Proca equation and vector field quantization in a rotating system*

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Abstract: A strong background field drastically changes the vacuum structure and proper basis of a system in both classical and quantum mechanics, e.g., the Landau levels in a background magnetic field. This is true even for a rotating system. In such a system, the usual set of plane-wave states would no longer be suitable as a starting point of perturbation. Alternatively and straightforwardly, in a rapidly and globally rotating system, it is better to reformulate the perturbation computation in principle. In this study, we completed the first step for the spin-1 field, which includes solving the Proca equation in the presence of a background rotation and completing its canonical quantization. We show that because of the symmetry, the eigen states are actually the same as those of Maxwell equations in cylindrical coordinates. The propagator as well as the near-central approximation were obtained by assuming that the vorticity areas are very small in the relativistic QGP.

Keywords: vector field, quantization in rotating frame, Proca equation

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I. INTRODUCTION

As the measurements at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) are processed [1-3], the polarization behaviors of vector mesons become more complex and difficult to understand. These measurements, which were motivated by explorations of large background fields in the fire ball of Quark Gluon Plasma (QGP) produced in relativistic heavy ion collisions, include magnetic and vorticity fields, and have driven studies on the QGP phenomenological properties into a more complicated and mysterious situation. For the magnetic field, although the mechanism of the chiral magnetic effect (CME) is clear theoretically [4-6], its detection is difficult because of large non-relevant fluctuations [7-10]. Regarding vorticity, after inspiring qualitative agreements on the simulations of the Λ polarization dependence on the collision energy and centrality with the STAR measurements [11, 12], studies have indicated that the vorticity distribution and evolution in QGP may be much more out of expectation in terms of both the quantitative analysis on the global polarization and the qualitative mismatch of the local polarization profiles [13-15].

The dependence of the vorticity amplitude on the collision energy and centrality was studied in [16] for the initial state of QGP and further simulated in [17] with a multi-phase transport model (AMPT) for the whole period of QGP evolution. Given that the qualitative predictions are reasonable, it is believed that the traditional understanding of the vorticity may not be completely wrong. Thus, according to concepts from kinetic theory, quantum Wigner functions, and hydrodynamics, various attempts [5, 14, 18-25] have been made to fix the conventional theory, which is based on the straightforward scenario of parton collisions, such as different vorticity definitions by considering the thermal environment [15, 26] and novel vacuum structures by the strongly rotating system [27]. In most of these studies, quarks, which serve as the visible spin-carriers of the final-state hadrons, have attracted notable interest [28, 29]. Gluons, which carry double spins and thus suffer double polarization effects, are neglected because of technical problems in most cases. Given that gluons are closely related with the fundamental problem of quantum chromodynamics (QCD), in this study, we took the first step toward the inclusion of gluons, that is, we studied the vector field in the presence of a background rotation field.

A strong background field changes the vacuum structure of a system drastically in both classical and quantum mechanics [30]. The Landau level is the most famous example in which the system is in a background magnetic

field. In such a case, the usual set of plane-wave states would no longer be suitable as a starting point of perturbation. However, the standard quantum field theory on textbooks roots in assuming that the in and out states are plane-wave, which is natural in high energy collisions. Therefore, in a rapidly and globally rotating system, a difficult but straightforward alternative is to reformulate the perturbation computation in principle. In this study, we completed the first step, which includes solving the Proca equation in presence of a background rotation and completing its canonical quantization. We show that because of the symmetry, the eigen states are actually the same as the ones of Maxwell equations in cylindrical coordinates [31-33]. The propagator as well as the near-central approximation were obtained by assuming that the vorticity areas are very small in the relativistic QGP. We will discuss the subtle zero-mass and gauge-symmetry topics in such a curved space-time system in future studies.

