Systematic description of nuclear electric quadrupole moments *

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Abstract: The nuclear electric quadrupole moment (NQM) is one of the fundamental bulk properties of the nucleus with which nuclear deformations can be investigated. The number of measured NQMs is significantly less than that of known masses, and there is still no global NQM formula for all bound nuclei. In this paper, we propose an analytical formula, which includes the shell corrections and which is the function of the charge number, mass number, spin, charge radius, and nuclear deformation, for calculating the NQMs of all bound nuclei. Our calculated NQMs of 524 nuclei in their ground states are reasonable compared to the experimental data based on the nuclear deformation parameters derived from the Weizsäcker-Skyrme (WS) nuclear mass models. Smaller rms deviations between the calculated NQMs and experimental data indicate that the deformation parameters predicted from the WS mass models are reasonable. In addition, 161 unmeasured NQMs with known spins are also predicted with the proposed formula.

Keywords: nuclear electric quadrupole moment, analytical formula, deformation parameter

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1 Introduction

The nuclear electric quadrupole moment (NQM) as one of the fundamental bulk properties can provide information about the non-spherical shape of the nuclear charge distribution [1]. The study of NQMs is helpful for understanding nuclear exotic structures, such as the shape coexistence [2–5], nucleon-nucleon interaction [6], and shell evolution [7]. Recently, NQM has been regarded as a microscopic probe to study the motion of atomic tunneling systems in amorphous solids [8], as a reason of the Cotton-Mouton effect in molecules and noble gas atoms [9, 10].

Several tables of experimental NQMs were compiled over the last several decades [11–24]. Together with some new and updated measurements for NQMs in recent years, there are about five hundred recommended experimental NQMs in the ground states. Obviously, the number of experimental NQMs is significantly less than that of the known masses (AME2012 [25] and AME2016 [26]). More theoretical studies, as well as experimental measurements, are still needed.

Recently, the NQMs of some medium mass nuclei or several isotope chains have been calculated using the shell models [27–29], self-consistent theory of finite Fermi systems based on the energy density functional [30–32], covariant density functional theory [3], and configuration-constrained potential energy surface method [33]. Although these theoretical results are in good agreement with experimental NQMs, there is lack of general analytical formula for calculating the NQMs of all bound nuclei. In this paper, a new analytical formula is proposed that involves the charge number, mass number, spin, charge radius, and deformation parameters.

The structure of this paper is as follows. In Sec. 2, an analytical formula of NQM is introduced. In Sec. 3, the calculated NQMs are compared with experimental data and previous theoretical results. Finally, a summary is given in Sec. 4.

2 Theoretical formula

Assuming a nucleus with charge density $\rho(\boldsymbol{r}')$ and an axially symmetric static deformation, its electric potential ϕ can be expressed as

$$\phi(\boldsymbol{r})=\frac{1}{4\pi\varepsilon_0}\int_V \rho(\boldsymbol{r}')\frac{\mathrm{d}\boldsymbol{r}'}{\boldsymbol{r}-\boldsymbol{r}'}, \tag{1}$$

where ε_0 is the vacuum dielectric constant. $\boldsymbol{r}$ and $\boldsymbol{r}'$ are position vectors from the origin of the coordinate system located at the center of mass to the observation point and the charge volume element $\rho(\boldsymbol{r}')\mathrm{d}\boldsymbol{r}'$, respectively. Obviously, there is $r \gg r'$, so the distance between $\boldsymbol{r}$ and $\boldsymbol{r}'$

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can be expressed as [34]

$$\frac{1}{|\boldsymbol{r}-\boldsymbol{r}'|}=\frac{1}{r}\sum_{n=0}^{\infty}P_n(\cos\theta)\left(\frac{r'}{r}\right)^n, \tag{2}$$

where θ is the angle between $\boldsymbol{r}$ and $\boldsymbol{r}'$, and $P_n(\cos\theta)$ is the Legendre polynomial.

