Calculation of dissociation temperature of quarkonium using Gaussian Expansion Method[*]

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Abstract: The dissociation temperatures of quarkonium states in a thermal medium are obtained in the framework of the quark model with the help of the Gaussian Expansion Method (GEM). This is the first time this method has been applied to the dissociation problem of mesons. The temperature-dependent potential is obtained by fitting the lattice results. Solving the Schrödinger equation with the GEM, the binding energies and corresponding wave functions of the ground states and the excited states are obtained at the same time. The accuracy and efficiency of the GEM provide a great advantage for the dissociation problem of mesons. The results show that the ground states 1^1S_0 and 1^3S_1 have much higher dissociation temperatures than other states, and the spin-dependent interaction has a significant effect on the dissociation temperatures of 1^3S_1 and 1^1S_0. We also suggest using the radius of the bound state as a criterion of quarkonium dissociation. This can help to avoid the inaccuracy caused by the long tail of quarkonium binding energy dependence on temperature.

Keywords: dissociation temperature, quarkonium, gaussian expansion method

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1   Introduction

One important question in relativistic heavy ion collisions is the presence of quark-gluon plasma (QGP). As bound states, heavy quarkonium states can survive in QGP where the light hadrons are already dissociated. However, since there are dissociated light quarks and light gluons in a thermal environment, the interaction between the constituents of a quarkonium state is weakened by the color screening produced by light quarks and light gluons [1]. When the interaction within a quarkonium state is weak enough, the binding energy will become low enough and eventually leads to the dissociation of quarkonium. Since the interaction potential is temperature dependent, we can obtain the dissociation temperature of quarkonium when the binging energy reaches zero.

In 1986, Matsui and Satz pointed out that charmonium state suppression can be recognized as a signature of QGP formation in relativistic heavy ion collisions [2], and the existence of charmonium states carries information about the QGP properties. There have been numerous theoretical and experimental studies from the Super Proton Synchrotron (SPS) to the Large Hadron Collider (LHC) on this subject. In 2006, Satz pointed out that the the properties of QGP can be investigated by studying the in-medium behavior of heavy quark bound states [1]. Qu et al. calculated the dissociation temperature of charmonium states by defining the maximum distance between two constituents of the states [3]. The melting temperatures of charmonium and bottomonium states are also discussed by Satz [4].

In the present work, the quark potential model is used to calculate the binding energies of heavy quarkonia. The temperature dependent potential between two constituents of the quarkonia is obtained by fitting the lattice QCD results on the free energies of the system. The Gaussian Expansion Method (GEM) [5], a power-

ful few-body method, is used to solve the Schrödinger equation.

The structure of this article is as follows. The potential model is presented in Section 2. Section 3 is devoted to the calculation method. Results and discussions are given in Section 4. In Section 5, the spin-dependent interaction is taken into account. Section 6 gives a criterion for the dissociation temperature, the radius of the quarkonium. A brief summary is given in the last section.

2 Potential model

Lattice quantum chromodynamics (QCD) indicates that the ground state of charmonium can survive in QGP up to $1.5T_c$ [6–10]. In contrast, excited states are dissociated at $1.1T_c$ [9], where T_c is the critical temperature of the deconfinement phase transition.

Because of the large masses of the charm quark and bottom quark, it is appropriate to apply the non-relativistic potential model to the study of charmonium states and bottomonium states. By solving the Schrödinger equation with the temperature-dependent heavy quark potential V, we can get the dissociation temperature of the ground state and excited states. The most frequently used potential for quarkonium states is the Cornell potential,

$$V(r) = -\frac{\alpha}{r} + \sigma r, \tag{1}$$

where α is the coupling constant and σ is the string tension linking the two heavy quarks. It combines a linear part at large distance and a Coulomb part at short distance. However, this version of the Cornell potential is independent of temperature. In QGP, considering that a non-vanishing effective mass is given to the exchanged gluon, which used to be massless, the functional form of screening can be obtained using a generalized Debye-Hückel formalism [11–14].