In this study, the Latin and Greek indexes represent components of vectors/tensors in the local rest/inertial frame and curved frame respectively, such as X^a and X^μ. X^a respects the usual Lorentz transformation, while X^μ obeys the coordinate transformation in its curved space-time. For any vector, these two types of components are connected by tetrads $e^a_{\ \mu}$ as

$$X^a = e^a_{\ \mu} X^\mu. \tag{1}$$

II. ROTATING PROCA FIELD

We consider a rotating system in a local rest/inertial frame, up to local Lorentz transformation tetrads $e^\mu_{\ a}$ and $e^a_{\ \mu}$, which bridge the vectors in curved and flat space-times and can be chosen as

$$e^a_{\ \mu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ v_1 & 1 & 0 & 0 \\ v_2 & 0 & 1 & 0 \\ v_3 & 0 & 0 & 1 \end{pmatrix}, \quad e^\mu_{\ a} = \begin{pmatrix} 1 & -v_1 & -v_2 & -v_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

By definition, they are connected with the global metrics and should satisfy the following relations, which approximately means defining a local Minkowski frame by absorbing the space-time curvature into local coordinates:

$$\eta_{ab} = g_{\mu\nu} e^\mu_{\ a} e^\nu_{\ b}, \tag{3}$$

$$e^a_{\ \mu} e^\mu_{\ b} = \delta^a_b, \tag{4}$$

$$e^\mu_{\ a} e^a_{\ \nu} = \delta^\mu_\nu, \tag{5}$$

where $\eta_{ab} = \{+,-,-,-\}$ and the rotating metric is

$$g_{\mu\nu} = \begin{pmatrix} 1 - v_1^2 - v_2^2 - v_3^2 & -v_1 & -v_2 & -v_3 \\ -v_1 & -1 & 0 & 0 \\ -v_2 & 0 & -1 & 0 \\ -v_3 & 0 & 0 & -1 \end{pmatrix}. \tag{6}$$

Evidently, there are infinite equivalent choices of tetrads satisfying the relation 3, 4, 5 up to a local Lorentz transformation. Here, we consider the uniform rotation case, which means that the linear velocity is $\vec{v} = \vec{\omega} \times \vec{x}$, where $\vec{\omega}$, i.e., the angular velocity, is a constant. For such globally rotating system, the boundary condition is necessary because of the light-speed limit. As the sharp cut boundary will only result in a discrete set of radial wave functions, no structural changes are included in the following formalism. We do not discuss this explicitly in this work and keep the summation over k_t as integration. Keep in mind that for different boundary conditions, this will be discretized into different series. Realistically, in the QGP, the rotation areas or volumes are so small that the light-speed limit problem is not important [17].

We extracted the rotation effects by constructing the Proca Lagrangian density with respect to the field in the local inertial frame. According to the general relativistic principle, it should be the same as the flat one except for all the quantities that are in the curved space-time. By using relation 3, the vector Lagrangian density could be written as

$$\mathcal{L}_v = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} m^2 A^2 = -\frac{1}{4} F^{ab} F_{ab} + \frac{1}{2} m^2 A^2, \tag{7}$$

where the covariant derivative in the flat space-time is

$$D_a A_b = e^\mu_{\ a} D_\mu A_b = e^\mu_{\ a} (\partial_\mu A_b + \Gamma_{\mu bc} A^c), \tag{8}$$

and the tensor of field strength is given by $F_{ab} = D_a A_b - D_b A_a$. The reason we need Eq. (8) is that we aim to study the system in an inertial frame, that is, we observed or measured the quantities constructed by the field in the flat space-time. Operation D_a can be defined via D_μ, which operates on the quantities as functions of global coordinates, i.e., Eq. (8). According to these space-time shifts, we now have a local-rest frame (up to a local Lorentz transformation) and the rotation effects whose effect is bending or twisting the global space-time are assumed as some background interactions appearing in the local Lagrangian density as an additional term, e.g., the explicit polarization effect term for the fermionic case. Here, note that connection $\Gamma_{\mu bc}$ is not the usual Christoffel connection. It includes derivatives of the local inertial basis along the world line of the curved space-time. It is expressed as

