Doubly coupled matter fields in massive bigravity *

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Abstract: In the context of massive (bi-)gravity, non-minimal matter couplings have been proposed. These couplings are special in the sense that they are free of the Boulware-Deser ghost below the strong coupling scale and can be used consistently as an effective field theory. Furthermore, they enrich the phenomenology of massive gravity. We consider these couplings in the framework of bimetric gravity and study the cosmological implications for background and linear tensor, vector, and scalar Previous works have investigated special branches of solutions. Here we perform a complete perturbation analysis for the general background equations of motion, completing previous analyses.

Keywords: modified gravity, cosmological perturbation theory, inflation

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1 Introduction

High precision cosmological observations have made it possible to test the underlying fundamental theory of gravity. Together with the assumption of General Relativity (GR) being the right theory, and the cosmological principle, the universe is well described by the ΛCDM model. It constitutes a predominant amount of dark energy in form of a cosmological constant and dark matter. Aside from negligible reported anomalies [1], the model is still the best fit to current cosmological data [2–4]. In spite of its observational triumph, the model suffers from serious theoretical problems, the most persistent being the cosmological constant problem [5].

An alternative scenario for dark energy can be provided by infrared modifications of gravity. The simplest case corresponds to modifications in the form of an additional scalar field [6–10]. The presence of self-interactions of the scalar field and the non-minimal couplings to gravity yield interesting cosmological scenarios [11–19]. Other interesting dark energy scenarios can be accommodated by considering a vector field as an additional field. The question about the consistent self-interactions of the vector field, or similarly its non-minimal coupling to gravity, has been receiving renewed interest lately [20–28].

An unavoidable question is whether the graviton could be massive, which would correspond to a natural infrared modification of gravity, since the mediated force by a massive graviton would be suppressed at large scales. The weakening of the graviton could be put on an equal footing with recent cosmological acceleration. At the linear level the theory is described by the Fierz and Pauli mass terms [29] without introducing the ghostly sixth mode. This linear model, however, suffers from the vDVZ discontinuity [30, 31] when the mass of the graviton is set to zero, since General Relativity is not recovered in that limit. Actually, very soon after that, Vainshtein realized that the linear approximation breaks down at some distance far from the source and that non-linear interactions become appreciable close to the source [32]. Usually, these non-linear interactions reintroduce the ghostly six mode, the Boulware-Deser ghost [33], and it was a challenging task to construct potential interactions which would propagate only five physical degrees of freedom [34–38]. This ghost-free theory of massive gravity is also technically natural and does not obtain strong renormalization by quantum corrections [39, 40].

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In the context of quantum stability of the theory, new ways of coupling the matter fields have been explored [41–43] The classical potential interactions had to be tuned in a very specific way to keep the Boulware-Deser ghost absent, and if one wants to keep this property also at the quantum level, only very restricted matter couplings through an effective composite metric are allowed. This effective metric is built out of the two metrics in such a way that the matter quantum loops would only introduce a running of the cosmological constant for the effective metric, which in other words correspond exactly to the allowed potential interactions. These doubly coupled matter fields already introduce the Boulware-Deser ghost at the classical level [41, 44], but the coupling through the effective metric is special in the sense that the decoupling limit of the theory below the strong coupling scale is maintained ghost-free [45, 46]. Therefore, this coupling can be used as a consistent effective field theory. In the unconstrained vielbein formulation of the theory one can construct yet more types of effective metrics to which the matter fields can couple as well and the decoupling limit would still be free of the Boulware-Deser ghost [47]. Actually, the hope using the unconstrained vielbein formulation was to preserve the ghost freedom fully non-linearly with the original effective vielbein [48]. Unfortunately, this resulted in a negative result and also in this formulation the Boulware-Deser ghost is reintroduced [49]. However, it is worth mentioning that if one is willing to break the local Lorentz symmetry, one can indeed achieve this fully non-linearly [50]. The inclusion of the doubly coupled matter fields has very important implications for cosmological applications [41, 51–60] as well as for dark matter phenomenology [61–63].

The analysis of cosmological perturbations of the doubly coupled matter fields in massive gravity revealed that ghost and gradient instabilities can be successfully avoided together with the strong coupling issues, since the vector and scalar perturbations maintain their kinetic terms [52]. The application to massive bimetric gravity yielded gradient instability in the vector sector and ghost instability in the scalar sector for one of the branches of solutions, whereas the other branch of solutions was free of any ghost instability. It is still an open question whether this second branch of solutions is also free from any gradient instabilities. The main purpose of the present work is to investigate the perturbation analysis of the bimetric gravity theory in the presence of the doubly coupled matter fields on top of general background equations of motion, without specifying the branch and providing also the full quadratic action for the scalar perturbations. Thus, our work completes the analysis started in Ref. [56].

2 Dynamical composite metric

A consistent coupling of some extra scalar field ϕ to both metrics simultaneously was introduced in Ref. [41] through a composite metric $\tilde{g}_{\mu\nu}$,

$$\tilde{g}_{\mu\nu}\equiv\alpha^2 g_{\mu\nu}+2\alpha\beta g_{\mu\lambda}X^{\lambda}{}_{\nu}+\beta^2 f_{\mu\nu}, \tag{1}$$

with $X^{\mu}{}_{\nu}$ defined by

$$X^{\mu}{}_{\lambda}X^{\lambda}{}_{\nu}\equiv g^{\mu\lambda}f_{\lambda\nu}. \tag{2}$$

We consider the same action as in Ref. [56],

$$S=S^g+S^f+S^{\rm pot}+S^{\rm com}, \tag{3}$$

with

$$S^g = \int {\rm d}^4x\sqrt{-g}\left(\frac{M_g^2}{2}R[g]+\mathcal{L}^{\rm matter}[g]\right), \tag{4}$$

$$S^f = \int {\rm d}^4x\sqrt{-f}\left(\frac{M_f^2}{2}R[f]+\mathcal{L}^{\rm matter}[f]\right), \tag{5}$$

$$S^{\rm pot} = \int {\rm d}t{\rm d}^3x\sqrt{-g}M_g^2m^2\sum_{n=0}^{4}c_n e_n(\boldsymbol{X}), \tag{6}$$

$$S^{\rm com} = \int {\rm d}^4x\sqrt{-\tilde{g}}P(\tilde{X},\phi), \tag{7}$$

where $R[g]$ and $R[f]$ are Ricci scalars for $g_{\mu\nu}$ and $f_{\mu\nu}$, respectively. As in Ref. [56], in this work we consider the matter contents of the $g_{\mu\nu}$ and $f_{\mu\nu}$ metrics to be two cosmological constants: $\mathcal{L}^{\rm matter}[g]=-M_g^2\Lambda_g$ and $\mathcal{L}^{\rm matter}[f]=-M_f^2\Lambda_f$. $S^{\rm pot}$ denotes the non-derivative potential interactions $S^{\rm pot}$ of the two metrics, $\boldsymbol{X}$ stands for $X^{\mu}{}_{\nu}$, and for a matrix $M^{\mu}{}_{\nu}$, $e_n(\boldsymbol{M})$ are the elementary symmetric polynomials defined by

$$e_n(\boldsymbol{M})\equiv n!M^{\mu_1}_{[\mu_1}M^{\mu_2}_{\mu_2}\cdots M^{\mu_n}_{\mu_n]}, \tag{8}$$

where the antisymmetrization is unnormalized. In Eq. (7), $\tilde{X}$ denotes the canonical kinetic term of ϕ in terms of the composite metric,

$$\tilde{X}\equiv-\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi. \tag{9}$$

In the following we will study this action on the FLRW background and establish our parametrization for linear perturbations.

