Neutral regular black hole solution in generalized Rastall gravity*

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Abstract: We investigate the static, spherically symmetric regular black hole solutions in the generalized Rastall gravity. In particular, the prescription of Rastall gravity implies that the present approach does not necessarily involve nonlinear electrodynamics. Subsequently, the resulting regular black hole solutions can be electrically and magnetically neutral. The general properties of the regular black hole solutions are explored. Moreover, specific solutions are derived and discussed, particularly regarding the parameter related to the degree of violation of the energy-momentum conservation in the Rastall theory.

Keywords: regular black hole, generalized Rastall gravity, static spherically symmetrical spacetime

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1 Introduction

The black hole is an interesting prediction of general relativity, a compact celestial body with enormous mass and immense gravity. Since 2015, LIGO and Virgo collaborations have detected gravitational waves emanating from the coalescence of black holes [1-6], which is one of the most crucial proofs of the existence of black holes. According to the Penrose-Hawking singularity theorems [7], a singularity might accompany black hole solutions. In the vicinity of a singularity, as the spacetime curvature approaches and exceeds the Planck value, the notion of classical spacetime ceases to be valid. Fortunately, in view of the cosmic censorship hypothesis proposed by Penrose, it is understood that the singularity is typically hidden behind the event horizon. The latter is a one-way membrane which prevents the singularity from affecting any observer located outside the horizon. Nonetheless, the concept of singularity in black hole physics plays a vital role concerning the information problem [8, 9], as well as the final fate of a black hole, associated with the evaporation due to the Hawking radiation [10, 11]. Alternatively, a somewhat conservative approach is to pursue a regular and effective geometric description of a region of black hole spacetime, which otherwise would be singular. One might aim for a moderate framework for analyzing many crucial physical problems without introducing significant deviations from the standard model. In this regard, in 1968, Bardeen proposed a regular black holes metric, where a region of nonsingular spacetime substitutes the central singularity [12]. On the other hand, this metric asymptotically coincides with the Schwarzschild solution at infinity. Subsequently, following this line of thought, various models for regular black holes were proposed [13-17]. It was argued later that black hole solutions could be physically interpreted in terms of nonlinear electrodynamics with magnetic monopole [18, 19]. Further investigations of the quasinormal modes of the regular black hole metrics have also been carried out [20-24], and the corresponding metrics were shown to be stable against various types of perturbations. However, nonlinear electrodynamics encounters several potential difficulties. First, the astronomical objects are by and large electrically neutral, or do not carry a substantial amount of charge. Second, while the (linear) quantum electrodynamics is one of the most validated theories in physics, there is still a lack of strong experimental support for nonlinear electrodynamics. Finally, the construc-

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ted regular black hole solution usually requires the concept of magnetic monopole, which is a hypothetical elementary particle. Moreover, owing to cosmic inflation, only an insignificant amount of magnetic monopoles might persist in the observable Universe.

In view of the above discussion, the present study is an attempt to investigate regular black hole solutions in the framework of the Rastall theory. The derived solutions can be electrically and magnetically neutral. Rastall gravity was proposed in 1972 by Rastall [25], as a generalization of Einstein's general relativity. It is proposed that the conservation of energy-momentum tensor in curved spacetime can be relaxed, and it attains the form

$$T^{\nu}_{\mu;\nu} = a_\mu, \tag{1}$$

where a_μ should vanish in a flat spacetime so that in this case the theory restores Einstein's gravity.

In his original work, Rastall assumes

$$a_\mu = \lambda \nabla_\mu R, \tag{2}$$

and therefore the field equation becomes

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa (T_{\mu\nu} - \lambda g_{\mu\nu} R), \tag{3}$$

where $\kappa = 8\pi G/c^4$ and λ is a constant. As a theory of modified gravity, Rastall gravity has received increasing attention lately [26-44], particularly due to recent findings in cosmology [45-54].

