^2} T(w) & 2\pi\alpha' E & 0 & 0 \\ -2\pi\alpha' E & \frac{r^2}{R^2} X(w) & 0 & 0 \\ 0 & 0 & \frac{r^2}{R^2} X(w) & 0 \\ 0 & 0 & 0 & \frac{r^2}{R^2} X(w) \end{pmatrix}, \quad (29)$$

which yields

$$\det(G_{\mu\nu} + \mathcal{F}_{\mu\nu}) = -\left(\frac{r}{R}\right)^4 X^2(w) \left[\frac{r^4(1-w^{-4})}{R^4} T(w) X(w) - (2\pi\alpha' E)^2 \right], \quad (30)$$

where we consider the total static-potential by turning on an electric field E along the x^1 -direction [26].

Combining (30) with (27) and setting the D3-brane at $r=r_0$, one gets

$$S_{\text{DBI}} = -T_{D3} \frac{r_0^4}{R^4} \int d^4x \sqrt{X^2(w_0) \left[(1-w_0^{-4}) T(w_0) X(w_0) - \frac{(2\pi\alpha')^2 E^2 R^4}{r_0^4} \right]}, \quad (31)$$

with

$$T(w_0) = 1 - k \left(75b^4 + \frac{1225}{16} b^8 - \frac{695}{16} b^{12} \right) + \dots,$$

$$X(w_0) = 1 - \frac{25k}{16} b^8 (1+b^4) + \dots, \quad (32)$$

where $w_0 = r_0/r_c = 1/b$.

It is clear that the square root in (31) is non-negative

$$(1-w_0^{-4}) T(w_0) X(w_0) - \frac{(2\pi\alpha')^2 E^2 R^4}{r_0^4} \geq 0, \quad (33)$$

which leads to

$$E_c \leq T_F \frac{r_0^2}{R^2} \sqrt{(1-w_0^{-4}) T(w_0) X(w_0)}. \quad (34)$$

It is clear that the critical field E_c in the AdS black hole background with R^4 correction equals

$$E_c = T_F \frac{r_0^2}{R^2} \sqrt{(1-b^4) T(w_0) X(w_0)}. \quad (35)$$

Next, we define a dimensionless electric field α that measures E in a unit of the critical field E_c ,

$$\alpha \equiv \frac{E}{E_c}. \quad (36)$$

With the electrostatic potential E , one finds the total potential V_{tot} as

$$V_{\text{tot}} = V_{\text{CP+SE}} - Ex = 2T_F r_0 \left[a \int_1^{1/a} dy \frac{\sqrt{T(w)R(w)}}{\sqrt{1 - \frac{(1-w_c^{-4})T(w_c)X(w_c)}{y^4(1-w^{-4})T(w)X(w)}}} - \frac{\alpha}{a} \sqrt{(1-b^4)T(w_0)X(w_0)} \int_1^{1/a} \frac{dy}{y^2 \sqrt{1-w^{-4}}} \sqrt{\frac{R(w)}{X(w)}} \frac{1}{\sqrt{\frac{y^4(1-w^{-4})T(w)X(w)}{(1-w_c^{-4})T(w_c)X(w)} - 1}} \right]. \quad (37)$$

For comparing with the Einstein case in [7], we set the fixed temperature $b = 0.5$. In Fig. 1, we plot the total potential V_{tot} as a function of the inter-distance x with coupling parameter $\lambda = 8$ (left panel) and $\lambda = 20$ (right panel). The shapes of the potential are plotted for $\alpha = 0.8, 0.9, 1.0$ and 1.1 from top to bottom, where

$T_F r_0 = L^2/r_0 = 1$. From the figures, we can see that the potential barrier vanishes for $\alpha \geq 1.0$ and the critical field is $\alpha = 1.0 (E = E_c)$, which is in agreement with the DBI result.

