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Triply heavy baryons in the constituent quark model*

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Abstract: The constituent quark model is used to compute the ground and excited state masses of QQQ baryons containing either c or b quarks. The quark model parameters previously used to describe the properties of charmonium and bottomonium states were used in this analysis. The non-relativistic three-body bound state problem is solved by means of the Gaussian expansion method which provides sufficient accuracy and simplifies the subsequent evaluation of the matrix elements. Several low-lying states with quantum numbers $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$ are reported. We compare the results with those obtained by the other theoretical formalisms. There is a general agreement for the mass of the ground state in each sector of triply heavy baryons. However, the situation is more puzzling for the excited states, and appropriate comments about the most relevant features of our comparison are given.

Keywords: Quantum Chromodynamics, quark models, properties of baryons, exotic baryons

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1 Introduction

Mesons containing only heavy valence quarks, either $c\bar{c}$ (charmonium) or $b\bar{b}$ (bottomonium), have contributed to the understanding of the quantum chromodynamics (QCD) due to their approximately non-relativistic nature and the clean spectrum of narrow states, at least below the open-flavor threshold. It is a fact that many precise experimental results are available for conventional heavy quarkonia and their analysis has significantly contributed to the understanding of, for instance, the quark-antiquark forces [1, 2]. On the other hand, tens of charmonium- and bottomonium-like XYZ states have been identified by the experiments at B-factories (BaBar, Belle, and CLEO), τ-charm facilities (CLEO-c and BES) and hadron colliders (CDF, D0, LHCb, ATLAS, and CMS). So far, there is no definite conclusion about the nature of these exotic states (see Refs. [1, 3, 4] for reviews of the experimental and theoretical status of the subject). The analysis and new determinations will continue with the upgrade of the experiments such as BES III [5], Belle II [6] and HL- and HE-LHC [7]. This will provide a sustained progress in the field, as well as the breadth and depth necessary for a vibrant heavy quarkonium research environment.

As they are baryonic analogues of heavy quarkonia, triply heavy baryons may provide a complementary window for the understanding of the strong interaction between quarks without taking into account the usual light quark complications. Moreover, as in heavy quarkonia, there is no restriction for finding exotic structures in the triply heavy baryon spectra, and thus a reliable prediction of conventional QQQ baryons[5)] is interesting by

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5) For now on we will denote Q as the heavy quarks c or b, being QQQ a baryon composed of any combination of these heavy quarks.

itself as it could provide a template for comparison of future experimental findings.

The production of triply heavy baryons is extremely difficult, and no experimental signal for any of them has yet been reported. Baranov et al. estimated that triply charmed baryons may not be observed in e^+e^- collisions, and the expectations for bbb baryons would be even worse [8]. This conclusion was also reached by Bjorken in the 1980s [9], and he proposed hadron induced fixed target experiments as the best strategy to observe the ground state triply charmed baryon Ω_{ccc}^{++}. First estimates of the production cross-section of triply heavy baryons at the LHC were evaluated in Refs. [10–13]. In a more recent calculation [14], it was estimated that around 10^4-10^5 events of triply heavy baryons with ccc and ccb quark content, could be accumulated for $10\,\mathrm{fb}^{-1}$ of integrated luminosity. Some investigations of the production rate of multi-charmed hadrons in heavy-ion collisions at high energies were presented in Refs. [15–17]. Finally, it was suggested in Refs. [18, 19] to look for triply heavy baryons in their semi-leptonic and non-leptonic decays.

From a theoretical point of view, the first study of heavy baryon spectroscopy was carried out to our knowledge in Ref. [20] using the QCD motivated bag model. More recently, there have been other theoretical mass determinations that included the non-relativistic quark models [21–25] and its relativistic variants [26, 27], the Faddeev formalism using a non-relativistic reduction of the quark-quark interaction [28], front-form formulation of the effective QCD Hamiltonian [29], QCD sum rules [30–33], non-relativistic effective field theories [34–37], continuum approach to QCD based on the Dyson-Schwinger equations [38, 39], and the lattice gauge theories [40–45].

In this work, we compute the spectrum of triply heavy baryons, including the ground and excited states with quantum numbers $J^P=\frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$. Our theoretical formalism is the constituent quark model (CQM) proposed in Ref. [46] (see references [47] and [48] for reviews). This model was recently successfully applied to the study of the spectra of mesons containing heavy quarks [49–52], their electromagnetic, weak and strong decays and reactions [49, 52–57], their coupling with meson-meson thresholds [58–61], and recently to the phenomenological exploration of multiquark structures [62, 63]. Therefore, this study entails a first step towards a unified description of heavy mesons and baryons using the same quark model, with the parameters and technical formalism presented in Refs. [49, 52]. Moreover, the predicted spectrum of triply heavy baryons could also be seen as a contribution to the template for comparing the future experimental findings, and for discerning between the conventional and exotic structures, since the potential models are expected to describe triply heavy baryons to a similar degree of accuracy as those obtained in the charmonium and bottomonium sectors.