In a recent study [44], it is pointed out that in accordance with Rastall's original proposal, a_μ can adopt other forms besides Eq. (2). This is because the only requirement is that a_μ vanishes in flat spacetime [25], as this does not lead to any conflict with present observations. In this context, we assume

$$a_\mu = \nabla^\nu \mathcal{A}_{\nu\mu}, \tag{4}$$

where $\mathcal{A}_{\nu\mu} = \mathcal{A}_{\mu\nu}$, and the field equation can be written as

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa (T_{\mu\nu} - \mathcal{A}_{\mu\nu}). \tag{5}$$

Here, $\mathcal{A}_{\mu\nu}$ as well as its derivatives must be sufficiently small where the curvature of the spacetime vanishes. It is straightforward to show that one can formally express various modified theories of gravity, such as $f(R)$ gravity and quadratic gravity, with the above generalized form of Rastall gravity [41, 44].

In this work, we investigate Rastall gravity with

$$\mathcal{A}_{\mu\nu} = \lambda g_{\mu\nu} H(R), \tag{6}$$

where $H = H(R)$ is an arbitrary function of the Ricci scalar. According to the above discussion, one requires that $H = 0$ in flat spacetime, where $R = 0$. In the remainder of the paper, it is shown that one may derive regular black hole solutions for Rastall gravity determined by Eq. (6).

The paper is organized as follows. In the next section, we discuss the general properties of regular black holes. The specific form of the line element is given explicitly. In Section 3, we provide a detailed account of the construction of the regular black hole solution in Rastall gravity, which assumes Eq. (6). Further discussions and concluding remarks are given in the last section.

2 Properties of regular black holes

In this section, we discuss the general properties of the metric of a regular black hole and derive the relevant requirements that need to be fulfilled. We start by considering the following form of the static, spherically symmetric metric in four-dimensional spacetime,

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\varphi^2), \tag{7}$$

where

$$f(r) = 1 - \frac{2\mathcal{M}(r)}{r} = 1 - \frac{2M_0}{r} C_M(r). \tag{8}$$

Here, M_0 is the mass of a black hole as measured by an inertial observer sitting at infinity, and the parameter C_M is introduced in the last equality which satisfies $C_M \to 1$ as $r \to \infty$. In what follows, we derive the condition for a regular solution of the black hole at $r = 0$ in terms of $C_M(r)$.

To avoid a singularity, the curvature of spacetime must not diverge. In practice, this is achieved by investigating the convergence of the relevant scalar quantities, namely, $R = g^{\mu\nu}R_{\mu\nu}$, $\mathbb{R} = R_{\alpha\beta}R^{\alpha\beta}$ and $\mathcal{R} = R_{\alpha\beta\gamma\sigma}R^{\alpha\beta\gamma\sigma}$. For a static regular black hole, we find

$$\begin{aligned} R &= \frac{2M_0}{r^2}\left(2C'_M + rC''_M\right), \quad \mathbb{R} = \frac{2M_0^2}{r^4}\left(4C'^2_M + r^2C''^2_M\right), \\ \mathcal{R} &= \frac{4M_0^2}{r^6}\left[12C_M^2 + 4rC_M\left(rC''_M - 4C'_M\right)\right. \\ &\quad \left. + r^2\left(r^2C''^2_M - 4rC'_MC''_M + 8C'^2_M\right)\right], \end{aligned} \tag{9}$$

where $C'_M \equiv \frac{\mathrm{d}C_M}{\mathrm{d}r}$ and $C''_M \equiv \frac{\mathrm{d}^2C_M}{\mathrm{d}r^2}$. In the vicinity of $r = 0$, we assume $C_M \sim r^{\alpha_{\text{center}}} + O(r^{\alpha_{\text{center}}})$. By substituting this condition into the above scalars and expanding around $r = 0$, one finds that the requirement of convergence implies a condition $\alpha_{\text{center}} \geqslant 3$. On the other hand, one has $C_M \to 1$ as $r \to \infty$, which usually does not lead to any divergence of the curvature. The above condition should be satisfied for any regular black hole solution.

Let us now discuss the properties of the metric near the black hole horizon. If one denotes the horizon by r_p, for $f(r_p) = 0$ one has

$$f(r) = 1 - \frac{r_pC_M(r)}{rC_M(r_p)}. \tag{10}$$

Therefore, the condition $r_p > 0$ implies $C_M(r_p) > 0$. The temperature at the horizon is

$$T_H = \frac{1 - r_p C_p}{4\pi r_p}, \tag{11}$$

where $C_p = C'_M / C_M\big|_{r=r_p}$.

