Cosmological scenario based on particle creation and holographic equipartition*

Fei-Quan Tu(屠飞泉)[1;1)] Yi-Xin Chen(陈一新)[2] Qi-Hong Huang(黄启洪)[1]

[1]School of Physics and Electronic Science, Zunyi Normal University, Zunyi 563006, China
[2]Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou 310027, China

Abstract: We propose a cosmological scenario that describes the evolution of the universe based on particle creation and holographic equipartition. The model attempts to solve the inflation of the early universe and the accelerated expansion of the present universe without introducing the dark energy from the thermodynamical perspective. Throughout the evolution of the universe, we assume that the universe consistently creates particles, and that the holographic equipartition is always satisfied. Further, we set the creation rate of particles proportional to H^2 in the early universe and to H in the present and late universe, where H depicts the Hubble parameter. Consequently, we obtain the solutions $a(t) \propto e^{\alpha t/3}$ and $a(t) \propto t^{1/2}$ for the early universe and solutions $a(t) \propto t^{\delta}$ and $a(t) \propto e^{Ht}$ for the present and late universe, respectively, where α and δ are the parameters. Finally, we obtain and analyze two important thermodynamic properties for the present model.

Keywords: particle creation, holographic equipartition, non-equilibrium thermodynamics

DOI: 10.1088/1674-1137/44/6/065103

1 Introduction

Accelerated expansion of the universe has been confirmed via observations by the Type Ia supernovae [1, 2], cosmic microwave background radiation [3, 4] and baryon acoustic oscillations [5, 6] since the 1990s. To explain this phenomenon, the concept of dark energy was introduced, and various models were proposed. From the perspective of the Einstein field equation, there are two main approaces to describe dark energy. One is to modify the matter sector of the Einstein field equation. These proposed models include the lambda cold dark matter (ΛCDM) model [7], $\Lambda(t)$CDM (i.e. the dark energy varies with time t) model [8, 9], particle creation model [10, 11], etc. Another is to modify the geometrical sector of the Einstein field equation. These theories are also referred to as the modified gravity, for example Lanczos-Lovelock gravity [12], $f(R)$ gravity [13, 14], etc. Irrespective of the approach, however, one of the main goals of these models is to describe the current accelerated expansion of the universe [15, 16]. Among them, the particle creation model has some of the following advantages [11, 17]: (i) A description of the non-equilibrium thermodynamics exists, as the process of particle creation is an irreversible process accompanied by entropy production [18, 19]. (ii) The particle creation mechanism unifies the dark energy and dark matter, as it contains only a single free parameter, namely the particle creation rate. The one-parameter model is preferred by statistical Bayesian analysis [20]. In Ref. [21], the authors found that the bulk viscous model (the one-parameter model) has some advantages over the ΛCDM model through the Bayesian analysis of Supernovae Type Ia data. In particular, the particle creation model can be described by the gravitationally induced particle creation mechanism. The particle number was shown not to represent a constant of the motion, even though the gravitational field is not quantized considering the covariant quantized free field equations of elementary particles in an expanding universe [22-24]. In addition to the above microscopic description, Prigogine et al. provided a macroscopic description of the particle creation mechanism induced by the gravitational field [10].

In the past 30 years, significant progress has been made in the study of gravity from the viewpoint of ther-

Received 7 January 2020 Published online 10 April 2020

* Supported by Doctoral Foundation of Zunyi Normal University (BS[2016]03), Education Department Foundation of Guizhou Province (QianjiaoheKYzi[2017]247), Major Research Project for Innovative Group of Education Department of Guizhou Province (KY[2018]028) and the NNSF of China (11775187, 11847031, 11865018 and 11865019)

1) E-mail: fqtuzju@foxmail.com

modynamics. In 1995, Jacobson showed that the Einstein field equation can be derived based on the Clausius relation and entropy-area relation [25]. Subsequently, Clausius and his collaborators investigated the properties of non-equilibrium thermodynamics of spacetime by introducing the term of entropy generation [26]. In contrast, Padmanabhan derived the important thermodynamic relation $S=2\beta E$ [27] and the first law of thermodynamics [28], which are consistent with the conventional form of thermodynamics from a similar but different thermodynamic perspective. Further, there is a simple correspondence between the surface $L_{\rm sur}$ and bulk $L_{\rm bulk}$ terms when the Lagrangian of gravity L is decomposed into $L=L_{\rm sur}+L_{\rm bulk}$ [29, 30], which shows that the gravitational action is holographic, as the same information is encoded in both terms. These results indicate a deep connection between gravitational dynamics and horizon thermodynamics.

