

Pair production in Reissner-Nordström-Anti de Sitter black holes*

Jun Zhang(张俊)^{1†} Yi-Yu Lin(林乙裕)^{1‡} Hao-Chun Liang(梁灏淳)^{1§} Ke-Jia Chi(匙可佳)^{1¶} Chiang-Mei Chen^{2#}
Sang Pyo Kim^{3¶} Jia-Rui Sun(孙佳睿)^{1b}

¹School of Physics and Astronomy, Sun Yat-Sen University, Guangzhou 510275

²Department of Physics, and Center for High Energy and High Field Physics (CHiP), National Central University, Chungli 32001

³Department of Physics, Kunsan National University, Kunsan 54150

Abstract: We studied the pair production of charged scalar particles of a five-dimensional near extremal Reissner-Nordström-Anti de Sitter (RN-AdS₅) black hole. The pair production rate and the absorption cross section ratio in full spacetime are obtained and are shown to have a concise relation with their counterparts in the near horizon region. In addition, the holographic descriptions of the pair production, both in the IR CFT in the near horizon region and the UV CFT at the asymptotic spatial boundary of the RN-AdS₅ black hole, are analyzed in the AdS₂/CFT₁ and AdS₅/CFT₄ correspondences, respectively. This work gives a complete description of scalar pair production in a near extremal RN-AdS₅ black hole.

Keywords: black holes, pair production, AdS/CFT correspondence

DOI: 10.1088/1674-1137/abf4f6

I. INTRODUCTION

The Schwinger pair production of charged particles is an important QED phenomenon that is related to the vacuum instability and persistence in the presence of strong external electromagnetic fields [1]. Another important spontaneous pair production phenomenon is the Hawking radiation from black holes, which can be viewed as a tunneling process through the black hole horizon [2]. A charged black hole thus provides a natural lab in which both the Schwinger pair production and the Hawking radiation can occur and mix with each other. Usually the equation of motions (EoMs) of quantum fields in a general black hole background is difficult to solve analytically in full spacetime. However, when the symmetry of the spacetime geometry is enhanced under some conditions, the problem becomes manageable; for this reason, in a series of recent studies, the spontaneous pair production of charged particles has been systematically studied in near extremal charged black holes, including the RN

black hole [3-5] and the Kerr-Newman (KN) black hole [6, 7], in which the near horizon geometry is enhanced into AdS₂ or warped AdS₃ in the near extremal limit. Owing to the enhanced near horizon symmetry, the explicit forms of the pair production rate and other 2-point correlation functions have been obtained and their holographic descriptions have been found based on the RN/CFT [8-13] and KN/CFTs dualities [14-16]. In addition to charged black hole backgrounds, pair production has also been investigated in pure AdS or dS spacetime, see, e.g., [17-20], whereas in the absence of a gravitational field, the pure Schwinger effect has been efficiently analyzed by using the phase-integral method [21-24].

However, previous studies mainly focused on analyzing spontaneous pair production in the near horizon region of black holes in an asymptotically flat spacetime. A charged black hole in AdS spacetime has an additional AdS symmetry at the asymptotical boundary. From the holographic point of view, the CFT description of pair production has been revealed only in the near horizon re-

Received 1 February 2021; Accepted 6 April 2021; Published online 11 May 2021

* Supported by the National Natural Science Foundation of China (11675272), the Ministry of Science and Technology of the R.O.C. under the grant MOST 108-2112-M-008-007, National Research Foundation of Korea (NRF) funded by the Ministry of Education (2019R111A3A01063183)

[†] E-mail: zhangj626@mail2.sysu.edu.cn

[‡] E-mail: linyy27@mail2.sysu.edu.cn

[§] E-mail: lianghch3@mail2.sysu.edu.cn

[¶] E-mail: chikj@mail2.sysu.edu.cn

[#] E-mail: cmchen@phy.ncu.edu.tw

[¶] E-mail: sangkim@kunsan.ac.kr

^b E-mail: sunjiarui@sysu.edu.cn



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP³ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

gion in terms of AdS₂/CFT₁ (or warped AdS₃/CFT₂). Although particle pairs produced in the near horizon region of black holes indeed provide important contributions to those in full spacetime, an understanding of the whole picture is still lacking. In the present paper, we extend the study of pair production to a full near extremal RN-AdS₅ black hole background, which possesses an AdS₅ geometry at the asymptotic spatial boundary as well as an AdS₂ structure in the near horizon region. It is shown that the radial equation of the charged scalar field propagating in this spacetime can be transformed into a Heun-like differential equation and thus be solved by matching its solutions in the near and far spacetime regions, using the low temperature limit. Consequently, analytical forms of the full solutions for the pair production rate, the absorption cross section ratio, and the retarded Green's functions are obtained, and they are shown to have concise relations with their counterparts calculated in the near horizon region. Based on these concise relations, numerical analysis can easily be performed, and the pair production rate in full spacetime is shown to be smaller than that in the near horizon region, which is consistent with the assumption that pair production mainly comes from the black hole near horizon region.

A near extremal RN-AdS₅ black hole is also a very useful background for studying holographic dualities. As the near horizon AdS₂ (or warped AdS₃) spacetime is dual to a 1D CFT (or chiral CFT₂), while asymptotical AdS₅ spacetime is dual to another 4D CFT, the former is called IR CFT, while the latter is called UV CFT, and they are connected with each other via the holographic renormalization group (RG) flow along the radial direction [25-27]. For example, it has been shown that a near extremal RN-AdS black hole acts as a holographic model in describing typical properties of a (non)Fermi liquid at the quantum critical point [28-31]. It is thus natural and interesting to find holographic descriptions of pair production in an RN-AdS₅ black hole both in the IR CFT₁ in the near horizon region and the UV CFT₄ at the asymptotical AdS₅ boundary. We show that the picture in the IR CFT₁ is very similar to those in the near extremal RN and KN black holes, and that the pair production rate and the absorption cross section ratio calculated from the AdS₂ spacetime can be matched with those from the dual IR CFT. Regarding the UV 4D CFT, a direct comparison of calculations between the bulk and the boundary in terms of the AdS₅/CFT₄ is not made due to a lack of information on the dual finite temperature CFT₄ side. However, from the bulk gravity side, the condition for pair production in the full near extremal RN-AdS₅ spacetime is the violation of the Breitenlohner-Freedman (BF) bound [32, 33] in AdS₅ spacetime. This, on the dual 4D CFT side, corresponds to a complex conformal weight for the scalar operator dual to the bulk charged scalar field, which indeed indicates instabilities for the scalar operator on the

boundary and is consistent with the situation in the IR CFT. Furthermore, we determined an interesting relation between the full pair production rate and the absorption cross section ratio via changing the roles of sources and operators simultaneously both in the IR and the UV CFTs.

The rest of the paper is organized as follows. In Sec. II, we provide a brief review of the bulk theory and consider the near horizon geometry of an RN- AdS_{*d*+1} black hole and the EoMs of the probe charged scalar field. In Sec. III, spontaneous pair production in the near horizon region of near extremal RN- AdS_{*d*+1} black holes is discussed, and the 2-point functions of the charged scalar field (such as the retarded Green's function), pair production rate, and absorption cross section ratio are calculated. In Sec. IV, the full analytical solution for the radial equation of the charged scalar field in RN-AdS₅ black holes is obtained by applying the matching technique. Consequently, the full analytical forms of the pair production rate, absorption cross section ratio, and retarded Green's function are found, and the connections with their counterparts in the near horizon region of the black hole are discussed. Then, in Sec. V, the dual CFTs descriptions of spontaneous pair production are both analyzed in terms of the AdS₂/CFT₁ correspondence in the IR region and the AdS₅/CFT₄ correspondence in the UV region, and their connections are also revealed. Finally, the conclusion and physical implications are provided in Sec. VI.

II. BULK THEORY

A. RN- AdS_{*d*+1} black hole

The *d*+1 dimensional Einstein-Maxwell theory has an action (in units of $c = \hbar = 1$) as

$$I = \int d^{d+1}x \sqrt{-g} \left[\frac{1}{16\pi G_{d+1}} \left(R + \frac{d(d-1)}{L^2} \right) - \frac{1}{g_s^2} F_{\mu\nu} F^{\mu\nu} \right], \quad (1)$$

where L is the curvature radius of the asymptotical AdS_{*d*+1} spacetime, and g_s is the dimensionless coupling constant of the $U(1)$ gauge field. The dynamical equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{d(d-1)}{2L^2} g_{\mu\nu} = \frac{8\pi G_{d+1}}{g_s^2} (4F_{\mu\lambda} F_{\nu}{}^{\lambda} - g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}),$$

$$\partial_{\mu} (\sqrt{-g} F^{\mu\nu}) = 0, \quad (2)$$

admit the Reissner-Nordström-Anti de Sitter (RN-AdS_{*d*+1}) black brane (or the planar black hole) solution [34]

$$\begin{aligned}
 ds^2 &= \frac{L^2}{r^2 f(r)} dr^2 + \frac{r^2}{L^2} (-f(r) dt^2 + dx_i^2), \\
 A &= \mu \left(1 - \frac{r_0^{d-2}}{r^{d-2}} \right) dt,
 \end{aligned} \tag{3}$$