In the Debye-Hückel formalism, when $T=0$, free energy $F(r,T=0)$, which can be taken as the interacting potential $V(r)$, has the form

$$F(r,T=0) = \beta r^\eta, \tag{2}$$

which can be derived from a corresponding Poisson equation

$$-\frac{\nabla^2 F}{r^{\eta+1}} + \frac{\eta+1}{r^{\eta+2}} \nabla F \cdot \hat{r} = 4\pi\beta\delta(r), \tag{3}$$

where β and η are parameters which determine the form of free energy. In the medium, the source term $\delta(r)$ is substituted by $\delta(r)+AF$, and all the effects of the medium are in the factor A. Defining the screening mass as $\mu = (4\pi\beta A)^{\frac{1}{\eta+3}}$, one has the following Poisson equation,

$$\frac{1}{r^{\eta+1}} \frac{\mathrm{d}^2 F}{\mathrm{d}r^2} + \frac{1-\eta}{r^{\eta+2}} \frac{\mathrm{d}F}{\mathrm{d}r} - \mu^{\eta+3} F = -4\pi\beta\delta(r). \tag{4}$$

The solution of this equation gives the form of free energy in the medium with interaction in the form of Eq. (2).

We consider Eq. (1) separately. For the Coulomb part, the parameters are $\beta=-\alpha$, $\eta=-1$, and the free energy is

$$F_c(r,T) = -\frac{\alpha}{r}[\mathrm{e}^{-\mu r} + \mu r]. \tag{5}$$

For the string part, the parameters are $\beta=\sigma$, $\eta=1$ and the free energy is

$$F_s(r,T) = \frac{\sigma}{\mu}\left[\frac{\Gamma(1/4)}{2^{3/2}\Gamma(3/4)} - \frac{\sqrt{\mu r}}{2^{3/4}\Gamma(3/4)} K_{1/4}\left[(\mu r)^2\right]\right]. \tag{6}$$

The free energy of Eq. (1) in the medium is given by

$$F(r,T) = F_c(r,T) + F_s(r,T). \tag{7}$$

All the effects of the medium are in the factor A and A leads to screening mass μ. So in this form of $F(r,T)$, only the parameter μ is temperature-dependent, while α and σ are not.

The temperature-dependent free energy $F(r,T)$ between two heavy quarks at finite temperature has been calculated by lattice QCD [15, 16]. Satz obtained the temperature dependence of screen masses by fitting this function with the results of lattice QCD.

To solve the Schrödinger equation for the quarkonium, the interacting potential between quark and antiquark is needed. The temperature-dependent potential can be obtained from the free energy of the system. A model has been applied which relates the internal energy $U(r,T)$ of a $q\bar{q}$ pair at separation distance r to the free energy of the system. In this model [1],

$$U(r,T) = F(r,T) - T\frac{\partial F(r,T)}{\partial T}, \tag{8}$$

and it is assumed that $U(r,T)$ is equal to the interacting potential $V(r,T)$ between two constituents.

For the sake of simplicity, we consider this quarkonium as the problem of a central force field and solve the Schrödinger equation (this is also a common approximation used for solving the dissociation temperature of heavy quarkonium in the past),

$$\left[-\frac{1}{2\mu_m}\nabla^2 + V(r,T) - V(\infty,T)\right]\psi_i(\boldsymbol{r},T) = \epsilon_i(T)\psi_i(\boldsymbol{r},T), \tag{9}$$

where μ_m is the reduced mass and $\mu_m = m_q/2$ in the quarkonium case. m_q is the mass of the charm quark or bottom quark. The binding energy of the quarkonium state at finite temperature is defined as

$$\Delta E = -\epsilon_i(T) = -(m_\psi - 2m_q - V(\infty,T)). \tag{10}$$

The quark potential we use is the internal energy $U(r,T)$ in Eq. (8).

3   Numerical method

In order to solve the Schrödinger equation, we use a variational method, GEM [5], which was first proposed by Kamimura in 1988 to solve the nonadiabatic three-body problem of muonic molecules and muon-atomic collisions [17, 18], and developed by Hiyama in the last two decades. This method has been extensively applied to hypernuclei physics [19–22]. Its accuracy has been proved in applications in nuclear physics [5], hadron physics [23], and cold atomic systems [21, 22, 24].