Further it is assumed that the charge density is constant, i.e., $\rho(\boldsymbol{r}')=\rho$. In terms of Eq. (2), Eq. (1) can be rewritten as

$$\begin{aligned}\phi(r) &= \sum_{n=0}^{\infty}\phi_n(r)\\ &= \frac{\rho}{4\pi\varepsilon_0}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\int_V r'^n P_n(\cos\theta)\mathrm{d}\boldsymbol{r}'.\end{aligned} \tag{3}$$

The first term $\phi_{n=0}$ and the second term $\phi_{n=1}$ are called monopole potential and dipole potential, respectively. The third term $\phi_{n=2}$, a quadrupole electric potential, can be expressed as

$$\phi_{n=2}=\frac{\rho}{4\pi\varepsilon_0}\frac{1}{r^3}\int_V\frac{1}{2}(3z'^2-r'^2)\mathrm{d}\boldsymbol{r}'. \tag{4}$$

It is generally considered that the quadrupole electric potential $\phi_{n=2}$ is caused by the intrinsic electric quadrupole moment Q_0. Thus, Q_0 is defined as

$$Q_0=\frac{\rho}{e}\int_V(3z'^2-r'^2)\mathrm{d}\boldsymbol{r}', \tag{5}$$

where e is the charge element. It is obvious that the intrinsic electric quadrupole moment Q_0 is closely related to the nuclear shape. Therefore, we define the radius $R(\theta)$ as the distance from the origin of the coordinate system to a point on the nuclear surface

$$R(\theta)=c_0R_c[1+\beta_2Y_{20}(\theta)+\beta_4Y_{40}(\theta)], \tag{6}$$

where the scale factor $c_0\approx1$ represents the incompressibility of the nuclear matter and is determined by the so-called constant volume condition [35, 36]. $Y_{20}(\theta)$ and $Y_{40}(\theta)$ are spherical harmonic functions. β_2 and β_4 are the quadrupole and octupole deformation parameters, respectively. In this paper, the effects of the higher order deformations are neglected.

Considering the influence of the nuclear shell effect and the isospin effect based on the Weizsäcker-Skyrme (WS) nuclear mass models [37–40], the nuclear charge radius R_c in Eq. (6) is expressed as [41]

$$R_c=1.226A^{1/3}+2.86A^{-2/3}-1.09(I-I^2)+0.99\frac{\Delta E}{A}, \tag{7}$$

where $I=\frac{N-Z}{A}$ is the isospin asymmetry, and ΔE denotes the shell correction energies of the nuclei at their ground states. The root-mean-square (rms) deviation derived from Eq. (7) with respect to 885 measured charge radii falls to 0.022 fm [41]. With the expression of the charge radii in Eq. (7), the intrinsic electric quadrupole moment can be rewritten as

$$Q_0=\frac{\rho}{e}\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}(3\cos^2\theta-1)\sin\theta\mathrm{d}\theta\int_0^{R(\theta)}r'^4\mathrm{d}r'. \tag{8}$$

After computing multiple and complex integral, Eq. (8) can be explicitly expressed as

$$Q_0=\frac{2\pi\rho}{5e}c_0^5R_c^5f(\beta_2,\beta_4). \tag{9}$$

Neglecting higher order terms, the factor $f(\beta_2,\beta_4)$ can be expressed as

$$f(\beta_2,\beta_4)=2\sqrt{\frac{5}{\pi}}\beta_2+\frac{20\beta_2^2}{7\pi}+\frac{24\sqrt{5}\beta_2\beta_4}{7\pi}+\frac{200\beta_4^2}{77\pi}. \tag{10}$$

In addition, the proton number Z can also be expressed as

$$\begin{aligned}Z &= \frac{\rho}{e}\int_0^{2\pi}\mathrm{d}\varphi\int_0^{\pi}\sin\theta\mathrm{d}\theta\int_0^{R(\theta)}r'^2\mathrm{d}r'\\ &= \frac{2}{3}\pi\frac{\rho}{e}c_0^3R_c^3k(\beta_2,\beta_4),\end{aligned} \tag{11}$$

with

$$k(\beta_2,\beta_4)=2+\frac{3}{2\pi}(\beta_2^2+\beta_4^2). \tag{12}$$

It is obvious that $k(\beta_2,\beta_4)=2$ for spherical nuclei.