with

$$\begin{aligned}
 f(r) &= 1 - \frac{G_{d+1} L^2 M}{r^d} + \frac{G_{d+1} L^2 Q^2}{r^{2d-2}}, \\
 \mu &= \sqrt{\frac{d-1}{2(d-2)} \frac{g_s Q}{r_0^{d-2}}},
 \end{aligned} \tag{4}$$

where r_0 is the radius of the outer horizon ($f(r_0) = 0$), μ is the chemical potential with dimension $[\mu] = \text{length}^{-(d-1)/2}$, M is the mass, and Q is the charge of the black brane. We may find an explicit expression of r_0 for $d = 4$ from a solution of the cubic equation, which is complicated, but r_0 has a general expression in the extremal case, i.e., r_* in IIB. The condition $f(r_0) = 0$ gives $M = \frac{r_0^d}{G_{d+1} L^2} + \frac{Q^2}{r_0^{d-2}}$ (which is the Smarr-like relation related to the first law of thermodynamics of the black brane); temperature T and “surface” entropy density s of the black brane are, respectively,

$$\begin{aligned}
 T &= \frac{r_0 d}{4\pi L^2} \left(1 - \frac{d-2}{d} \frac{G_{d+1} L^2 Q^2}{r_0^{2d-2}} \right), \\
 s &= \frac{1}{4G_{d+1}} \left(\frac{r_0}{L} \right)^{d-1}.
 \end{aligned} \tag{5}$$

Moreover, the first law of thermodynamics of the dual boundary d -dimensional quantum field is

$$\delta\epsilon = T\delta s + \mu\delta\rho_c, \tag{6}$$

where the “surface” energy and charge densities are, respectively,

$$\begin{aligned}
 \epsilon &= \frac{d-1}{16\pi L^{d-1}} M, \\
 \rho_c &= \frac{\sqrt{2(d-1)(d-2)}}{8\pi g_s L^{d-1}} Q.
 \end{aligned} \tag{7}$$

Then, it is straightforward to check the Euler relation

$$\left(\frac{d}{d-1} \right) \epsilon = \epsilon + p = Ts + \mu\rho_c, \tag{8}$$

where the pressure is $p = \frac{\epsilon}{d-1}$, which shows that the dual d -dimensional quantum field theory on the asymptotic boundary is conformal, as expected.

B. Near-horizon near-extremal geometry

To make the following analysis convenient, let us introduce the length scale $r_*^{2d-2} \equiv \frac{d-2}{d} G_{d+1} L^2 Q^2$; then, the temperature can be rewritten as

$$T = \frac{r_0 d}{4\pi L^2} \left(1 - \frac{r_*^{2d-2}}{r_0^{2d-2}} \right). \tag{9}$$

Note that r_* may be treated as the “effective” radius of the inner black hole horizon though $f(r_*) \neq 0$ in general and $r_* < r_0$. The extremal condition for a degenerate horizon at $r_0 = r_*$ is $M = M_0 \equiv \frac{2(d-1)}{d-2} \frac{r_*^d}{G_{d+1} L^2}$. The near extremal limit of the near horizon is obtained by taking the limit $\epsilon \rightarrow 0$ of the transformations

$$\begin{aligned}
 M - M_0 &= \frac{d(d-1)r_*^{d-2}}{G_{d+1} L^2} \epsilon^2 \rho_0^2, \\
 r_0 - r_* &= \epsilon \rho_0, \quad r - r_0 = \epsilon(\rho - \rho_0), \quad t = \frac{\tau}{\epsilon},
 \end{aligned} \tag{10}$$

where in general ρ_0 is finite and $\rho \in [\rho_0, \infty)$.

Expanding $f(r)$ around $r = r_0$, we have

$$f(r) \simeq \frac{d(d-1)}{r_0^2} (\rho^2 - \rho_0^2) \epsilon^2 + O(\epsilon^3), \tag{11}$$

the near horizon geometry is given by

$$\begin{aligned}
 ds^2 &= -\frac{\rho^2 - \rho_0^2}{\ell^2} d\tau^2 + \frac{\ell^2 d\rho^2}{\rho^2 - \rho_0^2} + \frac{r_0^2}{L^2} dx_i^2, \\
 A &= \frac{(d-2)\mu}{r_0} (\rho - \rho_0) d\tau,
 \end{aligned} \tag{12}$$

where $\ell^2 \equiv \frac{L^2}{d(d-1)}$ is defined as the square of the curvature radius of the effective AdS_2 geometry. The limit $\rho_0 \rightarrow 0$ yields the extremal limit.

The solution in Eq. (12) can also be written in the Poincaré coordinates in terms of $\xi = \ell^2/\rho$, ($|\xi| \leq \xi_0 = \ell^2/\rho_0$),

$$\begin{aligned}
 ds^2 &= \frac{\ell^2}{\xi^2} \left(-\left(1 - \frac{\xi^2}{\xi_0^2} \right) d\tau^2 + \frac{d\xi^2}{1 - \frac{\xi^2}{\xi_0^2}} \right) + \frac{r_0^2}{L^2} dx_i^2, \\
 A &= \frac{(d-2)\mu\ell^2}{r_0} \left(\frac{1}{\xi} - \frac{1}{\xi_0} \right) d\tau.
 \end{aligned} \tag{13}$$

The above geometry is a black brane with both local and asymptotical topology $\text{AdS}_2 \times \mathbf{R}^{d-1}$ (AdS_2 has the $SL(2, \mathbf{R})_R$ symmetry). The horizons of the new black

brane are located at $\xi = \pm\xi_0$, and its temperature is $T_n = \frac{1}{2\pi\xi_0}$. Note that if we adopt the new coordinates $z \equiv \xi/\xi_0$ with $|z| \leq 1$ and $\eta = \tau/\xi_0$, the metric becomes

$$ds^2 = \frac{\ell^2}{z^2} \left(-(1-z^2)d\eta^2 + \frac{dz^2}{1-z^2} \right) + \frac{r_0^2}{L^2} dx_r^2, \quad (14)$$

and the temperature associated with the inverse period of η is normalized to $\tilde{T}_n = \frac{1}{2\pi}$.

C. Charged scalar field probe

The action of a bulk probe charged scalar field Φ with mass m and charge q is

$$S = \int d^{d+1}x \sqrt{-g} \left(-\frac{1}{2} D_\alpha^* \Phi^* D^\alpha \Phi - \frac{1}{2} m^2 \Phi^* \Phi \right), \quad (15)$$

where $D_\alpha \equiv \nabla_\alpha - iqA_\alpha$ with ∇_α being the covariant derivative in curved spacetime. The corresponding Klein-Gordon (KG) equation is

$$(\nabla_\alpha - iqA_\alpha)(\nabla^\alpha - iqA^\alpha)\Phi = m^2\Phi. \quad (16)$$

Moreover, the radial flux of the probe field is

$$\mathcal{F} = i \sqrt{-g} g^{rr} (\Phi D_r^* \Phi^* - \Phi^* D_r \Phi). \quad (17)$$

In the RN-AdS $_{d+1}$ background (3), assuming $\Phi(t, \vec{x}, r) = \phi(r)e^{-i\omega t + i\vec{k}\cdot\vec{x}}$, the KG Eq. (16) has the radial form

$$\left(\frac{L}{r} \right)^{d-1} \partial_r \left(\frac{r^{d+1}}{L^{d+1}} f(r) \partial_r \right) \phi(r) + \left(\frac{L^2(\omega + qA_r)^2}{r^2 f(r)} - m^2 - \frac{L^2}{r^2} \vec{k}^2 \right) \phi(r) = 0. \quad (18)$$

The solutions to Eq. (18) cannot be directly found in terms of special functions in the full spacetime region. In what follows, we solve it in different regions and match these solutions to obtain the full solution.

III. PAIR PRODUCTION IN THE INNER ADS $_2$

A. Near-horizon solutions

Firstly, we analyze the near horizon, near extreme region (13) and solve the KG Eq. (16) by expanding the scalar field as

$$\Phi(\tau, \vec{x}, \xi) = \phi(\xi) e^{-i\omega\tau + i\vec{k}\cdot\vec{x}}. \quad (19)$$

Then, the KG equation reduces to¹⁾

$$\xi^2 \left(1 - \frac{\xi^2}{\xi_0^2} \right) \phi''(\xi) - \frac{2\xi^3}{\xi_0^2} \phi'(\xi) + \xi^2 \frac{(w + qA_r)^2}{1 - \frac{\xi^2}{\xi_0^2}} \phi(\xi) = m_{\text{eff}}^2 \ell^2 \phi(\xi), \quad (20)$$

where the effective mass square is defined as $m_{\text{eff}}^2 = m^2 + \frac{L^2 \vec{k}^2}{r_0^2}$, or the KG equation can be expressed in the z coordinate as

$$z^2(1-z^2)\phi''(z) - 2z^3\phi'(z) + \frac{z^2}{1-z^2} \left[\left(w\xi_0 + q_{\text{eff}}\ell \frac{1-z}{z} \right)^2 - m_{\text{eff}}^2 \ell^2 \frac{1-z^2}{z^2} \right] \phi(z) = 0, \quad (21)$$

where the effective charge of the probe field is $q_{\text{eff}} \equiv (d-2)\frac{\mu\ell}{r_0}q$. The singularities of Eq. (21) are located at $z=0, z=\pm 1$ and $z=\infty$.