Consider the two-body Schrödinger equation

$$\left[\widehat{H}-E\right]\Psi_{JM_J}=0, \tag{11}$$

$$\Psi_{JM} = \sum_{m_s,m} C^{JM_J}_{sm_s,lm}\psi_{lm}(\boldsymbol{r})\chi_{sm_s} \tag{12}$$

where J,l,s are the total, orbital and spin angular momenta of the two-body system, respectively. An efficient way to solve the Schrödinger equation is the Rayleigh-Ritz variational principle. The key of the Rayleigh-Ritz variational principle is how to choose the trial wave function. Generally, the trial wave function is expanded into a set of wave functions,

$$\psi_{lm}(\boldsymbol{r})=\sum_{n=1}^{n_{\max}} C_{nl}\phi_{nlm}(\boldsymbol{r}). \tag{13}$$

The variational principle leads to a generalized matrix eigenvalue problem,

$$\sum_{n'=1}^{n_{\max}}(H_{nn'}-EN_{nn'})C_{n'l}=0, \quad (n=1-n_{\max}). \tag{14}$$

Therefore, we can obtain the eigenvalues and wave functions of both ground state and excited states. A good set of basis functions can make the calculation more accurate and less laborious. In this case, we use a set of Gaussian basis functions with ranges in geometric progression [5],

$$\phi_{nlm}(\boldsymbol{r}) = \phi_{nl}(\boldsymbol{r})Y_{lm}(\hat{\boldsymbol{r}}), \tag{15}$$

$$\phi_{nl}(\boldsymbol{r}) = N_{nl}r^l\mathrm{e}^{-\nu_n r^2}, \tag{16}$$

where the N_{nl} is for normalization and Gaussian range ν_n is given by

$$\nu_n=\frac{1}{r_n^2}, \quad r_n=r_{\min}\left(\frac{r_{\max}}{r_{\min}}\right)^{\frac{n-1}{n_{\max}-1}}. \tag{17}$$

4   Results

In Fig. 1, we show the resulting binding energy behavior for different charmonium states, obtained with $m_c=1.25$ GeV and $\sqrt{\sigma}=0.445$ GeV and $\alpha=\pi/12$. T_d is the dissociation temperature.
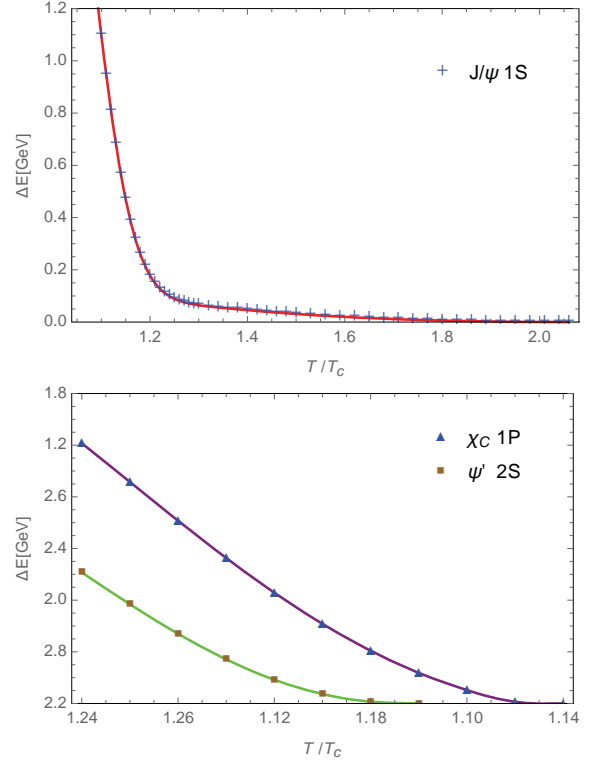

Fig. 1.   (color online) Dependence of binding energy of (top) J/$\psi(1S)$, and (bottom) $\chi_c(1P)$ and $\psi'(2S)$ on T.
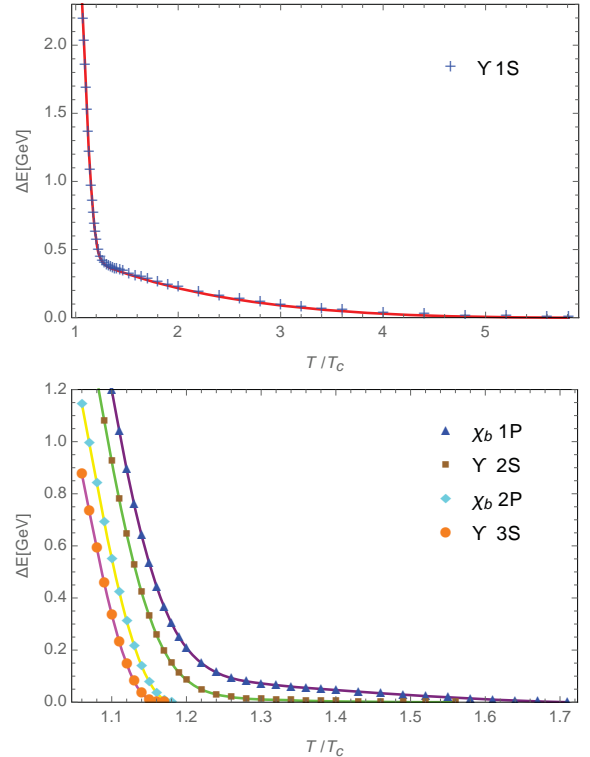