$$\begin{aligned}
\Gamma_{\mu ab} =& \eta_{ac}[e^c_{\ \nu}e_b^{\ \lambda}G^\nu_{\ \mu\lambda} - e_b^{\ \lambda}\partial_\mu e^c_{\ \lambda}]\\
=& \eta_{ac}(\delta^c_{\ \nu}+\xi^c_{\ \nu})(\delta_b^{\ \lambda}+\xi_b^{\ \lambda})\frac{1}{2}g^{\nu\alpha}(\partial_\mu h_{\alpha\lambda}+\partial_\lambda h_{\alpha\mu}-\partial_\alpha h_{\mu\lambda})\\
& -\eta_{ac}(\delta_b^{\ \lambda}+\xi_b^{\ \lambda})\partial_\mu(\delta^c_{\ \lambda}+\xi^c_{\ \lambda})\\
=& \eta_{ac}(\delta^c_{\ \nu})(\delta_b^{\ \lambda})\frac{1}{2}\eta^{\nu\alpha}(\partial_\mu h_{\alpha\lambda}+\partial_\lambda h_{\alpha\mu}-\partial_\alpha h_{\mu\lambda})\\
& -\eta_{ac}(\delta_b^{\ \lambda})\partial_\mu(\xi^c_{\ \lambda})\\
=& \frac{1}{2}(\partial_\mu h_{ab}+\partial_b h_{a\mu}-\partial_a h_{\mu b})-\partial_\mu\xi_{ab},
\end{aligned} \tag{9}$$

where $G^\nu_{\ \mu\lambda}$ is the usual Christoffel connection in the curved space-time. Its nonzero elements are

$$\begin{aligned}
\Gamma_{0ij} =& \frac{1}{2}(-\partial_j v_i+\partial_i v_j)=\epsilon_{ijm}\omega_m,\\
\Gamma_{i0j} =& -\frac{1}{2}(\partial_j v_i+\partial_i v_j),\\
\Gamma_{ij0} =& \frac{1}{2}(\partial_j v_i+\partial_i v_j).
\end{aligned} \tag{10}$$

Evidently, the last two equations are actually zero in the uniform rotation case.

Substituting the covariant derivation into the Lagrangian density, a straightforward calculation shows that in the local flat frame the field strength tensor is modified by the rotation as

$$\begin{aligned}
F_{0i} =& -F^{0i}=(\partial_0 A_i-\partial_i A_0)-v_j\partial_j A_i-\epsilon_{ijk}\omega_k A_j,\\
F_{ij} =& F^{ij}=\partial_i A_j-\partial_j A_i.
\end{aligned} \tag{11}$$

With our choice of tetrads, only the "electric" parts are changed while the "magnetic" parts are invariant. Evidently, this conclusion is tetrads-dependent because the "electric" and "magnetic" parts are transferable under a local Lorentz transformation. Hence, it is easy to compute the rotation-modified Lagrangian density and split it into the free and rotation-polarization parts as

$$\begin{aligned}
\mathcal{L}_v(\vec\omega) =& \mathcal{L}_v(\vec\omega=0)+\delta\mathcal{L}_v(\vec\omega)\\
=& \mathcal{L}_v(\vec\omega=0)-v_j f_{ji}f_{0i}+\frac{1}{2}(v_j\partial_j A_i+\epsilon_{ijm}\omega_m A_j)^2,
\end{aligned} \tag{12}$$

where $\mathcal{L}_v(\vec\omega=0)=-\frac{1}{2}(\sum_i -f_{0i}^2+\sum_{i<j}f_{ij}^2)$ and $f_{ab}=\partial_a A_b-\partial_b A_a$. Here, we have lowered all the indices. Evidently, the $O(\omega)$ term is the polarization form of $\vec{v}\cdot(\vec{E}\times\vec{B})=\vec\omega\cdot\vec{J}$, where the angular momentum is $\vec{J}=\vec{r}\times\vec{P}$ and $\vec{P}=\vec{E}\times\vec{B}$. However, unlike the fermionic case, the $O(\omega^2)$ terms are non-negligible because the eigen equations of the vector field are second order, i.e., ∂_0^2. The $O(\omega^2)$ terms actually correspond to the $O(\omega)$ correc-

tions to the eigen energies. This is shown next.