In particular, Padmanabhan proposed that the spacetime is composed of microscopic degrees of freedom, and the de Sitter universe satisfies the holographic equipartition (an especial holographic principle) [31, 32]. Further, he thought that our universe is asymptotically de Sitter and derived the evolution equations of the universe based on the difference between the number of degrees of freedom on the surface and those in the bulk. This model was subsequently extensively studied [33-41]. Therefore, there is a solid physical foundation to study the evolution of the universe from the perspective of holographic equipartition.

In this paper, we propose a cosmological scenario that describes the evolution history of the universe based on particle creation and holographic equipartition. Throughout the evolution of the universe, we assume that the universe consistently creates particles, and holographic equipartition is always satisfied. In our scenario, the creation rate of the radiation is $\Gamma=\alpha H^2$ in the early universe, and the creation rate of the pressureless matter is $\Gamma=\alpha H$ in the present and late universe, where α is a positive parameter, and H is the Hubble parameter. The entire evolution history of the universe may be explained as follows. The universe starts from an unstable de Sitter universe ($a(t)\propto {\rm e}^{\alpha t/3}$) and evolves into a standard radiation stage ($a(t)\propto t^{1/2}$) because of the creation of radiation. With the expansion of the universe, the pressureless matter, whose creation rate is $\Gamma=\alpha H$, begins to dominate the universe. The negative creation pressure of the matter accelerates the expansion of the universe and drives the present accelerated universe ($a(t)\propto t^{\delta}\,(\delta>1)$) to the de Sitter universe ($a(t)\propto {\rm e}^{Ht}$).

This paper is organized as follows. In Section 2, we introduce non-equilibrium thermodynamics, which describes particle creation. In Section 3, we obtain the energy of the universe enclosed by the Hubble horizon using the laws of energy and holographic equipartition. Then, we obtain the laws of evolution for the early universe as well as the present and late universe by choosing suitable creation rates of particles in Section 4. In Section 5, we investigate thermodynamical properties for this model. The summary is provided in Section 6. We employ units $c=\hbar=1$.

2 Particle creation in cosmology

The first law of thermodynamics in a closed system that only performs volume work can be expressed as

$$ {\rm d}E=\delta Q-p{\rm d}V, \tag{1}$$

where δQ is the amount of heat exchanged by the system; p is the pressure; ${\rm d}E$ is the change of internal energy, and ${\rm d}V$ is the change in volume. In a reversible thermodynamic process, the amount of heat exchanged by the system can be expressed as $\delta Q=T{\rm d}S$, where T is the temperature, and ${\rm d}S$ is the entropy change.

For an adiabatic open system, where the particle number is not conserved due to particle creation, the first law of thermodynamics is written as [10]

$$ {\rm d}(\rho V)+p{\rm d}V=\frac{h}{n}{\rm d}(nV), \tag{2}$$

where n is the particle number per unit volume; $h=\rho+p$ is the enthalpy, and ρ is the energy density.

To calculate the creation pressure attributed to particle creation, we consider the universe as a sphere with a radius $a(t)$. Thus, the volume is $V=\frac{4\pi}{3}a^3(t)$, and the right term of Eq. (2) is expressed as

$$ \frac{h}{n}{\rm d}(nV)=(\rho+p)V\left(3H+\frac{\dot{n}}{n}\right){\rm d}t, \tag{3}$$

where $H=\dfrac{\dot{a}(t)}{a(t)}$ is the Hubble parameter. Inserting Eq. (3) and the relation $\dot{n}+3Hn=n\Gamma$ [10, 42-44] into Eq. (2), where Γ is the particle creation rate, we obtain

$$ \dot{\rho}+3H(\rho+p+p_{\rm e})=0, \tag{4}$$

where the effect of particle creation is expressed as an extra pressure

$$ p_{\rm e}=-\frac{(\rho+p)\Gamma}{3H}. \tag{5}$$

Thus, we can deem that Eq. (4) is the modified continuity equation attributed to the particle creation.

Particle creation in the universe generates entropy, whose change is given by ${\rm d}S=\frac{s}{n}{\rm d}(nV)$, where $s=S/V$ [10], which provides an explanation for the origin of cosmological entropy. Furthermore, the negative pressure caused by particle creation can explain the current accelerated expansion. In fact, bulk viscous fluids are introduced to study the accelerated expansion of the universe also due to the negative pressure effect of viscous fluids

[45-48].