To find the solutions, we determine the indices at each singular point. For $z \rightarrow 0$, setting $\phi(z) \sim z^{\bar{\alpha}}$, the leading terms in Eq. (21) are

$$z^2 \phi''(z) + (q_{\text{eff}}^2 - m_{\text{eff}}^2) \ell^2 \phi(z) = 0, \quad (22)$$

which gives

$$\begin{aligned} \bar{\alpha} &= \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + 4(m_{\text{eff}}^2 - q_{\text{eff}}^2) \ell^2} \\ &\equiv \frac{1}{2} \pm \frac{1}{2} \sqrt{1 + 4\tilde{m}_{\text{eff}}^2} \ell^2 \equiv \frac{1}{2} \pm \nu. \end{aligned} \quad (23)$$

For $z \rightarrow -1$, setting $\phi(z) \sim (1+z)^{\bar{\beta}}$, Eq. (21) reduces to

$$2(1+z)\phi''(z) + 2\phi'(z) + \frac{(w\xi_0 - 2q_{\text{eff}}\ell)^2}{2(1+z)} \phi(z) = 0, \quad (24)$$

and the index is

$$\bar{\beta} = \pm i \left(\frac{w\xi_0}{2} - q_{\text{eff}}\ell \right) = \pm i \left(\frac{w}{4\pi T_n} - q_{\text{eff}}\ell \right). \quad (25)$$

Finally, for $z \rightarrow 1$, setting $\phi(z) \sim (1-z)^{\bar{\gamma}}$, Eq. (21) reduces to

1) Note that from Eq. (10), one has $\omega = \varepsilon w$, which may indicate that if one considered the finite frequency probe field in the τ, ξ coordinates, the probe field in the original t, r coordinates is automatically driven into the low frequency region.

$$2(1-z)\phi''(z) - 2\phi'(z) + \frac{(w\xi_0)^2}{2(1-z)}\phi(z) = 0, \quad (26)$$

from which

$$\tilde{\gamma} = \pm i \frac{w\xi_0}{2} = \pm i \frac{w}{4\pi T_n} = \pm i \frac{\omega/\varepsilon}{4\pi/(2\pi\xi_0)} = \pm i \frac{\omega}{2\varepsilon\rho_0/\ell^2} = \pm i \frac{\omega}{4\pi T} \quad (27)$$

is obtained. Further, imposing the ingoing boundary condition at the black brane horizon $z = 1$ requires $\tilde{\gamma} = -i \frac{w\xi_0}{2} = -i \frac{w}{4\pi T_n}$.

Also, note that Eq. (21) can be rewritten in a more explicit form as

$$\phi''(z) + \left(\frac{1}{z+1} + \frac{1}{z-1} \right) \phi'(z) + \left(\frac{\tilde{m}_{\text{eff}}^2 \ell^2}{z} + \frac{\frac{1}{2}(w\xi_0 - 2q_{\text{eff}}\ell)^2}{z+1} + \frac{\frac{1}{2}w^2\xi_0^2}{z-1} \right) \frac{\phi(z)}{z(z+1)(z-1)} = 0, \quad (28)$$

which becomes the Fuchs equation with three canonical singularities a_1, a_2 and a_3 , as follows:

$$\phi''(z) + \left(\frac{1 - \bar{\alpha}_1 - \bar{\alpha}_2}{z - a_1} + \frac{1 - \bar{\beta}_1 - \bar{\beta}_2}{z - a_2} + \frac{1 - \bar{\gamma}_1 - \bar{\gamma}_2}{z - a_3} \right) \phi'(z) + \left(\frac{\bar{\alpha}_1 \bar{\alpha}_2 (a_1 - a_2)(a_1 - a_3)}{z - a_1} + \frac{\bar{\beta}_1 \bar{\beta}_2 (a_2 - a_3)(a_2 - a_1)}{z - a_2} + \frac{\bar{\gamma}_1 \bar{\gamma}_2 (a_3 - a_1)(a_3 - a_2)}{z - a_3} \right) \frac{\phi(z)}{(z - a_1)(z - a_2)(z - a_3)} = 0, \quad (29)$$

where $a_1 = 0, a_2 = -1$, and $a_3 = 1$ and

$$\begin{aligned} \bar{\alpha}_1 &= \frac{1}{2} \pm \nu, & \bar{\alpha}_2 &= \frac{1}{2} \mp \nu, & \bar{\beta}_1 &= -\bar{\beta}_2 = \pm i \frac{w\xi_0 - 2q_{\text{eff}}\ell}{2}, \\ \bar{\gamma}_1 &= -\bar{\gamma}_2 = \pm i \frac{w\xi_0}{2}, \end{aligned} \quad (30)$$

and $\bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\beta}_1 + \bar{\beta}_2 + \bar{\gamma}_1 + \bar{\gamma}_2 = 1$ is satisfied. The Fuchs Eq. (29) can be transformed into the standard hypergeometric function

$$\zeta(1-\zeta)\psi''(\zeta) + [\tilde{\gamma} - (1 + \tilde{\alpha} + \tilde{\beta})\zeta]\psi'(\zeta) - \tilde{\alpha}\tilde{\beta}\psi(\zeta) = 0, \quad (31)$$

via the conformal coordinate transformation

$$\zeta = \frac{(a_2 - a_3)(z - a_1)}{(a_2 - a_1)(z - a_3)}, \quad \phi(z) = \left(\frac{z - a_1}{z - a_3} \right)^{\bar{\alpha}_1} \left(\frac{z - a_2}{z - a_3} \right)^{\bar{\beta}_1} \psi(\zeta), \quad (32)$$

where $\tilde{\alpha} = \bar{\alpha}_1 + \bar{\beta}_1 + \bar{\gamma}_1, \tilde{\beta} = \bar{\alpha}_1 + \bar{\beta}_1 + \bar{\gamma}_2$ and $\tilde{\gamma} = 1 + \bar{\alpha}_1 - \bar{\alpha}_2$. (Note that one can freely choose the indices $i = 1, 2$ for $\bar{\alpha}_i, \bar{\beta}_i$ and $\bar{\gamma}_i$.)

For Eq. (28), we have $\zeta = 2z/(z-1), \tilde{\alpha} = \frac{1}{2} \pm \nu + iw\xi_0 - iq_{\text{eff}}\ell, \tilde{\beta} = \frac{1}{2} \pm \nu - iq_{\text{eff}}\ell, \tilde{\gamma} = 1 \pm 2\nu$. Therefore, the explicit solutions in the near horizon near extreme region are

$$\begin{aligned} \phi(z) &= c_1 \left(\frac{z}{z-1} \right)^{\frac{1}{2} + \nu} \left(\frac{z+1}{z-1} \right)^{i \frac{w\xi_0}{2} - iq_{\text{eff}}\ell} {}_2F_1 \left(\frac{1}{2} + \nu + iw\xi_0 - iq_{\text{eff}}\ell, \frac{1}{2} + \nu - iq_{\text{eff}}\ell; 1 + 2\nu; \frac{2z}{z-1} \right) \\ &+ c_2 \left(\frac{z}{z-1} \right)^{\frac{1}{2} - \nu} \left(\frac{z+1}{z-1} \right)^{i \frac{w\xi_0}{2} - iq_{\text{eff}}\ell} {}_2F_1 \left(\frac{1}{2} - \nu + iw\xi_0 - iq_{\text{eff}}\ell, \frac{1}{2} - \nu - iq_{\text{eff}}\ell; 1 - 2\nu; \frac{2z}{z-1} \right). \end{aligned} \quad (33)$$

B. 2-point correlators from AdS₂

At the horizon of the AdS₂ black brane, $z = 1$, Eq. (33) is expanded as follows:

$$\phi(z) = c_H^{(\text{in})} (1-z)^{-i \frac{w}{4\pi T_n}} + c_H^{(\text{out})} (1-z)^{i \frac{w}{4\pi T_n}}, \quad (34)$$

where

$$c_H^{(\text{in})} = c_1 (-)^{-\frac{1}{2} - \nu - i \frac{w}{2\pi T_n} + iq_{\text{eff}}\ell} 2^{-\frac{1}{2} - \nu + i \frac{w}{4\pi T_n}} \frac{\Gamma(1 + 2\nu) \Gamma\left(i \frac{w}{2\pi T_n}\right)}{\Gamma\left(\frac{1}{2} + \nu + iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} + \nu + i \frac{w}{2\pi T_n} - iq_{\text{eff}}\ell\right)}$$

$$+c_2(-)^{-\frac{1}{2}+\nu-i\frac{w}{2\pi T_n}+iq_{\text{eff}}\ell} 2^{-\frac{1}{2}+\nu+i\frac{w}{4\pi T_n}} \frac{\Gamma(1-2\nu)\Gamma\left(i\frac{w}{2\pi T_n}\right)}{\Gamma\left(\frac{1}{2}-\nu+iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}-\nu+i\frac{w}{2\pi T_n}-iq_{\text{eff}}\ell\right)}, \quad (35)$$

and

$$c_H^{(\text{out})} = c_1(-)^{-\frac{1}{2}-\nu-i\frac{w}{2\pi T_n}+iq_{\text{eff}}\ell} 2^{-\frac{1}{2}-\nu-i\frac{w}{4\pi T_n}} \frac{\Gamma(1+2\nu)\Gamma\left(-i\frac{w}{2\pi T_n}\right)}{\Gamma\left(\frac{1}{2}+\nu-iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}+\nu-i\frac{w}{2\pi T_n}+iq_{\text{eff}}\ell\right)} + c_2(-)^{-\frac{1}{2}+\nu-i\frac{w}{2\pi T_n}+iq_{\text{eff}}\ell} 2^{-\frac{1}{2}+\nu-i\frac{w}{4\pi T_n}} \frac{\Gamma(1-2\nu)\Gamma\left(-i\frac{w}{2\pi T_n}\right)}{\Gamma\left(\frac{1}{2}-\nu-iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}-\nu-i\frac{w}{2\pi T_n}+iq_{\text{eff}}\ell\right)}. \quad (36)$$