Fig. 2.   (color online) Dependence of binding energy of (top) $\Upsilon(1S)$, and (bottom) $\chi_b(1P)$, $\Upsilon(2S)$, $\chi_b(2P)$, and $\Upsilon(3S)$ on T.

The same formalism, with $m_{\rm b} = 4.65$ GeV replacing $m_{\rm c}$, leads to the resulting binding energy behavior for bottomonium states. The results are shown in Fig. 2. In Fig. 3, we show the comparison between the ground states of charmonium and bottomonium.
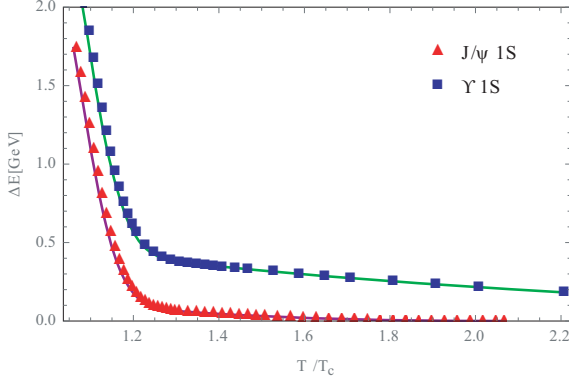

Fig. 3. (color online) Comparison between the ground states of charmonium and bottomonium.

Table 1.   Dissociation temperatures $T_{\rm d}/T_{\rm c}$ of charmonium, compared with the results in Ref. [1]

Charmonium	$1S$	$1P$	$2S$
Ref. [1]	2.10	1.16	1.12
our results	2.06	1.16	1.13

Table 2.   Dissociation temperatures $T_{\rm d}/T_{\rm c}$ of bottomonium, compared with the results in Ref. [1]

bottomonium	$1S$	$1P$	$2S$	$2P$	$3S$
Ref. [1]	> 4.0	1.76	1.60	1.19	1.17
our results	5.81	1.71	1.56	1.18	1.17

Table 3.   Dissociation temperature $T_{\rm d}/T_{\rm c}$ of charmonium for different potentials, compared with the results in Ref. [3]

potential	charmonium	$1S$	$1P$	$2S$
V_1	Ref. [3]	2.34	1.14	1.14
	Our Results	2.23	1.17	1.14
V_2	Ref. [3]	2.57	1.15	1.15
	our results	2.32	1.17	1.15

The results for dissociation temperatures of charmonium and bottomonium in Ref. [1] and our calculation results are listed in Table 1 and Table 2 respectively.

We also calculate the dissociation temperature with different parameters for the potential in order to make a further comparison with the previous work. In Ref. [3], two different potentials, the Cornell potential and modified Cornell potential, have been applied to the quarkonium system. The parameters in the potential model, α, σ, and $m_{\rm c}$, are determined by fitting the charmonium

masses $m_{\rm J/\psi} = 3.10$ GeV, $m_{\chi_{\rm c}} = 3.53$ GeV, and $m_{\psi'} = 3.68$ GeV [25]. Solving the Schrödinger equation with the Cornell potential V_1 and with the modified Cornell potential V_2 [3], almost the same results are obtained, and are listed in Table 3, with the results of Ref. [3] for comparison.