3 Cosmological parametrization

We parametrize the two metrics $g_{\mu\nu}$ and $f_{\mu\nu}$ to be

$$g_{\mu\nu}{\rm d}x^\mu{\rm d}x^\nu = -N_g^2\left({\rm e}^{2A}-\left({\rm e}^{-\boldsymbol{H}}\right)^{ij}B_iB_j\right){\rm d}t^2 +2N_ga_gB_i{\rm d}t{\rm d}x^i+a_g^2\left({\rm e}^{\boldsymbol{H}}\right)_{ij}{\rm d}x^i{\rm d}x^j, \tag{10}$$

$$f_{\mu\nu}{\rm d}x^\mu{\rm d}x^\nu = -N_f^2\left({\rm e}^{2\varphi}-\left({\rm e}^{-\boldsymbol{\Gamma}}\right)^{ij}\Omega_i\Omega_j\right){\rm d}t^2 +2N_fa_f\Omega_i{\rm d}t{\rm d}x^i+a_f^2\left({\rm e}^{\boldsymbol{\Gamma}}\right)_{ij}{\rm d}x^i{\rm d}x^j, \tag{11}$$

where N_g, a_g, N_f and a_f are functions of time only, and the matrix exponentials are defined perturbatively as $(\mathrm{e}^{\boldsymbol{H}})_{ij}\equiv\delta_{ij}+H_{ij}+\frac{1}{2}H_i{}^kH_{kj}+\mathcal{O}(H^3)$ and $(\mathrm{e}^{-\boldsymbol{H}})^{ij}=\delta^{ij}-H^{ij}+\frac{1}{2}H^i{}_kH^{kj}+\mathcal{O}(H^3)$, etc. Throughout this paper, spatial indices are raised and lowered by δ_{ij} and δ^{ij}. We further decompose (with $\partial^2\equiv\delta^{ij}\partial_i\partial_j$),

$$B_i \equiv \partial_iB+S_i, \tag{12}$$

$$H_{ij} \equiv 2\zeta\delta_{ij}+\left(\partial_i\partial_j-\frac{1}{3}\delta_{ij}\partial^2\right)E+\partial_{(i}F_{j)}+h_{ij}, \tag{13}$$

$$\Omega_i \equiv \partial_i\omega+\sigma_i, \tag{14}$$

$$\Gamma_{ij} \equiv 2\psi\delta_{ij}+\left(\partial_i\partial_j-\frac{1}{3}\delta_{ij}\partial^2\right)\chi+\partial_{(i}\xi_{j)}+\gamma_{ij}, \tag{15}$$

with $\partial_{(i}F_{j)}\equiv\frac{1}{2}(\partial_iF_j+\partial_jF_i)$, etc, and

$$\partial^iS_i=\partial^iF_i=\partial^i\sigma_i=\partial^i\xi_i=0, h^i{}_i=\gamma^i{}_i=0, \partial^ih_{ij}=\partial^i\gamma_{ij}=0. \tag{16}$$

Accordingly, it is convenient to parametrize the composite metric to be

$$\tilde{g}_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu = -\tilde{N}^2\left(\mathrm{e}^{2\tilde{A}}-(\mathrm{e}^{-\tilde{\boldsymbol{H}}})^{ij}\tilde{B}_i\tilde{B}_j\right)\mathrm{d}t^2 +2\tilde{N}\tilde{a}\tilde{B}_i\mathrm{d}t\mathrm{d}x^i+a^2(\mathrm{e}^{\tilde{\boldsymbol{H}}})_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{17}$$

where

$$\tilde{N}\equiv\alpha N+\beta N_f, \qquad \tilde{a}\equiv\alpha a+\beta a_f. \tag{18}$$

Similar to Eqs. (12)-(15), we may also decompose

$$\tilde{B}_i\equiv\partial_i\tilde{B}+\tilde{S}_i, \quad \tilde{H}_{ij}\equiv2\tilde{\zeta}\delta_{ij}+\left(\partial_i\partial_j-\frac{1}{3}\delta_{ij}\partial^2\right)\tilde{E}+\partial_{(i}\tilde{F}_{j)}+\tilde{h}_{ij}, \tag{19}$$

with $\partial^i\tilde{S}_i=\partial^i\tilde{F}_i=\partial^i\tilde{h}_{ij}=\delta^{ij}\tilde{h}_{ij}=0$. Note $\tilde{A}$ etc. are expressed in terms of $\{A,B_i,H_{ij},\varphi,\Omega_i,\Gamma_{ij}\}$ as

$$\tilde{A}=\sum_{n=1}\tilde{A}^{(n)}(A,B_i,H_{ij},\varphi,\Omega_i,\Gamma_{ij}) \tag{20}$$

etc., where n denotes the order in $\{A,B_i,H_{ij},\varphi,\Omega_i,\Gamma_{ij}\}$. At the linear order, we have, for the scalar modes,

$$\tilde{A}^{(1)} = \alpha\frac{N}{\tilde{N}}A+\beta\frac{N_f}{\tilde{N}}\varphi, \tag{21}$$

$$\tilde{B}^{(1)} = \alpha r_1B+\beta r_2\omega, \tag{22}$$

$$\tilde{\zeta}^{(1)} = \alpha\frac{a}{\tilde{a}}\zeta+\beta\frac{a_f}{\tilde{a}}\psi, \tag{23}$$

$$\tilde{E}^{(1)} = \alpha\frac{a}{\tilde{a}}E+\beta\frac{a_f}{\tilde{a}}\chi, \tag{24}$$

with

$$r_1\equiv\frac{aN\left(N_f\tilde{a}+a_f\tilde{N}\right)}{(Na_f+aN_f)\tilde{a}\tilde{N}}, \quad r_2\equiv\frac{a_fN_f\left(N\tilde{a}+a\tilde{N}\right)}{(Na_f+aN_f)\tilde{a}\tilde{N}}, \tag{25}$$

for the vector modes,

$$\tilde{S}_i^{(1)}=\alpha r_1S_i+\beta r_2\sigma_i, \qquad \tilde{F}_i^{(1)}=\alpha\frac{a}{\tilde{a}}F_i+\beta\frac{a_f}{\tilde{a}}\xi_i, \tag{26}$$

and for the tensor modes,

$$\tilde{h}_{ij}^{(1)}=\alpha\frac{a}{\tilde{a}}h_{ij}+\beta\frac{a_f}{\tilde{a}}\gamma_{ij}. \tag{27}$$