Although the particle creation model is widely employed in cosmology to investigate the accelerated expansion of the universe, and has made great progress therein, a question arises naturally on whether the effect of particle creation can be observed. Lima et al. [49, 50] performed kinematic tests for the flat, closed, and open FRW universes with matter creation by investigating the look-back time, age of the universe, luminosity distance, angular diameter and galaxy number counts redshift relations, and finally obtained results that agreed with the observation. In particular, research by Pigozzo et al. [51] indicated a cosmological, late-time dark matter creation at 95% confidence level by analyzing the linear matter power spectrum, distance measurements from Type Ia supernovae, and the position of the first peak in the anisotropy spectrum of the cosmic microwave background.

3 Energy equipartition and holographic equipartition

Research in recent decades has uncovered an important fact, namely that spacetime can be heated up like matter. Unruh discovered that an observer can measure a temperature $T = \frac{\kappa}{2\pi}$ when they accelerate through the inertial vacuum with a proper acceleration κ in 1976 [52]. Furthermore, the temperature measured by the accelerated observer in the vacuum is as real, as that measured by an inertial thermometer in ordinary matter [53]. Hence, it can be concluded that the spacetime, just as ordinary matter, is made up of microscopic degrees of freedom. In the description of the evolution of the universe based on the emergent perspective of gravity, the energy equipartition law

$$E = \frac{1}{2}N_{\rm bulk}kT \tag{6}$$

holds, where E is the energy in volume V, $N_{\rm bulk}$ is the number of degrees of freedom in volume V, and T is the local acceleration temperature.

In Ref. [31], Padmanabhan proposed that our universe is asymptotically, rather than exactly, de Sitter, and the expansion of the universe is being driven by the difference $(N_{\rm sur} - N_{\rm bulk})$, where $N_{\rm sur}$ is the number of surface degrees of freedom on the horizon. However, in this study, we assume that the universe always obeys holographic equipartition because of particle creation. Therefore, the number of bulk degrees of freedom enclosed by the horizon is

$$N_{\rm bulk} = N_{\rm sur}. \tag{7}$$

In addition, the number of surface degrees of freedom can be expressed as [54]

$$N_{\rm sur} = \frac{A}{L_p^2}, \tag{8}$$

where A is the area of the horizon. Hence, the total energy in the volume V is

$$E = \frac{A}{2L_p^2}kT. \tag{9}$$

Considering a universe enclosed by the Hubble horizon, which is a sphere with the radius H^{-1}, the volume, temperature, and area of the Hubble horizon can be written as $V = \frac{4\pi}{3H^3}$, $kT = \frac{H}{2\pi}$, and $A = \frac{4\pi}{H^2}$, respectively. Inserting these physical quantities into Eq. (9), we obtain

$$E = \frac{1}{HL_p^2}. \tag{10}$$

This is an explicit relation between the energy enclosed by the Hubble horizon and the Hubble parameter. The relation of Eq. (10) implies that the universe enclosed by the Hubble horizon has a negative specific heat, as $E \propto T^{-1}$. This is also the expected result, even in Newtonian gravitating systems [55].

There are two relevant important energy contents in the flat universe, namely the Misner-Sharp energy and Komar energy. Thus, a question naturally arises as to whether the energy E in the energy equipartition law represents the Misner-Sharp energy or the Komar energy. We followingly investigate the two energy conditions.

3.1 Misner-Sharp energy

The Misner-Sharp energy is [56]

$$E = \int T_{\mu\nu}u^\mu u^\nu {\rm d}V, \tag{11}$$

where $u^\mu = \delta_0^\mu$ is the four-velocity, and $T_{\mu\nu} = (\rho, p+p_{\rm e}, p+p_{\rm e}, p+p_{\rm e})$ is the energy-momentum tensor. Then, the energy inside the Hubble horizon is

$$E = \int_0^{H^{-1}} 4\pi r^2 \rho {\rm d}r = \frac{4\pi\rho}{3H^3}, \tag{12}$$

where we have assumed that the energy density ρ is homogeneous. Comparing Eq. (10) with Eq. (12), we obtain

$$\rho = \frac{3H^2}{4\pi L_p^2}. \tag{13}$$

Thus, we obtain one of the evolution equations of the universe from the energy equipartition law. The equipartition law is an equation of state, while Eq. (13) can be deemed as a dynamic evolution equation of the universe. Thus, the evolution of the universe can be derived from an equation of state, namely the equipartition law.