In literature, in order to derive a metric for the regular black hole, one introduces a nonlinear electrodynamic field in the system. Alternatively, it is shown in the following section that a static regular black hole solution can be found in Rastall gravity in terms of $H(R)$.

3 Regular black holes in Rastall gravity

In the static, spherically symmetric black hole metric, the energy-momentum tensor of a given type of fluid surrounding the black hole can be written as [26, 55]

$$T^t_t = -\rho(r), \qquad T^i_j = -\rho(r)\alpha\left[\beta\delta^i_j - (1+3\beta)\frac{r_i r^j}{r_n r^n}\right], \tag{12}$$

where ρ and p are the energy density and pressure of the matter field. After averaging over the angle in an isotropic system, the spatial components read

$$\langle T^i_j\rangle = \frac{\alpha}{3}\rho\delta^i_j = p\delta^i_j, \tag{13}$$

where we have used $\langle r^i r_j\rangle = \frac{1}{3}\delta^i_j r^n r_n$. For the barotropic equation of state, we have [26, 55]

$$p = \omega\rho, \qquad \omega = \frac{\alpha}{3}, \qquad \beta = -\frac{1+3\omega}{6\omega}. \tag{14}$$

The corresponding energy-momentum tensor takes the following form

$$T^t_t = T^r_r = -\rho(r), \qquad T^\theta_\theta = T^\varphi_\varphi = \frac{1}{2}(1+3\omega)\rho(r). \tag{15}$$

Now, by substituting the static black hole metric, Eq. (7), into the field equations of the Rastall theory, Eq. (5) and Eq. (6), one obtains

$$\begin{aligned} rf'(r) + f(r) - 1 + \kappa\left[r^2\rho(r) + \lambda r^2 H\right] &= 0, \\ rf''(r) + 2f'(r) + \kappa\left[2\lambda r H - (1+3\omega) r\rho(r)\right] &= 0, \end{aligned} \tag{16}$$

while the equation for the energy-momentum tensor, Eq. (1), becomes

$$\rho'(r) + 3\frac{1+\omega}{r}\rho(r) + \lambda\frac{\mathrm{d}H}{\mathrm{d}r} = 0. \tag{17}$$

We note that Eq. (17) is not independent since it is implied by Eq. (16). Here, $H = H(R)$ is a function of the Ricci scalar, which determines the specific form of the metric while satisfying Eq. (17).

By solving Eqs. (16) and (17), one finds

$$\begin{aligned} H &= -\frac{(1+3\omega)(f(r)-1) + 3(1+\omega)rf'(r) + r^2 f''(r)}{3r^2\lambda\kappa(1+\omega)}, \\ \rho(r) &= \frac{r^2 f''(r) - 2f(r) + 2}{3\kappa(1+\omega)r^2}, \end{aligned} \tag{18}$$

which can be rewritten in terms of C_M by making use of Eq. (8)

$$\begin{aligned} H &= 2M_0\frac{(1+3\omega)C'_M(r) + rC''_M(r)}{3\kappa\lambda(1+\omega)r^2}, \\ \rho(r) &= 2M_0\frac{2C'_M(r) - rC''_M(r)}{3\kappa(1+\omega)r^2}. \end{aligned} \tag{19}$$

We are now in a position to study the conditions under which the above solution is indeed regular. According to the discussion above, in flat spacetime $H(R)$ satisfies the condition

$$H \to 0 \text{ as } R \to 0. \tag{20}$$

This also implies that $H(R) \to 0$ as $r \to \infty$. On the other hand, it is also required that $\rho(r)$ does not possess any singularity in the entire range $r \in [0, +\infty)$. By making use of the properties of the curvature scalars and $C_M(r)$ discussed previously, we find the desired conditions

$$\begin{aligned} C_M(r\to 0) &\to r^{\alpha_{\text{center}}} \text{ with } \alpha_{\text{center}} \geqslant 3, \\ C_M(r\to\infty) &\to 1 + r^{\alpha_{\text{infinity}}} \text{ with } \alpha_{\text{infinity}} < 0. \end{aligned} \tag{21}$$