Therefore, the intrinsic electric quadrupole moment Q_0 can be simplified as

$$Q_0=\frac{3}{5}c_0^2R_c^2Z\frac{f(\beta_2,\beta_4)}{k(\beta_2,\beta_4)}. \tag{13}$$

Obviously, if to retain only the 1st and 2nd terms in $f(\beta_2,\beta_4)$, and only the 1st term in $k(\beta_2,\beta_4)$, Eq. (13) has an expression similar to Refs. [42–45].

It can be seen that the intrinsic electric quadrupole moment Q_0 directly depends on the nuclear charge radius. So, for an axially symmetric deformed nucleus with a spin S of the ground state, the observable electric quadrupole moment is empirically expressed as

$$Q=\frac{S(2S-1)}{(S+1)(2S+3)}Q_0. \tag{14}$$

From Eq. (14) it can be seen that the values of Q for nuclei in the ground states with $S=0$ or $1/2$ are zero [24]. Here we consider only nuclei with $S>1/2$. In addition, the latest recommended experimental NQM values [24] are adopted for comparison. It is worth mentioning that the data of Ref. [24] are not only recently updated, but also three nuclei (^{23}Al and ^{121,123}Ba) from Ref. [22] are omitted, as shown in Table 1.

3 Results and discussions

Theoretically, according to Eqs. (13) and (14), it is possible to calculate the NQMs of arbitrary nuclei in both the ground and excited states. However, it is

Table 1. The rms deviations σ with respect to nuclear quadrupole moments. The 1st column denotes the year of the experimental data compilations, and the 2nd column indicates the amount of experimental data. Residual columns denote the rms deviations (in barn) based on the deformation parameters from different WS nuclear mass models.

year	number	WS3.2	WS3.3	WS3.6	WS4
2001 [14]	389	0.626	0.613	0.615	0.618
2005 [15]	463	0.616	0.605	0.607	0.610
2011 [16]	509	0.574	0.564	0.565	0.568
2013 [17]	525	0.566	0.553	0.554	0.558
2014 [22]	527	0.555	0.544	0.546	0.552
2016 [24]	524	0.492	0.477	0.479	0.486

difficult to precisely measure the deformation parameters of the nuclei. Fortunately, in recent years, some nuclear mass models have been successfully proposed, such as the macro-microscopic WS mass models (WS3.2 [37], WS3.3 [38], WS3.6 [39], and WS4 [40]), from which the deformation parameters of the ground states can be obtained. Based on the deformation parameters derived from the WS mass models, the rms deviations $\sigma^2=\frac{1}{M}\sum_{i=1}^{M}[Q^i_{\rm exp}-Q^i_{\rm th}]^2$ between the predictions of Eq. (14) and the experimental data compiled in different years are listed in Table 1. The 1st column indicates the years of compilation of experimental data, and the 2nd column indicates the amount of experimental data. Residual columns denote the rms deviations using deformation parameters from different WS mass models. These WS mass models are based on the macroscopic-microscopic method, in which the isospin and mass dependence of model parameters are investigated with the Skyrme energy density functional. The rms deviation of WS3.2 model is 0.516 MeV with respect to 2149 known nuclear masses (N and $Z\geq 8$), and the new magic number $N=16$ in light neutron-rich nuclei and the shape coexistence phenomena for some nuclei have been examined with this model [37]. Further, considering the mirror nuclei constraint, the rms deviation with respect to 2149 known nuclear masses is reduced to 0.441 MeV, and the rms deviation of $\alpha-$decay energies of 46 superheavy nuclei is reduced to 0.263 MeV in WS3.3 model [38]. For WS3.6 model, the rms deviation with respect to 2149 known nuclear masses is significantly reduced to 336 keV by considering some residual corrections [39]. Further, taking into account the surface diffuseness correction for unstable nuclei in WS4 model [40], the rms deviations fall to 298 keV with respect to the 2353 known masses. From Table 1, it can be seen that the predictions of NQMs using deformation parameters from the WS models are closer to the experimental data recommended in recent years. This indicates that the deformation parameters obtained from WS mass models are reasonable. In particular, the rms deviations using deformation parameters from WS3.3 model are the smallest for the same experimental data compilation. However, considering the smallest rms deviation with respect to the nuclear masses, neutron separation energies, and α-decay Q-values [40], the deformation parameters obtained from the WS4 mass model are used for the analysis in the following calculations.