5   Spin-dependent potential

In the past, the dissociation temperature of heavy quarkonium was limited to $1S,1P$ and $2S$ states without invoking the spin-dependent interaction. Now we are extending this study to other states, such as $^1S_0$ and $^3S_1$. In order to do this, we need to introduce the spin-dependent interaction $V_{\rm SD}$ in addition to the confinement $V_{\rm conf}$ and color Coulomb interactions,

$$V = V_{\rm Cornell} + V_{\rm SD}. \tag{18}$$

The form of the spin-dependent part is given in Ref. [26],

$$\begin{aligned}
V_{\rm SD}(r) = & \left(\frac{\boldsymbol{\sigma}_{\rm q}}{4m_{\rm q}^2} + \frac{\boldsymbol{\sigma}_{\bar{\rm q}}}{4m_{\bar{\rm q}}^2} \right) \cdot \boldsymbol{L} \left(\frac{1}{r} \frac{{\rm d}\varepsilon}{{\rm d}r} + \frac{2}{r} \frac{{\rm d}V_1}{{\rm d}r} \right) \\
& + \left(\frac{\boldsymbol{\sigma}_{\rm q} + \boldsymbol{\sigma}_{\bar{\rm q}}}{2m_{\rm q} m_{\bar{\rm q}}} \right) \cdot \boldsymbol{L} \left(\frac{1}{r} \frac{{\rm d}V_2}{{\rm d}r} \right) \\
& + \frac{1}{12 m_{\rm q} m_{\bar{\rm q}}} (3\boldsymbol{\sigma}_{\rm q} \cdot \boldsymbol{r} \boldsymbol{\sigma}_{\bar{\rm q}} \cdot \boldsymbol{r} - \boldsymbol{\sigma}_{\rm q} \cdot \boldsymbol{\sigma}_{\bar{\rm q}}) V_3 \\
& + \frac{1}{12 m_{\rm q} m_{\bar{\rm q}}} \boldsymbol{\sigma}_{\rm q} \cdot \boldsymbol{\sigma}_{\bar{\rm q}} V_4 \\
& + \frac{1}{2} \left[\left(\frac{\boldsymbol{\sigma}_{\rm q}}{m_{\rm q}^2} - \frac{\boldsymbol{\sigma}_{\bar{\rm q}}}{m_{\bar{\rm q}}^2} \right) \cdot \boldsymbol{L} + \left(\frac{\boldsymbol{\sigma}_{\rm q} - \boldsymbol{\sigma}_{\bar{\rm q}}}{m_{\rm q} m_{\bar{\rm q}}} \right) \cdot \boldsymbol{L} \right] V_5, \tag{19}
\end{aligned}$$

where $\varepsilon = \varepsilon(r)$ is the static potential given by Gromes in Ref. [27],

$$\frac{{\rm d}\varepsilon}{{\rm d}r} = \frac{{\rm d}V_2}{{\rm d}r} - \frac{{\rm d}V_1}{{\rm d}r}. \tag{20}$$

We apply a simple nonrelativistic potential model, the BGS model [28]. Its Cornell part is

$$V_{\rm Cornell} = -\frac{4}{3} \frac{\alpha_{\rm s}}{r} + br. \tag{21}$$

For quarkonium, the quark and antiquark have the same mass. Then the spin-dependent parts, corresponding to the vector one-gluon-exchange and scalar confinement ansatz, are:

$$V_1 = -br, \tag{22}$$

$$V_2 = -\frac{4}{3} \frac{\alpha_{\rm s}}{r}, \tag{23}$$

$$V_3 = 4\frac{\alpha_{\rm s}}{r^3}, \tag{24}$$

$$V_4 = \frac{32}{3} \pi \alpha_{\rm s} \left(\frac{\beta}{\sqrt{\pi}} \right)^3 {\rm e}^{-\beta^2 r^2}, \tag{25}$$

$$V_5 = 0, \tag{26}$$

where the parameters are determined by fitting the charmonium spectrum: $\alpha_s = 0.5462$, $b = 0.1425$ GeV^2, $\beta = 1.0946$ GeV, and $m_c = 1.4794$ GeV.