The present paper is arranged as follows. We describe briefly in Sec. 2 the constituent quark model, the triply heavy baryon wave function and the computational formalism based on the Gaussian expansion method. Sec. 3 is devoted to the analysis and discussion of the obtained results. We summarize and give some prospects in Sec. 4.

2 Theoretical framework

The Hamiltonian which describes the triply heavy baryon bound system can be written as

$$H=\sum_{i=1}^{3}\left(m_i+\frac{\vec{p}_i^{\,2}}{2m_i}\right)-T_{\rm CM}+\sum_{j>i=1}^{3}V(\vec{r}_{ij}), \tag{1}$$

where $T_{\rm CM}$ is the center-of-mass kinetic energy. Since chiral symmetry is explicitly broken in the heavy quark sector, the two-body potential can be deduced from the one-gluon exchange and confining interactions. The one-gluon exchange potential is given by

$$V_{\rm OGE}(\vec{r}_{ij})=\frac{1}{4}\alpha_s(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\times\left[\frac{1}{r_{ij}}-\frac{1}{6m_im_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}\right], \tag{2}$$

where m_i is the quark mass, λ^c are the $SU(3)$ color Gell-Mann matrices, and the Pauli matrices are denoted by $\vec{\sigma}$. The contact term of the central potential has been regularized as

$$\delta(\vec{r}_{ij})\sim\frac{1}{4\pi r_0^2}\frac{e^{-r_{ij}/r_0}}{r_{ij}}, \tag{3}$$

where $r_0(\mu_{ij})=\tilde{r}_0/\mu_{ij}$ is a regulator that depends on the reduced mass of the quark-quark pair, μ_{ij}.

The wide energy range needed to provide a consistent description of light, strange and heavy mesons requires an effective scale-dependent strong coupling constant. We use the frozen coupling constant [46]

$$\alpha_s(\mu_{ij})=\frac{\alpha_0}{\ln\left(\frac{\mu_{ij}^2+\mu_0^2}{\Lambda_0^2}\right)}, \tag{4}$$

in which α_0, μ_0 and Λ_0 are parameters of the model.

Color confinement should be encoded in the non-Abelian character of QCD. Studies on a lattice have demonstrated that multi-gluon exchanges produce an attractive linearly rising potential proportional to the distance between infinitely heavy quarks [64]. However, the spontaneous creation of light quark pairs from QCD vacuum may give rise, at the same scale, to the breakup of the created color flux-tube [64]. We have tried to mimic these two phenomenological observations by the expression:

$$V_{\rm CON}(\vec{r}_{ij}) = [-a_c(1-e^{-\mu_c r_{ij}})+\Delta](\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c), \quad (5)$$

where a_c and μ_c are model parameters. One can see in Eq. (5) that the potential is linear at short inter-quark distances with an effective confinement strength $\sigma=-a_c\mu_c(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)$, while it becomes constant at large distances.

Let us mention that the associated tensor and spin-orbit terms of the potentials presented above appear not to be essential for a global description of baryons [28]. Therefore, they have been neglected since the main purpose of our approach is to get a first and reliable unified description of heavy mesons and baryons. The quark model parameters relevant for this work are shown in Table 1.

Table 1. Quark model parameters.

Quark masses	m_c (MeV)	1763
	m_b (MeV)	5110
OGE	$\hat{r}_0$ (MeV fm)	28.327
	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
Confinement	a_c (MeV)	507.4
	μ_c (fm^{-1})	0.576
	Δ (MeV)	184.432

The triply heavy baryon wave function is constructed as a product of four terms: the color, flavor, spin and space wave functions. The color wave function can be written as usual for a baryon. The spin wave function of a 3-quark system was worked out, for instance, in Ref. [65], and the flavor wave function of a full heavy quark baryon is trivial.

The spatial wave function of the 3-body system can be written as a sum of amplitudes of three rearrangement channels

$$\psi_{LM_L}=\Phi^{(c=1)}_{LM_L}(\vec{\rho}_1,\vec{\lambda}_1)+\Phi^{(c=2)}_{LM_L}(\vec{\rho}_2,\vec{\lambda}_2)+\Phi^{(c=3)}_{LM_L}(\vec{\rho}_3,\vec{\lambda}_3), \quad (6)$$

where $\vec{\rho}_i$ and $\vec{\lambda}_i$ are the internal Jacobi coordinates

$$\vec{\rho}_i=\vec{x}_j-\vec{x}_k, \quad \vec{\lambda}_i=\vec{x}_i-\frac{m_j\vec{x}_j+m_k\vec{x}_k}{m_j+m_k}, \quad (7)$$

with $i, j, k = 1,\cdots,3$ and $i\neq j\neq k$. Note that we work with a triply heavy baryon in which either the three quarks are the same, and then only one rearrangement channel is needed, or two of the three quarks are equal, and thus two rearrangement channels must been incorporated.