The corresponding equations of motion are obtained as

$$\partial_i f_{i0}-m^2 A_0=\Delta A_0-m^2 A_0=0, \tag{13}$$

$$\begin{aligned}
& \partial_0^2 A_i-\Delta A_i-2v_j\partial_0\partial_j A_i+(\partial_j v_i-\partial_i v_j)\partial_0 A_j\\
& +v_j\partial_j(v_n\partial_n A_i+2\epsilon_{inm}\omega_m A_n)-(\omega^2 A_i-\omega_n\omega_i A_n)\\
& +m^2 A_i=0.
\end{aligned} \tag{14}$$

There are three polarization components for the massive vector field. We could first adopt the transverse constraint $\nabla\cdot\vec{A}=0$ to extract the two transverse ones. Thus, $\partial_i f_{i0}$ is reduced to ΔA_0. Considering that operator $\Delta=\nabla^2$ is semi-positive, the first equation gives $A_0=0$. By choosing the angular velocity direction as the z axis, i.e., $\vec\omega=\omega e_z$, in cylindrical coordinates, the second one degenerates to the ordinary form in the $\omega=0$ case $\partial_0^2 A_i-\Delta A_i=0$. The solutions are the well-known cylindrical waves that can be found in any electrodynamics textbook. Adopting the same symbols as in a recent study [33], in the cylindrical system the solutions are

$$A_{TE}=\begin{pmatrix} A_0\\ A_\rho^{TE}\\ A_\phi^{TE}\\ A_z^{TE}\end{pmatrix}=\begin{pmatrix} 0\\ \dfrac{n}{k_t\rho}J_n(k_t\rho)\\ \dfrac{i}{k_t}\partial_\rho J_n(k_t\rho)\\ 0\end{pmatrix}e^{i(n\phi+k_z z-E_{nk_t k_z}t)},$$

$$A_{TM}=\begin{pmatrix} A_0\\ A_\rho^{TM}\\ A_\phi^{TM}\\ A_z^{TM}\end{pmatrix}=\begin{pmatrix} 0\\ \dfrac{k_z}{E_k k_t}\partial_\rho J_n(k_t\rho)\\ i\dfrac{nk_z}{E_k k_t\rho}J_n(k_t\rho)\\ -i\dfrac{k_t}{E_k}J_n(k_t\rho)\end{pmatrix}e^{i(n\phi+k_z z-E_{nk_t k_z}t)}, \tag{15}$$

where $E_k=\sqrt{k_t^2+k_z^2}$. In the static case, the eigen energies are $E_{nk_z k_z}=E_k$. For the longitudinal part, we choose $A_\rho=A_\phi=0$ and the equation gives the solution for the third polarization direction

$$A_L=\begin{pmatrix} \dfrac{k_z}{E_{kt}}\\ 0\\ 0\\ -\dfrac{E_k}{E_{kt}}\end{pmatrix}J_n(k_t\rho)e^{in\phi}e^{ik_z z}e^{-iE_{nk_t k_z}t}, \tag{16}$$

where $E_{kt}=\sqrt{k_t^2+m^2}$.

As the uniform rotation preserves the cylindrical symmetry, we expect that these solutions are eigen states of the rotation case as well, with a modified energy. By substituting them into Eq. (14) we find that the equations are satisfied if the energy is solved from

$$(-E_{nk,k_z}^2 + E_k^2 - 2E_{nk,k_z}\omega n - \omega^2 n^2)\vec{A} = 0. \qquad (17)$$

This equation implies that the new eigen energies are $E_{nk,k_z} = \pm E_k - n\omega$, which, as expected, leads to $n\omega$ polarization corrections for the spin-1 field on top of the rotation background.