3.2 Komar energy

The Komar energy is defined as [57, 58]

$$E = \int (2T_{\mu\nu} - Tg_{\mu\nu})u^\mu u^\nu {\rm d}V, \tag{14}$$

where T is the trace of the energy-momentum source $T_{\mu\nu}$. Then, the energy inside the Hubble horizon can be reduced to

$$E=\int_0^{H^{-1}}4\pi r^2|\rho+3(p+p_{\rm e})|{\rm d}r=\frac{4\pi}{3H^3}|\rho+3(p+p_{\rm e})|. \quad (15)$$

Thus, Eq. (10) changes to

$$|\rho+3(p+p_{\rm e})|=\frac{3H^2}{4\pi L_p^2}. \quad (16)$$

This demonstrates that the change of the Hubble parameter is related to the creation pressure of particles when the Komar energy is chosen. Interestingly, Padmanabhan suggested that the Komor energy is the active gravitational mass-energy [27] and derived the standard Friedmann equation taking into consideration the Komar energy [31, 32]. Therefore, we investigate the evolution of the universe using the Komar energy as the energy of the universe enclosed by the Hubble horizon in this study. Evidently, we can also use the Misner-Sharp energy as the energy of the universe and obtain the evolution laws of the universe consistent with astronomical observations. However, this is outside of the scope of the present study.

Conventionally, the cosmic fluid is deemed as an ideal fluid, whose equation of state is

$$p=\omega\rho, \quad (17)$$

where ω is constant.

Here, we would like to argue the validity of Eq. (8) from the viewpoint of gravitational dynamics. Suppose that the gravity can be quantized and has a minimum quantum of area on the order of L_p^2 [54], then the horizon with the area A can be divided into $N=\frac{A}{c_1L_p^2}$ cells, where c_1 is a numerical factor. Further, if there are c_2 microscopic states for the every cell, every cell has c_2 degrees of freedom. According to the energy equipartition law (6), we obtain the total energy

$$E^{'}=\frac{c_2}{2}NkT=\frac{c_2}{c_1}\frac{A}{2L_p^2}kT. \quad (18)$$

From Eq. (18), we find that Eq. (9) can be recovered by choosing the relation $\frac{c_2}{c_1}=1$. In contrast, Eq. (18) can be reduced to $\rho=\frac{3H^2}{8\pi L_p^2}$, which is the standard Friedmann equation for the flat FRW spacetime, if we choose $\frac{c_2}{c_1}=\frac{1}{2}$ and the Misner-Sharp energy of Eq. (12). Hence, the energy equipartition law and Einstein equation are equivalent to some extent when we take the Misner-Sharp energy as the energy of the universe and choose $\frac{c_2}{c_1}=\frac{1}{2}$. Thus, the Misner-Sharp energy seems to be a good choice for the energy of the universe. However, we employ the Komar energy as the energy of the universe in this paper. The reasons are as follows. First, the Komar energy is the true gravitational energy, as shown by Padmanabhan. Second, the accelerated expansion of the present universe cannot be described by the Einstein equation without a cosmological constant, whereas it can be explained by a modified gravitational field equation. In contrast, Eq. (16) can be derived by a modified gravitational field equation. Third, Eq. (8), rather than the Einstein equation, is the ansatz from the perspective of thermodynamics. The above discussions show only the validity of Eq. (8) from the viewpoint of gravitational dynamics and do not imply that only the Minser-Sharp energy can explain the validity of Eq. (8). The reason we use the Misner-Sharp energy to argue the validity of Eq. (8) is that it is convenient to compare with the standard evolution equation derived from the general relativity. Indeed, the modified Friedmann equation can be obtainedby choosing the Komar energy.

A question arises naturally: How does the holographic equipartition hold true in non-equilibrium? The holographic equipartition implies that the number of degrees of freedom on the surface equals that in the bulk. With the expansion of the universe, the Hubble horizon is increasing. The number of surface degrees of freedom is likewise increasing, as it is proportional to the area of Hubble horizon. Furthermore, the number of bulk degrees of freedom also rises because of the creation of particles. Therefore, the holographic equipartition is satisfied in the universe, where the particles are being created.

4 Physical process of universe evolution

In this section, we provide two specific cases as examples to illustrate the validity of the present model. Furthermore, we analyze the specific physical process of the evolution of the universe from the two cases.

In every epoch, we assume that the rate of particle creation has the following formula

$$\Gamma=\gamma\left(\frac{H}{H_0}\right)^{n-1}H=\alpha H^n, \quad (19)$$

where γ is a positive constant of the order one, and n is a non-negative integer, H_0 is a quantity with the same dimension as H, and $\alpha=\frac{\gamma}{H_0^{n-1}}$. The purpose of introducing H_0 is to ensure that the dimension of Γ is consistent with that of H. Combining Eq. (4), Eq. (5), and Eq. (17), we obtain the specific modified continuity equation

$$\dot{\rho}+3(1+\omega)H\rho\left(1-\frac{\alpha}{3}H^{n-1}\right)=0. \quad (20)$$

Further, the dynamic evolution equation of the universe (16) can be reduced to

$$|\,\alpha(1+\omega)H^{n-1}-(1+3\omega)\,|\,\rho=\frac{3H^2}{4\pi L_p^2}. \quad (21)$$

Thus Eq. (20) and Eq. (21) constitute the fundamental equations of the evolution of the universe. As long as the two equations are combined, some basic physical quantities of the evolution of the universe, such as the scale factor $a(t)$ and energy density ρ, can be solved.