In contrast, at the AdS₂ boundary, $z \rightarrow 0$, the asymptotic expansion of Eq. (33) is

$$\begin{aligned} \phi(z) &= c_2(-)^{\frac{1}{2}-\nu+i\frac{w}{4\pi T_n}-iq_{\text{eff}}\ell} z^{\frac{1}{2}-\nu} + c_1(-)^{\frac{1}{2}+\nu+i\frac{w}{4\pi T_n}-iq_{\text{eff}}\ell} z^{\frac{1}{2}+\nu} \\ &= \mathcal{A}(w, \vec{k}) z^{\frac{1}{2}-\nu} + \mathcal{B}(w, \vec{k}) z^{\frac{1}{2}+\nu}, \end{aligned} \quad (37)$$

where \mathcal{A} is the source of the charged scalar field in the bulk AdS₂, while \mathcal{B} is the response or the operator $\hat{O}(w, \vec{k})$ (in the momentum space) of the boundary CFT₁ (i.e., the IR CFT) dual to the charged scalar field in the bulk AdS₂ background. Note that in order to obtain the propagating modes, ν should be purely imaginary, which can be set as $\nu \equiv i|\nu|$, i.e., $\phi(z) = c_B^{(\text{out})} z^{\frac{1}{2}-i|\nu|} + c_B^{(\text{in})} z^{\frac{1}{2}+i|\nu|}$. It was shown in [3] that the condition of an imaginary ν is equivalent to the violation of the BF bound in AdS₂ spacetime, namely

$$\tilde{m}_{\text{eff}}^2 < -\frac{1}{4\ell^2}, \quad (38)$$

which corresponds to a complex conformal weight of the scalar operator in the dual IR CFT.

1. Pair production rate and absorption cross section ratio

The Schwinger pair production rate $|b|^2$ and the ab-

sorption cross section ratio σ_{abs} can be calculated from the radial flux by imposing different boundary conditions

$$\mathcal{F} = i\left(\frac{r_0}{L}\right)^{d-1} (1-z^2)(\Phi\partial_z\Phi^* - \Phi^*\partial_z\Phi), \quad (39)$$

which gives

$$\begin{aligned} \mathcal{F}_B^{(\text{in})} &= 2|\nu|\left(\frac{r_0}{L}\right)^{d-1} |c_B^{(\text{in})}|^2, \\ \mathcal{F}_B^{(\text{out})} &= -2|\nu|\left(\frac{r_0}{L}\right)^{d-1} |c_B^{(\text{out})}|^2, \\ \mathcal{F}_H^{(\text{in})} &= \frac{w}{2\pi T_n} \left(\frac{r_0}{L}\right)^{d-1} |c_H^{(\text{in})}|^2, \\ \mathcal{F}_H^{(\text{out})} &= -\frac{w}{2\pi T_n} \left(\frac{r_0}{L}\right)^{d-1} |c_H^{(\text{out})}|^2, \end{aligned} \quad (40)$$

where $\mathcal{F}_B^{(\text{in})}$ and $\mathcal{F}_B^{(\text{out})}$ are the ingoing and outgoing fluxes at the AdS₂ boundary, while $\mathcal{F}_H^{(\text{in})}$ and $\mathcal{F}_H^{(\text{out})}$ are the ingoing and outgoing fluxes at the AdS₂ black brane horizon, respectively.

The Schwinger pair production rate $|b^{\text{AdS}_2}|^2$ can be computed either by choosing the inner boundary condition or the outer boundary condition, which gives the same result [3], e.g., by adopting the outer boundary condition, i.e., $\mathcal{F}_B^{(\text{in})} = 0$, ($c_B^{(\text{in})} = 0 \Rightarrow c_1 = 0$),

$$\begin{aligned} |b^{\text{AdS}_2}|^2 &= \frac{\mathcal{F}_B^{(\text{out})}}{\mathcal{F}_H^{(\text{in})}} = \frac{4\pi T_n |\nu|}{w} \left| \frac{c_B^{(\text{out})}}{c_H^{(\text{in})}} \right|^2 = \frac{8\pi T_n |\nu|}{w} \left| \frac{\Gamma\left(\frac{1}{2}-i|\nu|+iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}-i|\nu|+i\frac{w}{2\pi T_n}-iq_{\text{eff}}\ell\right)}{\Gamma(1-2i|\nu|)\Gamma\left(i\frac{w}{2\pi T_n}\right)} \right|^2 \\ &= \frac{2\sinh(2\pi|\nu|)\sinh\left(\frac{w}{2T_n}\right)}{\cosh\pi(|\nu|-q_{\text{eff}}\ell)\cosh\pi\left(|\nu|-\frac{w}{2\pi T_n}+q_{\text{eff}}\ell\right)}. \end{aligned} \quad (41)$$

Similarly, by adopting the outer boundary condition, the absorption cross section ratio is computed as

$$\begin{aligned} \sigma_{\text{abs}}^{\text{AdS}_2} &= \frac{\mathcal{F}_B^{(\text{out})}}{\mathcal{F}_H^{(\text{out})}} = \frac{4\pi T_n |\nu|}{w} \left| \frac{c_B^{(\text{out})}}{c_H^{(\text{out})}} \right|^2 = \frac{8\pi T_n |\nu|}{w} \left| \frac{\Gamma\left(\frac{1}{2} - i|\nu| - iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} - i|\nu| - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell\right)}{\Gamma(1 - 2i|\nu|) \Gamma\left(-i\frac{w}{2\pi T_n}\right)} \right|^2 \\ &= \frac{2 \sinh(2\pi|\nu|) \sinh\left(\frac{w}{2T_n}\right)}{\cosh \pi(|\nu| + q_{\text{eff}}\ell) \cosh \pi\left(|\nu| + \frac{w}{2\pi T_n} - q_{\text{eff}}\ell\right)}. \end{aligned} \tag{42}$$

The pair production rate and the absorption cross section ratio are connected by the simple relation

$$|\mathfrak{b}^{\text{AdS}_2}|^2 = -\sigma_{\text{abs}}(|\nu| \rightarrow -|\nu|). \tag{43}$$

It was shown that the abovementioned relation also holds for a charged scalar field [11] and for a charged spinor field [4], both in a four-dimensional near extremal RN black hole.

2. Retarded Green's function

The two-point retarded Green's function of the boundary operator dual to the bulk charged scalar field is computed through

$$\begin{aligned} G_R^{\text{AdS}_2}(w, \vec{k}) &\equiv \langle \hat{\mathcal{O}} \hat{\mathcal{O}} \rangle_R \\ &= -2\mathcal{F}|_{z \rightarrow 0} \sim \frac{\mathcal{B}(w, \vec{k})}{\mathcal{A}(w, \vec{k})} \\ &\quad + \text{contact terms} \end{aligned} \tag{44}$$

by taking the inner boundary condition, i.e., $\mathcal{F}_H^{(\text{out})} = 0$, which gives

$$\frac{c_2}{c_1} = (-)^{1-2\nu} 2^{-2\nu} \frac{\Gamma(1+2\nu) \Gamma\left(\frac{1}{2} - \nu - iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} - \nu - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell\right)}{\Gamma(1-2\nu) \Gamma\left(\frac{1}{2} + \nu - iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} + \nu - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell\right)}. \tag{45}$$

Thus, the two-point retarded Green's function is

$$G_R^{\text{AdS}_2}(w, \vec{k}) \sim \frac{\mathcal{B}(w, \vec{k})}{\mathcal{A}(w, \vec{k})} = (-)^{2\nu} \frac{c_1}{c_2} = (-)^{4\nu-1} 2^{2\nu} \frac{\Gamma(1-2\nu) \Gamma\left(\frac{1}{2} + \nu - iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} + \nu - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell\right)}{\Gamma(1+2\nu) \Gamma\left(\frac{1}{2} - \nu - iq_{\text{eff}}\ell\right) \Gamma\left(\frac{1}{2} - \nu - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell\right)}. \tag{46}$$

In addition, the corresponding boundary condition ($\mathcal{F}_B^{(\text{in})} = 0$ and $\mathcal{F}_H^{(\text{out})} = 0$) is used to obtain the quasinormal modes of the charged scalar field in AdS₂ spacetime, which correspond to the poles of the retarded Green's function of dual operators (with complex conformal weight $h_R = \frac{1}{2} + \nu$) in the IR CFT, namely

$$\begin{aligned} \frac{1}{2} + \nu - i\frac{w}{2\pi T_n} + iq_{\text{eff}}\ell &= -N \Rightarrow w \\ &= 2\pi T_n (q_{\text{eff}}\ell - iN - ih_R), \quad N = 0, 1, \dots \end{aligned} \tag{47}$$

Eq. (47) gives the quasinormal modes of the charged

scalar field perturbation.