Since the quarkonium free energy is fitted with the Cornell potential, we only replace the Cornell part V_{Cornell} in the BGS model with the temperature-dependent potential $V(r,T)$ in Eq. (8). We solve the Schrödinger equation with the Hamiltonian

$$\widehat{H} = -\frac{1}{m_c}\nabla^2 + V(r,T) - V(\infty,T) + V_{\text{SD}}(r). \quad (27)$$

The wave functions are:

$$\Psi_{JM_J} = \sum_{\alpha=1}^{\alpha_{\max}} A_\alpha \Phi_\alpha(\boldsymbol{r}), \quad (28)$$

$$\Phi_\alpha(\boldsymbol{r}) = [\phi_{nl}(\boldsymbol{r})\chi_{sm_s}]_{JM_J}, \quad (29)$$

and with α specifying $\alpha \equiv \{s,n,l\}$, we can get the eigen-energies and the corresponding wavefunctions. Furthermore, the dissociation temperatures can be obtained. Applying the GEM to solve the Schrödinger equation, accurate results can be obtained. The dissociation temperatures for the S-, P- and D-states can be found in Table 4 with charm quark mass $m_c = 1.4794$ GeV.

From Table 4, we can see that the ground states 1^1S_0, 1^3S_1 have much higher dissociation temperatures than other states. The experimental spectrum of charmonium states gives $m(1^3S_1) - m(1^1S_0) \simeq 113.5$ MeV and $m(2^3S_1) - m(2^1S_0) \simeq 46.9$ MeV [29]. However, the dissociation temperatures of 1^3S_1 and 1^1S_0 have a significant difference and the difference in dissociation temperatures of 2^3S_1 and 2^1S_0 is less than $0.01T_c$. The binding energies of all states reduce rapidly before $1.2T_c$. For states which have low dissociation temperatures (less than $1.2T_c$), the differences in dissociation temperature caused by the spin-orbit coupling effect are very small.

Table 4. Dissociation temperature T_d/T_c of charmonium states.

state	T_d/T_c	state	T_d/T_c	state	T_d/T_c
1^1S_0	2.79	1^1P_1	1.168	2^1P_1	1.088
1^3S_1	2.22	1^3P_0	1.194	2^3P_0	1.099
2^1S_0	1.149	1^3P_1	1.170	2^3P_1	1.089
2^3S_1	1.141	1^3P_2	1.161	2^3P_2	1.082

state	T_d/T_c	state	T_d/T_c
1^1D_2	1.106	2^1D_2	1.044
1^3D_1	1.112	2^3D_1	1.049
1^3D_2	1.107	2^3D_2	1.044
1^3D_3	1.103	2^3D_3	1.041

6   Bound state radii

The results show that the dependence of binding energy of bound states on temperature T has a long tail (see Fig. 4). With the increase of temperature, the reduction in binding energy becomes slower, especially for the states with high dissociation temperature. So instead of binding energy, we suggest that bound state radii can be used as a criterion of dissociation. The root mean square (RMS) radii of the bound states can be defined as [30]

$$\sqrt{\langle r^2 \rangle} = \left[\int \Psi_{JM_J}^* r^2 \Psi_{JM_J} \mathrm{d}\tau \right]^{1/2}. \quad (30)$$

The temperature dependence of bound state radius for each state is shown in Fig. 5 and Fig. 6 for charmonium and bottomonium respectively, corresponding to Fig. 1 and Fig. 2.
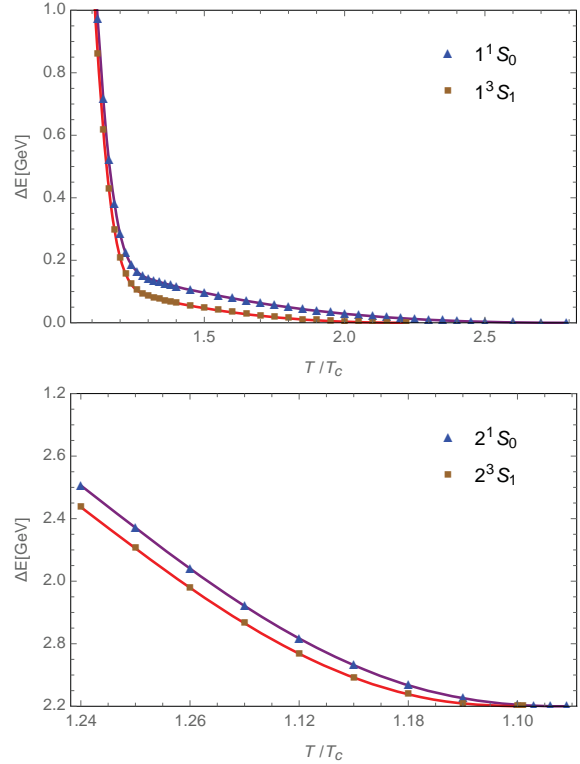

Fig. 4. (color online) Dependence of binding energy of 1^1S_0, 1^3S_1, 2^1S_0, and 2^3S_1 on T.