The nontrivial difference between these solutions and the usual plane-wave case is that the following properties are satisfied

$$\nabla \times \vec{A}_{TE} = E_k\vec{A}_{TM}, \qquad (18)$$

$$\nabla \times \vec{A}_{TM} = E_k\vec{A}_{TE}. \qquad (19)$$

For comparison, we list the similar relations for plane-wave $\vec{A}_{1,2}^{pw}$

$$\nabla \times \vec{A}_1^{pw} = i|\vec{k}||\vec{A}_1^{pw}|/|\vec{A}_2^{pw}|\vec{A}_2^{pw}, \qquad (20)$$

$$\nabla \times \vec{A}_2^{pw} = -i|\vec{k}||\vec{A}_2^{pw}|/|\vec{A}_1^{pw}|\vec{A}_1^{pw}. \qquad (21)$$

The same signs in Eqs. (18) and (19) allow us to construct two new modes, i.e., $\vec{A}_\pm = \vec{A}_{TM} \pm \vec{A}_{TE}$, for which $\vec{E} \cdot \vec{B} \neq 0$. They clearly satisfy

$$\nabla \times \vec{A}_+ = E_k\vec{A}_+, \qquad (22)$$

$$\nabla \times \vec{A}_- = -E_k\vec{A}_-. \qquad (23)$$

For the sake of clarity, we list $\vec{E}$ and $\vec{B}$ as follows:

$$\vec{E}_+ = \partial_0 A_i^+ = -iE_{nk,k_z}A_i^+, \qquad (24)$$

$$\vec{B}_+ = -\epsilon_{ijk}\partial_j A_k^+ = -E_k A_i^+, \qquad (25)$$

and

$$\vec{E}_- = \partial_0 A_i^- = -iE_{nk,k_z}A_i^-, \qquad (26)$$

$$\vec{B}_- = -\epsilon_{ijk}\partial_j A_k^- = E_k A_i^-. \qquad (27)$$

Clearly, this will not cause problems because $\vec{E}_+ \cdot \vec{B}_+ + \vec{E}_- \cdot \vec{B}_- = 0$ in the equally polarized case. If these

two modes could be separated by some potential or mechanism, the non-zero topological charge would be generated simultaneously and induce further chiral balance between left and right chiral fermions. Given that there is no interaction between photons, the polarization choice could only be achieved by gratings. For non-abelian gauge bosons, such as gluons, a topological nontrivial vacuum could more likely be achieved by introducing a gluon-gluon interaction in a special channel.

III. PROPAGATOR

In this section, we complete the canonical quantization for the spin-1 vector field. First, we check the orthogonality of the three eigen states. Evidently, A_{TM} includes a mixed part of A_L. In order to construct the propagator of vector field, we first orthogonalize it as

$$B_L = A_L - \alpha A_{TM}, \qquad (28)$$

$$B_L \cdot A_{TM} = 0. \qquad (29)$$

The orthogonalization constraint gives $\alpha = -ik_t\sqrt{k^2+m^2}/ (k\sqrt{k_t^2+m^2})$. In the following, we rename B_L as A_L, and all the three eigen states are normalized as

$$\int \rho d\rho d\phi dz A_{TE,TM,L}^\mu A_{\mu,TE,TM,L}$$
$$= -(2\pi)^2 \delta_{mn}\delta(k_z - p_z)\frac{1}{k_t}\delta(k_t - p_t). \qquad (30)$$

The canonical quantization gives the vector field as

$$A_\mu = \sum_{n,\lambda} \int \frac{dk_z dk_t}{2\pi\sqrt{2E_k}} e^{-iE_n t}e^{ik_z z}e^{in\phi}a_{nk,k_z}A_{\mu,\lambda} + h.c., \qquad (31)$$

where $\lambda \in \{L, TM, TE\}$ is the polarization index. Starting from the quantization assumption $[A_\mu(x), \Pi_\nu(y)] = g_{\mu\nu}\delta^4(x-y)$, the commutation relation of a and $a^\dagger$ is derived as

$$[a_{nk,k_z}, a_{mp,p_z}^\dagger] = \delta_{mn}\delta(k_z - p_z)k_t\delta(k_t - p_t). \qquad (32)$$