IV. PAIR PRODUCTION IN THE ASYMPTOTICAL AdS₅

In this section, we describe our study of the pair production for the whole spacetime of RN-AdS₅. Like before, we need to solve the corresponding radial Klein equation for the scalar field.

To find the solution in the full region, we focus on $d = 4$ and the near extremal cases. By introducing the coordinate transformation $\varrho = \frac{r^2}{M'}$ (and denoting $M' =$

$G_{d+1}L^2M$, $\varrho_0 = \frac{r_0^2}{M'}$ and $\varrho_* = \frac{r_*^2}{M'}$, the radial Eq. (18) can be expressed as

$$\phi''(\varrho) + \left(\frac{1}{\varrho - \varrho_1} + \frac{1}{\varrho - \varrho_2} + \frac{1}{\varrho - \varrho_0} \right) \phi'(\varrho) + \left(\frac{\varrho(\tilde{\omega}\varrho - \tilde{q}\mu\varrho_0)^2}{(\varrho - \varrho_1)^2(\varrho - \varrho_2)^2(\varrho - \varrho_0)^2} - \frac{\tilde{m}^2\varrho + \tilde{k}^2}{(\varrho - \varrho_1)(\varrho - \varrho_2)(\varrho - \varrho_0)} \right) \phi(\varrho) = 0, \quad (48)$$

where the parameters are $\tilde{\omega} = \frac{L^2(\omega + q\mu)}{2\sqrt{M'}}$, $\tilde{q} = \frac{L^2q}{2\sqrt{M'}}$, $\tilde{m} = \frac{Lm}{2}$, $\tilde{k} = \frac{L^2|\tilde{k}|}{2\sqrt{M'}}$ and

$$\begin{aligned} \varrho_1 &= -\frac{1}{2}\varrho_0 - \frac{1}{2}\sqrt{\varrho_0^2 + 8\frac{\varrho_*^3}{\varrho_0}}, \\ \varrho_2 &= -\frac{1}{2}\varrho_0 + \frac{1}{2}\sqrt{\varrho_0^2 + 8\frac{\varrho_*^3}{\varrho_0}}. \end{aligned} \quad (49)$$

Further, defining another coordinate

$$y \equiv \frac{\varrho - \varrho_0}{\varrho_0}, \quad a \equiv \frac{\varrho_2 - \varrho_0}{\varrho_0}, \quad b \equiv \frac{\varrho_1 - \varrho_0}{\varrho_0}, \quad (50)$$

the metric of the RN-AdS₅ black hole becomes

$$\begin{aligned} ds^2 &= \frac{L^2 dy^2}{4(1+y)^2 f(y)} + \frac{r_0^2}{L^2} (1+y) (-f(y) dt^2 + dx_i^2), \\ A &= \frac{\mu y}{1+y} dt, \end{aligned} \quad (51)$$

where

$$\begin{aligned} f(y) &= 1 - \frac{M'}{r_0^4} (1+y)^{-2} + \frac{Q^2}{r_0^6} (1+y)^{-3}, \\ Q^2 &= G_{d+1} L^2 Q^2. \end{aligned} \quad (52)$$

Moreover, Eq. (48) transforms into

$$\begin{aligned} \phi''(y) + \left(\frac{1}{y} + \frac{1}{y-a} + \frac{1}{y-b} \right) \phi'(y) \\ + \left(\frac{(\tilde{\omega}(y+1) - \tilde{q}\mu)^2 (y+1)}{y^2 (y-a)^2 (y-b)^2} \right. \\ \left. - \frac{\tilde{m}^2 (y+1)\varrho_0 + \tilde{k}^2}{y(y-a)(y-b)} \right) \frac{\phi(y)}{\varrho_0} = 0. \end{aligned} \quad (53)$$

To solve $\phi(y)$, first, we determine its exponents at the corresponding singularities 0, a , b , and ∞ , which are $\alpha_{1,2}$, $\beta_{1,2}$, $\gamma_{1,2}$, and $\delta_{1,2}$, respectively,

$$\begin{aligned} \alpha_{1,2} &= \pm i \frac{(\tilde{\omega} - \tilde{q}\mu)}{ab\sqrt{\varrho_0}} = \pm \frac{i\omega}{4\pi T}, \\ \beta_{1,2} &= \pm i \frac{(\tilde{\omega}(1+b) - \tilde{q}\mu)\sqrt{1+b}}{(a-b)b\sqrt{\varrho_0}}, \\ \gamma_{1,2} &= \pm i \frac{(\tilde{\omega}(1+a) - \tilde{q}\mu)\sqrt{1+a}}{(b-a)a\sqrt{\varrho_0}}, \end{aligned} \quad (54)$$

where the index “1” corresponds to the “+” sign, and the index “2” corresponds to the “-” sign. Then, decomposing $\phi(y)$ as

$$\phi(y) = \left(\frac{y}{y-b} \right)^{\alpha_1} \left(\frac{y-a}{y-b} \right)^{\gamma_1} R(y), \quad (55)$$

we obtain

$$\begin{aligned} R''(y) + \left(\frac{1}{y} + \frac{1}{y-a} + \frac{1}{y-b} - \frac{2b\alpha_1}{y(y-b)} + \frac{2(a-b)\gamma_1}{(y-a)(y-b)} \right) R'(y) \\ + V_2 R(y) = 0, \end{aligned} \quad (56)$$

where

$$V_2 \equiv -\frac{(2+3a-a^2y)\tilde{\omega}^2 - 4(a+1)\tilde{\omega}\tilde{q}\mu + (a+2)\tilde{q}^2\mu^2}{\varrho_0 a^2 y (y-a)(y-b)^2} - M_1, \quad (57)$$

and

$$\begin{aligned} M_1 &= \frac{(b-a)\gamma_1}{y(y-a)(y-b)} + \frac{2b(a-b)\alpha_1\gamma_1}{y(y-a)(y-b)^2} + \frac{\tilde{m}^2(y+1)\varrho_0 + \tilde{k}^2}{\varrho_0 y (y-a)(y-b)} \\ &\quad + \frac{b\alpha_1}{y(y-a)(y-b)}. \end{aligned} \quad (58)$$

A. Solutions in the near and far regions

We divide the regions into a near region

$$y = \frac{\varrho - \varrho_0}{\varrho_0} \ll 1, \quad (59)$$

and a far region

$$y = \frac{\varrho - \varrho_0}{\varrho_0} \gg -a, \tag{60}$$

and an overlapping region, in which

$$-a \ll 1. \tag{61}$$

The physical reasoning of $-a \ll 1$ relies on the observation that the temperature of a black hole is

$$T = \frac{r_0}{\pi L^2} \left(1 - \frac{\varrho_*^3}{\varrho_0^3} \right), \tag{62}$$

which gives $-a \rightarrow 0$ for $T \rightarrow 0$, as

$$-a = \frac{3}{2} - \frac{1}{2} \sqrt{9 - \frac{8\pi L^2 T}{r_0}}. \tag{63}$$

We want to point out that the matching condition in Eq. (61) indicates that the near extremal condition is essential for matching the solutions in the near and far regions. It is not necessary for the frequency to be infinitely small; however, the frequency should definitely not be very large compared with the temperature T ; otherwise, the backreaction to the background geometry cannot be ignored.

Now we find the approximate solutions in different regions. First, by using the near region condition ($y \ll 1$), Eq. (53) reduces to

$$\begin{aligned} \phi''(y) + \left(\frac{1}{y} + \frac{1}{y-a} \right) \phi'(y) \\ + \left(-\frac{(a\alpha_1 - iq_{\text{eff}}\ell)^2}{y^2(y-a)^2} + \frac{\tilde{m}^2\varrho_0 + \tilde{k}^2}{\varrho_0 b y(y-a)} \right) \phi(y) = 0. \end{aligned} \tag{64}$$

Obviously Eq. (64) can be solved by the hypergeometric function as

$$\begin{aligned} \phi(y) = (y-a)^{\alpha_1 - iq_{\text{eff}}\ell} \left(c_3 y^{\alpha_1} {}_2F_1 \left(\alpha, \beta; \gamma; \frac{y}{a} \right) \right. \\ \left. + c_4 y^{-\alpha_1} {}_2F_1 \left(1 - \gamma + \alpha, 1 - \gamma + \beta; 2 - \gamma; \frac{y}{a} \right) \right), \end{aligned} \tag{65}$$

where $\alpha = \frac{1}{2} + \nu + 2\alpha_1 - iq_{\text{eff}}\ell$, $\beta = \frac{1}{2} - \nu + 2\alpha_1 - iq_{\text{eff}}\ell$, and $\gamma = 1 + 2\alpha_1$, and $\ell = L/\sqrt{12}$ is the radius of the effective AdS₂ geometry in the near horizon region of the RN-AdS₅ black hole. Second, in the far region, by using the condition ($y \gg -a$), Eq. (56) can turn into

$$R''(y) + \left(\frac{2}{y} + \frac{1}{y-b} - \frac{4b\alpha_1}{y(y-b)} + \frac{2ibq_{\text{eff}}\ell}{y(y-b)} \right) R'(y) + V_3 R(y) = 0, \tag{66}$$

where

$$\begin{aligned} V_3 \equiv \frac{\tilde{\omega}^2}{\varrho_0 y(y-b)^2} + \frac{4b^2\alpha_1(\alpha_1 - iq_{\text{eff}}\ell)}{y^2(y-b)^2} \\ - \frac{b(2\alpha_1 - iq_{\text{eff}}\ell)}{y^2(y-b)} - \frac{\tilde{m}^2(y+1)\varrho_0 + \tilde{k}^2}{\varrho_0 y^2(y-b)}, \end{aligned} \tag{67}$$