Each amplitude in Eq. (6) is expanded in terms of an infinitesimally shifted Gaussian basis functions [66]:

$$\Phi^{(c)}_{LM_L}(\vec{\rho}_c,\vec{\lambda}_c)=\sum_{n_1l_1,n_2l_2}A^{(c)}_{n_1l_1,n_2l_2}\left[\phi_{n_1l_1}(\vec{\rho}_c)\varphi_{n_2l_2}(\vec{\lambda}_c)\right]_{LM_L} \quad (8)$$

where

$$\begin{aligned}\phi_{n_1l_1m_1}(\vec{\rho}) &= N_{n_1l_1}\rho^{l_1}e^{-\nu_{n_1}\rho^2}Y_{l_1m_1}(\hat{\rho})\\ &= N_{n_1l_1}\lim_{\varepsilon\to0}\frac{1}{(\nu_{n_1}\varepsilon)^{l_1}}\sum_{k=1}^{k_{\max}}C_{l_1m_1,k}\,e^{-\nu_{n_1}(\vec{\rho}-\varepsilon\vec{D}_{l_1m_1,k})^2}, \quad (9)\end{aligned}$$

$$\begin{aligned}\varphi_{n_2l_2m_2}(\vec{\lambda}) &= N_{n_2l_2}\lambda^{l_2}e^{-\nu_{n_2}\lambda^2}Y_{l_2m_2}(\hat{\lambda})\\ &= N_{n_2l_2}\lim_{\varepsilon\to0}\frac{1}{(\nu_{n_2}\varepsilon)^{l_2}}\sum_{t=1}^{t_{\max}}C_{l_2m_2,t}\,e^{-\nu_{n_2}(\vec{\lambda}-\varepsilon\vec{D}_{l_2m_2,t})^2}. \quad (10)\end{aligned}$$

The spherical harmonics are denoted by $Y_{l_1m_1}(\hat{\rho})$ and $Y_{l_2m_2}(\hat{\lambda})$; $N_{n_1l_1}$ and $N_{n_2l_2}$ are the normalization constants. The basis parameters $\{C_{l_1m_1,k},\ D_{l_1m_1,k};\ k=1,\cdots,k_{\max}\}$ as well as $\{C_{l_2m_2,t},\ D_{l_2m_2,t};\ t=1,\cdots,t_{\max}\}$, are determined, for instance, in Appendix A.2 of Ref. [66]. The limit $\varepsilon\to0$ must be carried out after the matrix elements have been calculated analytically. This new set of basis functions makes the calculation of the 3-body matrix elements easier, without resorting to the laborious Racah algebra. Following Ref. [66], the Gaussian ranges ν_{n_i} with $i=1,2$, are taken as a geometric progression, which enables their optimization using a small number of free parameters. Moreover, the geometric progression is dense for short distances, which allows a description of the dynamics mediated by short range potentials. The fast damping of the Gaussian tail is not a problem, since we can choose the maximal range to be much longer than the hadronic size.

The Rayleigh-Ritz variational principle is used to solve the Schrödinger equation

$$[H-E]\Psi_{JM_J}=0, \quad (11)$$

and to determine the eigenenergies E and coefficients $A^{(c)}_{n_1l_1,n_2l_2}$. Note that the complete wave-function is written as

$$\Psi_{JM_J}=\mathcal{A}\left\{\left[\psi_{LM_L}\chi^{\sigma}_{SM_S}(3)\right]_{JM_J}\chi^f\chi^c\right\}, \quad (12)$$

where χ^c, χ^f, $\chi^{\sigma}_{SM_S}(3)$ and ψ_{LM_L} are the color, flavor, spin and space wave functions, respectively. In order to fulfill the Pauli principle, the antisymmetric operator $\mathcal{A}$ is the same for Ω_{ccc} and Ω_{bbb}, i.e. $\mathcal{A}=1-(13)-(23)$ in a system with three identical particles.[1)] However, $\mathcal{A}=1$ for Ω_{ccb}, and $\mathcal{A}=1-(23)$ for Ω_{cbb}. This is needed because we have constructed the antisymmetric wave function for the first two quarks of the 3-quark cluster, and the remaining quark is added to the wave function simply by considering the appropriate Clebsch-Gordan coefficient.

1) Note here that each (ij)-term in $\mathcal{A}$ express an interchange operator of S_3 permutation group for QQQ-clusters, with Q either a c- or b-quark.