Figure 1 shows the difference between theoretical NQMs in the ground states and 524 experimental data ($Z\geq 8$, $A\geq 16$). In Fig. 1, the red circles show the results for the deformation parameters derived from WS4 mass model, for which the rms deviation is 0.486 barn. It is worth mentioning that the scales of horizontal and vertical coordinates are different on two panels. It can be seen that the differences between the calculated NQMs and measured data are smaller for the light nuclei, which is also observed in Refs. [46–48].

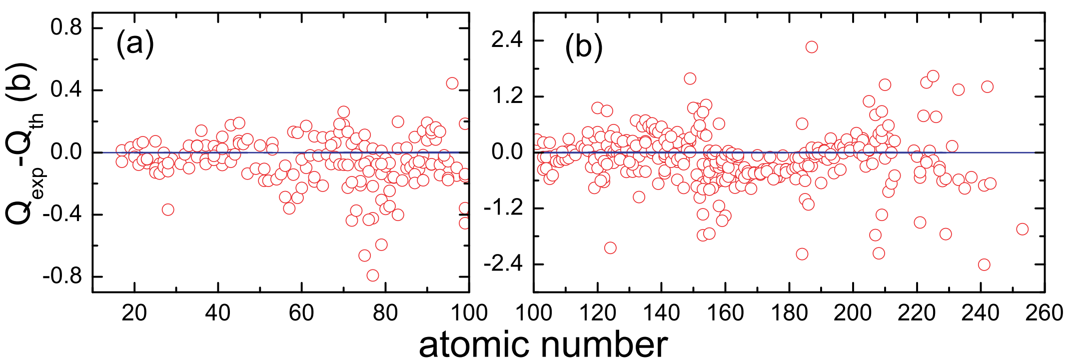

Fig. 1. (color online) Difference between the calculated NQMs and experimental data for the ground states of 524 nuclei ($Z\geq 8$, $A\geq 16$). (a) For nuclei with $A<100$, and (b) for nuclei with $A\geq 100$.

Figure 2 shows the difference between the theoretical results and experimental data for a) the odd Z - even N nuclei (179 isotopes, σ=0.411 b), b) even Z - odd N nuclei (180 isotopes, σ = 0.541 b), and c) odd Z - odd N nuclei (165 isotopes, σ=0.568 b). It can be seen that the discrepancies are relatively large for odd Z - odd N nuclei, which is similar to the cases in the description of nuclear masses in ground states [37–40]. For even Z - even N nuclei with zero spin, their NQMs are zero, which has been verified by the measurements and Eq. (14).