(where the relation $\gamma_1 \approx \alpha_1 + \frac{i\tilde{q}\mu}{b\sqrt{\varrho_0}} = \alpha_1 - iq_{\text{eff}}\ell$ when $|a| \ll 1$ is used). Similarly, Eq. (66) has a solution in terms of the hypergeometric function as

$$\begin{aligned} \phi(y) = \left(\frac{y}{b} - 1 \right)^\lambda \left(c_5 y^{\nu - \frac{1}{2}} {}_2F_1 \left(\alpha', \beta'; \gamma'; \frac{y}{b} \right) \right. \\ \left. + c_6 y^{-\nu - \frac{1}{2}} {}_2F_1 \left(1 - \gamma' + \alpha', 1 - \gamma' + \beta'; 2 - \gamma'; \frac{y}{b} \right) \right) \end{aligned} \tag{68}$$

in which $\alpha' = \frac{1}{2} + \nu + \Delta + \lambda$, $\beta' = \frac{1}{2} + \nu - \Delta + \lambda$, $\gamma' = 1 + 2\nu$,

$$\Delta = \sqrt{1 + \tilde{m}^2}, \text{ and } \lambda = \sqrt{(iq_{\text{eff}}\ell)^2 - \frac{\tilde{\omega}^2}{\varrho_0 b}}.$$

B. Near-far matching

In the overlapping region, one has the inequalities

$$-a \ll y \ll 1 < -b \tag{69}$$

($1 < -b$ since $-b = \frac{3}{2} + \frac{1}{2} \sqrt{9 - \frac{8\pi L^2 T}{r_0}} \rightarrow 3$, as $T \rightarrow 0$),

which means $\left| \frac{a}{y} \right| \rightarrow 0$ and $\left| \frac{y}{b} \right| \rightarrow 0$, which transforms Eqs. (65) and (68) into the following forms: the near region-solution

$$\begin{aligned} \phi(y) = \left((-1)^{-\alpha} c_3 \frac{\Gamma(\gamma)\Gamma(\beta-\alpha)}{\Gamma(\beta)\Gamma(\gamma-\alpha)} \right. \\ \left. + (-1)^{-1+\gamma-\alpha} c_4 \frac{\Gamma(2-\gamma)\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \right) y^{-\frac{1}{2}-\nu} \\ + \left((-1)^{-\beta} c_3 \frac{\Gamma(\gamma)\Gamma(\alpha-\beta)}{\Gamma(\alpha)\Gamma(\gamma-\beta)} \right. \\ \left. + (-1)^{-1+\gamma-\beta} c_4 \frac{\Gamma(2-\gamma)\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \right) y^{-\frac{1}{2}+\nu} \end{aligned} \tag{70}$$

and the far region solution

$$\phi(y) \rightarrow (-1)^\lambda c_5 y^{-\frac{1}{2}+\nu} + (-1)^\lambda c_6 y^{-\frac{1}{2}-\nu}. \quad (71)$$

Comparing these two identities, one finds the connection relations

$$c_5 = (-1)^{-\lambda} \left((-1)^{-\beta} c_3 \frac{\Gamma(\gamma)\Gamma(\alpha-\beta)}{\Gamma(\alpha)\Gamma(\gamma-\beta)} + (-1)^{-1+\gamma-\beta} c_4 \frac{\Gamma(2-\gamma)\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \right), \quad (72)$$

$$c_6 = (-1)^{-\lambda} \left((-1)^{-\alpha} c_3 \frac{\Gamma(\gamma)\Gamma(\beta-\alpha)}{\Gamma(\beta)\Gamma(\gamma-\alpha)} + (-1)^{-1+\gamma-\alpha} c_4 \frac{\Gamma(2-\gamma)\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \right). \quad (73)$$

B. 2-point correlators from AdS₅

1. Pair production and absorption cross section

Now we denote the radial flux of the charged scalar field in metric (51) as \mathcal{D} :

$$\mathcal{D} = \frac{2ir_0^4(1+y)^3 f(y)}{L^5} \left(\phi(y)\partial_y \phi^*(y) - \phi^*(y)\partial_y \phi(y) \right). \quad (74)$$

In the near horizon limit, i.e., $y \rightarrow 0$, Eq. (65) reduces to

$$\phi(y) = c_3 y^{\alpha_1} (y-a)^{\alpha_1 - iq_{\text{eff}}\ell} + c_4 y^{-\alpha_1} (y-a)^{\alpha_1 - iq_{\text{eff}}\ell}, \quad (75)$$

where the first part is the outgoing mode, and the second part is the ingoing mode. Further, the asymptotic form of $\phi(y)$ at the boundary ($y \rightarrow \infty$) of the AdS₅ spacetime results in the form

$$\phi(y) = A(\tilde{\omega}, \tilde{k}) y^{-1+\Delta} + B(\tilde{\omega}, \tilde{k}) y^{-1-\Delta}, \quad (76)$$

where $A(\tilde{\omega}, \tilde{k})$ is the source of the charged scalar field in the bulk RN-AdS₅ black hole, while $B(\tilde{\omega}, \tilde{k})$ is the response (the operator) of the boundary CFT₄ (i.e., the UV CFT) dual to the charged scalar field in the bulk. As in the case of the AdS₂ spacetime, the condition for the propagating modes requires an imaginary Δ , i.e., $\Delta = i|\Delta|$,

which means

$$m^2 \leq -\frac{4}{L^2}, \quad (77)$$

namely, the violation of the BF bound in AdS₅ spacetime.

Therefore, the corresponding outgoing and ingoing fluxes at the horizon and the boundary of the near extremal RN-AdS₅ black brane are

$$\begin{aligned} \mathcal{D}_H^{(\text{out})} &= \frac{4\pi\omega T r_0^2}{abL} |c_3|^2 = \frac{2r_0^3\omega}{L^3} |c_3|^2, \\ \mathcal{D}_H^{(\text{in})} &= -\frac{4\pi\omega T r_0^2}{abL} |c_4|^2 = -\frac{2r_0^3\omega}{L^3} |c_4|^2, \\ \mathcal{D}_B^{(\text{out})} &= \frac{4|\Delta|r_0^4}{L^5} |A(\tilde{\omega}, \tilde{k})|^2, \\ \mathcal{D}_B^{(\text{in})} &= -\frac{4|\Delta|r_0^4}{L^5} |B(\tilde{\omega}, \tilde{k})|^2. \end{aligned} \quad (78)$$

The absorption cross section ratio $\sigma_{\text{abs}}^{\text{AdS}_5}$ and the Schwinger pair production rate $|b^{\text{AdS}_5}|^2$ can be calculated by choosing the inner boundary condition $\mathcal{D}_H^{(\text{out})} = 0$ and ($c_3 = 0$) and are given by

$$\sigma_{\text{abs}}^{\text{AdS}_5} = \left| \frac{\mathcal{D}_H^{(\text{in})}}{\mathcal{D}_B^{(\text{in})}} \right| = \frac{2TL^2 |\nu| \sinh(2\pi|\Delta|)}{r_0 \left| H(\nu; \Delta; \lambda) (G_R^{\text{AdS}_5})^{-1} + H(-\nu; \Delta; \lambda) \right|^2} \sigma_{\text{abs}}, \quad (79)$$

and

$$|b^{\text{AdS}_5}|^2 = \left| \frac{\mathcal{D}_H^{(\text{in})}}{\mathcal{D}_B^{(\text{out})}} \right| = \frac{2TL^2 |\nu| \sinh(2\pi|\Delta|)}{r_0 \left| H(-\nu; -\Delta; \lambda) G_R^{\text{AdS}_5} + H(\nu; -\Delta; \lambda) \right|^2} |b^{\text{AdS}_2}|^2, \quad (80)$$

where $H(x; y; z)$ denotes a function

$$H(x; y; z) \equiv (-1)^{2x} 2^x \frac{\Gamma(1+2x)}{\Gamma\left(\frac{1}{2}+x-y+z\right)\Gamma\left(\frac{1}{2}+x-y-z\right)}, \quad (81)$$

and

$$G_R^{\text{AdS}_2} = (-1)^{4\nu-1} 2^{2\nu} \frac{\Gamma(1-2\nu)}{\Gamma(1+2\nu)} \frac{\Gamma\left(\frac{1}{2}+\nu-iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}+\nu-i\frac{\omega}{2\pi T}+iq_{\text{eff}}\ell\right)}{\Gamma\left(\frac{1}{2}-\nu-iq_{\text{eff}}\ell\right)\Gamma\left(\frac{1}{2}-\nu-i\frac{\omega}{2\pi T}+iq_{\text{eff}}\ell\right)}, \quad (82)$$

which is exactly the retarded Green's function in Eq. (46) of the IR CFT in the near horizon, near extremal region. Furthermore, σ_{abs} and $|\mathfrak{b}^{\text{AdS}_2}|^2$ are exactly the absorption cross section ratio and the mean number of produced pairs of the corresponding IR CFT obtained from Eqs. (42) and (41). We find a relationship

$$|\mathfrak{b}^{\text{AdS}_5}|^2 = -\sigma_{\text{abs}}^{\text{AdS}_5} (|\nu| \rightarrow -|\nu|, |\Delta| \rightarrow -|\Delta|), \quad (83)$$

which is similar to Eq. (43) except for a combined change in signs in both $|\nu|$ and $|\Delta|$.

