Impact of $a_0^0(980) - f_0(980)$ mixing on the localized *CP* violations of the $B^- \to K^- \pi^+ \pi^-$ decay*

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Abstract: In the framework of the QCD factorization approach, we study the localized *CP* violations of the $B^- \to K^- \pi^+ \pi^-$ decay with and without the $a_0^0(980) - f_0(980)$ mixing mechanism and observe that the localized *CP* violation can be enhanced by this mixing effect when the mass of the $\pi^+ \pi^-$ pair is in the vicinity of the $f_0(980)$ resonance. The corresponding theoretical prediction results are $\mathcal{A}_{CP}(B^- \to K f_0 \to K^- \pi^+ \pi^-) = [0.126, 0.338]$ (0.232 ± 0.106 with the central value form) and $\mathcal{A}_{CP}(B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-) = [0.230, 0.615]$ (0.423 ± 0.193 with the central value form), respectively. Meanwhile, we also calculate the branching fraction of the $B^- \to K^- f_0(980) \to K^- \pi^+ \pi^-$ decay, which is consistent with the experimental results. We suggest that the $a_0^0(980) - f_0(980)$ mixing mechanism should be considered when theoretically and experimentally studying the *CP* violation of the *B* or *D* meson decays.

Keywords: *CP* violation, heavy quark physics, *B* meson decay

DOI: 10.1088/1674-1137/ac6d4e

I. INTRODUCTION

The research on *CP* violation is vital for accurately testing the Standard Model (SM), searching for the origin of *CP* violation, and exploring new physics. *CP* violation has been favored by scientists since its discovery in 1964 [1, 2]. The decays of *B* mesons are important events for us to study the *CP* violation. In recent years, with the operation of large-scale experimental devices such as LHC and BESIII, breakthroughs have been achieved in the experimental and theoretical research on the *CP* violation of the heavy-flavor meson system. In 2013 and 2019, the LHC Collaboration discovered large *CP* violations in the three-body decay of the *B* mesons ($B^\pm \to \pi^+ \pi^+ \pi^-$ and $B^\pm \to K^\pm \pi^+ \pi^-$ decays) [3, 4], which attracted many theorists to explore the mysteries.

The nature of the light scalar mesons has attracted significant interest for decades since its discovery [5–11].

Because of sharing the same quantum numbers, light scalar mesons have an important role in elucidating the QCD vacuum. The $a_0^0(980) - f_0(980)$ mixing mechanism has been a popular research topic because of its potential to reveal the structure of scalar mesons. In the late 1970s, the $a_0^0(980) - f_0(980)$ mixing effect was first suggested theoretically [12]. $a_0^0(980)$ and $f_0(980)$ have the same spin parity quantum numbers but different isospins. Because of the isospin breaking effect, when they decay into $K\bar{K}$, a difference of 8 MeV exists between the charged and neutral kaon thresholds. To date, $a_0^0(980)$ and $f_0(980)$ mixing has been studied extensively in various processes and with respect to its different aspects [13–33]. The signal of this effect was observed for the first time by the BESIII Collaboration in the $J/\psi \to \phi f_0(980) \to \phi a_0^0(980) \to \phi \eta \pi^0$ and $\chi_{c1} \to a_0^0(980)\pi^0 \to f_0(980)\pi^0 \to \pi^+ \pi^- \pi^0$ decays [34]. Inspired by the fact that $\rho - \omega$ mixing (also due

to isospin breaking effect) can induce large *CP* violations when the invariant mass of the $\pi\pi$ pair is in the $\rho - \omega$ mixing effective area [35–37], we aimed to study the $a_0^0(980) - f_0(980)$ mixing effect on the localized *CP* violations in three-body decays of the *B* meson.

In this paper, we investigate the localized *CP* violation by $a_0^0(980) - f_0(980)$ mixing and the branching fraction of the $B^- \to K f_0 \to K^- \pi^+ \pi^-$ decay in the QCDF approach. The remainder of this paper is organized as follows. In Sect. II, we present the formalism for *B* decays in the QCDF approach. In Sect. III, we present the $a_0^0(980) - f_0(980)$ mixing mechanism, calculations of the localized *CP* violation, and the branching fraction of the $B^- \to K f_0 \to K^- \pi^+ \pi^-$ decay. The numerical results are provided in Sect. IV, and we summarize and discuss the paper in Sect V.