4.1 Case study of $\Gamma/H = \alpha H$: evolution of early universe

A natural choice is $n = 2$, namely $\Gamma/H = \alpha H$. According to astronomical observations and cosmological theories, we know that the Hubble parameter H has been decreasing since the very early stage of the universe except for the period of inflation. In this case, the particle creation rate is particularly high in the early universe and decreases rapidly over time, hence choosing such a particle creation rate to study the evolution of the early universe is reasonable. In this subsection, ω is set to $\frac{1}{3}$, as the early universe is dominated by radiation.

With the above assumption, Eq. (20) and Eq. (21) are reduced to

$$\dot{\rho} + 4H\rho - \frac{4\alpha}{3}H^2\rho = 0 \tag{22}$$

and

$$\left|\frac{4\alpha}{3}H - 2\right|\rho = \frac{3H^2}{4\pi L_p^2}. \tag{23}$$

Combining the above equations, we obtain the result

$$(\alpha H - 3)(-6H^3 + 4\alpha H^4 - 3H\dot{H}) = 0. \tag{24}$$

This equation has two solutions

$$H = \alpha/3 \tag{25}$$

and

$$t = \frac{\alpha}{3}\ln\left|\frac{4\alpha H - 6}{H}\right| + \frac{1}{2H}, \tag{26}$$

where the integration constant is chosen as zero.

The solution in Eq. (25) implies that there is an inflation solution $a(t) \propto \mathrm{e}^{\frac{\alpha t}{3}}$ in the early universe. In contrast, the evolution law of the universe is $H = \frac{1}{2t}$ when t is large from Eq. (26), which can be reduced to the standard evolution law $a(t) \propto t^{1/2}$ in the radiation dominated stage. These solutions can be explained as follows. The universe starts from an unstable de Sitter space ($a(t) \propto \mathrm{e}^{\frac{\alpha t}{3}}$), after which it evolves to the standard radiation phase ($a(t) \propto t^{1/2}$). During the evolution of the early universe, the creation rate of the radiation is consistently $\Gamma = \alpha H^2$.

4.2 Case study of $\Gamma/H = \alpha$: evolution of present and late universe

Another natural choice is $n = 1$, namely $\Gamma/H = \alpha$. With this choice, Eq. (20) and Eq. (21) are reduced to

$$\dot{\rho} + (1+\omega)(3-\alpha)H\rho = 0 \tag{27}$$

and

$$|(\alpha-1)+(\alpha-3)\omega|\rho = \frac{3H^2}{4\pi L_p^2}. \tag{28}$$

From Eq. (27) and Eq. (28), we obtain the results $\rho = \text{constant}$ and $H^2 = 8\pi L_p^2\rho/3$ if the parameter ω is set to -1. The results are consistent with the evolution laws of the universe when the universe is dominated by vacuum energy in general relativity. In contrast, the same results are obtained if $\alpha = 3$. This implies that the evolution law of the universe is $a(t) \propto \mathrm{e}^{Ht}$ and independent of the nature of matter when the particle creation rate Γ equals $3H$. Therefore, we can describe the exponentially expanding de Sitter universe without introducing vacuum energy, which is equivalent to the negative pressure matter when $\alpha = 3$.

When $\omega \neq -1$ and $\alpha \neq 3$, by combining Eq. (27) and Eq. (28), we obtain

$$\dot{H} = -\frac{1}{\delta}H^2. \tag{29}$$

Further, we obtain

$$a(t) = t^{\delta}, \tag{30}$$

where $\delta = \frac{2}{(1+\omega)(3-\alpha)}$. The parameter δ satisfies the inequality $\delta > 1$, as the expansion of the universe is speeding up according to astronomical observations. If the universe is dominated by pressureless matter at present, namely $\omega = 0$, we obtain the range of the parameter α as $1 < \alpha < 3$. From this result, we can describe the accelerated expansion of the universe without introducing dark energy. For example, the authors of Ref. [59] pointed out that the rate of expansion, which is consistent with supernova observations, is currently $a(t) = t^2$. This rate of expansion can be obtained when $\alpha = 2$ for the present universe, dominated by pressureless matter. Hence, we can describe the accelerated expansion of the current universe and the evolution of the de Sitter universe at a later time using these settings.