With increasing temperature, the bound state radii increase rapidly and approach infinity when the state dissociates. Since the average free path of QGP is estimated to be 1.0 fm, we take 2.0 fm as the criterion. When the RMS bound state radius of a state is larger than 2.0 fm, we think it dissociates. The results obtained are listed in Tables 5–7.

The results show that the dissociation temperatures obtained by RMS bound state radii are lower than or the same as those obtained by binding energy. For the states which have low dissociation temperature, the decrease of binding energy and increase of RMS bound state radii are both rapid with increasing temperature, so they reach the criterion points at nearly the same time. For states which have high dissociation temperature, however, when temperature becomes higher, the
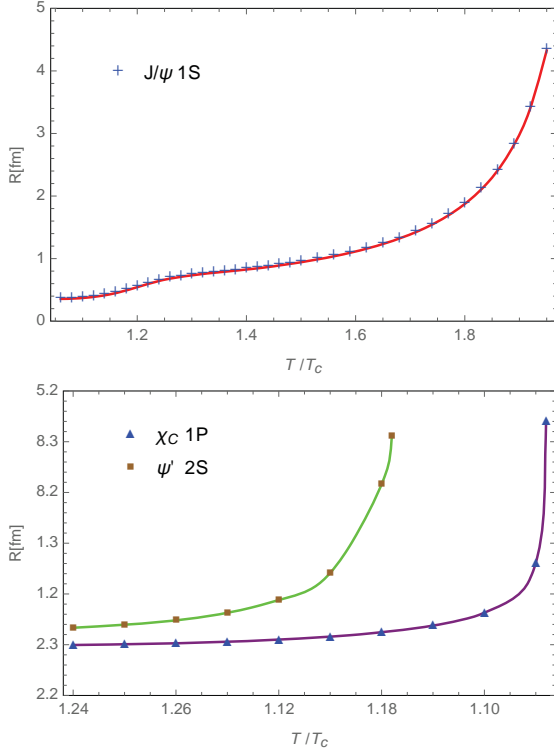
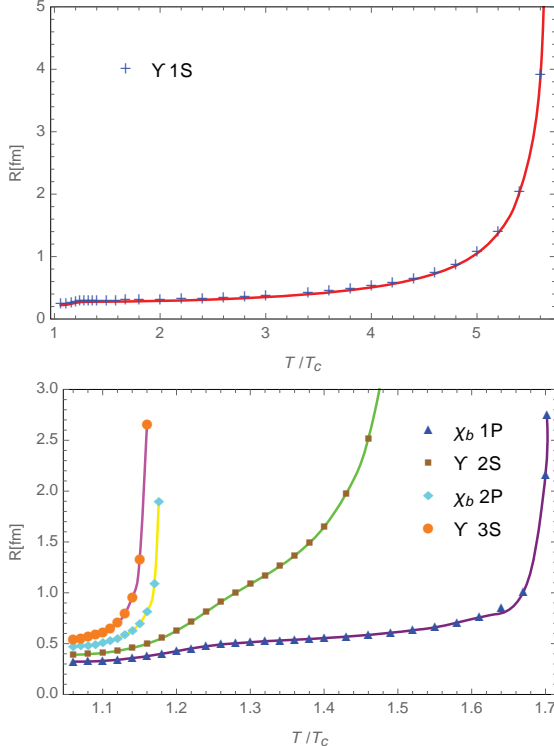Fig. 5. (color online) Dependence of bound state radii of (top) J/ψ(1S), and (bottom) χ$_c$(1P), and ψ′(2S) on T.

Fig. 6. (color online) Dependence of bound state radii of (top) Υ(1S), and (bottom) χ$_b$(1P), Υ(2S), χ$_b$(2P), and Υ(3S) on T.