II. B DECAYS IN THE QCD FACTORIZATION APPROACH

In the framework of the QCD factorization approach [38, 39], we can obtain the matrix element *B* decaying to two mesons M_1 and M_2 by matching the effective weak Hamiltonian onto a transition operator, which is summarized as follows ($\lambda_p^{(D)} = V_{pb}V_{pD}^*$ with $D = d$ or s)

$$\langle M_1 M_2 | \mathcal{H}_{\text{eff}} | B \rangle = \sum_{p=u,c} \lambda_p^{(D)} \langle M_1 M_2 | \mathcal{T}_A^p + \mathcal{T}_B^p | B \rangle, \quad (1)$$

where $\mathcal{T}_A^p$ and $\mathcal{T}_B^p$ describe the contributions from non-annihilation and annihilation topology amplitudes, respectively, which can be expressed in terms of the parameters a_i^p and b_i^p, respectively, both of which are defined in detail in Ref. [38].

With the operator product expansion, the effective weak Hamiltonian can be expressed as [38]

$$\mathcal{H}_{\text{eff}} = \frac{G_{\text{F}}}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(D)} \Big(c_1 Q_1^p + c_2 Q_2^p + \sum_{i=3}^{10} c_i Q_i$$
$$+ c_{7\gamma} Q_{7\gamma} + c_{8g} Q_{8g} \Big) + \text{h.c.}, \quad (2)$$

where G_{F} represents the Fermi constant, $\lambda_p^{(D)} = V_{pb}V_{pD}^*$ (V_{pb} and V_{pD} are the CKM matrix elements), $D = d, s$ can be a down or strange quark, and $c_i (i = 1, 2, \cdots, 10)$ are the Wilson coefficients. The operators Q_i are given by [39]

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{s}p)_{V-A}, \qquad Q_2^p = (\bar{p}_\alpha b_\beta)_{V-A}(\bar{s}_\beta p_\alpha)_{V-A},$$
$$Q_3 = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V-A}, \quad Q_4 = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V-A},$$
$$Q_5 = (\bar{s}b)_{V-A}\sum_q (\bar{q}q)_{V+A}, \quad Q_6 = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q (\bar{q}_\beta q_\alpha)_{V+A},$$

$$Q_7 = (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V+A},$$
$$Q_8 = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_\beta q_\alpha)_{V+A},$$
$$Q_9 = (\bar{s}b)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}q)_{V-A},$$
$$Q_{10} = (\bar{s}_\alpha b_\beta)_{V-A}\sum_q \frac{3}{2}e_q(\bar{q}_\beta q_\alpha)_{V-A},$$
$$Q_{7\gamma} = \frac{-e}{8\pi^2}m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b,$$
$$Q_{8g} = \frac{-g_s}{8\pi^2}m_b \bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b, \quad (3)$$

where $Q_{1,2}^p$ and Q_{3-10} are the tree and penguin operators, respectively, $Q_{7\gamma}$ is the electromagnetic dipole operator, Q_{8g} is the chromomagnetic dipole operators, α and β are color indices, and $q = u, d, s, c, b$ quarks.

Generally, a_i^p includes the contributions from the naive factorization, vertex correction, penguin amplitude, and spectator scattering terms, which have the following expressions [38]:

$$a_i^p(M_1 M_2) = \left(c_i + \frac{c_{i\pm1}}{N_c}\right)N_i(M_2) + \frac{c_{i\pm1}}{N_c}\frac{C_{\text{F}}\alpha_s}{4\pi}$$
$$\times \left[V_i(M_2) + \frac{4\pi^2}{N_c}H_i(M_1 M_2)\right] + P_i^p(M_2), \quad (4)$$

where $N_i(M_2)$ are the leading-order coefficients, $V_i(M_2)$, $H_i(M_1 M_2)$, and $P_i^p(M_1 M_2)$ are from one-loop vertex corrections, hard spectator interactions with a hard gluon exchange between the emitted meson, and the spectator quark of the *B* meson and penguin contractions, respectively, and their specific forms and derivations are presented in Refs. [5, 38, 40], $C_{\text{F}} = (N_c^2 - 1)/2N_c$ with $N_c = 3$ [38].

The weak annihilation contributions can be expressed in terms of b_i and $b_{i,\text{EW}}$, which are

$$b_1 = \frac{C_{\text{F}}}{N_c^2}c_1 A_1^i, \quad b_2 = \frac{C_{\text{F}}}{N_c^2}c_2 A_1^i,$$
$$b_3^p = \frac{C_{\text{F}}}{N_c^2}\Big[c_3 A_1^i + c_5(A_3^i + A_3^f) + N_c c_6 A_3^f\Big],$$
$$b_4^p = \frac{C_{\text{F}}}{N_c^2}\Big[c_4 A_1^i + c_6 A_2^i\Big],$$
$$b_{3,\text{EW}}^p = \frac{C_{\text{F}}}{N_c^2}\Big[c_9 A_1^i + C_7(A_3^i + A_3^f) + N_c c_8 A_3^f\Big],$$
$$b_{4,\text{EW}}^p = \frac{C_{\text{F}}}{N_c^2}\Big[c_{10} A_1^i + c_8 A_2^i\Big], \quad (5)$$

where the subscripts 1, 2, 3 of $A_n^{i,f}$ ($n = 1, 2, 3$) denote the annihilation amplitudes induced from $(V-A)(V-A)$,