Thus far, we analyzed the specific physical processes of the evolution of the universe from the cases of $\Gamma/H = \alpha H$ and $\Gamma/H = \alpha$ based on particle creation and holographic equipartition. These results are consistent with the conclusions of the analysis of Λ dark energy model, which is generally accepted. Therefore, the current model can efficiently explain the entire evolution of the universe without introducing dark energy.

At the end of this section, we discuss the evolution mechanism of Γ. We observe that H^2 evolves to H naturally if the creation rate of particles is chosen as $\Gamma = \gamma(\frac{H^2}{H_0} + H)$ in this section. At the early universe, the term $\gamma\frac{H^2}{H_0}$ dominates. The Hubble parameter H decreases as the universe evolves, the second term γH begins to dominate. Thus, the behaviour of Γ from H^2 to H changes naturally at the matter-radiation equality.

5 Thermodynamical properties based on holographic equipartition

We investigate the thermodynamical properties of the universe based on the holographic equipartition in this model. To observe the energy equipartition, we calculate the physical quantity

$$\frac{1}{2}\beta E = \frac{1}{2}\frac{2\pi}{kH}\frac{1}{HL_p^2} = \frac{\pi}{H^2L_p^2}. \tag{31}$$

Hence, the relation

$$S = \frac{1}{2}\beta E \tag{32}$$

holds in this model, as $S = \frac{A}{4L_p^2} = \frac{\pi}{H^2L_p^2}$. This is an important relation of thermodynamics that has been shown by Padmanabhan in the cases of static spacetime in general relativity [27] and a wider class of gravity theories, like the Lanczos-Lovelock gravity [60]. This relation is indeed the energy equipartition law in the bulk when the holographic equipartition $N_{\rm sur} = N_{\rm bulk}$ is satisfied, as $E = \frac{2}{\beta}S = \frac{kT}{2}\frac{A}{L_p^2} = \frac{1}{2}N_{\rm sur}kT = \frac{1}{2}N_{\rm bulk}kT$. Hence, the ansatz that the holographic equipartition remains satisfied throughout the evolution of the universe is consistent with the conclusion that the relation $S = \frac{1}{2}\beta E$ depicts the energy equipartition in static spacetime. Moreover, the application of the energy equipartition law is generalized to dynamic spacetime. Consequently, the validity of relation presented in Eq. (32) also indicates that it is a reasonable ansatz that the universe always obeys the holographic equipartition.

Furthermore, differentiating Eq. (10), we obtain the change of total energy enclosed by the Hubble horizon during a time interval dt

$$\mathrm{d}E = -\frac{\dot{H}}{H^2L_p^2}\mathrm{d}t. \tag{33}$$

On the other hand, we obtain

$$T\mathrm{d}S = \frac{H}{2\pi}\left(-\frac{2\pi\dot{H}}{H^3L_p^2}\right)\mathrm{d}t = -\frac{\dot{H}}{H^2L_p^2}\mathrm{d}t, \tag{34}$$

where the temperature $T = \frac{H}{2\pi}$ and the entropy-area relation $S = \frac{A}{4L_p^2}$ are used. Comparing Eq. (33) with Eq. (34), we obtain the relation

$$\mathrm{d}E = T\mathrm{d}S. \tag{35}$$

This relation depicts the energy conservation relation for the universe enclosed by the Hubble horizon. However, the first law of thermodynamics is expressed as $-\mathrm{d}E = T\mathrm{d}S$ in Ref. [61] (the relation is also used in some references, e.g., Refs. [44, 62-66]). We explain the difference between the notion dE used in Ref. [61] and that used in the present paper. In Ref. [61], $-\mathrm{d}E$ is the heat flux crossing the horizon during time dt. However, in this case dE is the change of the energy inside the Hubble horizon. Moreover, $-\mathrm{d}E$ is defined by $-\mathrm{d}E = 4\pi R^2 T_{\mu\nu}k^\mu k^\nu \mathrm{d}t$, where k^μ is the future directed ingoing null vector field in Ref. [61]. As dE is mainly composed of two parts, one related to the particle creation and the other to the particles across the Hubble horizon, the change of entropy in this study is mainly caused by the change in the number of particles. Further, the energy E in the bulk can be also considered as the energy on the horizon due to the holographic equipartition. Further, the entropy change on the horizon dS is the irreversible entropy change attributed to particle creation. Therefore, non-equilibrium thermodynamics accompanying the entropy production can be expressed in the form of Eq. (35) of the equilibrium thermodynamics on the Hubble horizon.