The background equations of motion can be determined by requiring the vanishing of the first order action of A, ζ, φ, ψ and $\delta\phi$, which is given by

$$S_1=\int\mathrm{d}t\mathrm{d}^3x N_ga_g^3\left(\mathcal{E}_AA+\mathcal{E}_\zeta3\zeta+\mathcal{E}_\varphi\varphi+\mathcal{E}_\psi3\psi+\frac{\tilde{N}\tilde{a}^3}{N_ga_g^3}\mathcal{E}_\phi\delta\phi\right). \tag{28}$$

The set of equations of motion are thus given by

$$\mathcal{E}_A \equiv M_g^2\left(3H_g^2-\Lambda_g\right)+\mathcal{E}_A^{\mathrm{pot}}+\alpha\frac{\tilde{a}^3}{a_g^3}\left(P-2\tilde{X}P_{,\tilde{X}}\right)=0, \tag{29}$$

$$\mathcal{E}_\zeta \equiv M_g^2\left(3H_g^2+\frac{2}{N_g}\frac{\mathrm{d}H_g}{\mathrm{d}t}-\Lambda_g\right)+\mathcal{E}_\zeta^{\mathrm{pot}}+\alpha\frac{\tilde{N}\tilde{a}^2}{N_ga_g^2}P=0, \tag{30}$$

$$\mathcal{E}_\varphi \equiv \frac{N_fa_f^3}{N_ga_g^3}M_f^2\left(3H_f^2-\Lambda_f\right)+\mathcal{E}_\varphi^{\mathrm{pot}} +\beta\frac{N_f\tilde{a}^3}{N_ga_g^3}\left(P-2\tilde{X}P_{,\tilde{X}}\right)=0, \tag{31}$$

$$\mathcal{E}_\psi \equiv \frac{N_fa_f^3}{N_ga_g^3}M_f^2\left(3H_f^2+\frac{2}{N_f}\frac{\mathrm{d}H_f}{\mathrm{d}t}-\Lambda_f\right)+\mathcal{E}_\psi^{\mathrm{pot}} +\beta\frac{\tilde{N}\tilde{a}^2a_f}{N_ga_g^3}P=0, \tag{32}$$

where $P_{,\tilde{X}}$ is the shorthand for $\partial P/\partial\tilde{X}$, and H_g and H_f are the Hubble parameters associated with the two metrics respectively, i.e.,

$$H_g\equiv\frac{1}{N_ga_g}\frac{\mathrm{d}a_g}{\mathrm{d}t}, \qquad H_f\equiv\frac{1}{N_fa_f}\frac{\mathrm{d}a_f}{\mathrm{d}t}. \tag{33}$$

In the above,

$$\mathcal{E}_A^{\mathrm{pot}} = M_g^2m^2\left(c_0+3\frac{a_f}{a_g}c_1+6\frac{a_f^2}{a_g^2}c_2+6\frac{a_f^3}{a_g^3}c_3\right), \tag{34}$$

$$\mathcal{E}_\zeta^{\mathrm{pot}} = b_1+\frac{a_gN_f}{N_ga_f}b_2, \tag{35}$$

$$\mathcal{E}_\varphi^{\mathrm{pot}} = M_g^2m^2\frac{N_f}{N_g}\left(c_1+6\frac{a_f}{a_g}c_2+18\frac{a_f^2}{a_g^2}c_3+24\frac{a_f^3}{a_g^3}c_4\right), \tag{36}$$

$$\mathcal{E}_\psi^{\mathrm{pot}} = b_2+b_3, \tag{37}$$

where we have introduced

$$b_1 \equiv M_g^2m^2\left(c_0+2\frac{a_f}{a_g}c_1+2\frac{a_f^2}{a_g^2}c_2\right), \tag{38}$$

$$b_2 \equiv M_g^2m^2\frac{a_f}{a_g}\left(c_1+4\frac{a_f}{a_g}c_2+6\frac{a_f^2}{a_g^2}c_3\right), \tag{39}$$

$$b_3 \equiv 2M_g^2m^2\frac{N_fa_f}{N_ga_g}\left(c_2+6\frac{a_f}{a_g}c_3+12\frac{a_f^2}{a_g^2}c_4\right), \tag{40}$$

for later convenience. The equation of motion for the scalar field is given by

$$\mathcal{E}_\phi\equiv P_{,\phi}-\frac{1}{\tilde{N}\tilde{a}^3}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\tilde{a}^3}{\tilde{N}}\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}P_{,\tilde{X}}\right), \tag{41}$$

4 Cosmological perturbations

The quadratic action for the two tensor perturbations h_{ij} and γ_{ij} is given by

$$\begin{aligned} S_2^{\rm tensor} = & \frac{1}{8}\int {\rm d}t \frac{{\rm d}^3k}{(2\pi)^3}\left[N_g a_g^3 M_g^2\left(\frac{1}{N_g^2}\dot{h}_{ij}^2-\frac{k^2}{a^2}h_{ij}^2\right)\right. \\ & +N_f a_f^3 M_f^2\left(\frac{1}{N_f^2}\dot{\gamma}_{ij}^2-\frac{k^2}{a_f^2}\gamma_{ij}^2\right) \\ & \left.+N_g a_g^3 \mathcal{M}^2(h_{ij}-\gamma_{ij})(h^{ij}-\gamma^{ij})\right]. \end{aligned} \tag{42}$$

where a dot denotes the derivative with respect to t,

$$\begin{aligned} \mathcal{M}^2 \equiv & \frac{a_f}{a_g}\left[M_g^2 m^2\left(c_1+2\frac{a_f}{a_g}c_2+2\frac{N_f}{N_g}\left(c_2+3\frac{a_f}{a_g}c_3\right)\right)\right. \\ & \left.+\alpha\beta\frac{\tilde{N}\tilde{a}}{N_g a_g}P\right]. \end{aligned} \tag{43}$$