Table 5. Dissociation temperature T_d/T_c of charmonium and bottomonium with $m_c = 1.25$ GeV, $m_b = 4.65$ GeV, obtained by RMS bound state radii.

charmonium	T_d/T_c	bottomonium	T_d/T_c
$1S$	1.83	$1S$	5.40
$1P$	1.16	$1P$	1.70
$2S$	1.13	$2S$	1.44
		$2P$	1.18
		$3S$	1.16

Table 6. Dissociation temperature T_d/T_c of charmonium with Cornell potential V_1 and Cornell potential V_2, obtained by RMS bound state radii.

cornell potential V_1	T_d/T_c	cornell potential V_2	T_d/T_c
$1S$	1.98	$1S$	2.08
$1P$	1.17	$1P$	1.17
$2S$	1.13	$2S$	1.14

Table 7. Dissociation temperature T_d/T_c of charmonium with spin-dependent potential, obtained by RMS bound state radii.

state	T_d/T_c	state	T_d/T_c	state	T_d/T_c
1^1S_0	2.47	1^1P_1	1.167	2^1P_1	1.087
1^3S_1	1.98	1^3P_0	1.194	2^3P_0	1.098
2^1S_0	1.139	1^3P_1	1.170	2^3P_1	1.088
2^3S_1	1.131	1^3P_2	1.159	2^3P_2	1.080

state	T_d/T_c	state	T_d/T_c
1^1D_2	1.105	2^1D_2	1.044
1^3D_1	1.112	2^3D_1	1.049
1^3D_2	1.107	2^3D_2	1.044
1^3D_3	1.103	2^3D_3	1.041

Table 8. Dissociation temperature T_d/T_c of charmonium with $m_c = 1.25$ GeV and $m_c = 1.4794$ GeV. The results in (a) are obtained by binding energy and results in (b) are obtained by RMS bound state radii. In the case of $m_c = 1.25$ GeV, the three states are J/ψ(1S), χ$_c$(1P), and ψ′(2S); in the case of $m_c = 1.4794$ GeV, the three states are J/ψ(1^3S_1), χ$_c$($1^3P_0, 1^3P_1, 1^3P_2$), and ψ′(2^3S_1).

(a)			
$m_c = 1.25$ GeV	T_d/T_c	$m_c = 1.4794$ GeV	T_d/T_c
$1S$	2.06	1^3S_1	2.22
$1P$	1.16	1^3P_0	1.194
		1^3P_1	1.170
		1^3P_2	1.161
$2S$	1.13	2^3S_1	1.141
(b)			
$m_c = 1.25$ GeV	T_d/T_c	$m_c = 1.4794$ GeV	T_d/T_c
$1S$	1.83	1^3S_1	1.98
$1P$	1.16	1^3P_0	1.194
		1^3P_1	1.170
		1^3P_2	1.159
$2S$	1.13	2^3S_1	1.131

decrease of binding energy becomes very slow while the increase of RMS bound state radii is still rapid. In this case, the difference between the two criteria is obvious and the results obtained by RMS bound state radii are always lower. This shows the RMS bound state radii is more valid than binding energy as a dissociation temperature criterion.

In Table 8, we show the comparison for J/ψ, χ$_c$, and ψ′ between the cases of $m_c = 1.25$ GeV and $m_c = 1.4794$ GeV.

These two different potentials (with or without spin-dependent interaction) can both lead to the vector charmonium spectrum at zero temperauture, but only the dissociation temperatures of the ground state have obvious differences. For excited states, the dissociation temperatures are very close even when the difference of the two charm quark masses is larger than 15%.

7   Summary

Using a temperature-dependent interacting potential between quark and antiquark, the dissociation temperatures of charmonium and bottomonium have been calculated. The results show that the ground state has a much higher dissociation temperature than the excited states for both charmonium and bottomonium. This means that the ground state is much more insoluble than the excited states. In addition, it is found that the binding energies drop rapidly before $1.2T_c$. Here we need to point out that the function form of free energy obtained from Debye-Hückel theory can be fitted well with lattice data only in the range of temperature $0 \leqslant T \leqslant 2T_c$ [14].

The binding energies and bound state radii of ground states and excited states for charmonium and bottomonium at finite temperature are obtained by solving the Schrödinger equation with the help of the GEM. The binding energies of all the ground states and excited states obtained at each temperature are convergent with enough Gaussians. The GEM gives accurate numerical results even when the binding energy is very close to zero. Another advantage is that the diagonalization of the Hamiltonian automatically gives the lowest eigenstate and many excited eigenstates with the same spin and parity at the same time.

Because of the long tail of the dependence of the binding energy on temperature, we also use RMS bound state radii as a criterion of dissociation. In this way, the dissociation temperature of the heavy quarkonium can be fixed more accurately.

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