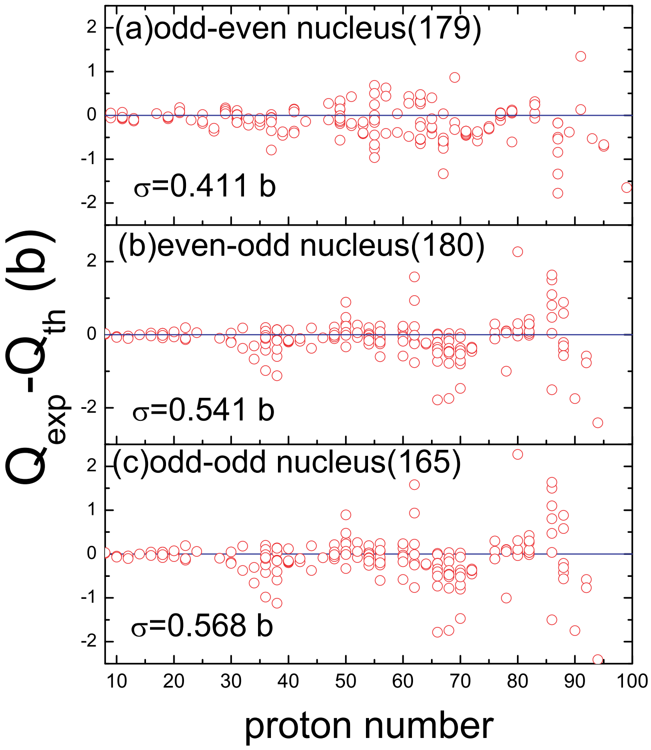

Fig. 2. (color online) The same as Fig. 1, but for the odd Z - even N nuclei (179), even Z - odd N nuclei (180), and odd Z - odd N nuclei (165), respectively.

Figure 3 shows comparisons of experimental NQMs with the results of the shell model calculations and those of this paper. The theoretical values of $Q_{\rm th1}$ (black squares) are derived from the pf-shell model calculations [28]. The theoretical values of $Q_{\rm WS4}$ (red circles) are the results of this work, obtained with deformation parameters from the WS4 mass model. The experimental data are taken from Ref. [22]. Fig. 4 shows comparisons of experimental NQMs with the least-squares fitting of experimental data $Q_{\rm th2}$ (black squares). In Figs. 3 and 4, it can be seen that the model predictions of $Q_{\rm WS4}$ in this work are comparable with the predictions of the shell model and the least-squares fitting method. In this work, we also predict 161 unmeasured NQMs of nuclei with known spins using deformation parameters derived from the WS4 mass model, and the results are listed in Table 2.

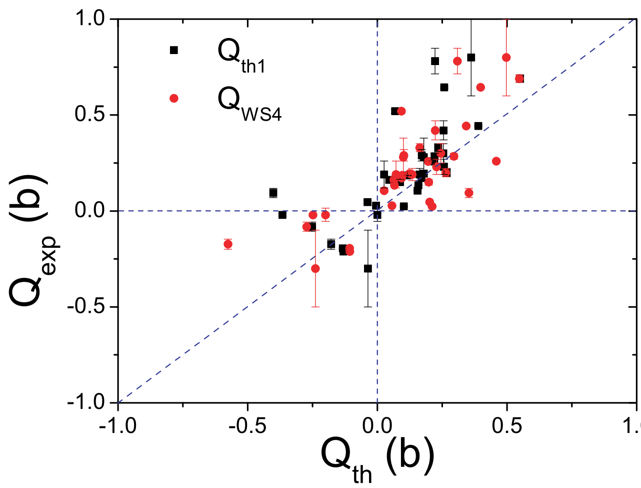

Fig. 3. (color online) Comparisons of experimental NQMs with the results of the shell model calculations and this paper. The theoretical values of $Q_{\rm th1}$ (black squares) are derived from the pf-shell model calculations [28], and the theoretical values of $Q_{\rm WS4}$ (red circles) are the results of this paper with deformation parameters derived from mass model WS4. The experimental data are taken from Ref. [24].

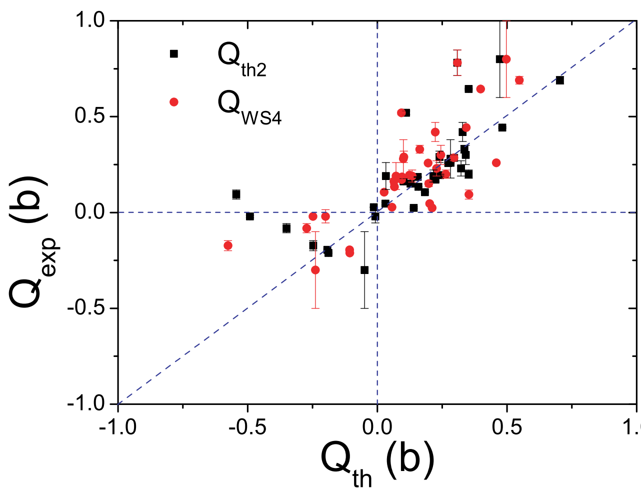

Fig. 4. (color online) The same as Fig. 3, but the theoretical values of $Q_{\rm th2}$ (black squares) are derived from the least-squares fitting of the experimental data [28].