6 Conclusions

In this study, we analyze the evolution of the universe based on particle creation and holographic equipartition. We assume that the universe always obeys the holographic equipartition $N_{\rm bulk} = N_{\rm sur}$ throughout its evolution because of the particle creation. This is a reasonable assumption, as there is a simple and explicit holographic correspondence between the surface and bulk terms of the Lagrangian of gravity. We obtain an evolution equation of the universe by the energy equipartition and taking the Komar energy as the energy of gravity. In the early universe, we choose the creation rate of the radiation $\Gamma = \alpha H^2$ and obtain two solutions, $a(t) \propto \mathrm{e}^{\alpha t/3}$ and $a(t) \propto t^{1/2}$. In the present and late universe, we obtain two solutions, $a(t) \propto t^\delta$ and $a(t) \propto \mathrm{e}^{Ht}$, by setting the creation rate of the pressureless matter as $\Gamma = \alpha H$. These solutions are in good agreement with astronomical observations.

Based on the above results, we believe that the evolution history of the universe can be explained as follows. The universe starts from an unstable de Sitter universe ($a(t) \propto \mathrm{e}^{\alpha t/3}$) and evolves into a standard radiation stage ($a(t) \propto t^{1/2}$) as a result of the creation of radiation. Then, pressureless matter starts to dominate the evolution of the universe because of its expansion. In the present universe dominated by pressureless matter, the universe expands according to the law $a(t) \propto t^\delta$. Thus, we can explain the current accelerated expansion as long as the parameter δ is greater than 1. Finally, the universe evolves to the de Sitter Universe ($a(t) \propto \mathrm{e}^{Ht}$) in the late stage. Although the mechanisms of the transition from $a(t) \propto \mathrm{e}^{\alpha t/3}$ to $a(t) \propto t^{1/2}$ in the early universe and the transition from $a(t) \propto t^\delta$ to $a(t) \propto \mathrm{e}^{Ht}$ in the present and late universe are not clear, we should be able to test the particle creation effect of the current model based on the observation data of the large-scale clustering of galaxies, the distance measurement of Type Ia supernovae, and the position of the first acoustic peak in the anisotropy spectrum of the cosmic microwave background, as discussed in Section 2. This test will be considered further in future studies. For completeness, we also obtain and discuss the thermodynamic relation $\mathrm{d}E = T\mathrm{d}S$ and conclude that the relation $S = \frac{1}{2}\beta E$ is indeed the energy equipartition law in our scenario.