With Eq. (80) at hand we can easily investigate the relationship between the pair production rate in the near horizon and that for the whole spacetime of RN-AdS₅. As shown in Fig. 1, we can see that the mean number of produced pairs for the whole spacetime is less than that from near horizon region. Moreover, with increasing charge of the scalar field, the corresponding ratio becomes smaller, which is consistent with previous assumptions stating that

the Schwinger effect mainly occurs in the near horizon region.

2. Retarded Green's function

To calculate the retarded Green's function, an ingoing boundary condition is required, namely $c_3 = 0$. Then, from Eqs. (72) and (73), the connection relations are

$$c_5 = (-1)^{-1+\gamma-\beta-\lambda} c_4 \frac{\Gamma(2-\gamma)\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)}, \quad (84)$$

$$c_6 = (-1)^{-1+\gamma-\alpha-\lambda} c_4 \frac{\Gamma(2-\gamma)\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)}. \quad (85)$$

Substituting Eqs. (84) and (85) into Eq. (68) and taking $y \rightarrow \infty$, namely the boundary of the AdS₅ spacetime, one obtains

$$A(\tilde{\omega}, \tilde{k}) = (-1)^{iq_{\text{eff}}\ell - 2\lambda - 1 + \Delta} c_4 \left(\frac{\Gamma(2-\gamma)\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \frac{\Gamma(\gamma')\Gamma(\alpha'-\beta')}{\Gamma(\alpha')\Gamma(\gamma'-\beta')} + \frac{\Gamma(2-\gamma)\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \frac{\Gamma(2-\gamma')\Gamma(\alpha'-\beta')}{\Gamma(1-\gamma'+\alpha')\Gamma(1-\beta')} \right),$$

$$B(\tilde{\omega}, \tilde{k}) = (-1)^{iq_{\text{eff}}\ell - 2\lambda - 1 - \Delta} c_4 \left(\frac{\Gamma(2-\gamma)\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \frac{\Gamma(\gamma')\Gamma(\beta'-\alpha')}{\Gamma(\beta')\Gamma(\gamma'-\alpha')} + \frac{\Gamma(2-\gamma)\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \frac{\Gamma(2-\gamma')\Gamma(\beta'-\alpha')}{\Gamma(1-\gamma'+\beta')\Gamma(1-\alpha')} \right). \quad (86)$$

Therefore, the retarded Green's function of the boundary CFT₄ is given by

$$G_R^{\text{AdS}_5} \sim \frac{B(\tilde{\omega}, \tilde{k})}{A(\tilde{\omega}, \tilde{k})} = (-1)^{-2\Delta} \frac{\frac{\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \frac{\Gamma(\gamma')\Gamma(\beta'-\alpha')}{\Gamma(\beta')\Gamma(\gamma'-\alpha')} + \frac{\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \frac{\Gamma(2-\gamma')\Gamma(\beta'-\alpha')}{\Gamma(1-\gamma'+\beta')\Gamma(1-\alpha')}}{\frac{\Gamma(\alpha-\beta)}{\Gamma(1-\gamma+\alpha)\Gamma(1-\beta)} \frac{\Gamma(\gamma')\Gamma(\alpha'-\beta')}{\Gamma(\alpha')\Gamma(\gamma'-\beta')} + \frac{\Gamma(\beta-\alpha)}{\Gamma(1-\gamma+\beta)\Gamma(1-\alpha)} \frac{\Gamma(2-\gamma')\Gamma(\alpha'-\beta')}{\Gamma(1-\gamma'+\alpha')\Gamma(1-\beta')}}}, \quad (87)$$

which is further simplified into

$$G_R^{\text{AdS}_5} \sim (-1)^{-2\Delta} \frac{\Gamma(-2\Delta)}{\Gamma(2\Delta)} \frac{H(\nu; \Delta; \lambda) + H(-\nu; \Delta; \lambda) G_R^{\text{AdS}_2}}{H(\nu; -\Delta; \lambda) + H(-\nu; -\Delta; \lambda) G_R^{\text{AdS}_2}}. \quad (88)$$

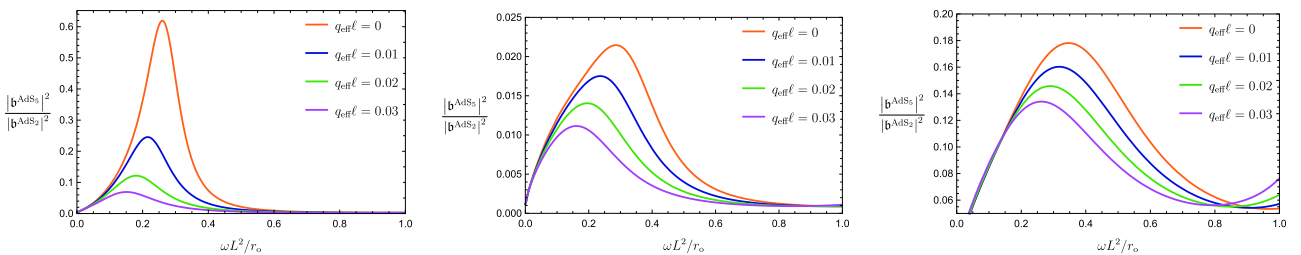


Fig. 1. (color online) Ratio of mean number of produced pairs for the whole spacetime to that in the near horizon region as a function of $\omega L^2/r_0$ for different values of $q_{\text{eff}}\ell$ with $TL^2/r_0 = 0.001$, $\nu = 0.1i$, and $\Delta = 0.1i$ (left); $TL^2/r_0 = 0.001$, $\nu = 0.01i$, and $\Delta = 0.01i$ (middle); $TL^2/r_0 = 0.01$, $\nu = 0.1i$, and $\Delta = 0.1i$ (right).

V. CFTS DESCRIPTIONS IN IR AND UV REGIONS

From the AdS/CFT correspondence, the IR CFT₁ in the near horizon, near extremal limit and the UV CFT₄ at the asymptotic boundary of the RN-AdS₅ black hole can be connected by the holographic RG flow [26, 27]. The CFT description of the Schwinger pair production in the IR region of charged black holes has been systematically studied in a series of previous works [3, 4, 6, 7]. Herein, we address the dual CFTs descriptions in the UV region and compare them with those in the IR region.

The IR CFT₁ of the RN-AdS black hole is very similar to that of the RN black hole in an asymptotically flat spacetime, as CFT₁ can be viewed as a chiral part of CFT₂, which has the universal structures in its correlation functions. For instance, the absorption cross section of a scalar operator O in 2D CFT has the universal form

$$\sigma \sim \frac{(2\pi T_L)^{2h_L-1}}{\Gamma(2h_L)} \frac{(2\pi T_R)^{2h_R-1}}{\Gamma(2h_R)} \sinh\left(\frac{\omega_L - q_L \Omega_L}{2T_L} + \frac{\omega_R - q_R \Omega_R}{2T_R}\right) \times \left| \Gamma\left(h_L + i \frac{\omega_L - q_L \Omega_L}{2\pi T_L}\right) \right|^2 \left| \Gamma\left(h_R + i \frac{\omega_R - q_R \Omega_R}{2\pi T_R}\right) \right|^2, \quad (89)$$

where (h_L, h_R) , (ω_L, ω_R) , and (q_L, q_R) are the left- and right-hand conformal weights, excited energies, charges associated with operator O , respectively, while (T_L, T_R) and (Ω_L, Ω_R) are the temperatures and chemical potentials of the corresponding left- and right-hand sectors of the 2D CFT. Further identifying the variations in the black hole area entropy with those of the CFT microscopic entropy, namely $\delta S_{\text{BH}} = \delta S_{\text{CFT}}$, one derives

$$\frac{\delta M}{T_H} - \frac{\Omega_H \delta Q}{T_H} = \frac{\omega_L - q_L \Omega_L}{T_L} + \frac{\omega_R - q_R \Omega_R}{T_R}, \quad (90)$$

where the left hand side of Eq. (90) is calculated with coordinate (14), for which $\delta M = \xi_o w$, $\delta Q = q$, $T_H = \tilde{T}_n$, and $\Omega_H = 2\mu\ell^2/r_o$, and thus, it is equal to $w/T_n - 2\pi q_{\text{eff}}\ell$. Moreover, the violation of the BF bound in AdS₂ makes the conformal weights of the scalar operator O dual to ϕ a complex, which can be chosen as $h_L = h_R = \frac{1}{2} + i|\nu|$, even without further knowledge about the central charge and (T_L, T_R) of the IR CFT dual to the near extremal RN-AdS₅ black hole. One can also see that the absorption cross section ratio (42) in the AdS₂ spacetime has the form of Eq. (89) up to some coefficients, depending on the mass and charge of the scalar field.