The above conditions, Eq. (21), can be satisfied when C_M is in the form of a fraction, where both numerator and denominator are polynomials in r. The first line of Eq. (21) dictates that the lowest degree of the monomials in the numerator is at least three orders larger than that in the denominator. The second line of Eq. (21) implies that the highest degree of the monomials in the numerator must be smaller than that in the denominator. As a simple illustration, a possible solution reads

$$C_M(r) = \frac{r^3}{r^3 + 2\sigma^2}, \tag{22}$$

where σ is a constant. If $\sigma = 0$, the corresponding metric is not regular. For $\sigma \neq 0$, we have a regular black hole solution

$$\begin{aligned} f(r) &= 1 - \frac{2M_0 r^2}{r^3 + 2\sigma^2}, \qquad \rho(r) = \frac{24M_0\sigma^2 r^3}{\kappa(1+\omega)(r^3+2\sigma^2)^3}, \\ H &= 12M_0\sigma^2\frac{(\omega-1)r^3 + 2(\omega+1)\sigma^2}{\kappa\lambda(1+\omega)(r^3+2\sigma^2)^3}, \\ R &= 24M_0\sigma^2\frac{4\sigma^2 - r^3}{(r^3+2\sigma^2)^3}. \end{aligned} \tag{23}$$

In fact, the above solution can readily be identified as the Hayward regular black hole [13]. Moreover, the above results imply that $r = r(R)$. Subsequently, one finds the following relation

$$H(R) = 3B^2R^2\frac{(\omega-1)\left[B^2M_0^{1/3} - 2RM_0^{2/3}\right] - 2\sigma BR}{\kappa\lambda(1+\omega)\left(B^2 - 2RM_0^{1/3}\right)^3}, \tag{24}$$

where $B = \left(R^{3/2}\sqrt{81\sigma^2 R + 8M_0} - 9\sigma R^2\right)^{1/3}$. Obviously, Eq. (24) satisfies the condition that $H(R) \to 0$ as $R \to 0$. Therefore, we have constructed regular black hole solutions in Rastall gravity, and in particular, we note that

$\omega \neq -1$.

4 Further discussions and concluding remarks

Following the original idea proposed by Rastall, the conservation of the energy-momentum tensor is generalized in this work to the form

$$\nabla_\mu T^\mu_\nu = \lambda \nabla_\nu H(R), \tag{25}$$

while the corresponding field equation reads

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa \left(T_{\mu\nu} - \lambda g_{\mu\nu} H(R) \right), \tag{26}$$

where $H(R)$ vanishes in flat spacetime.

In previous sections, we have shown that the regular black hole solutions can be derived in the framework of Rastall gravity. As discussed above, the obtained black hole spacetime can be electrically neutral, which does not involve nonlinear electrodynamics nor the associated theoretical speculations. In this sense, the present study provides a novel possibility for models of regular black holes.

The constructed black hole spacetime is surrounded by a matter field, described by the energy-momentum tensor, Eq. (15). However, to guarantee that the solution is nonsingular, one finds that the equation of state of the matter field has to satisfy the condition $\omega \neq -1$. In other words, the dark energy model in terms of the cosmological constant cannot be a candidate for hosting the regular black hole solution considered.

On the other hand, when assuming $T_{\mu\nu} = 0$, the static black hole solution found in the present model is no longer regular. However, interestingly enough, in this case one can show that the resulting metric is equivalent to an (anti-)de Sitter spacetime. To demonstrate this point, one first contracts both sides of Eq. (26) by $g^{\mu\nu}$ to obtain

$$4\lambda\kappa H(R) - R = 0. \tag{27}$$

This is an algebraic equation, and in general it has a non-vanishing root at R, besides the one at the origin. Thus, one may rewrite Eq. (26) as follows

$$R_{\mu\nu} = g_{\mu\nu} \left(\frac{1}{2} R - \kappa\lambda H(R) \right) \equiv g_{\mu\nu} \Lambda_{\rm eff}, \tag{28}$$

where R is a non-zero root of Eq. (27). For the reason which will shortly become clear, $\Lambda_{\rm eff}$ is defined as the effective cosmological constant, and one has

$$\begin{aligned} R &= 4\Lambda_{\rm eff}, \\ H &= \frac{\Lambda_{\rm eff}}{\kappa\lambda}. \end{aligned} \tag{29}$$

As a result, the metric Eq. (7) can be written as

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda_{\rm eff}}{3} r^2. \tag{30}$$