Therefore, the propagator from $x = (t, \vec{x}) = (t, \rho, \phi, z)$ to $y = (s, \vec{y}) = (s, r, \theta, \zeta)$ is defined as

$$D_{\mu\nu}(x,y) = \langle 0|TA_\mu(x)A_\nu(y)|0\rangle$$
$$= \theta(t-s)\langle 0|A_\mu A_\nu|0\rangle + \theta(s-t)\langle 0|A_\nu A_\mu|0\rangle, \qquad (33)$$

where T is the time-ordering operator. Substituting the field operators into it and changing the summation over n

into $-n$ in one of the two terms, the propagator is reduced to

$$
\begin{aligned}
D_{\mu\nu}(x,y) =& \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)}e^{in(\phi-\theta)}\\
&\times e^{-i(k_0-n\omega)(t-s)}\frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{k_0^2-E_k^2+i\eta}\\
=& \frac{i}{(2\pi)^3}\sum_{n,\lambda}\int dk_0 dk_z k_t dk_t e^{ik_z(z-\zeta)}e^{in(\phi-\theta)}\\
&\times e^{-ik_0(t-s)}\frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{(k_0-n\omega)^2-E_k^2+i\eta}
\end{aligned}
$$

$$
\times e^{-ik_0(t-s)}\frac{A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r)}{(k_0-n\omega)^2-E_k^2+i\eta}, \tag{34}
$$

which is similar to the result in flat space-time. For the second equation, we shifted the integration over k_0 to $k_0-n\omega$ to make the poles locate at $E_k-n\omega$, which correspond to the eigen energies of the vector field. Again, we find that the rotation effects serve as an additional chemical potential rather than a straightforward energy shift as $k_0^2-E_{nk_t,k_z}^2$ A. This is the same as the fermionic case. A straightforward computation can yield the following for the Lorentz structure part:

$$
\begin{aligned}
D_{\mu\nu}^n(k_t,k_z;\rho,r) =& \sum_\lambda A_{\mu,\lambda}(k_t,n,k_z;\rho)A^*_{\nu,\lambda}(k_t,n,k_z;r) = \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n^{++} & -iM_n^{+-} & 0 \\ 0 & iM_n^{-+} & M_n & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\\
&+\frac{E_{kt}^2}{4m^2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & M_n & -iM_n^{-+} & -2i\frac{k_t k_z}{E_{kt}^2}N_n^{1-} \\ 0 & iM_n^{+-} & M_n^{++} & 2\frac{k_t k_z}{E_{kt}^2}N_n^{1+} \\ 0 & 2i\frac{k_t k_z}{E_{kt}^2}N_n^{2-} & 2\frac{k_t k_z}{E_{kt}^2}N_n^{2+} & 4\frac{E_z^2}{E_{kt}^2}\Pi_n \end{pmatrix} + \frac{k_t E_k}{2m^2}\begin{pmatrix} \frac{2k^2}{k_t E_k}\Pi_n & -iN_n^{2-} & -N_n^{2+} & -2\frac{k_z}{k_t}\Pi_n \\ iN_n^{1-} & 0 & 0 & 0 \\ -N_n^{1+} & 0 & 0 & 0 \\ -2\frac{k_z}{k_t}\Pi_n & 0 & 0 & 0 \end{pmatrix}. \tag{35}
\end{aligned}
$$

In order to make the expression more compact, we use the following symbols

$$
\begin{aligned}
M_n^{++} =& Z_n^+(\rho,\phi)Z_n^{+*}(r,\theta), \quad M_n^{+-} = Z_n^+(\rho,\phi)Z_n^{-*}(r,\theta),\\
M_n^{-+} =& Z_n^-(\rho,\phi)Z_n^{+*}(r,\theta), \quad M_n = Z_n^-(\rho,\phi)Z_n^{-*}(r,\theta),
\end{aligned} \tag{36}
$$

$$
\begin{aligned}
N_n^{1+} =& Z_n^+(\rho,\phi)J_n(r), \quad N_n^{1-} = Z_n^-(\rho,\phi)J_n(r),\\
N_n^{2+} =& J_n(\rho)Z_n^{+*}(r,\theta), \quad N_n^{2-} = J_n(\rho)Z_n^{-*}(r,\theta),
\end{aligned} \tag{37}
$$