3 Results

Table 2 shows the total spin and parity J^P of the triply heavy baryons whose masses are calculated. In the non-relativistic approximation, the total angular momentum L and spin S are good quantum numbers, and they couple to the total spin J.[1)] The total angular momentum is the result of coupling of the two possible excitations along the Jacobi coordinates, i.e. l_1 and l_2 of Eq. (8), to obtain L. In this analysis, l is never greater than L , and the possible channels are listed in the first column of Table 2. Since a baryon is a 3-quark bound system, its total spin can only take values $1/2$ and $3/2$, and its parity is given by $(-1)^{l_1+l_2}$, since the parity of a quark is positive by convention.

Table 2. Possible J^P quantum numbers of the studied triply heavy quark bound systems.

(l_1,l_2)	L	$S=\frac{1}{2}$	$S=\frac{3}{2}$
(0,0)	0	$\frac{1}{2}^+$	$\frac{3}{2}^+$
(0, 1)			
(1, 0)	1	$\frac{1}{2}^-,\frac{3}{2}^-$	$\frac{1}{2}^-,\frac{3}{2}^-,\frac{5}{2}^-$
(0, 2)			
(1, 1)	2	$\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+$	$\frac{1}{2}^+,\frac{3}{2}^+,\frac{5}{2}^+,\frac{7}{2}^+$
(2, 0)			

Tables 3, 4, 5 and 6 show, respectively, the spectra of the Ω_{ccc}, Ω_{ccb}, Ω_{cbb} and Ω_{bbb} baryon sectors computed in our formalism following the pattern of quantum numbers shown in Table 2. Since there are no experimental data for triply heavy baryons, we first compare our results with the available predictions of lattice QCD. However, lattice regularized computations have their own issues, such as the use of the non-relativistic QCD (NRQCD) actions for heavy quarks, which are not ideally suited for charm quarks. Also, they do not address all systematic uncertainties. Therefore, the predictions of other theoretical approaches are also reported in these Tables.

3.1 The Ω_{ccc} baryon sector

Table 3 shows our predicted masses of Ω_{ccc} baryons with the total spin and parity $J^P=\frac{1}{2}^{\pm},\frac{3}{2}^{\pm},\frac{5}{2}^{\pm}$ and $\frac{7}{2}^+$. We report the S-, P- and D-wave ground and radial-excited states for all channels mentioned. Since the total wave function of a ccc baryon must be totally antisymmetric in order to fulfill the Pauli principle, there is no S-wave bound state with the total spin and parity $J^P=\frac{1}{2}^+$. From Table 3, we predict a mass of 4.80 GeV for the ground

Table 3. Predicted masses, in MeV, of Ω_{ccc} baryons with the total spin and parity $J^P=\frac{1}{2}^{\pm},\frac{3}{2}^{\pm},\frac{5}{2}^{\pm}$ and $\frac{7}{2}^+$. We compare our results with those obtained by the other theoretical approaches, in particular the recent lattice QCD results in Ref. [42].

J^P	nL	This work	[42]	[22]	[26]	[27]	[28]	[29]	[31]	[33]	[35]	[38]	[39]	[43]	[44]
$\frac{1}{2}^+$	1D	5376	5395(13)	5412	–	5216	5324	5358	–	–	–	–	–	–	–
	2D	5713	–	–	–	–	–	–	–	–	–	–	–	–	–
$\frac{3}{2}^+$	1S	4798	4759(6)	4763	4803	4773	4799	4797	4990(140)	4720(120)	4760(60)	4760	5000	4789(22)	4796(20)
	2S	5286	5313(31)	5317	–	–	5243	5309	–	–	–	5150	–	–	–
$\frac{3}{2}^+$	1D	5376	5426(13)	5412	–	–	5324	5358	–	–	–	–	–	–	–
	2D	5713	–	–	–	–	–	–	–	–	–	–	–	–	–
$\frac{5}{2}^+$	1D	5376	5402(15)	5412	–	–	5324	5358	–	–	–	–	–	–	–
	2D	5713	–	–	–	–	–	–	–	–	–	–	–	–	–
$\frac{7}{2}^+$	1D	5376	5393(49)	5412	–	–	5324	5358	–	–	–	–	–	–	–
	2D	5713	–	–	–	–	–	–	–	–	–	–	–	–	–
$\frac{1}{2}^-$	1P	5129	5116(9)	5132	–	5109	5094	5103	–	–	–	–	–	–	–
	2P	5525	5608(31)	5610	–	–	5456	–	–	–	–	–	–	–	–
$\frac{3}{2}^-$	1P	5129	5120(13)	5132	–	5014	5094	5103	5110(150)	4900(100)	–	5027	–	–	–
	2P	5525	5658(31)	5610	–	–	5456	–	–	–	–	–	–	–	–
$\frac{5}{2}^-$	1P	5558	5512(64)	5637	–	–	5494	–	–	–	–	–	–	–	–
	2P	5846	5705(25)	–	–	–	–	–	–	–	–	–	–	–	–

1) Since spin-orbital interactions are not employed in the present work, triply-heavy baryons with different J^P but equal L and S will be degenerated in mass.

state, which has quantum numbers $nL(J^P) = 1S\,(\frac{3}{2}^+)$. The mass of the same state is predicted by the lattice QCD [42] to be 4.76 GeV, which compares reasonably well with our result. A similar level of agreement between the lattice and our calculations is achieved for the other ground states of the reported J^P channel in Table 3.