Even though the cosmological constant is not an *ad hoc* assumption in the field equation, the properties of the resulting metric indicate that it is a de Sitter spacetime. Therefore, it arises naturally from the consistency of the theory. For instance, let us consider the case where $\Lambda_{\rm eff} > 0$. The de Sitter background is realized in the context of the present generalized Rastall gravity in terms of the effective cosmological constant $\Lambda_{\rm eff} = \frac{R}{4}$. If one chooses $H(R) = R^n$ with $n > 1$ and $\lambda > 0$, one finds

$$R = 4\Lambda_{\rm eff} = \left(\frac{1}{4\kappa\lambda} \right)^{\frac{1}{n-1}}, \qquad H = \frac{\Lambda_{\rm eff}}{\kappa\lambda} = \left(\frac{1}{4\kappa\lambda} \right)^{\frac{n}{n-1}},$$
$$\Lambda_{\rm eff} = \frac{1}{4} \left(\frac{1}{4\kappa\lambda} \right)^{\frac{1}{n-1}}. \tag{31}$$

Moreover, it is not difficult to show that one can further extend the above considerations to rotating black holes in the presence of the (linear) Maxwell field. In this case, the gravitational field equation is given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \kappa\lambda g_{\mu\nu} R - 2F_{\mu\alpha} F^\alpha_\nu + \frac{1}{2} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} = 0, \tag{32}$$

where the electromagnetic tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ satisfies the Maxwell equations

$$\partial_\mu \left(\sqrt{-g} F^{\mu\nu} \right) = 0. \tag{33}$$

We note that Eq. (32) still reduces to Eq. (27) when both sides are contracted by $g^{\mu\nu}$, as the two terms involving the electromagnetic tensor cancel out. This implies that we again get an equation similar to Eq. (28)

$$R_{\mu\nu} - 2F_{\mu\alpha} F^\alpha_\nu + \frac{1}{2} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} = g_{\mu\nu} \left(\frac{1}{2} R - \kappa\lambda H(R) \right) \equiv g_{\mu\nu} \Lambda_{\rm eff}, \tag{34}$$

where R and $H(R)$ still satisfy Eq. (29).

This result gives the Kerr-Newman (anti-)de Sitter black hole metric in Rastall gravity, namely,

$$\begin{aligned} {\rm d}s^2 = &-\frac{\Delta_r}{\Xi^2 \rho^2} \left({\rm d}t - a \sin^2\theta {\rm d}\varphi \right)^2 + \frac{\Delta_\theta}{\Xi^2 \rho^2} \sin^2\theta \\ &\times \left(a{\rm d}t - \left(r^2 + a^2 \right) {\rm d}\varphi \right)^2 + \frac{\rho^2}{\Delta_r} {\rm d}r^2 + \frac{\rho^2}{\Delta_\theta} {\rm d}\theta^2, \end{aligned} \tag{35}$$

where the electromagnetic potential is $A_\mu = \frac{Qr}{\Xi\rho^2} \left(\delta^t_\mu - a \sin^2\theta \delta^\varphi_\mu \right)$, and Q and a are the electrical charge and angular momentum per unit mass, respectively. Also,

$$\begin{aligned} \rho^2 &= r^2 + a^2 \cos^2\theta, \qquad \Xi = 1 + \frac{a^2}{3} \Lambda_{\rm eff}, \\ \Delta_r &= \left(1 - \frac{r^2}{3} \Lambda_{\rm eff} \right) \left(r^2 + a^2 \right) - 2Mr + Q^2, \\ \Delta_\theta &= 1 + \frac{a^2 \cos^2\theta}{3} \Lambda_{\rm eff}. \end{aligned} \tag{36}$$

The above (anti-)de Sitter black hole solution is physically intriguing, since it emerges entirely from the vacuum Rastall equation without the cosmological constant, which probably may lead to further implications in cosmology. More effort along this line of thought is under progress.

To summarize, we studied the static, spherically symmetric black hole solutions in generalized Rastall gravity. An essential feature of the derived solutions is that the black holes can be electrically and magnetically neutral, which is distinct from most literature on this topic where the nonlinear electrodynamic field is involved. We also discussed the general properties of the regular black hole solutions.

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