$(V − A)(V + A)$, and $(S − P)(S + P)$ operators, respectively, the superscripts i and f refer to gluon emission from the initial- and final-state quarks, respectively. The explicit expressions for $A_n^{i,f}$ are provided in Ref. [41].

Concretely, $\mathcal{T}_A^p$ contains the contributions from naive factorization, vertex correction, penguin amplitude, and spectator scattering and can be expressed as

$$
\begin{aligned}
\mathcal{T}_A^p =& \delta_{pu}\alpha_1(M_1 M_2)A([\bar{q}_s u][\bar{u}D]) \\
&+ \delta_{pu}\alpha_2(M_1 M_2)A([\bar{q}_s D][\bar{u}u]) \\
&+ \alpha_3^p(M_1 M_2)\sum_q A([\bar{q}_s D][\bar{q}q]) \\
&+ \alpha_4^p(M_1 M_2)\sum_q A([\bar{q}_s q][\bar{q}D]) \\
&+ \alpha_{3,\mathrm{EW}}^p(M_1 M_2)\sum_q \frac{3}{2}e_q A([\bar{q}_s D][\bar{q}q]) \\
&+ \alpha_{4,\mathrm{EW}}^p(M_1 M_2)\sum_q \frac{3}{2}e_q A([\bar{q}_s q][\bar{q}D]),
\end{aligned}
\tag{6}
$$

where the sums extend over $q = u, d, s$, and $\bar{q}_s (= \bar{u}, \bar{d}\,\text{or}\,\bar{s})$ denotes the spectator antiquark. The coefficients $\alpha_i^p(M_1 M_2)$ and $\alpha_{i,\mathrm{EW}}^p(M_1 M_2)$ contain all dynamical information and can be expressed in terms of the coefficients a_i^p.

For the power-suppressed annihilation part, we can parameterize it into the following form:

$$
\mathcal{T}_B^p = \delta_{pu}b_1(M_1 M_2)\sum_{q'} B([\bar{u}q'][\bar{q}'u][\bar{D}b])
$$

$$
\begin{aligned}
&+ \delta_{pu}b_2(M_1 M_2)\sum_{q'} B([\bar{u}q'][\bar{q}'D][\bar{u}b]) \\
&+ b_3^p(M_1 M_2)\sum_{q,q'} B([\bar{q}q'][\bar{q}'D][\bar{q}b]) \\
&+ b_4^p(M_1 M_2)\sum_{q,q'} B([\bar{q}q'][\bar{q}'q][\bar{D}b] \\
&+ b_{3,\mathrm{EW}}^p(M_1 M_2)\sum_{q,q'} \frac{3}{2}e_q B([\bar{q}q'][\bar{q}'D][\bar{q}b]) \\
&+ b_{4,\mathrm{EW}}^p(M_1 M_2)\sum_{q,q'} \frac{3}{2}e_q B([\bar{q}q'][\bar{q}'q][\bar{D}b]),
\end{aligned}
\tag{7}
$$

where $q, q' = u, d, s$ and the sums extend over q, q'. The sum over q' occurs because a quark-antiquark pair must be created via $g \to \bar{q}'q'$ after the spectator quark is annihilated.

III. $a_0^0(980)−f_0(980)$ MIXING MECHANISM, CALCULATION OF *CP* VIOLATION, AND BRANCHING FRACTION

A. $a_0^0(980) - f_0(980)$ **mixing mechanism**

In the condition of turning on the $a_0^0(980) - f_0(980)$ mixing mechanism, we can obtain the propagator matrix of $a_0^0(980)$ and $f_0(980)$ by summing up all the contributions of $a_0^0(980) \to f_0(980) \to \cdots \to a_0^0(980)$ and $f_0(980) \to a_0^0(980) \to \cdots \to f_0(980)$, respectively, which are expressed as [33]

$$
\begin{pmatrix} P_{a_0}(s) & P_{a_0 f_0}(s) \\ P_{f_0 a_0}(s) & P_{f_0}(s) \end{pmatrix} = \frac{1}{D_{f_0}(s)D_{a_0}(s) - |\Lambda(s)|^2} \begin{pmatrix} D_{a_0}(s) & \Lambda(s) \\ \Lambda(s) & D_{f_0}(s) \end{pmatrix},
\tag{8}
$$