Lattice QCD [42] reports two almost degenerate states in each channel with quantum numbers $J^P = \frac{1}{2}^+, \frac{3}{2}^+$ and $\frac{5}{2}^+$. They correspond to a different spin excitation because the three J^P quantum numbers can be obtained when coupling a D-wave component with either $S = \frac{1}{2}$ or $\frac{3}{2}$. Table 3 shows the eigenstate of the lowest mass, which corresponds to the coupling $L\otimes S = 2\otimes 3/2$. Our prediction for the other case, $L\otimes S = 2\otimes 1/2$, is 5407 MeV , which compares reasonably well with the lattice results 5401 ± 14 MeV, 5461 ± 13 MeV and 5460 ± 15 MeV for $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$, respectively. In the positive parity sector, the only radial excitation that can be compared is the $2S\,(\frac{3}{2}^+)$ state. Our prediction, 5.29 GeV, is in fair agreement with the lattice result 5.31 GeV. There is a mismatch of ~ 0.1 GeV between our calculations and the lattice regularized QCD for negative parity excited states (see Table 3).

Let us mention that the level of agreement should be taken with some caution because our constituent quark model suffers from theoretical uncertainties that can be estimated at ± 50 MeV when the most sensitive model parameter is modified by 10% . On the other hand, systematic uncertainties are not estimated in Ref. [42], which is to say that the lattice errors given in Table 3 are just statistical. With the lattice NRQCD action and the parameters used in [42], the systematic errors may be significant, especially for the spin dependent energy splitting. A calculation of the charmonium spectrum with the same lattice formulation is given in Ref. [67], which can give an idea of the typical systematic uncertainties.

Table 3 also compares our predictions with the results reported by the oher theoretical formulations. For the ground state $nL(J^P) = 1S\,(\frac{3}{2}^+)$, our results agree with the general trend, except for the few cases where the predicted mass is around 5.0 GeV. For the rest of the spectrum, the data reported by the other approaches is quite sparse, with big uncertainties in some cases, making it difficult to perform a quantitative comparison. However, there are some theoretical calculations [22, 27–29] where the reported spectrum is as complete as ours and where the level of agreement is quite reasonable.

3.2 The Ω_{ccb} baryon sector

Table 4 shows our spectrum of Ω_{ccb} baryons. We predict two almost degenerate states with quantum numbers $nL(J^P) = 1S\,(\frac{1}{2}^+)$ and $1S\,(\frac{3}{2}^+)$, and masses of the order of 8.0 GeV, which are the two ground states of positive parity Ω_{ccb} baryons. The agreement with the recent lattice-QCD prediction [45] is remarkable. In this case, the lattice computations are based on: (i) three different lattice spacings, allowing precise results at the continuum limit; (ii) the relativistic formulation for the light, strange and charm quarks; (iii) the lattice NRQCD action for bottom quarks with non-perturbative tuned coefficients up to $\mathcal{O}(\alpha_s v^4)$; and (iv) control of the statistical errors below a percent level.

Let us now turn to the results reported by the other approaches. As can be seen, there is no agreement in the masses of the $nL(J^P) = 1S\,(\frac{1}{2}^+)$ and $1S\,(\frac{3}{2}^+)$ states. The mass splitting seems to be small, of the order of tens of MeV, but their absolute masses cluster around two different mean values, 8.0 GeV and around $8.2-8.3$ GeV. This is quite puzzling. As we do not have a reasonable answer for this issue, we continue to investigate this sector. It is fair to note that there are some theoretical computations, mostly the QCD sum rule predictions, where the reported masses of the $1S\,(\frac{1}{2}^+)$ and $1S\,(\frac{3}{2}^+)$ states are quite different, and large error bands are reported.

There are few computations [28, 29, 38] that provide a spectrum as complete as ours. The results presented in Ref. [28] are in reasonable agreement with our calculated masses, as shown in Table 4. This could be because the formalism and quark-quark interactions are very similar despite the differences in numerical tools and model parameters. The relativistic effects may have been implemented in Refs. [29, 38]. The predicted states reported in [29] are $0.2-0.3$ GeV higher than ours and those reported by lattice QCD [45]. In Ref. [38], the spectrum agrees with ours if all values are lifted by about 0.1 GeV. This indicates at least that the mass splittings could be considered as similar, while the Ω_{ccb} spectrum needs to be disentangled experimentally.