To study the effect of R^4 corrections on the potential barrier, we plot V_{tot} against x at $\alpha = 0.8$, with different

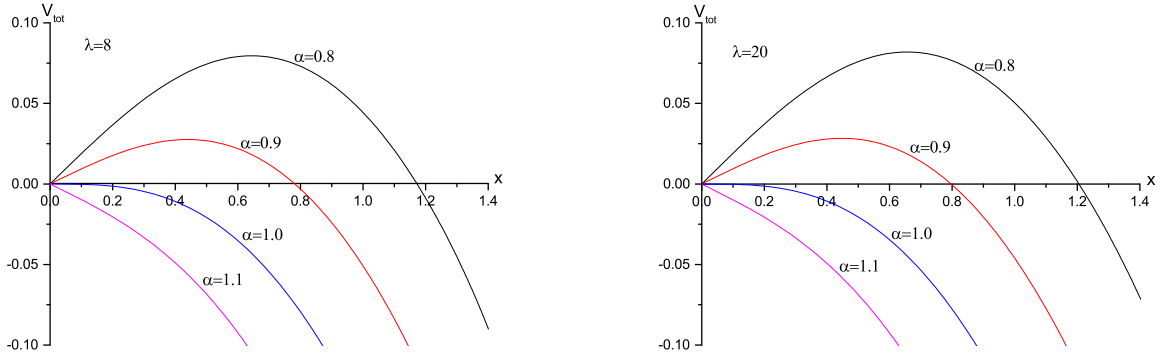


Fig. 1. (color online) V_{tot} against x in the presence of R^4 corrections. Left: $\lambda = 8$; Right: $\lambda = 20$. In all of the plots from top to bottom $\alpha = 0.8, 0.9, 1.0, 1.1$, respectively.

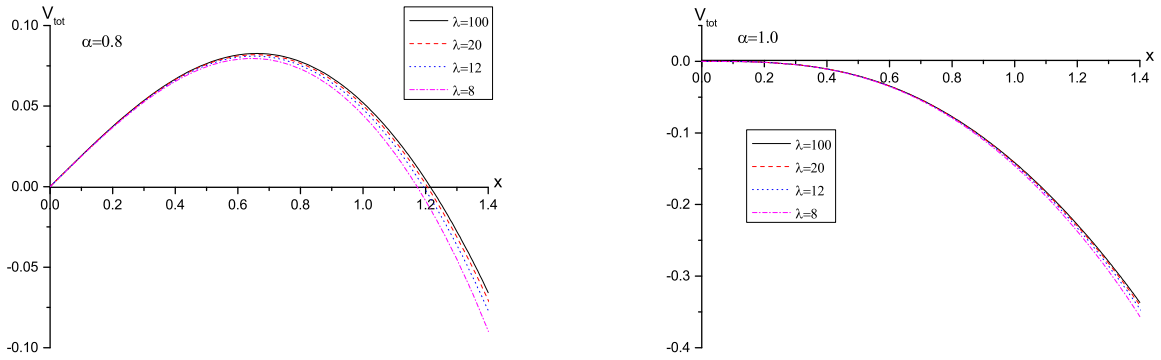


Fig. 2. (color online) V_{tot} against x in the presence of R^4 corrections. Left: $\alpha = 0.8$; Right: $\alpha = 1.0$. In all of the plots from top to bottom $\lambda = 100, 20, 12, 8$, respectively.

values of λ in the left panel of Fig. 2. It is found that decreasing λ leads to decreasing height and width of the barrier, and it is known that a higher barrier means the more difficult the pair production. This proves a way to enhance the Schwinger effect by decreasing the value of λ .

Further, we plot the shape of V_{tot} against x at $\alpha = 1.0$ with different λ in the right panel of Fig. 2, to show the effect of R^4 corrections on E_c . It implies that the vacuum becomes unstable when the barrier starts to vanish (at $\alpha \geq 1.0$) for each λ .

These results are consistent with the anticipation of the expression of production rate in section-1.

4 Confining D3-brane background

In this section, we study the Schwinger effect in a confining D3-brane background with R^4 corrections. The

induced metric is [27]

$$ds^2 = -\frac{r^2 T(w) dt^2}{R^2} + \frac{r^2 X(w)}{R^2} [(dx^1)^2 + (dx^2)^2 + (1-w^{-4})(dx^3)^2] + \frac{R^2 R(w) dr^2}{(1-w^{-4})r^2}, \quad (38)$$

with

$$\begin{aligned} T(w) &= 1 - k \left(75w^{-4} + \frac{1225}{16}w^{-8} - \frac{695}{16}w^{-12} \right) + \dots, \\ X(w) &= 1 - \frac{25k}{16}w^{-8}(1+w^{-4}) + \dots, \\ R(w) &= 1 + k \left(75w^{-4} + \frac{1175}{16}w^{-8} - \frac{4585}{16}w^{-12} \right) + \dots, \end{aligned} \quad (39)$$

and r_h represents the inverse compactification radius in the x^3 -direction.