References

1 A.G. Riess *et al*., Astron. J., **116**: 1009 (1998)
2 S. Perlmutter *et al*., Astrophys. J., **517**: 565 (1999)
3 E. Komatsu *et al*., Astrophys. J. Suppl., **192**: 18 (2011)
4 C. L. Bennett *et al*., Astrophys. J. Suppl. Ser., **208**: 20 (2013)
5 D. J. Eisenstein *et al*., Astrophys. J., **633**: 560 (2005)
6 A. G. Sanchez *et al*., Mon. Not. R. Astron. Soc., **425**: 415 (2012)
7 S. Weinberg, Rev. Mod. Phys., **61**: 1 (1989)
8 K. Freese, F. C.Adams, J. A. Frieman *et al*., Nucl. Phys. B, **287**: 797 (1987)
9 S. Nojiri and S. D. Odintsov, Phys. Lett. B, **639**: 144 (2006)
10 I. Prigogine, J. Geheniau, E. Gunzig *et al*., Gen. Rel. Grav., **21**: 767 (1989)
11 J. A. S. Lima, S. Basilakos, and F. E. M. Costa, Phys. Rev. D, **86**: 103534 (2012)
12 T. Padmanabhan and D. Kothawala, Physics Reports, **531**: 115 (2013)
13 T.P. Sotiriou and V. Faraoni, Rev. Modern Phys., **82**: 451 (2010)
14 A. De Felice and S. Tsujikawa, Living Rev. Rel., **13**: 3 (2010)
15 N. Tamanini, Dynamical Systems in Dark Energy Models (Ph.D. thesis), U. Coll. London, 2014a, URL http://discovery.ucl.ac.uk/1456304/
16 S. Bahamonde, C. G. Bohmer, S. Carloni *et al*., Physics Reports, **775**: 1 (2018)
17 S. Pan and S. Chakraborty, Adv. High Energy Phys., **2015**: 654025 (2015)
18 M. O. Calvao, J. A. S. Lima, and I. Waga, Phys. Lett. A, **162**: 223 (1992)
19 J. A. S. Lima and A. S. M. Germano, Phys. Lett. A, **170**: 373 (1992)
20 A. C. C. Guimaras, J. V. Cunha, and J. A. S. Lima, J. Cosmol. Astropart. Phys., **10**: 010 (2009)
21 A. Sasidharan, N. D. J. Mohan, M. V. John *et al*., Eur. Phys. J. C, **78**: 628 (2018)
22 L. Parker, Phys. Rev. Lett., **21**: 562 (1968)
23 V. Mukhanov and S. Winitzki, *Introduction to quantum effects in gravity*, (Cambridge University Press, 2007)
24 L.Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Field and Gravity*, (Cambridge University Press, 2009)
25 T. Jacobson, Phys. Rev. Lett., **75**: 1260 (1995)
26 C. Eling, R. Guedens, and T. Jacobson, Phys. Rev. Lett., **96**: 121301 (2006)
27 T. Padmanabhan, Class. Quant. Grav., **21**: 4485 (2004)
28 T. Padmanabhan, Phys. Rep., **406**: 49 (2005)
29 A. Mukhopadhyay and T. Padmanabhan, Phys. Rev. D, **74**: 124023 (2006)
30 T. Padmanabhan, *Gravitation: Foundations and Frontiers*, (Cambridge University Press, 2010)
31 T. Padmanabhan, *Emergence and Expansion of Cosmic Space as due to the Quest for Holographic Equipartition*, arXiv: 1206.4916
32 T. Padmanabhan, Res. Astron. Astrophys., **12**: 891 (2012)
33 R. G. Cai, J. High Energy Phys., **11**: 016 (2012)
34 A. Sheykhi, M. H. Dehghani, and S. E. Hosseini, Phys. Lett. B, **726**: 23 (2013)
35 F. -Q. Tu and Y. -X. Chen, J. Cosmol. Astropart. Phys., **05**: 024 (2013)
36 A. F. Ali, Phys. Lett. B, **732**: 335 (2014)
37 F. -Q. Tu and Y. -X. Chen, Gen. Relativ. Gravit., **47**: 87 (2015)
38 N. Komatsu, Phys. Rev. D, **96**: 103507 (2017)
39 N. Komatsu, Eur. Phys. J. C, **77**: 229 (2017)
40 F. -Q Tu, Y. -X. Chen, B. Sun *et al*., Phys. Lett. B, **784**: 411 (2018)
41 N. Komatsu, Phys. Rev. D, **99**: 043523 (2019)
42 S. Chakraborty, Phys. Lett. B, **732**: 81 (2014)
43 A. C. Balfagon, Gen. Relativ. Gravit., **47**: 111 (2015)
44 S. Saha and A. Mondal, Eur. Phys. J. C, **77**: 196 (2017)
45 M. Cruz, S. Lepe, and S. D. Odintsov, Phys. Rev. D, **98**: 083515 (2018)
46 R. Maartens, *Causal Thermodynamics in Relativity*, arXiv: astro-ph/9609119
47 W. Zimdahl, Phys. Rev. D, **53**: 5483 (1996)
48 I. Brevik and O. Gorbunova, Gen. Relativ. Gravit., **37**: 2039 (2005)
49 J.A.S. Lima and J.S. Alcaniz, Astron. Astrophys., **348**: 1 (1999)
50 J.S. Alcaniz and J.A.S. Lima, Astron.Astrophys, **349**: 729 (1999)
51 C.Pigozzo, S. Carneiro, J. S. Alcaniz *et al*., J. Cosmol. Astropart. Phys., **05**: 022 (2016)
52 W. G. Unruh, Phys. Rev. D, **14**: 870 (1976)
53 S. Kolekar and T. Padmanabhan, Class. Quant. Grav., **32**: 202001 (2015)
54 T. Padmanabhan, Mod. Phys. Lett. A, **25**: 1129 (2010)
55 T. Padmanabhan, Phys. Rep., **188**: 285 (1990)
56 C. W. Misner and D. H. Sharp, Phys. Rev., **136**: B571 (1964)
57 A. Komar, Phys. Rev., **113**: 934 (1959)
58 A. Komar, Phys. Rev., **129**: 1873 (1963)
59 T. Padmanabhan and T. R. Choudhury, Phys. Rev. D, **66**: 081301(R) (0813)
60 T. Padmanabhan, Rep. Prog. Phys., **73**: 046901 (2010)
61 R. Bousso, Phys. Rev. D, **71**: 064024 (2005)
62 K. Karami, A. Abdolmaleki, N. Sahraeia *et al*., J. High Energy Phys., **08**: 150 (2011)
63 A. V. Frolov and L. Kofman, J. Cosmol. Astropart. Phys., **05**: 009 (2003)
64 U. H. Danielsson, Phys. Rev. D, **71**: 023516 (2005)
65 M. Akbar and R. -G. Cai, Phys. Lett. B, **635**: 7 (2006)
66 R. -G. Cai and S. P. Kim, J. High Energy Phys., **02**: 050 (2005)