where $P_{a_0}(s)$ and $P_{f_0}(s)$ are the propagators of a_0 and f_0, respectively, $P_{a_0 f_0}(s)$, $P_{f_0 a_0}(s)$, and $\Lambda(s)$ occur due to the $a_0^0(980) - f_0(980)$ mixing effect, and $D_{a_0}(s)$ and $D_{f_0}(s)$ are the denominators for the propagators of a_0 and f_0 when the $a_0^0(980) - f_0(980)$ mixing effect is absent, respectively, which can be expressed as follows in the Flatté parametrization:

$$
\begin{aligned}
D_{a_0}(s) =& m_{a_0}^2 - s - \mathrm{i}\sqrt{s}[\Gamma_{\eta\pi}^{a_0}(s) + \Gamma_{K\bar{K}}^{a_0}(s)], \\
D_{f_0}(s) =& m_{f_0}^2 - s - \mathrm{i}\sqrt{s}[\Gamma_{\pi\pi}^{f_0}(s) + \Gamma_{K\bar{K}}^{f_0}(s)],
\end{aligned}
\tag{9}
$$

where m_{a_0} and m_{f_0} are the masses of the a_0 and f_0 mesons, respectively, with the decay width Γ_{bc}^a expressed as

$$
\Gamma_{bc}^a(s) = \frac{g_{abc}^2}{16\pi\sqrt{s}}\rho_{bc}(s) \quad \text{with}
$$

$$
\rho_{bc}(s) = \sqrt{\left[1 - \frac{(m_b - m_c)^2}{s}\right]\left[1 + \frac{(m_b - m_c)^2}{s}\right]}.
\tag{10}
$$

Scholars have indicated that the contribution from the amplitude of $a_0^0(980) - f_0(980)$ mixing is convergent and can be expressed as an expansion in the $K\bar{K}$ phase space when only $K\bar{K}$ loop contributions are considered [12, 42],

$$
\begin{aligned}
\Lambda(s)_{K\bar{K}} =& \frac{g_{a_0 K^+ K^-}g_{f_0 K^+ K^-}}{16\pi}\Big\{\mathrm{i}\Big[\rho_{K^+ K^-}(s) - \rho_{K^0 \bar{K}^0}(s)\Big] \\
&- O(\rho_{K^+ K^-}^2(s) - \rho_{K^0 \bar{K}^0}^2(s))\Big\},
\end{aligned}
\tag{11}
$$

where $g_{a_0 K^+ K^-}$ and $g_{f_0 K^+ K^-}$ are the effective coupling constants. Since the mixing mainly results from the $K\bar{K}$ loops, we can adopt $\Lambda(s) \approx \Lambda_{K\bar{K}}(s)$.

B. Decay amplitudes, localized *CP* violation, and branching fraction

With the $a_0^0(980) - f_0(980)$ mixing being considered, the process of the $B^- \to K^- \pi^+ \pi^-$ decay is shown in Fig. 1 and the amplitude can be expressed as

$$\mathcal{M} = \langle K^- \pi^+ \pi^- | \mathcal{H}^T | B^- \rangle + \langle K^- \pi^+ \pi^- | \mathcal{H}^P | B^- \rangle, \quad (12)$$

in which $\mathcal{H}^T$ and $\mathcal{H}^P$ are the tree and penguin operators, respectively, and we obtain

$$\langle K^- \pi^+ \pi^- | \mathcal{H}^T | B^- \rangle = \frac{g_{f_0 \pi \pi} T_{f_0}}{D_{f_0}} + \frac{g_{f_0 \pi \pi} T_{a_0} \Lambda}{D_{a_0} D_{f_0} - \Lambda^2},$$

$$\langle K^- \pi^+ \pi^- | \mathcal{H}^P | B^- \rangle = \frac{g_{f_0 \pi \pi} P_{f_0}}{D_{f_0}} + \frac{g_{f_0 \pi \pi} P_{a_0} \Lambda}{D_{a_0} D_{f_0} - \Lambda^2}, \quad (13)$$

where $T_{a_0(f_0)}$ and $P_{a_0(f_0)}$ represent the tree and penguin diagram amplitudes for $B \to K a_0(f_0)$ decay, respectively. Substituting Eq. (13) into Eq. (12), the total amplitude of the decay $B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-$ can be expressed as

$$\mathcal{M}(B^- \to K^- \pi^+ \pi^-) = \frac{g_{f_0 \pi \pi}}{D_{f_0}} \mathcal{M}(B^- \to K^- f_0)$$