Taking the previous steps, we obtained the inter-distance x and the sum of potential energy and static

energy $V_{\text{CP+SE}}$, respectively. One goes

$$x = \frac{2R^2}{ar_0} \int_1^{1/a} \sqrt{\frac{R(w)}{X(w)(1-w^{-4})}} \frac{dy}{y^2 \sqrt{\frac{y^4 T(w) X(w)}{T(w_c) X(w_c)} - 1}}, \quad (40)$$

$$V_{\text{CP+SE}} = 2T_F ar_0 \int_1^{1/a} \sqrt{\frac{T(w)R(w)}{(1-w^{-4})}} \frac{y^2 dy}{\sqrt{y^4 - \frac{T(w_c)X(w_c)}{T(w)X(w)}}}. \quad (41)$$

where $T(w_c)$ and $X(w_c)$ is defined in (24).

Now, we study the critical electric field as we did before, and the induced metric is:

$$\begin{aligned} G_{00} &= -\frac{r^2}{R^2} T(w) & G_{11} &= \frac{r^2}{R^2} X(w) \\ G_{22} &= \frac{r^2}{R^2} X(w) & G_{33} &= \frac{r^2}{R^2} (1-w^{-4}) X(w) \end{aligned} \quad (42)$$

Then, we obtain

$$G_{\mu\nu} + \mathcal{F}_{\mu\nu} = \begin{pmatrix} -\frac{r^2}{R^2} T(w) & 2\pi\alpha' E & 0 & 0 \\ -2\pi\alpha' E & \frac{r^2}{R^2} X(w) & 0 & 0 \\ 0 & 0 & \frac{r^2}{R^2} X(w) & 0 \\ 0 & 0 & 0 & \frac{r^2(1-w^{-4})}{R^2} X(w) \end{pmatrix} \quad (43)$$

which yields

$$\begin{aligned} \det(G_{\mu\nu} + \mathcal{F}_{\mu\nu}) &= -\frac{r^4}{R^4} X^2(w)(1-w^{-4}) \left[\frac{r^4}{R^4} T(w) X(w) - (2\pi\alpha' E)^2 \right], \end{aligned} \quad (44)$$

where we assume the electric field E is turned on along the x^1 -direction as well.

Substituting (44) into (27) and setting the D3-brane at $r=r_0$, one can obtain

$$S_{\text{DBI}} = -T_{D3} \frac{r_0^4}{R^4} \int d^4x \sqrt{X^2(w_0)(1-w_0^{-4}) \left[T(w_0) X(w_0) - \frac{(2\pi\alpha')^2 E^2 R^4}{r_0^4} \right]}, \quad (45)$$

then, obviously,

$$X^2(w_0)(1-w_0^{-4}) > 0. \quad (46)$$

For ensuring the square root in (45) is non-negative

$$T(w_0) X(w_0) - \frac{(2\pi\alpha')^2 E^2 R^4}{r_0^4} \geq 0, \quad (47)$$

which means

$$E \leq T_F \frac{r_0^2}{R^2} \sqrt{T(w_0) X(w_0)}. \quad (48)$$

Obviously, the critical field E_c in the confining D3-brane background with R^4 corrections is obtained

$$E = T_F \frac{r_0^2}{R^2} \sqrt{T(w_0) X(w_0)}. \quad (49)$$

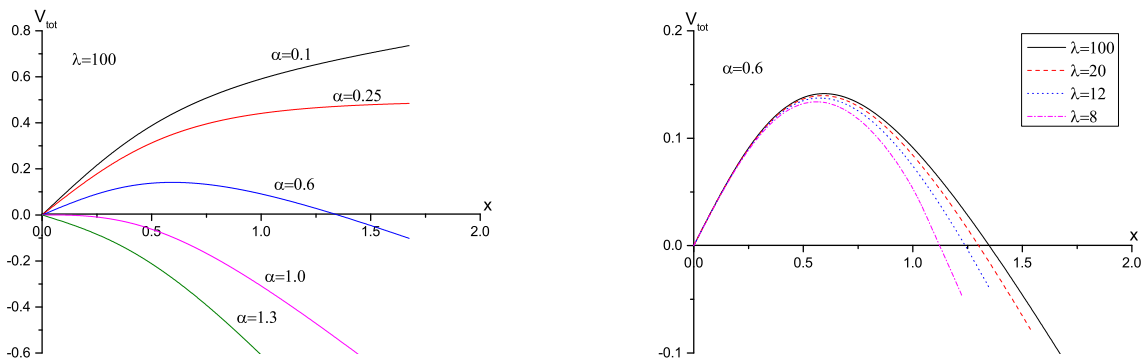


Fig. 3. (color online) V_{tot} against X . Left: $\lambda=100$, from top to bottom $\alpha=0.1, 0.25, 0.6, 1.0, 1.3$; Right: $\alpha=0.6$, from top to bottom $\lambda=100, 20, 12, 8$, respectively.