The quadratic action for the four vector modes S_i, F_i, σ_i and ξ_i is given by

$$\begin{aligned} S_2^{\rm vector} = & \int {\rm d}t\frac{{\rm d}^3k}{(2\pi)^3}\left\{\frac{1}{4}N_g a_g^3 M_g^2 k^2\left(\frac{1}{a_g}S_i-\frac{1}{2N_g}\dot{F}_i\right)^2\right. \\ & +\frac{1}{4}N_f a_f^3 M_f^2 k^2\left(\frac{1}{a_f}\sigma_i-\frac{1}{2N_f}\dot{\xi}_i\right)^2 \\ & -\frac{1}{2}N_g a_g^3\mathcal{C}\left(S_i-\frac{a_g N_f}{N_g a_f}\sigma_i\right)^2 \\ & \left.+\frac{N_g a_g^3}{16}\mathcal{M}^2 k^2(F_i-\xi_i)^2\right\}, \end{aligned} \tag{44}$$

where $\mathcal{M}^2$ is given in Eq. (43) and we also introduce

$$\begin{aligned} \mathcal{C} \equiv & \frac{1}{1+\frac{a_g N_f}{N_g a_f}}b_2+\frac{\alpha\beta}{\left(1+\frac{a_g N_f}{N_g a_f}\right)^2}\frac{\tilde{N}\tilde{a}a_f}{N_g a_g^2} \\ & \times\left[\left(1+\frac{\tilde{a}N_f}{\tilde{N}a_f}+\frac{N_g\tilde{a}}{\tilde{N}a_g}\right)\left(P-2\tilde{X}P_{,\tilde{X}}\right)-P\right], \end{aligned} \tag{45}$$

with b_2 given in Eq. (39) for short. Since the vector modes S_i and σ_i have no dynamics in Eq. (44), we may solve them in terms of F_i and ξ_i and arrive at the reduced action for F_i and ξ_i, which is given by

$$\begin{aligned} S_2^{\rm vector} = & \frac{1}{16}\int {\rm d}t\frac{{\rm d}^3k}{(2\pi)^3}N_g a_g^3 k^2 \\ & \times\left[\mathcal{G}_{\rm v}\frac{1}{N_g^2}(\partial_t(F_i-\xi_i))^2+\mathcal{M}^2(F_i-\xi_i)^2\right], \end{aligned} \tag{46}$$

with

$$\mathcal{G}_{\rm v}=\left(\frac{a_g^3 N_f}{a_f^3 N_g}\frac{1}{M_f^2}+\frac{1}{M_g^2}-\frac{1}{2\mathcal{C}}\frac{k^2}{a_g^2}\right)^{-1}. \tag{47}$$

From Eq. (46) it is clear that there are two vectorial degrees of freedom given that $\beta\neq 0$, which can be identified as $F_i-\xi_i$. For the stability condition we have to impose $\mathcal{G}_{\rm v}>0$.

We study now the linear stability of the scalar modes in our model. Initially we have 9 scalar modes, of which four (A, B, ζ and E) are from $g_{\mu\nu}$, four (φ, ω, ψ and χ) are from $f_{\mu\nu}$, and one is the perturbation of the scalar field $\delta\phi$. In order to simplify the calculation, we choose a gauge in which $\delta\phi=\chi=0$. In the residual 7 modes, only 2 modes are dynamical, which can be conveniently chosen to be

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix}\equiv\begin{pmatrix} Q \\ E \end{pmatrix}. \tag{48}$$

with

$$Q=\zeta+\frac{k^2}{6}E+\frac{\beta H_g}{\alpha H_f}\psi. \tag{49}$$

After some manipulations, the final quadratic action for these two scalar modes takes the following general structure (in matrix form),

$$S_2^{\rm scalar}=\frac{1}{2}\int {\rm d}t\frac{{\rm d}^3k}{(2\pi)^3}\left(\dot{V}^T\boldsymbol{\mathcal{G}}\dot{V}+\dot{V}^T\boldsymbol{\mathcal{F}}V+V^T\boldsymbol{\mathcal{W}}V\right), \tag{50}$$

where $\mathcal{G}_{mn}$ and $\mathcal{W}_{mn}$ are symmetric while $\mathcal{F}_{mn}$ is anti-symmetric, which are given by

$$\mathcal{G}_{mn} = \Xi_{mn}-\frac{1}{\mathcal{D}}\mathcal{A}_m\mathcal{A}_n, \tag{51}$$

$$\mathcal{F}_{12} \equiv -\mathcal{F}_{21}=\mathcal{A}-\frac{1}{\mathcal{D}}(\mathcal{D}_1\mathcal{A}_2-\mathcal{D}_2\mathcal{A}_1), \tag{52}$$

$$\mathcal{W}_{mn}=\mathcal{B}_{mn}-\frac{1}{\mathcal{D}}\mathcal{D}_m\mathcal{D}_n-\frac{1}{2}\frac{\rm d}{{\rm d}t}\left[\frac{1}{\mathcal{D}}(\mathcal{D}_m\mathcal{A}_n+\mathcal{D}_n\mathcal{A}_m)\right], \tag{53}$$

with $m,n=1,2$. In Eqs. (51)-(52), we have

$$\begin{aligned} \mathcal{D} = & \frac{\beta^2}{\alpha^2}\frac{H_g^2}{H_f^2}\left[\left(\frac{\rm d}{{\rm d}t}\left(\ln\frac{H_g}{H_f}\right)\right)^2\Xi_{11}+\Xi_{44}-\frac{{\rm d}\Xi_{14}}{{\rm d}t}\right] \\ & -\frac{{\rm d}\Xi_{36}}{{\rm d}t}+\Xi_{66}+\frac{\beta}{\alpha}\frac{H_g}{H_f}\left[-\frac{\rm d}{{\rm d}t}\left(\ln\frac{H_g}{H_f}\right)(\Xi_{16}-\Xi_{34})\right. \\ & \left.+\frac{{\rm d}\Xi_{16}}{{\rm d}t}-2\Xi_{46}+\frac{{\rm d}\Xi_{34}}{{\rm d}t}\right], \end{aligned} \tag{54}$$

$$\mathcal{D}_1 \equiv \Xi_{46}-\frac{{\rm d}\Xi_{34}}{{\rm d}t}+\frac{\beta H_g}{\alpha H_f}\left(\frac{{\rm d}\Xi_{14}}{{\rm d}t}-\Xi_{44}\right), \tag{55}$$

$$\begin{aligned} \mathcal{D}_2 \equiv & -\frac{{\rm d}\Xi_{35}}{{\rm d}t}+\Xi_{56}+\frac{k^2}{6}\left(\frac{{\rm d}\Xi_{34}}{{\rm d}t}-\Xi_{46}\right) \\ & +\frac{\beta H_g}{\alpha H_f}\left[\frac{k^2}{6}\left(\Xi_{44}-\frac{{\rm d}\Xi_{14}}{{\rm d}t}\right)-\Xi_{45}+\frac{{\rm d}\Xi_{15}}{{\rm d}t}\right], \end{aligned} \tag{56}$$

$$\mathcal{A}_1 \equiv \Xi_{34}-\Xi_{16}+\frac{\beta}{\alpha}\frac{\rm d}{{\rm d}t}\left(\frac{H_g}{H_f}\right)\Xi_{11}, \tag{57}$$