and

$$
\Pi_n = J_n(\rho)J_n(r), \tag{38}
$$

where $Z_n^+(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} + J_{n+1}(\rho)e^{i\phi}$ and $Z_n^-(\rho,\phi) = J_{n-1}(\rho)e^{-i\phi} - J_{n+1}(\rho)e^{i\phi}$, and we have written $J_n(k_t\rho)$ as $J_n(\rho)$ for simplicity. Note that the propagator is in Cartesian coordinates, e.g., $D_{01}^n = D_{tx}^n$. Evidently, the propagator is not invariant under translation on the $X-Y$ plane. If we set the rotation speed $\omega = 0$, it reduces to the usual one, $D_{\mu\nu}(x-y)$, in the flat space-time. This can be verified by a straightforward computation by utilizing the following relations:

$$
\sum_n e^{in(\phi-\phi')}J_n(k_t\rho)J_n(k_t\rho') = J_0(k_t|\vec\rho-\vec\rho'|), \tag{39}
$$

$$
\begin{aligned}
\int_0^{2\pi}d\xi e^{ikr\cos\xi}\cos(\xi+\alpha) =& 2\pi i\cos\alpha J_1(kr),\\
\int_0^{2\pi}d\xi e^{ikr\cos\xi}\sin(\xi+\alpha) =& 2\pi i\sin\alpha J_1(kr),
\end{aligned} \tag{40}
$$

and

$$
\begin{aligned}
\int_0^{2\pi}d\xi e^{ikr\cos\xi}\cos(\xi+\alpha)\cos(\xi+\alpha) &= \pi J_0(kr) - \pi\cos2\alpha J_2(kr),\\
\int_0^{2\pi}d\xi e^{ikr\cos\xi}\sin(\xi+\alpha)\sin(\xi+\alpha) &= \pi J_0(kr) + \pi\cos2\alpha J_2(kr),\\
\int_0^{2\pi}d\xi e^{ikr\cos\xi}\cos(\xi+\alpha)\sin(\xi+\alpha) &= -\pi\sin2\alpha J_2(kr).
\end{aligned} \tag{41}
$$

IV. NEAR-CENTER EXPANSION

For a globally rotating system, the size is limited, especially for the QGP, which is believed rotating very rap-

idly as a fireball product in relativistic heavy ion collisions. AMPT's simulation indicates that the vorticity profile is more similar to a set of vorticity spots rather than to a rotating lemon. Therefore, we consider the propagat-

or in the near central case and set one of the space-time points as the origin, i.e., $r = 0$ and $\rho \to 0$. This will reduce it into three non-zero parts corresponding to $n = 0, \pm 1$ because only $J_0(0) \neq 0$. The explicit results are

$$
D_{\mu\nu}^{n=0}(k_t, k_z, \rho) = \begin{pmatrix}
\dfrac{k^2}{m^2} J_0 & 0 & 0 & -\dfrac{E_k k_z}{m^2} J_0 \\[2mm]
\mathrm{i}\dfrac{k_t E_k}{2m^2} Z_0^-(\rho, \phi) & 0 & 0 & -\mathrm{i}\dfrac{k_t E_k}{2m^2} Z_0^-(\rho, \phi) \\[2mm]
-\dfrac{k_t E_k}{2m^2} Z_0^+(\rho, \phi) & 0 & 0 & \dfrac{k_t E_k}{2m^2} Z_0^+(\rho, \phi) \\[2mm]
-\dfrac{E_k k_z}{m^2} J_0 & 0 & 0 & \dfrac{E_z^2}{m^2} J_0
\end{pmatrix}.
\tag{42}
$$

Although $n = -1$ and $n = 1$ could be related by a property of the Bessel function, i.e., $J_{-n} = (-1)^n J_n$, we still list the results here for future reference. As n increases, the corresponding contribution will be rapidly suppressed by

the decay of the Bessel function as $\rho^n/n!$ at a small ρ range, and it approaches $\cos(\rho)/\sqrt{\rho}$ at large distance, which is not a case of interest to us.