$$+ \frac{g_{f_0 \pi \pi} \Lambda}{D_{a_0} D_{f_0} - \Lambda^2} \mathcal{M}(B^- \to K^- a_0). \quad (14)$$
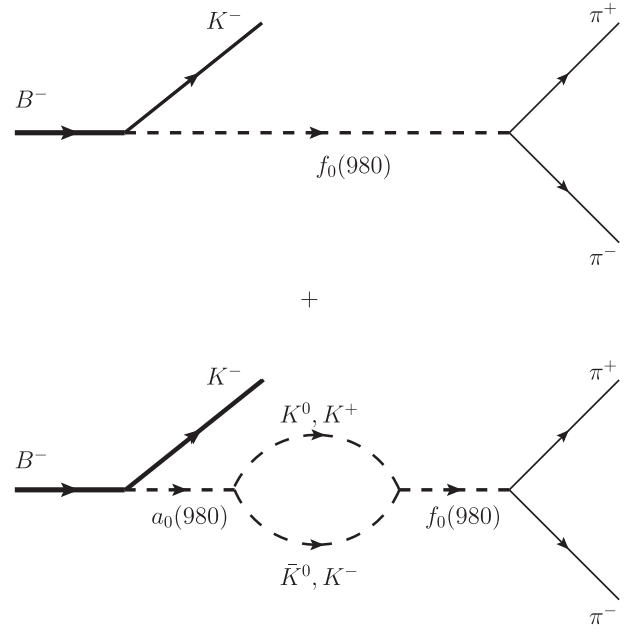
Fig. 1. Feynman diagram for the $B^- \to K^- \pi^+ \pi^-$ decay with the $a_0^0(980) - f_0(980)$ mixing mechanism.

In the QCD factorization approach, we derive the amplitudes of the $B^- \to K^- f_0$ and $B^- \to K^- a_0$ decays, which are

$$\mathcal{M}(B^- \to K^- f_0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ (\delta_{pu} a_1 + a_4^p - r_\chi^K a_6^p + a_{10}^p - r_\chi^K a_8^p)_{f_0^u K} (m_B^2 - m_{f_0}^2) f_K F_0^{B f_0^u}(m_K^2)$$

$$- \Big(\delta_{pu} a_2 + 2a_3^p + 2a_5^p + \frac{1}{2} a_9^p + \frac{1}{2} a_7^p \Big)_{K f_0^u} (m_B^2 - m_K^2) \bar{f}_{f_0^u} F_0^{BK}(m_{f_0}^2)$$

$$- \Big(a_3^p + a_5^p + a_4^p - r_\chi^f a_6^p - \frac{1}{2} a_9^p - \frac{1}{2} a_7^p - \frac{1}{2} a_{10}^p + \frac{1}{2} r_\chi^f a_8^p \Big)_{K f_0^s} (m_B^2 - m_K^2) \bar{f}_{f_0^s} F_0^{BK}(m_{f_0}^2)$$

$$+ (\delta_{pu} b_2 + b_3^p + b_{3,\mathrm{EW}})_{K f_0^u} f_B \bar{f}_{f_0^u} f_K + \Big(\delta_{p,u} b_2 + b_3^p - \frac{1}{2} b_{3,\mathrm{EW}} \Big)_{K f_0^s} f_B \bar{f}_{f_0^s} f_K \Big\}, \quad (15)$$

and

$$\mathcal{M}(B^- \to K^- a_0) = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \Big\{ (\delta_{pu} a_1 + a_4^p - r_\chi^K a_6^p + a_{10}^p - r_\chi^K a_8^p)_{a_0 K} (m_B^2 - m_{a_0}^2) F_0^{B a_0}(m_K^2) f_K$$

$$- \Big(\delta_{pu} a_2 + \frac{3}{2} a_9^p + \frac{3}{2} a_7^p \Big)_{K a_0} (m_B^2 - m_K^2) F_0^{B \to K}(m_{a_0}^2) \bar{f}_{a_0} + (\delta_{pu} b_2 + b_3^p + b_{3,\mathrm{EW}}^p)_{a_0 K} f_B \bar{f}_{a_0} f_K \Big\}, \quad (16)$$

respectively, where G_F represents the Fermi constant; f_B, f_K, $\bar{f}_{f_0}$, and $\bar{f}_{a_0}$ are the decay constants of B, K, f_0, and a_0, respectively; $F_0^{B f_0^u}(m_K^2)$, $F_0^{BK}(m_{f_0}^2)$, and $F_0^{B a_0}(m_K^2)$ are the form factors for the B to f_0, K and a_0 transitions, respectively.