$$\begin{aligned} \mathcal{A}_2 \equiv & \Xi_{35}-\Xi_{26}+\frac{k^2}{6}(\Xi_{16}-\Xi_{34}) \\ & +\frac{\beta H_g}{\alpha H_f}\left[\Xi_{24}-\Xi_{15}+\frac{\rm d}{{\rm d}t}\left(\ln\frac{H_g}{H_f}\right)\left(\Xi_{12}-\frac{k^2}{6}\Xi_{11}\right)\right], \end{aligned} \tag{58}$$

$$\mathcal{A} \equiv \Xi_{15}-\Xi_{24}, \quad (59)$$

and

$$\mathcal{B}_{11} \equiv \Xi_{44}-\frac{\mathrm{d}\Xi_{14}}{\mathrm{d}t}, \quad (60)$$

$$\mathcal{B}_{12} \equiv \mathcal{B}_{21}\equiv\Xi_{45}-\frac{k^2}{6}\Xi_{44}-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\Xi_{15}+\Xi_{24}-\frac{k^2}{3}\Xi_{14}\right), \quad (61)$$

$$\mathcal{B}_{22} \equiv \frac{k^4}{36}\Xi_{44}-\frac{k^2}{3}\Xi_{45}+\Xi_{55} -\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{k^4}{36}\Xi_{14}-\frac{k^2}{6}\Xi_{15}-\frac{k^2}{6}\Xi_{24}+\Xi_{25}\right), \quad (62)$$

where Ξ_{ij} with $i,j=1,\cdots,6$ are given in Appendix . Up to now, no approximation has been made in deriving the above expressions.

Unlike the tensor and vector modes, the lengthy expressions in the above make the analysis for the scalar modes rather cumbersome. In the following, we analyze the instabilities in the small scale limit $k\to\infty$. For the kinetic terms, we have

$$\mathcal{G}_{11}=\hat{\mathcal{G}}_{11}+\mathcal{O}(k^{-2}), \quad \text{and} \quad \mathcal{G}_{22}=k^2\hat{\mathcal{G}}_{22}+\mathcal{O}(k^0), \quad (63)$$

where

$$\hat{\mathcal{G}}_{11}=\frac{\alpha^2\left(\frac{\mathrm{d}\bar{\Phi}}{\mathrm{d}t}\right)^2\tilde{a}^3}{\tilde{N}^3H_g^2}\left(P_{,\tilde{X}}+2\tilde{X}P_{,\tilde{X}\tilde{X}}\right), \quad (64)$$

and

$$\hat{\mathcal{G}}_{22}=-\frac{\mathcal{C}a_g^5}{4N_g}-\frac{1}{\hat{\mathcal{D}}}\left(\hat{\mathcal{A}}_2\right)^2, \quad (65)$$

with

$$\hat{\mathcal{D}} = M_g^2N_ga_g\left[\frac{2\beta^2}{\alpha^2}\left(\frac{1}{H_f}\frac{2}{N_g}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H_g}{H_f}\right)+\frac{H_g^2}{H_f^2}\right) +2\frac{a_f}{a_g}\frac{M_f^2}{M_g^2}\frac{N_f}{N_g}-\frac{\mathcal{C}a_g^4N_f^2}{a_f^4H_f^2N_g^2M_g^2}\left(1+\frac{\beta a_f^2N_g}{\alpha a_g^2N_f}\right)^2 -\frac{1}{M_g^2a_g}\frac{2}{N_g}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{a_f}{H_f}M_f^2+\frac{\beta^2}{\alpha^2}\frac{a_gH_g}{H_f^2}M_g^2\right)\right], \quad (66)$$

and

$$\hat{\mathcal{A}}_2 = \frac{a_g^5N_f}{2\alpha a_f^2H_fN_g}\left[\mathcal{C}\left(\beta\frac{a_f^2N_g}{a_g^2N_f}+\alpha\right) +\beta\frac{a_f^2N_g}{a_g^2N_f}\left(b_1-M_g^2\Lambda_g+3M_g^2H_g^2\right)-\alpha b_2\frac{a_fN_g}{a_gN_f}\right]. \quad (67)$$

It can also be verified that $\mathcal{G}_{12}\sim\mathcal{O}(k^0)$. Thus in the large k limit, the no-ghost condition on the kinetic terms requires that $P_{,\tilde{X}}+2\tilde{X}P_{,\tilde{X}\tilde{X}}>0$ as well as

$$\frac{\mathcal{C}a_g^5}{4N_g}+\frac{1}{\hat{\mathcal{D}}}\left(\hat{\mathcal{A}}_2\right)^2<0. \quad (68)$$

These results can be compared with those derived in Ref. [56].

For the gradient terms, in the large k limit we have

$$\mathcal{W}_{11}=k^2\hat{\mathcal{W}}_{11}+\mathcal{O}(k^0), \quad \mathcal{W}_{22}=k^4\hat{\mathcal{W}}_{22}+\mathcal{O}(k^2), \quad (69)$$

and $\mathcal{W}_{12}\sim\mathcal{O}(k^2)$, where

$$\hat{\mathcal{W}}_{11}=\frac{a_gN_g}{H_g^2}\left(2M_g^2\frac{1}{N_g}\frac{\mathrm{d}H_g}{\mathrm{d}t}-\mathcal{C}\right)-\frac{1}{\hat{\mathcal{D}}}\hat{\mathcal{D}}_1\hat{\mathcal{D}}_2, \quad (70)$$

and

$$\hat{\mathcal{W}}_{22} = \frac{1}{4}a_g^3N_g\left[m^2M_g^2\left(c_0+c_1\left(\frac{a_f}{a_g}+\frac{N_f}{N_g}\right)+2c_2\frac{a_f}{a_g}\frac{N_f}{N_g}\right) +\alpha^2\frac{\tilde{a}\tilde{N}}{a_gN_g}P-M_g^2\Lambda_g +M_g^2\left(3H_g^2+\frac{2}{N_g}\frac{\mathrm{d}H_g}{\mathrm{d}t}\right)+\frac{1}{3}\mathcal{M}^2\right]. \quad (71)$$