3.3 The Ω_{cbb} baryon sector

Table 5 shows our spectrum of Ω_{cbb} baryons. Our predicted masses for the $nL(J^P) = 1S\,(\frac{1}{2}^+)$ and $1S\,(\frac{3}{2}^+)$ states are respectively 11200 MeV and 11221 MeV , and agree again with the recent lattice QCD results of Ref. [45], which are considered quite robust and precise.

We find in this sector a similar situation as already discussed for Ω_{ccb}. There is no clear consensus between the different approaches for the masses of the $nL(J^P) = 1S\,(\frac{1}{2}^+)$ and $1S\,(\frac{3}{2}^+)$ states. It looks like there is a convergence around 11.2 GeV, but quite different results with large uncertainties are again given by the QCD sum rules, introducing noise which is difficult to disentangle. As expected, the calculations of Ref. [28] is in fair agreement with ours. A comparison of our results with the relativistic formulations reveals an opposite situation to that in the Ω_{ccb} baryon sector. The computations in Ref. [29] seem to agree with our results, but we do not have the

Table 4. Predicted masses, in MeV, of Ω_{ccb} baryons with the total spin and parity $J^P=\frac{1}{2}^{\pm},\frac{3}{2}^{\pm},\frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$. We compare our results with those obtained by the other theoretical approaches, in particular the recent lattice QCD results in Ref. [45].

J^P	nL	This work	[45]	[26]	[28]	[29]	[31]	[32]	[33]	[35]	[38]	[39]	[44]
$\frac{1}{2}^+$	$1S$	8004	8005(13)	8018	8019	8301	8230(130)	8500(120)	–	7980(70)	7867	8190	8007(22)
	$2S$	8455	–	–	8450	8600	–	–	–	–	8337	–	–
$\frac{1}{2}^+$	$1D$	8536	–	–	8528	8647	–	–	–	–	–	–	–
	$2D$	8838	–	–	8762	–	–	–	–	–	–	–	–
$\frac{3}{2}^+$	$1S$	8023	8026(13)	8025	8056	8301	8230(130)	–	8070(100)	–	7963	8190	8037(22)
	$2S$	8468	–	–	8465	8600	–	–	–	–	8427	–	–
$\frac{3}{2}^+$	$1D$	8536	–	–	8528	8647	–	–	–	–	–	–	–
	$2D$	8838	–	–	8762	–	–	–	–	–	–	–	–
$\frac{5}{2}^+$	$1D$	8536	–	–	8528	8647	–	–	–	–	–	–	–
	$2D$	8838	–	–	8762	–	–	–	–	–	–	–	–
$\frac{7}{2}^+$	$1D$	8538	–	–	8528	8647	–	–	–	–	–	–	–
	$2D$	8839	–	–	8762	–	–	–	–	–	–	–	–
$\frac{1}{2}^-$	$1P$	8306	–	–	8316	8491	8360(130)	–	–	–	8164	–	–
	$2P$	8663	–	–	8579	–	–	–	–	–	–	–	–
$\frac{3}{2}^-$	$1P$	8306	–	–	8316	8491	8360(130)	–	8350(100)	–	8275	–	–
	$2P$	8663	–	–	8579	–	–	–	–	–	–	–	–
$\frac{5}{2}^-$	$1P$	8311	–	–	8331	8491	–	–	–	–	–	–	–
	$2P$	8667	–	–	8589	–	–	–	–	–	–	–	–

masses previously reported in Ref. [38].

It is interesting to mention that the mass reported in Ref. [35], which is a model independent prediction based on the non-relativistic effective field theory, agrees well with our result. Note too that the same level of agreement with the predictions of Ref. [35] exists in all Ω_{QQQ} sectors, with Q either a c or b quark.

3.4 The Ω_{bbb} baryon sector

Table 6 shows the spectrum of Ω_{bbb} baryons. We predict a mass of 14.40 GeV for the lowest state of the spectrum, which has quantum numbers $nL(J^P)=1S(\frac{3}{2}^+)$. The mass of this state predicted by the lattice calculations is 14.37 GeV [41]. The agreement between the lattice and our calculations for each ground state of the J^P-channel, reported in Table 6 , is worse (but not dramatically) than in the Ω_{ccc} sector. The only radial excitation that can be compared is the $2S\,(\frac{3}{2}^+)$ state, for which our prediction of 14.81 GeV is in fair agreement with the lattice result of 14.84 GeV. As in the Ω_{ccc} baryon sector, Table 6 shows the eigenstate of the lowest mass with quantum numbers $J^P=\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$, corresponding to the coupling $L\otimes S=2\otimes 3/2$. Our prediction for the case $L\otimes S=2\otimes 1/2$ is 14932 MeV, which compares reasonably well with the lattice results 14953 ± 18 MeV, 15005 ± 19 MeV and 15007 ± 19 MeV for $\frac{1}{2}^+$, $\frac{3}{2}^+$ and $\frac{5}{2}^+$, respectively.