By integrating the numerator and denominator of the differential *CP* asymmetry parameter, we can obtain the

localized integrated *CP* asymmetry, which can be measured using experiments and takes the following form in the region R:

$$A_{CP}^R = \frac{\int_R \mathrm{d}s \, \mathrm{d}s' (|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2)}{\int_R \mathrm{d}s \, \mathrm{d}s' (|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2)}, \quad (17)$$

where s and s' are the invariant masses squared of $\pi\pi$ or $K\pi$ pair in our case, and $\bar{\mathcal{M}}$ is the decay amplitude of the *CP*-conjugate process.

Since the decay process $B^- \to K^-\pi^+\pi^-$ has a three-body final state, the branching fraction of this decay can be expressed as [43]

$$\mathcal{B} = \frac{\tau_B}{(2\pi)^5 16 m_B^2} \int \mathrm{d}s |\mathbf{p}_1^*| |\mathbf{p}_3| \int \mathrm{d}\Omega_1^* \int \mathrm{d}\Omega_3 |\mathcal{M}|^2, \quad (18)$$

in which Ω_1^* and Ω_3 are the solid angles for the final π in the $\pi\pi$ rest frame and for the final K in the B meson rest frame, respectively, $|\mathbf{p}_1^*|$ and $|\mathbf{p}_3|$ are the norms of the three-momenta of final-state π in the $\pi\pi$ rest frame and K in the B rest frame, respectively, which take the following forms:

$$|\mathbf{p}_1^*| = \frac{\sqrt{\lambda(s, m_\pi^2, m_\pi^2)}}{2\sqrt{s}}, |\mathbf{p}_3| = \frac{\sqrt{\lambda(m_B^2, m_K^2, s)}}{2 m_B}, \quad (19)$$

where $\lambda(a,b,c)$ is the Källén function and with the form $\lambda(a,b,c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$.

IV. NUMERICAL RESULTS

When solving the contributions from the hard spectator and the weak annihilation, we encounter the singularity problem of infrared divergence $X = \int_0^1 \mathrm{d}x/(1-x)$. We can adopt the method in Refs. [5, 38, 39] to parameterize the endpoint divergence as $X_{H,A} = (1 + \rho_{H,A}\mathrm{e}^{\mathrm{i}\phi_{H,A}})$ $\ln\frac{m_B}{\Lambda_h}$, where Λ_h is a typical scale of order 0.5 GeV, $\rho_{H,A}$ is an unknown real parameter, and $\phi_{H,A}$ is the free strong phase in the range $[0, 2\pi]$. For convenience, we use the