$$
D_{\mu\nu}^{n=-1}(k_t, k_z, \rho) = \frac{1}{4} \begin{pmatrix}
0 & -2\mathrm{i}\dfrac{k_t E_k}{m^2} J_1 & 2\dfrac{k_t E_k}{m^2} J_1 & 0 \\[2mm]
0 & 2J_0 e^{\mathrm{i}\phi} - \dfrac{k_t^2}{m^2} Z_{-1}^-(\rho, \phi) & 2\mathrm{i}J_0 e^{\mathrm{i}\phi} - \mathrm{i}\dfrac{k_t^2}{m^2} Z_{-1}^-(\rho, \phi) & 0 \\[2mm]
0 & -2\mathrm{i}J_0 e^{\mathrm{i}\phi} - \mathrm{i}\dfrac{k_t^2}{m^2} Z_{-1}^+(\rho, \phi) & 2J_0 e^{\mathrm{i}\phi} + \dfrac{k_t^2}{m^2} Z_{-1}^+(\rho, \phi) & 0 \\[2mm]
0 & 2\mathrm{i}\dfrac{k_t k_z}{m^2} J_1 & -2\dfrac{k_t k_z}{m^2} J_1 & 0
\end{pmatrix},
$$

$$
D_{\mu\nu}^{n=1}(k_t, k_z, \rho) = \frac{1}{4} \begin{pmatrix}
0 & -2\mathrm{i}\dfrac{k_t E_k}{m^2} J_1 & -2\dfrac{k_t E_k}{m^2} J_1 & 0 \\[2mm]
0 & 2J_0 e^{-\mathrm{i}\phi} + \dfrac{k_t^2}{m^2} Z_1^-(\rho, \phi) & -2\mathrm{i}J_0 e^{-\mathrm{i}\phi} - \mathrm{i}\dfrac{k_t^2}{m^2} Z_1^-(\rho, \phi) & 0 \\[2mm]
0 & 2\mathrm{i}J_0 e^{-\mathrm{i}\phi} + \mathrm{i}\dfrac{k_t^2}{m^2} Z_1^+(\rho, \phi) & 2J_0 e^{-\mathrm{i}\phi} + \dfrac{k_t^2}{m^2} Z_1^+(\rho, \phi) & 0 \\[2mm]
0 & 2\mathrm{i}\dfrac{k_t k_z}{m^2} J_1 & 2\dfrac{k_t k_z}{m^2} J_1 & 0
\end{pmatrix}.
$$

V. SUMMARY AND OUTLOOK

We studied the massive vector field with respect to the Proca equation in the presence of rotation background by introducing the curved metrics in such an equation. Unlike the fermionic case, in addition to a explicit polarization term $\vec{\omega} \cdot \vec{J}$, there are $O(\omega^2)$ terms that are non-negligible. As we expected, because of the cylindrical symmetry in the rotating system, the eigen states are the same as those of the free Proca equation in these coordinates. The polarization effects were also obtained as the energy shifts in the eigen energies expressed as

$\Delta E = n\omega$. With canonical quantization, the propagator was also computed. Its poles indicate that the rotation serves as an additional chemical potential rather than a trivial energy modification. This coincides with the fermionic case. It is also shown that the translation symmetry is broken in the cylindrical coordinates and thus the propagator depends on two space-time points explicitly. Such a behavior will clearly make the further loop computation much more complicated. As a potential simplified case, we briefly studied the near-central result because of the size of the rotating area/volume is usually limited. We checked that, in zero-rotation limit, it can be

reduced to the usual form explicitly by using the summation relations of Bessel functions.

This is a preliminary study on the complete framework of solving the quantum field problems defined in a curved coordinate. In such a case, the translation symmetry will be broken and so will the usual energy-momentum conservation at interaction vertices because the momenta are no longer good quantum numbers when the space is not flat. We also showed that although the computation is much more complicated, the standard process is straightforward for the interaction-free case. However, it could be expected that the interaction terms will induce more fundamental problems, especially for the p-wave cases, e.g., $A^2\partial A$, which will suffer not only from non-flat eigen states modification but also additional terms from the covariant derivation. This will evidently appear in non-abelian gauge field models such as QCD. Therefore, before facing the interaction problem, we should conduct more studies on the gauge symmetry in such coordinates. We will continue to discuss these topics in future studies.

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