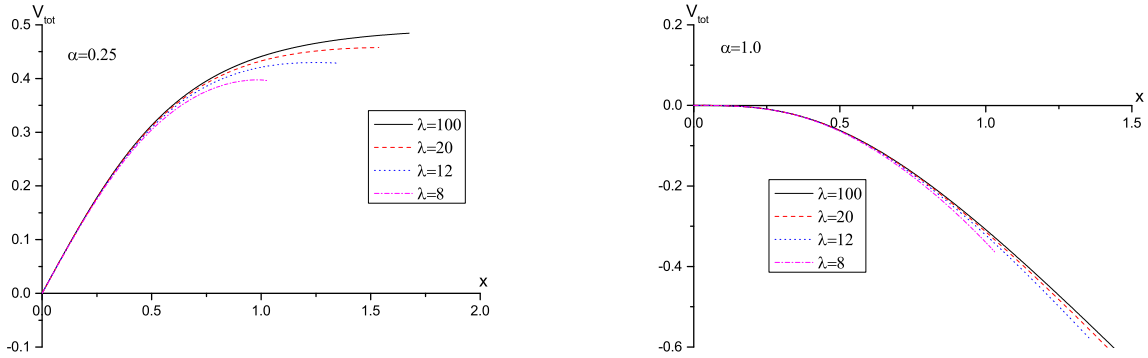


Fig. 4. (color online) V_{tot} against X . Left: $\alpha=0.25$; Right: $\alpha=1.0$. In the plots from top to bottom $\lambda=100,20,12,8$, respectively.

The total potential is

$$V_{\text{tot}} = V_{\text{CP+SE}} - Ex = 2T_F r_0 \left[a \int_1^{1/a} \sqrt{\frac{T(w)R(w)}{(1-w^{-4})}} \frac{y^2 dy}{\sqrt{y^4 - \frac{T(w_c)X(w_c)}{T(w)X(w)}}} - \frac{\alpha}{a} \sqrt{T(w_0)X(w_0)} \int_1^{1/a} \sqrt{\frac{R(w)}{X(w)(1-w^{-4})}} \frac{dy}{y^2 \sqrt{\frac{y^4 T(w)X(w)}{T(w_c)X(w_c)} - 1}} \right]. \quad (50)$$

where α is defined in (36).

We plot V_{tot} versus x with $\lambda=100$ by setting $b = 0.5$ and $2L^2/r_0 = 2T_F r_0 = 1$ in the left panel of Fig. 3, the same as [26], which is similar picture to other cases with different λ . As the figures shows, there exist two critical values for the electric field: one is at $\alpha=1(E=E_c)$, the other is at $\alpha=0.25(E=E_s=0.25E_c)$.

As we did before, to analyze the effect of R^4 corrections on the potential barrier, we plot V_{tot} versus x at $\alpha=0.6$ with different λ in the right panel of Fig. 3, which shows that as λ decreases both the height and width of the potential barrier decrease. Therefore, one could conclude that increasing λ enhances the Schwinger effect.

Similarly, to show the effect of R^4 corrections on the two critical fields, we plot V_{tot} versus x at $\alpha=0.25$ and $\alpha=1$ with different λ in Fig. 4. From the left panel, one can see that for various λ , the potential with $\alpha=0.25$ will saturate at large x . From the right panel, one can find that the barrier vanishes for each plot at $\alpha=1$, which is in agreement with the DBI result.

These results are consistent with our theoretical ex-

pectations in section-1.

5 Conclusion and discussion

We have investigated R^4 corrections to the holographic Schwinger effect in an AdS black hole background and a confining D3-brane background. The critical value for the electric field is obtained. It is shown that for both backgrounds, decreasing the parameter λ enhances the Schwinger effect.

Further, the study on the relation between Schwinger effect and η/s at strong coupling, as expressed in Eq. (10), shows that increasing λ leads to decreasing η/s , thus making the fluid conform more to the ideal case, and decreasing λ leads to increasing Schwinger effect. Thus, one can conclude that the Schwinger effect is enhanced while the η/s decreases at strong coupling.

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