notations $\rho = \rho_{H,A}$ and $\phi = \phi_{H,A}$. In our calculations, we adopt $\rho \in [0,1]$ and $\phi \in [0, 2\pi]$ for the two-body $B^- \to K^- f_0$ and $B^- \to K^- a_0$ decays. The first term of Eq. (14) is the amplitude of the $B^- \to K^-\pi^+\pi^-$ decay without the effect of the $a_0^0(980) - f_0(980)$ mixing when the mass of the $\pi^+\pi^-$ pair is in the vicinity of the $f_0(980)$ resonance. Substituting this term into Eq. (17), we can obtain the localized *CP* violation of the $B^- \to K^- f_0 \to K^-\pi^+\pi^-$ decay when we consider the integration interval as $[m_{f_0} - \Gamma_{f_0}, m_{f_0} + \Gamma_{f_0}]$, which is $\mathcal{A}_{CP}(B^- \to K f_0 \to K^-\pi^+\pi^-)$ $= [0.126, 0.338]$ (Fig. 2 (a)). Using the central value form, we can express it as $\mathcal{A}_{CP}(B^- \to K f_0 \to K^-\pi^+\pi^-) = 0.232 \pm 0.106$. Substituting Eqs. (15) and (16) into Eq. (14), we can also obtain the total amplitude of the $B^- \to K^- f_0(a_0) \to K^-\pi^+\pi^-$ decay with the $a_0^0(980) - f_0(980)$ mixing mechanism. Subsequently, by inserting it into Eq. (17), we can also obtain the result of the localized *CP* violation in the presence of $a_0^0(980) - f_0(980)$ mixing by integrating the same integration interval as above. The predicted result is $\mathcal{A}_{CP}(B^- \to K^- f_0(a_0) \to K^-\pi^+\pi^-) = [0.230, 0.615]$ (0.423 ± 0.193 with the central value form), which is plotted in Fig. 2 (b). The *CP* violating asymmetry in Fig. 2 (b) is significantly larger than that in Fig. 2 (a). Thus, we conclude that the $a_0^0(980) - f_0(980)$ mixing mechanism can induce a larger localized *CP* violation for the $B^- \to K^-\pi^+\pi^-$ decay. For consistency, we also calculate the branching fraction of the $B^- \to K^-\pi^+\pi^-$ decay with the $a_0^0(980) - f_0(980)$ mixing effect being considered and estimate the relative contribution strength of this effect, $\mathcal{R}$. We obtain $\mathcal{B}(B^- \to K^- f_0(a_0) \to K^-\pi^+\pi^-) = [6.852, 15.64] \times 10^{-6}$ and $\mathcal{R} = [0.0008, 0.017]$, respectively. Compared with the contribution from first term in Eq. (14), that from the second term is very small and can even be ignored when calculating the branching fraction;
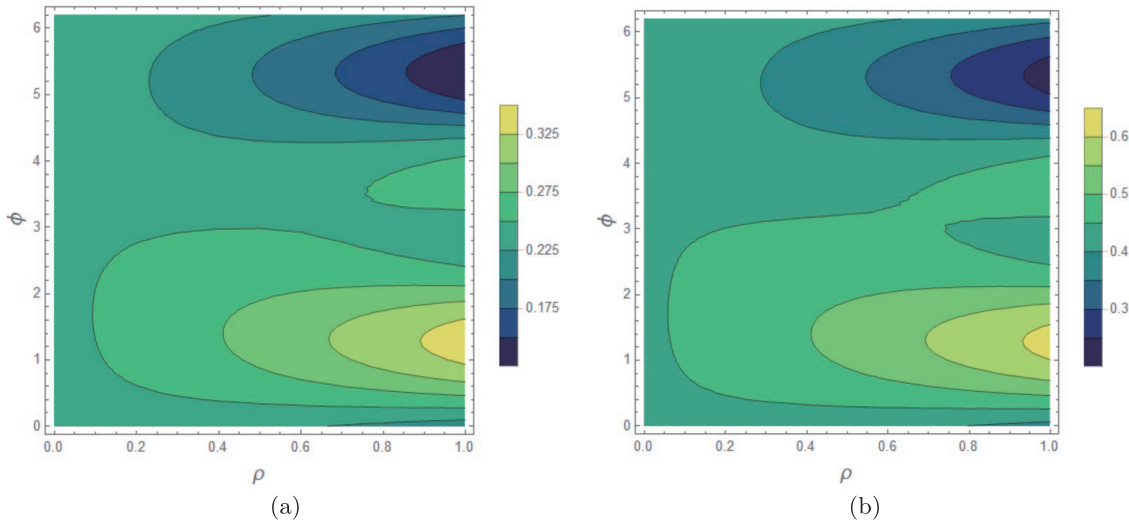
Fig. 2. (color online) Localized *CP* violation of the $B^- \to K^- f_0 \to K^-\pi^+\pi^-$ decay (a) without the $a_0^0(980) - f_0(980)$ mixing mechanism, (b) with the $a_0^0(980) - f_0(980)$ mixing mechanism.
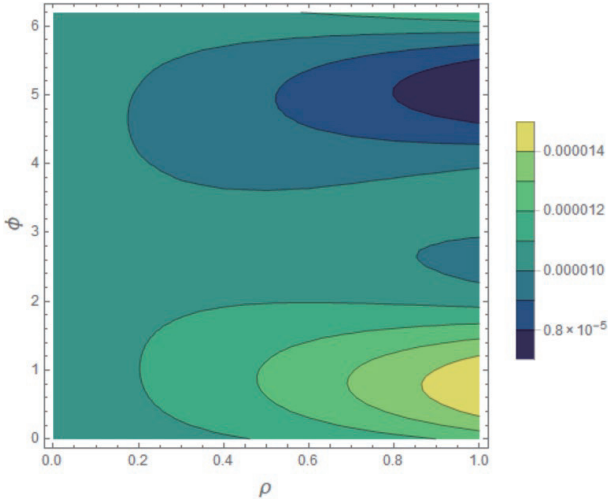
Fig. 3. (color online) Branching fraction of the $B^- \to K^- f_0 \to K^- \pi^+ \pi^-$ decay.

thus, we obtain $\mathcal{B}(B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-) \approx \mathcal{B}(B^- \to K f_0 \to K^- \pi^+ \pi^-)$. Subsequently, we calculate the branching fraction of the $B^- \to K f_0 \to K^- \pi^+ \pi^-$ decay combining the first term in Eq. (14) and Eqs. (15) and (18), the theoretical result is $\mathcal{B}(B^- \to K^- f_0 \to K^- \pi^+ \pi^-) = [7.150, 14.89] \times 10^{-6}$, which is plotted in Fig. 3. This result is consistent with the experimental result $\mathcal{B}(B^- \to K f_0 \to K^- \pi^+ \pi^-) = (9.4^{+1.0}_{-1.2}) \times 10^{-6}$ [44] when the divergence parameter ranges are used as $\rho \in [0,1]$ and $\phi \in [0, 2\pi]$.