4 Summary

An analytical formula that takes into account the shell corrections, which is a function of the charge number, mass number, spin, charge radius, and deformation parameters, is proposed to calculate the NQMs of arbitrary nuclei. Using the deformation parameters of the ground states derived from the WS mass models, the

Table 2. The predicted NQMs (Q_{WS4}, in barn) in the ground state, the deformation parameters of which are derived from the WS4 mass model. The 4th column denotes the spin S of the ground state derived from Ref. [22]. The 5th and 6th columns denote the deformation parameters derived from Ref. [40].

Z	N	A	S	β_2	β_4	Q_{WS4}
11	13	24	4.0	0.1979	0.0073	0.1331
11	19	30	2.0	0.1366	0.0032	0.0523
11	20	31	1.5	0.2863	0.0390	0.0795
12	9	21	2.5	0.1914	0.0112	0.1017
12	17	29	1.5	0.1475	-0.0069	0.0431
12	21	33	1.5	0.3085	0.0241	0.0962
13	10	23	2.5	0.2065	0.0199	0.1241
13	17	30	3.0	0.0983	-0.0045	0.0670
13	21	34	4.0	0.0961	0.0095	0.0838
14	19	33	1.5	-0.0412	-0.0012	-0.0144
14	21	35	3.5	-0.0520	0.0045	-0.0430
15	17	32	1.0	-0.0509	-0.0015	-0.0098
17	15	32	1.0	-0.0536	-0.0020	-0.0123
17	16	33	1.5	-0.0538	-0.0027	-0.0247
17	21	38	2.0	-0.0656	0.0027	-0.0447
17	27	44	2.0	0.1106	-0.0089	0.0792
19	16	35	1.5	-0.0374	-0.0015	-0.0206
19	17	36	2.0	-0.0561	-0.0034	-0.0442
19	19	38	3.0	-0.0443	-0.0014	-0.0516
19	23	42	2.0	0.0814	0.0105	0.0700
19	24	43	1.5	0.0936	0.0077	0.0562
19	25	44	2.0	0.0937	0.0003	0.0798
19	26	45	1.5	0.0851	-0.0065	0.0500
19	27	46	2.0	0.0575	-0.0045	0.0478
19	32	51	1.5	-0.0358	-0.0024	-0.0204
20	27	47	3.5	0.0389	-0.0023	0.0563
20	29	49	1.5	-0.0243	0.0003	-0.0147
21	27	48	6.0	0.0595	0.0042	0.1275
21	28	49	3.5	-0.0385	0.0038	-0.0586
22	21	43	3.5	-0.0632	0.0078	-0.1032
23	25	48	4.0	0.1463	0.0181	0.3015
23	26	49	3.5	0.1324	0.0113	0.2474
24	25	49	2.5	0.1555	0.0155	0.2378
24	27	51	3.5	0.1175	0.0087	0.2316
26	33	59	1.5	0.1050	-0.0056	0.1007
27	28	55	3.5	0.0481	-0.0035	0.1079
28	27	55	1.5	0.0487	-0.0033	0.0492
28	29	57	1.5	0.0385	0.0000	0.0391
28	37	65	2.5	-0.0814	-0.0041	-0.1484
29	28	57	3.5	0.0392	0.0000	0.0974
29	47	76	6.0	0.1059	0.0059	0.3989
29	48	77	2.5	0.0925	-0.0019	0.1959
31	42	73	1.5	-0.1697	-0.0182	-0.1956
33	36	69	2.5	-0.1251	-0.0019	-0.2886
33	41	74	2.0	-0.1777	-0.0122	-0.3208
33	44	77	1.5	0.1270	0.0111	0.1863
34	39	73	4.5	-0.1444	-0.0091	-0.5279
35	37	72	3.0	-0.1243	-0.0073	-0.3652
35	40	75	1.5	-0.1912	-0.0030	-0.2635
35	43	78	1.0	-0.2121	-0.0118	-0.1442
35	49	84	2.0	0.0962	-0.0170	0.2130
39	47	86	4.0	0.0500	-0.0033	0.2303
41	48	89	4.5	0.0450	-0.0010	0.2386
41	54	95	4.5	0.0378	0.0033	0.2056
41	55	96	6.0	0.0270	0.0041	0.1698
41	56	97	4.5	-0.0211	0.0042	-0.1137