In Eq. (70), $\hat{\mathcal{D}}$ is given in Eq. (66), and

$$\hat{\mathcal{D}}_1=\frac{a_g}{M_g^2H_f}\left[\frac{\mathcal{C}a_g^2N_f}{a_f^2H_gM_g^2}\left(1+\frac{\beta a_f^2N_g}{\alpha a_g^2N_f}\right)-2\frac{\beta}{\alpha}\frac{\mathrm{d}\ln H_g}{\mathrm{d}t}\right], \quad (72)$$

and

$$\hat{\mathcal{D}}_2 = a_g^3N_g\left\{\frac{\beta}{\alpha}\left[\frac{\alpha^2\tilde{a}^2}{a_g^2}\left(P-2\tilde{X}P_{,\tilde{X}}\right)+b_1+\mathcal{E}_A^g\right]\frac{1}{2H_g}\frac{1}{N_g}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{H_g}{H_f}\right)+\frac{1}{2}\left(1+\frac{\beta H_g}{\alpha H_f}\right)\mathcal{M}^2+\frac{3\beta H_g}{2\alpha H_f}M_g^2\left(3H_g^2+\frac{2}{N_g}\frac{\mathrm{d}H_g}{\mathrm{d}t}-\Lambda_g\right)\right.$$
$$+\frac{3\alpha\beta\tilde{a}\tilde{N}P}{2a_gN_g}\left(\frac{H_g}{H_f}-\frac{a_f}{a_g}\right)+\frac{3}{2}m^2M_g^2\left[\frac{\beta H_g}{\alpha H_f}\left(c_0+c_1\left(\frac{a_f}{a_g}+\frac{N_f}{N_g}\right)+2c_2\frac{a_f}{a_g}\frac{N_f}{N_g}\right)-\frac{a_f}{a_g}\left(c_1+2c_2\left(\frac{a_f}{a_g}+\frac{N_f}{N_g}\right)+6c_3\frac{a_f}{a_g}\frac{N_f}{N_g}\right)\right]$$
$$+\frac{\mathcal{C}}{4}\frac{1}{H_fH_gM_g^2}\left(\beta+\alpha\frac{N_f}{N_g}\frac{a_g^2}{a_f^2}\right)$$
$$\left.\times\left[-\alpha\frac{\tilde{a}^2}{a_g^2}\left(P-2\tilde{X}P_{,\tilde{X}}\right)-\frac{1}{\alpha}\left(b_1-M_g^2\Lambda_g+3M_g^2H_g^2\right)+\frac{a_g^3N_fH_gM_g^2}{a_f^3N_gH_fM_f^2}\left(\beta\frac{\tilde{a}^2}{a_g^2}\left(P-2\tilde{X}P_{,\tilde{X}}\right)+\frac{a_gb_2}{\alpha a_f}\right)\right]\right\}$$
$$-\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\left[a_g^3\left(\frac{\beta}{\alpha H_f}\left(b_1-M_g^2\Lambda_g+3M_g^2H_g^2\right)-\frac{a_gb_2}{a_fH_f}\right)\right]. \quad (73)$$

Thus, in the large k limit, the absence of gradient instability requires

$$\hat{\mathcal{W}}_{11}>0, \qquad \text{and} \qquad \hat{\mathcal{W}}_{22}>0. \tag{74}$$

The propagating speeds of the two scalar modes are given by the eigenvalues of $\boldsymbol{\mathcal{G}}^{-1}\boldsymbol{\mathcal{W}}$, which correspond to

$$c_1^2=\frac{\hat{\mathcal{W}}_{11}}{\hat{\mathcal{G}}_{11}} \qquad \text{and} \qquad c_2^2=\frac{\hat{\mathcal{W}}_{22}}{\hat{\mathcal{G}}_{22}} \tag{75}$$

in the same limit.

5 Conclusion

In this work, we have investigated the cosmological perturbation analysis of the bimetric theory with a scalar field coupled simultaneously to both metrics in terms of a composite metric. The scalar field represents the matter field that lives on both metrics.

The ghost and gradient instabilities of the tensor and vector modes as well as the ghost instabilities of the scalar modes of the same model have been analyzed in Ref. [56] for some concrete background evolution, while in this work we complete the analysis by presenting the full quadratic action for the scalar modes (Eq. (50)) as well as the conditions for the absence of gradient instabilities (Eq. (74)) on general background evolution in the presence of matter fields. Although in this work we focus on the small scale limit $k\to 0$ due to the lengthy expressions, the results presented in this work enable one to make further analysis in different limits as well as on concrete background solutions.

Moreover, we consider only the coupling of the scalar field to the composite metric in a minimal way, while in principle one may consider non-minimal derivative couplings, as was pointed out in Ref. [64]. This bimetric model with doubly coupled matter fields offers an interesting cosmological framework. In one branch of solutions, in which the Hubble rates are proportional to each other, this interesting phenomenology is plagued by the ghost and gradient instabilities, as was shown in Ref. [56]. However, in the other branch of background cosmology with the algebraical ratio between the scale factors of the two metrics, there are no ghost instabilities associated with the vector and scalar perturbations. Here, we also show the conditions for the absence of the gradient instabilities for the scalar perturbations, which were lacking in the literature. Fulfilling all these instability conditions, this branch of solutions still offers a promising dark energy model, which has a very rich phenomenology [65].

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Appendix A

Expressions of Ξ_{ab}

The expressions of Ξ_{ab} with $a,b=1,\cdots,6$ are given by:

$$\Xi_{11}=-\frac{1}{\Delta}\frac{16}{\tilde{N}^3}\alpha^2\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2 k^6\tilde{a}^3 a_f a_g^4 g_{\phi\phi}H_f^2 M_f^2 M_g^2 N_f^2 N_g\left[a_f^3 M_f^2 N_g\left(3\mathcal{C}a_g^2-2k^2M_g^2\right)+3\mathcal{C}a_g^5M_g^2N_f\right] \tag{A1}$$

$$\Xi_{12}=\frac{1}{\Delta}\frac{8}{3\tilde{N}^3}\alpha\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2 k^8\tilde{a}^3 a_f a_g^4 g_{\phi\phi}H_f M_f^2 M_g^4 N_f^2 N_g\left[2\alpha k^2 a_f^3 H_f M_f^2 N_g-3\mathcal{C}a_g^5 N_f(\alpha H_f+\beta H_g)\right], \tag{A2}$$

$$\begin{aligned}\Xi_{14} \;=\; &\frac{1}{\Delta}\frac{8k^6a_g^4M_g^2N_f^2}{\tilde{N}^3a_f^2}\Bigg\{\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3a_g^2g_{\phi\phi}(\alpha F_{24}N_g-\beta F_{14}N_f)\left(\alpha a_f^3H_fM_f^2N_g-\beta a_g^3H_gM_g^2N_f\right)\\&+F_{14}\tilde{N}^3a_f^3H_f^2H_gM_f^2\left(a_f^3M_f^2N_g\left(2k^2M_g^2-3\mathcal{C}a_g^2\right)-3\mathcal{C}a_g^5M_g^2N_f\right)\Bigg\},\end{aligned} \tag{A3}$$

$$\begin{aligned}\Xi_{15} \;=\; &\frac{1}{\Delta}\frac{8k^{10}a_g^5M_g^4N_f^2N_g}{3\tilde{N}^3a_f^2}\Bigg\{\beta\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3a_g^2g_{\phi\phi}N_f\left(\beta a_g^3H_gM_g^2N_f-\alpha a_f^3H_fM_f^2N_g\right)\\&+\tilde{N}^3a_f^3H_f^2H_gM_f^2\left[a_f^3M_f^2N_g\left(2k^2M_g^2-3\mathcal{C}a_g^2\right)-3\mathcal{C}a_g^5M_g^2N_f\right]\Bigg\},\end{aligned} \tag{A4}$$