In contrast, the absorption cross section and retarded Green's functions in a general 4D finite temperature CFT cannot be as easily calculated in momentum space as in the 2D CFT. Thus, it is not straightforward to compare

the calculations between the bulk gravity and the boundary CFT sides. Nevertheless, from Eqs. (79) and (80), both the absorption cross section ratio $\sigma_{\text{abs}}^{\text{AdS}_5}$ and the Schwinger mean number of produced pairs $|b^{\text{AdS}_5}|^2$ calculated from the bulk near extremal RN-AdS₅ black hole have a simple proportional relation with their counterparts in the near horizon region. Moreover, the violation of the BF bound (77) in AdS₅ spacetime indicates the complex conformal weights $\bar{\Delta} = 2 + 2i|\Delta|$ of the scalar operator \bar{O} in the UV 4D CFT at the asymptotic spatial boundary of the RN-AdS₅ black hole, which also indicates that, to have pair production in the full bulk spacetime, the corresponding operators in the UV CFT should be unstable. Interestingly, Eq. (83) shows that under the interchange between the roles of source and operator both in the IR and UV CFTs at the same time, namely $h_{L,R} = \frac{1}{2} + i|\nu| \rightarrow \frac{1}{2} - i|\nu|$ and $\bar{\Delta} = 2 + 2i|\Delta| \rightarrow 2 - 2i|\Delta|$, the full absorption cross section ratio $\sigma_{\text{abs}}^{\text{AdS}_5}$ and the Schwinger pair production rate $|b^{\text{AdS}_5}|^2$ are interchanged with each other only up to a minus sign. Note that both the charge and the mass of the scalar particle contribute to the conformal weights $h_{L,R}$ of the scalar operator in the dual IR CFT; however, only the mass contributes to the conformal weight $\bar{\Delta}$ of the scalar operator in the dual UV CFT. Actually, it can be seen from the expressions of the conformal weights that the non-zero charge and mass for the scalar field are crucial for the violation of the BF bound in the corresponding AdS spacetimes and hence guarantee the existence of the Schwinger pair production. However, when the charge of the particle is zero, there will be no Schwinger effect, except for an exponentially suppressed Hawking radiation in near extremal black holes.

VI. SUMMARY AND DISCUSSION

In this paper, we describe our study of the spontaneous scalar pair production in a near extremal RN-AdS₅ black hole that possesses an AdS₂ structure in the IR region and an AdS₅ geometry in the UV region.

We firstly calculated the mean number of produced pairs (see Eq. (41)) in the near horizon region, which has an AdS₂ structure. The retarded Green's function (see Eq. (46)) has also been obtained for this region. Then, we solved the equation for the whole spacetime of the near extremal RN-AdS₅ black hole by using the matching technique. The matching condition we chose is the low temperature limit, i.e., the near extremal limit of the black hole. Therefore, the greybody factor in Eq. (79) and the mean number of produced pairs in Eq. (80) for the whole spacetime are not merely valid for the low frequency limit, and one can easily apply our calculation to the RN-dS black hole, which was recently described in the low fre-

quency limit [35]. Moreover, the retarded Green's function for an RN-AdS black hole has been calculated (see Eq. (88)), which again is valid at finite frequency, and its corresponding value has only been investigated in the low frequency limit [31] before. Interestingly, we found that there exists a very explicit relationship between the mean number of produced pairs (see also Eq. (80)) for the whole spacetime and that in the near horizon region, which enables us to easily compare the pair production rates of these two regions. We showed that, for an near-extremal RN-AdS₅ black hole, the dominant contribution to the pair production rate mainly comes from the near horizon region, as expected.

Moreover, the CFT descriptions of the pair production are investigated both from the AdS₂/CFT₁ corres-

pondence in the IR and the AdS₅/CFT₄ duality in the UV regions, and consistent results and new connections between the pair production rate and the absorption cross section ratio are found, although the related information computed from the finite temperature 4D CFT is incomplete. This work has successfully generalized the study of pair production in charged black holes to the full spacetime and provided new insights for a complete understanding of the pair production process in curved spacetime.

ACKNOWLEDGEMENT

We would like to thank Shu Lin, Rong-Xin Miao, and Yuan Sun for useful discussions.

References

- [1] Julian Schwinger, *Physical Review* **82**(5), 664 (1951)
- [2] Maulik K Parikh and Frank Wilczek, *Physical Review Letters* **85**(24), 5042 (2000)
- [3] Chiang-Mei Chen, Sang Pyo Kim, I-Chieh Lin *et al.*, *Physical Review D* **85**(12), 124041 (2012)
- [4] Chiang-Mei Chen, Jia-Rui Sun, Fu-Yi Tang *et al.*, *Classical and Quantum Gravity* **32**(19), 195003 (2015)
- [5] Chiang-Mei Chen and Sang Pyo Kim, *Physical Review D* **101**(8), 085014 (2020)
- [6] Chiang-Mei Chen, Sang Pyo Kim, Jia-Rui Sun *et al.*, *Physical Review D* **95**(4), 044043 (2017)
- [7] Chiang-Mei Chen, Sang Pyo Kim, Jia-Rui Sun *et al.*, *Physics Letters B* **872**, 129-138 (2018)
- [8] Chiang-Mei Chen, Jia-Rui Sun, and Shou-Jyun Zou, *Journal of High Energy Physics* **2010**(1), 57 (2010)
- [9] Chiang-Mei Chen, Ying-Ming Huang, and Shou-Jyun Zou, *Journal of High Energy Physics* **2010**(3), 123 (2010)
- [10] Chiang-Mei Chen and Jia-Rui Sun, *Journal of High Energy Physics* **2010**(8), 34 (2010)
- [11] Chiang-Mei Chen, Ying-Ming Huang, Jia-Rui Sun *et al.*, *Physical Review D* **82**(6), 066003 (2010)
- [12] Bin Chen and Jia-ju Zhang, *Journal of High Energy Physics* **2013**(1), 155 (2013)
- [13] Bin Chen and Jia-ju Zhang, *Physical Review D* **87**(8), 081505 (2013)
- [14] Chiang-Mei Chen, Ying-Ming Huang, Jia-Rui Sun *et al.*, *Physical Review D* **82**(6), 066004 (2010)
- [15] Thomas Hartman, Keiju Murata, Tatsuma Nishioka *et al.*, *Journal of High Energy Physics* **2009**(04), 019 (2009)
- [16] Thomas Hartman, Wei Song, and Andrew Strominger, *Journal of High Energy Physics* **2010**(3), 118 (2010)
- [17] Jaume Garriga, *Physical Review D* **49**(12), 6343 (1994)
- [18] Boris Pioline and Jan Troost, *Journal of High Energy Physics* **2005**(03), 043 (2005)
- [19] Sang Pyo Kim and Don N Page, *Physical Review D* **78**(10), 103517 (2008)
- [20] Rong-Gen Cai and Sang Pyo Kim, *Journal of High Energy Physics* **2014**(9), 72 (2014)
- [21] Sang Pyo Kim and Don N Page, *Physical Review D* **65**(10), 105002 (2002)
- [22] Sang Pyo Kim and Don N Page, *Physical Review D* **75**(4), 045013 (2007)
- [23] Cesim K Dumlu and Gerald V Dunne, *Physical review letters* **104**(25), 250402 (2010)
- [24] Sang Pyo Kim and Don N Page. arXiv preprint arXiv:1904.09749, 2019.
- [25] Vijay Balasubramanian and Per Kraus, *Physical Review Letters* **83**(18), 3605 (1999)
- [26] Idse Heemskerk and Joseph Polchinski, *Journal of High Energy Physics* **2011**(6), 31 (2011)
- [27] Thomas Faulkner, Hong Liu, and Mukund Rangamani, *Journal of High Energy Physics* **2011**(8), 51 (2011)
- [28] Thomas Faulkner, Hong Liu, John McGreevy *et al.*, *Physical Review D* **83**(12), 125002 (2011)
- [29] Nabil Iqbal and Hong Liu, *Fortschritte der Physik* **57**(5-7), 367-384 (2009)
- [30] Hong Liu, John McGreevy, and David Vegh, *Physical Review D* **83**(6), 065029 (2011)
- [31] Tom Faulkner, Nabil Iqbal, Hong Liu *et al.*, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* **369**(1941), 1640-1669 (2011)
- [32] Peter Breitenlohner and Daniel Z Freedman, *Annals of Physics* **144**(2), 249-281 (1982)
- [33] Peter Breitenlohner and Daniel Z Freedman, *Physics Letters B* **115**(3), 197-201 (1982)
- [34] Xu Dianyan, *Classical and Quantum Gravity* **5**(6), 871 (1988)
- [35] Jamil Ahmed and K Saifullah, *The European Physical Journal C* **78**(4), 316 (2018)