We turn now to a comparison of the results of our quark model with the other theoretical approaches. There are again few results, mostly reported for the QCD sum rules, which contribute to the scattering of ground state masses. If these results are excluded from the average, then the $1S\,(\frac{3}{2}^+)$ state has a mass around 14.40 GeV, which agrees with our result. It is encouraging to observe a fair agreement between our results and those reported in Refs. [28, 29]. A spectrum as complete as ours is also reported in Ref. [22], and one can see that there is a global agreement with our predictions. However, Ref. [22] uses the linear confining interaction between quarks which in general produces larger masses for higher radial and orbital excitations.

We conclude this section by remarking that although the production of triply bottom baryons as well as their identification could be extremely difficult, we consider that the experimental search for these states must be pursued. The Ω_{bbb} system is theoretically the most interesting of all studied here because the triply bottom quark content makes it the most non-relativistic conventional few-body bound system that can be formed in QCD.

Table 5. Predicted masses, in MeV, of Ω_{cbb} baryons with the total spin and parity $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$. We compare our results with those obtained by the other theoretical approaches, in particular the recent lattice QCD results in Ref. [45].

J^P	nL	This work	[45]	[26]	[28]	[29]	[31]	[32]	[33]	[35]	[38]	[39]	[44]
$\frac{1}{2}^+$	$1S$	11200	11194(13)	11280	11217	11218	11500(110)	11730(160)	–	11190(80)	11077	11370	11195(22)
	$2S$	11607	–	–	11625	11585	–	–	–	–	11603	–	–
$\frac{1}{2}^+$	$1D$	11677	–	–	11718	11626	–	–	–	–	–	–	–
	$2D$	11955	–	–	11986	–	–	–	–	–	–	–	–
$\frac{3}{2}^+$	$1S$	11221	11211(13)	11287	11251	11218	11490(110)	–	11350(150)	–	11167	11380	11229(22)
	$2S$	11622	–	–	11643	11585	–	–	–	–	11703	–	–
$\frac{3}{2}^+$	$1D$	11677	–	–	11718	11626	–	–	–	–	–	–	–
	$2D$	11955	–	–	11986	–	–	–	–	–	–	–	–
$\frac{5}{2}^+$	$1D$	11677	–	–	11718	11626	–	–	–	–	–	–	–
	$2D$	11955	–	–	11986	–	–	–	–	–	–	–	–
$\frac{7}{2}^+$	$1D$	11688	–	–	11718	11626	–	–	–	–	–	–	–
	$2D$	11963	–	–	11986	–	–	–	–	–	–	–	–
$\frac{1}{2}^-$	$1P$	11482	–	–	11524	11438	11620(110)	–	–	–	11413	–	–
	$2P$	11802	–	–	11820	–	–	–	–	–	–	–	–
$\frac{3}{2}^-$	$1P$	11482	–	–	11524	11438	11620(110)	–	11500(200)	–	11523	–	–
	$2P$	11802	–	–	11820	–	–	–	–	–	–	–	–
$\frac{5}{2}^-$	$1P$	11569	–	–	11598	11601	–	–	–	–	–	–	–
	$2P$	11888	–	–	11899	–	–	–	–	–	–	–	–

4 Summary

The study of heavy quarkonia has become very useful for examining the relevant QCD properties and fundamental parameters, without the usual complications of the light quark systems. Hence, triply heavy baryons may provide a complementary window for the understanding of QCD. Moreover, as in charmonium and bottomonium, there is no restriction for finding exotic candidates in the spectra of QQQ baryons, and a reliable prediction of the conventional triply heavy baryons is important as it may serve as a template for comparing future experimental findings.

In the constituent quark model approach, we computed the ground and excited state masses of triply heavy baryons with quantum numbers $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$. The quark model parameters used in this study are those that were fitted during the last decade to reproduce a diverse array of heavy quarkonium properties, such as the masses, but also the electromagnetic, strong and weak decays and reactions.

We solved the non-relativistic 3-body bound state equation by means of a variational method in which the wave function is expanded using infinitesimally shifted Gaussians. This set of basis functions makes the calculation of 3-body matrix elements easier, without resorting to the laborious Racah algebra. The Gaussian range is taken as a geometric progression, enabling its optimization with a small number of free parameters. Moreover, the geometric progression is dense at short distances, which allows a description of the dynamics mediated by short range potentials. The fast damping of the Gaussian tail is not a problem, since we can choose the maximal range to be much longer than the hadronic size.