$$\begin{aligned}\Xi_{16} \;=\; &\frac{1}{\Delta}\frac{8k^6a_g^4M_g^2{N_f}^2}{\tilde{N}^3a_f^2}\Bigg\{\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3a_g^2g_{\phi\phi}(\alpha F_{26}N_g-\beta F_{16}N_f)\left(\alpha a_f^3H_fM_f^2N_g-\beta a_g^3H_gM_g^2N_f\right)\\&+F_{16}\tilde{N}^3a_f^3H_f^2H_gM_f^2\left[a_f^3M_f^2N_g\left(2k^2M_g^2-3\mathcal{C}a_g^2\right)-3\mathcal{C}a_g^5M_g^2N_f\right]\Bigg\},\end{aligned} \tag{A5}$$

$$\begin{aligned}\Xi_{22} =& -\frac{1}{\Delta}\frac{4}{9\tilde{N}^3}k^{10}a_f a_g^4 H_f M_f^2 M_g^2 N_f^2 N_g\bigg\{9\mathcal{C}\tilde{N}^3 a_f^3 a_g^5 H_f H_g^2 M_f^2 M_g^2 \\ &-\alpha\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 g_{\phi\phi}\left[\alpha a_f^3 H_f M_f^2 N_g\left(2k^2M_g^2+3\mathcal{C}a_g^2\right)-3\mathcal{C}a_g^5 M_g^2 N_f(\alpha H_f+2\beta H_g)\right]\bigg\},\end{aligned} \tag{A6}$$

$$\begin{aligned}\Xi_{24} =& \frac{1}{\Delta}\frac{4k^8 a_g^4 M_g^2 N_f^2}{3\tilde{N}^3 a_f^2}\bigg\{\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 a_g^2 g_{\phi\phi}\left(\alpha F_{24}N_g-\beta F_{14}N_f\right)\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right) \\ &+\tilde{N}^3 a_f^3 H_f H_g M_f^2 M_g^2\left[2F_{14}k^2 a_f^3 H_f M_f^2 N_g-3\mathcal{C}a_g^5\left(F_{14}H_f N_f+F_{24}H_g N_g\right)\right]\bigg\},\end{aligned} \tag{A7}$$

$$\begin{aligned}\Xi_{25} =& \frac{1}{\Delta}\frac{4}{9\tilde{N}^3 a_f^2}k^{12}a_g^5 M_g^4 N_f^2 N_g\bigg\{\beta\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 a_g^2 g_{\phi\phi}N_f\left(\beta a_g^3 H_g M_g^2 N_f-\alpha a_f^3 H_f M_f^2 N_g\right) \\ &+\tilde{N}^3 a_f^3 H_f^2 H_g M_f^2 M_g^2\left(2k^2 a_f^3 M_f^2 N_g-3\mathcal{C}a_g^5 N_f\right)\bigg\},\end{aligned} \tag{A8}$$

$$\begin{aligned}\Xi_{26} =& \frac{1}{\Delta}\frac{4}{3\tilde{N}^3 a_f^2}k^8 a_g^4 M_g^2 N_f^2\bigg\{\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 a_g^2 g_{\phi\phi}\left(\alpha F_{26}N_g-\beta F_{16}N_f\right)\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right) \\ &+\tilde{N}^3 a_f^3 H_f H_g M_f^2 M_g^2\left[2F_{16}k^2 a_f^3 H_f M_f^2 N_g-3\mathcal{C}a_g^5\left(F_{16}H_f N_f+F_{26}H_g N_g\right)\right]\bigg\},\end{aligned} \tag{A9}$$

$$\begin{aligned}\Xi_{34} =& -\frac{1}{\Delta}\frac{8}{\tilde{N}^3}k^6 a_f a_g^3 M_f^2 N_f N_g\bigg\{F_{24}\tilde{N}^3 a_g H_f H_g^2 M_g^2\left[a_f^3 M_f^2 N_g\left(3\mathcal{C}a_g^2-2k^2M_g^2\right)+3\mathcal{C}a_g^5 M_g^2 N_f\right] \\ &-\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 g_{\phi\phi}\left(\alpha F_{24}N_g-\beta F_{14}N_f\right)\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right)\bigg\},\end{aligned} \tag{A10}$$

$$\Xi_{35}=-\frac{1}{\Delta}\frac{8}{3\tilde{N}^3}\beta\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2 k^{10}\tilde{a}^3 a_f a_g^4 g_{\phi\phi}M_f^2 M_g^2 N_f^2 N_g^2\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right), \tag{A11}$$

$$\begin{aligned}\Xi_{36} =& -\frac{1}{\Delta}\frac{8}{\tilde{N}^3}k^6 a_f a_g^3 M_f^2 N_f N_g\bigg\{F_{26}\tilde{N}^3 a_g H_f H_g^2 M_g^2\left[a_f^3 M_f^2 N_g\left(3\mathcal{C}a_g^2-2k^2M_g^2\right)+3\mathcal{C}a_g^5 M_g^2 N_f\right] \\ &-\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 g_{\phi\phi}\left(\alpha F_{26}N_g-\beta F_{16}N_f\right)\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right)\bigg\},\end{aligned} \tag{A12}$$

$$\begin{aligned}\Xi_{44} =& -\frac{1}{\Delta}\frac{4k^6 a_g^3 N_f^2 N_g}{\tilde{N}^3 a_f^2}\bigg\{\tilde{N}^3\Big[a_f^6 H_f^2 M_f^4\left(\mathcal{C}F_{14}^2-4k^2 M_{44}a_g H_g^2 M_g^4 N_g+6\mathcal{C}M_{44}a_g^3 H_g^2 M_g^2 N_g\right) \\ &+2\mathcal{C}a_f^3 a_g^3 H_f H_g M_f^2 M_g^2\left(3M_{44}a_g^3 H_f H_g M_g^2 N_f-F_{14}F_{24}\right)+\mathcal{C}F_{24}^2 a_g^6 H_g^2 M_g^4\Big] \\ &-2\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2 M_{44}\tilde{a}^3 g_{\phi\phi}\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right)^2\bigg\}\end{aligned} \tag{A13}$$