Table 2 – continued from previous page

Z	N	A	S	β_2	β_4	Q_{WS4}
42	49	91	4.5	0.0429	0.0056	0.2366
42	61	103	1.5	-0.2091	-0.0008	-0.3997
42	63	105	2.5	-0.2094	-0.0092	-0.7148
43	50	93	4.5	-0.0439	0.0041	-0.2418
43	51	94	7.0	-0.0519	0.0044	-0.3520
43	52	95	4.5	0.0640	0.0190	0.3765
43	53	96	7.0	0.0686	0.0159	0.4972
44	51	95	2.5	0.0618	0.0162	0.2434
44	53	97	2.5	0.0799	0.0163	0.3200
44	61	105	1.5	-0.2139	-0.0007	-0.4347
45	60	105	3.5	0.1881	0.0224	1.0865
45	61	106	1.0	0.2056	0.0202	0.2559
46	55	101	2.5	0.1159	0.0081	0.4990
47	50	97	4.5	0.0534	-0.0018	0.3419
47	51	98	6.0	0.0631	0.0015	0.4714
47	51	98	5.0	0.0631	0.0015	0.4327
47	52	99	4.5	0.0801	0.0052	0.5266
47	53	100	5.0	0.0841	0.0009	0.5865
47	55	102	5.0	0.1130	0.0021	0.8041
47	59	106	1.0	0.1250	-0.0037	0.1570
47	61	108	1.0	0.1347	0.0038	0.1725
47	65	112	2.0	0.1822	0.0083	0.6869
51	63	114	3.0	0.0867	-0.0024	0.5128
51	64	115	2.5	-0.0982	0.0041	-0.4698
51	65	116	3.0	-0.1276	0.0088	-0.7080
51	67	118	1.0	-0.1395	0.0045	-0.1850
51	68	119	2.5	-0.1463	0.0019	-0.6906
51	69	120	1.0	-0.1500	-0.0011	-0.1980
51	69	120	8.0	-0.1500	-0.0011	-1.3893
51	74	125	3.5	0.1189	-0.0030	0.8179
51	75	126	8.0	0.1054	-0.0017	1.0901
51	76	127	3.5	0.0919	0.0009	0.6322
51	77	128	8.0	0.0791	0.0040	0.8191
51	78	129	3.5	0.0624	0.0080	0.4297
51	80	131	3.5	0.0442	0.0028	0.3014
51	82	133	3.5	0.0186	0.0081	0.1266
52	75	127	1.5	0.1252	0.0065	0.3846
52	79	131	1.5	0.0727	0.0123	0.2228
53	64	117	2.5	0.1951	0.0221	1.1023
53	65	118	2.0	0.1983	0.0183	0.8968
53	66	119	2.5	0.2016	0.0154	1.1406
53	67	120	2.0	0.2010	0.0104	0.9082
53	68	121	2.5	0.2000	0.0057	1.1274
53	69	122	1.0	0.1933	0.0046	0.3052
53	70	123	2.5	0.1817	0.0038	1.0235
53	71	124	2.0	0.1796	0.0008	0.8090
53	73	126	2.0	0.1649	0.0000	0.7437
53	77	130	5.0	-0.1173	-0.0096	-0.9688
53	82	135	3.5	-0.0481	0.0074	-0.3383
55	63	118	6.0	0.2102	0.0144	2.1929
56	65	121	2.5	0.2232	0.0032	1.3550
56	67	123	2.5	0.2273	-0.0047	1.3778