There are no experimental data related to triply heavy baryons. Our spectrum for the Ω_{ccc} and Ω_{bbb} sectors could be compared with the available lattice regularized QCD computations. One can state that there is a reasonable agreement with the lattice calculations for ground states of all J^P channels studied. However, discrepancies are found for excited states. Some of them can be related to our limitations and theoretical uncertainties, but the lattice regularized computations also have their own issues, such as the use of the NRQCD actions for heavy quarks. Also, they do not address all systematic uncertainties. We also compared our results with the other theoretical approaches, and reached the conclusion that there is a gen-

Table 6. Predicted masses, in MeV, of Ω_{bbb} baryons with the total spin and parity $J^P = \frac{1}{2}^{\pm}, \frac{3}{2}^{\pm}, \frac{5}{2}^{\pm}$ and $\frac{7}{2}^{+}$. We compare our results with those obtained by the other theoretical approaches, in particular the recent lattice QCD results in Ref. [41].

J^P	nL	This work	[41]	[22]	[26]	[28]	[29]	[31]	[33]	[35]	[38]	[39]	[44]
$\frac{1}{2}^{+}$	$1D$	14894	14938(18)	14954	–	14944	14896	–	–	–	–	–	–
	$2D$	15175	–	–	–	15304	–	–	–	–	–	–	–
$\frac{3}{2}^{+}$	$1S$	14396	14371(12)	14371	14569	14398	14347	14830(100)	14300(200)	14370(80)	14370	14570	14366(22)
	$2S$	14805	14840(14)	14848	–	14835	14832	–	–	–	14980	–	–
$\frac{3}{2}^{+}$	$1D$	14894	14958(18)	14954	–	14944	14896	–	–	–	–	–	–
	$2D$	15175	–	–	–	15304	–	–	–	–	–	–	–
$\frac{5}{2}^{+}$	$1D$	14894	14964(18)	14954	–	14944	14896	–	–	–	–	–	–
	$2D$	15175	–	–	–	15304	–	–	–	–	–	–	–
$\frac{7}{2}^{+}$	$1D$	14894	14969(17)	14954	–	14944	14896	–	–	–	–	–	–
	$2D$	15175	–	–	–	15304	–	–	–	–	–	–	–
$\frac{1}{2}^{-}$	$1P$	14688	14706(9)	14713	–	14738	14645	–	–	–	–	–	–
	$2P$	15016	–	15107	–	15052	–	–	–	–	–	–	–
$\frac{3}{2}^{-}$	$1P$	14688	14714(9)	14713	–	14738	14645	14950(110)	14900(200)	–	14771	–	–
	$2P$	15016	–	15107	–	15052	–	–	–	–	–	–	–
$\frac{5}{2}^{-}$	$1P$	15038	–	15125	–	15078	–	–	–	–	–	–	–
	$2P$	15284	–	–	–	15402	–	–	–	–	–	–	–

eral trend for the mass of the lowest state in the spectra of Ω_{ccc} and Ω_{bbb}. The predictions of higher excited states have not been performed in a systematic way in many theoretical formulations, and we commented on those cases that present a spectrum as complete as ours.

The Ω_{ccb} and Ω_{cbb} sectors have been less explored in the lattice QCD. Only masses for the $nL(J^P) = 1S\,(\frac{1}{2}^{+})$ and $1S\,(\frac{3}{2}^{+})$ states were reported and they agree with our results. A wide array of theoretical predictions is available for the Ω_{ccb} and Ω_{cbb} sectors. If one discards the results of the QCD sum rules, a general agreement between the different approaches is achieved for the average mass of the $nL(J^P) = 1S\,(\frac{1}{2}^{+})$ and $nL(J^P) = 1S\,(\frac{3}{2}^{+})$ states. These values are compatible with our predictions. Again, few theoretical formulations report a complete spectrum of low-lying excited states. When available, we have compared them with our calculations.

It is interesting to remark that the spectra of Ω_{ccc}, Ω_{ccb}, Ω_{cbb} and Ω_{bbb} baryons are quite reach in the energy region 1 GeV above the corresponding ground states. We encourage the design of experiments that would be able to detect such particles as the rewards could be high. As mentioned, triply heavy baryons are ideally suited for studies of QCD, as has been the case for heavy quarkonia.

Finally, following these calculations, a possible direction would be to study the coupling of triply heavy baryons close to their baryon-meson thresholds using the $^3P_0$ decay model, as this mechanism connects the 3- and 5-quark sectors. Mass shifts, decay widths and all kinds of scattering phenomena could be available to study these exotic structures. A similar procedure was followed by some of the present authors in the heavy quark meson sector with considerable success.

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