$$\begin{aligned}\Xi_{45} =& \frac{1}{\Delta}\frac{4}{3\tilde{N}^3 a_f^2}k^{10}a_g^4 M_g^2 N_f^2 N_g^2\bigg\{\tilde{N}^3 a_f^3 H_f M_f^2\Big[\mathcal{C}a_g^3 H_g M_g^2\left(F_{24}-6a_g^3 H_f H_g M_g^2 N_f\right) \\ &-a_f^3 H_f M_f^2\left(\mathcal{C}F_{14}-4k^2 a_g H_g^2 M_g^4 N_g+6\mathcal{C}a_g^3 H_g^2 M_g^2 N_g\right)\Big] \\ &+2\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2\tilde{a}^3 g_{\phi\phi}\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right)^2\bigg\},\end{aligned} \tag{A14}$$

$$\begin{aligned}\Xi_{46} =& \frac{1}{\Delta}\frac{4}{\tilde{N}^3 a_f^2}k^6 a_g^3 N_f^2 N_g\bigg\{\tilde{N}^3\Big[\mathcal{C}a_f^3 a_g^3 H_f H_g M_f^2 M_g^2\left(F_{14}F_{26}+F_{16}F_{24}-6M_{46}a_g^3 H_f H_g M_g^2 N_f\right) \\ &-a_f^6 H_f^2 M_f^4\left(\mathcal{C}F_{14}F_{16}-4k^2 M_{46}a_g H_g^2 M_g^4 N_g+6\mathcal{C}M_{46}a_g^3 H_g^2 M_g^2 N_g\right)-\mathcal{C}F_{24}F_{26}a_g^6 H_g^2 M_g^4\Big] \\ &+2\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2 M_{46}\tilde{a}^3 g_{\phi\phi}\left(\alpha a_f^3 H_f M_f^2 N_g-\beta a_g^3 H_g M_g^2 N_f\right)^2\bigg\},\end{aligned} \tag{A15}$$

$$\Xi_{55} = -\frac{1}{\Delta}\frac{4k^6a_g^3N_f^2N_g}{9\tilde{N}^3a_f^2}\left\{\tilde{N}^3a_f^3a_gH_f^2M_f^2M_g^2\left[a_f^3M_f^2N_g\left(\mathcal{C}k^8a_gM_g^2N_g+H_g^2\left(54\mathcal{C}M_{55}a_g^2-36k^2M_{55}M_g^2\right)\right)\right.\right.$$
$$\left.\left.+54\mathcal{C}M_{55}a_g^5H_g^2M_g^2N_f\right]-18\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2M_{55}\tilde{a}^3g_{\Phi\Phi}\left(\alpha a_f^3H_fM_f^2N_g-\beta a_g^3H_gM_g^2N_f\right)^2\right\}. \tag{A16}$$

$$\Xi_{56}=-\frac{1}{\Delta}\frac{4}{3}\mathcal{C}k^{10}a_fa_g^4H_fM_f^2M_g^2N_f^2N_g^2\left(F_{16}a_f^3H_fM_f^2-F_{26}a_g^3H_gM_g^2\right), \tag{A17}$$

$$\Xi_{66} = -\frac{1}{\Delta}\frac{4k^6a_g^3N_f^2N_g}{\tilde{N}^3a_f^2}\left\{\tilde{N}^3\left[a_f^6H_f^2M_f^4\left(\mathcal{C}F_{16}^2-4k^2M_{66}a_gH_g^2M_g^4N_g+6\mathcal{C}M_{66}a_g^3H_g^2M_g^2N_g\right)\right.\right.$$
$$\left.+2\mathcal{C}a_f^3a_g^3H_fH_gM_f^2M_g^2\left(3M_{66}a_g^3H_fH_gM_g^2N_f-F_{15}F_{25}\right)+\mathcal{C}F_{26}^2a_g^6H_g^2M_g^4\right]$$
$$\left.-2\mathcal{C}\left(\frac{\mathrm{d}\bar{\phi}}{\mathrm{d}t}\right)^2M_{66}\tilde{a}^3g_{\phi\phi}\left(\alpha a_f^3H_fM_f^2N_g-\beta a_g^3H_gM_g^2N_f\right)^2\right\}. \tag{A18}$$

In the above,

$$g_{\phi\phi}=\frac{1}{2}\left(P_{,\tilde{X}}+2\tilde{X}P_{,\tilde{X}\tilde{X}}\right), \tag{A19}$$

$\mathcal{C}$ and $\mathcal{M}$ are given in Eqs. (45) and (43), respectively, and

$$F_{14}=a_gN_g\left[2k^2M_g^2+3\alpha^2\tilde{a}^2\left(P-2\tilde{X}P_{,\tilde{X}}\right)+3a_g^2(b_1-M_g^2\Lambda_g+3M_g^2H_g^2)\right], \tag{A20}$$

$$F_{16}=3N_g\left[\alpha\beta\tilde{a}^2a_f\left(P-2\tilde{X}P_{,\tilde{X}}\right)+a_g^3b_2\right], \tag{A21}$$

$$F_{24}=\frac{a_gN_f}{a_fN_g}F_{16}, \tag{A22}$$

$$F_{26}=a_fN_f\left[2k^2M_f^2+3\beta^2\tilde{a}^2\left(P-2\tilde{X}P_{,\tilde{X}}\right)+3a_f^2\left(-M_f^2\Lambda_f+3M_f^2H_f^2\right)\right]+3a_g^3b_3N_g, \tag{A23}$$

$$M_{44} = 2k^2a_gM_g^2N_g+3a_g^2\left\{3\left(m^2a_fM_g^2(c_1N_g+2c_2N_f)+\alpha^2\tilde{a}\tilde{N}P\right)\right.$$
$$\left.+a_g\left[N_gM_g^2\left(3m^2c_0-3\Lambda_g+9H_g^2+3\frac{2}{N_g}\frac{\mathrm{d}H_g}{\mathrm{d}t}\right)+N_g\mathcal{M}^2+3m^2c_1M_g^2N_f\right]\right\}, \tag{A24}$$

$$M_{46} = 3a_g\left[3a_f\left(m^2a_gM_g^2(c_1N_g+2c_2N_f)+\alpha\beta\tilde{a}\tilde{N}P\right)+6m^2a_f^2M_g^2(c_2N_g+3c_3N_f)-a_g^2N_g\mathcal{M}^2\right], \tag{A25}$$

$$M_{55}=\frac{1}{18}k^6a_gM_g^2N_g+\frac{1}{6}k^2a_g^3N_g\mathcal{M}^2, \tag{A26}$$

$$M_{66} = 2k^2a_fM_f^2N_f+3a_g^3N_g\mathcal{M}^2+9a_f{}^2\left(2m^2a_gM_g^2(c_2N_g+3c_3N_f)+\beta^2\tilde{a}\tilde{N}P\right)$$
$$+9a_f^3\left[6m^2c_3M_g^2N_g+N_fM_g^2\left(24m^2c_4-\Lambda_f\right)+N_fM_f^2\left(3H_f^2+\frac{2}{N_f}\frac{\mathrm{d}H_f}{\mathrm{d}t}\right)\right]. \tag{A27}$$

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