Table 2 – continued from previous page

Z	N	A	S	β_2	β_4	Q_{WS4}
58	79	137	1.5	0.1135	-0.0122	0.4041
58	81	139	1.5	0.0867	-0.0047	0.3092
58	83	141	3.5	0.0844	0.0048	0.7122
58	85	143	1.5	0.1027	0.0045	0.3767
61	77	138	3.0	0.1456	-0.0287	1.1510
61	82	143	2.5	0.0566	-0.0113	0.3822
61	83	144	5.0	0.0631	-0.0076	0.6954
61	88	149	3.5	0.1830	0.0348	1.7868
63	75	138	6.0	0.1966	-0.0102	2.5119
63	76	139	5.5	0.1883	-0.0125	2.3096
64	83	147	3.5	-0.0220	0.0036	-0.2058
64	85	149	3.5	-0.0438	0.0017	-0.4096
64	87	151	3.5	0.1738	0.0185	1.7774
64	89	153	1.5	0.1952	0.0265	0.8715
64	95	159	1.5	0.2565	0.0371	1.1883
70	87	157	3.5	0.1209	0.0264	1.3952
73	104	177	3.5	0.2355	-0.0295	2.9340
73	110	183	3.5	0.2200	-0.0550	2.7156
74	111	185	1.5	0.2033	-0.0558	1.0938
74	113	187	1.5	0.1853	-0.0500	1.0030
75	104	179	2.5	0.2129	-0.0231	2.1093
75	105	180	1.0	0.2110	-0.0279	0.5840
75	106	181	2.5	0.2078	-0.0330	2.0480
79	104	183	2.5	-0.1581	0.0001	-1.5167
79	109	188	1.0	0.1497	-0.0254	0.4445
79	111	190	1.0	0.1357	-0.0261	0.4029
81	117	198	2.0	-0.0547	-0.0037	-0.4641
81	119	200	2.0	-0.0666	-0.0039	-0.5638
81	121	202	2.0	-0.0583	-0.0035	-0.4970
81	123	204	2.0	-0.0439	-0.0069	-0.3762
81	127	208	5.0	-0.0132	0.0025	-0.2354
82	101	183	1.5	-0.0370	0.0037	-0.2198
82	103	185	1.5	-0.0709	0.0101	-0.4199
82	102	184	6.5	-0.0580	0.0117	-1.1211
82	107	189	1.5	0.1087	0.0016	0.6856
83	116	199	4.5	-0.0422	0.0018	-0.7136
83	118	201	4.5	-0.0533	0.0054	-0.9030
83	128	211	4.5	-0.0266	0.0079	-0.4654
84	117	201	1.5	-0.0689	0.0079	-0.4334
84	119	203	2.5	-0.0738	0.0083	-0.8297
84	121	205	2.5	-0.0676	0.0079	-0.7640
84	123	207	2.5	-0.0511	0.0014	-0.5796
85	132	217	4.5	0.0760	0.0402	1.4998
87	115	202	3.0	0.1157	0.0061	1.6926
87	116	203	4.5	0.1088	0.0049	2.0806
87	117	204	3.0	-0.1175	0.0056	-1.5847
87	118	205	4.5	-0.1151	0.0079	-2.0415
87	119	206	3.0	-0.1122	0.0089	-1.5256
89	128	217	4.5	0.0573	0.0168	1.1558
91	137	228	3.0	0.1691	0.0754	3.0037
91	139	230	2.0	0.1802	0.0765	2.2115
96	147	243	2.5	0.2270	0.0491	3.7338
96	149	245	3.5	0.2265	0.0399	4.8454

measured NQMs can be well reproduced. The root-mean-square deviations with respect to 524 ($Z \geq 8$, $A \geq 16$) measured NQMs for the ground states are about 0.5 barn based on the nuclear deformation parameters derived from the WS mass models. The results show that the deformation parameters derived from the WS mass models are reasonable. The deviations from the measured data are smaller for odd Z - even N nuclei, similar to the cases in the description of nuclear masses. In addition, the results of this approach are comparable with the results of the microscopic shell model and the least-squares fitting of experimental data. With the formula proposed in this paper, 161 unmeasured NQMs of nuclei with known spins are also predicted.

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