In addition to the above specific calculations, from a logical perspective, we can perform the following analysis as in Ref. [45]. The width or the branching fraction of the cascade decay $B^- \to K f_0 \to K^- \pi^+ \pi^-$ can be split into the weak decay $B^- \to K f_0$ and strong decay $f_0 \to \pi^+ \pi^-$, and can be replaced by the width relationship as $\Gamma_{B^- \to K f_0 \to K^- \pi^+ \pi^-} \propto \Gamma_{B^- \to K f_0} \Gamma_{f_0 \to \pi^+ \pi^-}$ adopting the narrow width approximation, with $\Gamma_{B^- \to K f_0} \propto | M_{B^- \to K f_0} + \Delta_{a_0 f_0} M_{B^- \to K a_0} |^2$. The mixing term contribution is small, at most a few percent or less; hence, it is negligible to branching fraction. However, the mixing corrections behave differently for *CP* conjugate processes, which has been discussed in detail in Ref. [45]; $\mathcal{A}_{CP}(B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-) = \mathcal{A}_{CP}^0 + \Delta\mathcal{A}_{CP}$ with a Taylor expansion of $\mathcal{A}_{CP}$ up to $O(\text{mixing}-\text{term})$, where $\mathcal{A}_{CP}^0$ and $\Delta\mathcal{A}_{CP}$ are the *CP* violations without and only considering the $a_0^0(980) - f_0(980)$ mixing effect, respectively. From our calculation, we can derive $\Delta\mathcal{A}_{CP} = 0.191 \pm 0.087$ which is comparable to the value of $\mathcal{A}_{CP}^0 = 0.232 \pm 0.106$ and cannot be ignored. These analysis are consistent with the above calculations. Therefore, we propose that the $a_0^0(980) - f_0(980)$ mixing mechanism be extended and applied to other decays.

V. SUMMARY AND DISCUSSION

In this paper, we have studied the localized integ-

rated *CP* violation of the $B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-$ decay considering the $a_0^0(980) - f_0(980)$ mixing mechanism in the QCD factorization approach. We observe that the localized integrated *CP* violation is enlarged owing to the $a_0^0(980) - f_0(980)$ mixing effect. Without the $a_0^0(980) - f_0(980)$ mixing, the localized *CP* violation is observed to be $\mathcal{A}_{CP}(B^- \to K f_0 \to K^- \pi^+ \pi^-) = [0.126, 0.338]$ (0.232 ± 0.106 with the central value form), while $\mathcal{A}_{CP}(B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-) = [0.230, 0.615]$ (0.423 ± 0.193 with the central value form) when this mixing effect is considered. Meanwhile, the *CP* violation caused by the mixing term $\Delta\mathcal{A}_{CP} = 0.191 \pm 0.087$ is comparable to that of $\mathcal{A}_{CP}^0 = 0.232 \pm 0.106$ when the mixing effect is not considered; therefore, we cannot easily ignore the contribution from the $a_0^0(980) - f_0(980)$ mixing effect. In addition, we also calculate the branching fraction of the $B^- \to K^- f_0 \to K^- \pi^+ \pi^-$ decay and obtain $\mathcal{B}(B^- \to K f_0 \to K^- \pi^+ \pi^-) = [7.150, 14.89] \times 10^{-6}$ as shown in Fig. 3, which agrees well with the experimental result $\mathcal{B}(B^- \to K f_0 \to K^- \pi^+ \pi^-) = 9.4^{+1.0}_{-1.2} \times 10^{-6}$. The mixing corrections for the branching fraction behave differently for *CP* conjugate processes, as mentioned in Sect. IV. Since the mixing term is very small, often a few percent or even less, while calculating the branching fraction, we can ontain the approximation $\mathcal{B}(B^- \to K^- f_0(a_0) \to K^- \pi^+ \pi^-) \approx \mathcal{B}(B^- \to K f_0 \to K^- \pi^+ \pi^-)$ by ignoring the $a_0^0(980) - f_0(980)$ mixing effect. However, for *CP* violation, this mixing effect contributes significantly and cannot be neglected. The same scenario is also expended for other *B* or *D* mesons decay channels. Thus, we suggest that $a_0^0(980) - f_0(980)$ mixing mechanism should be considered when studying the heavy meson decays both theoretically and experimentally when this mixing effect can exist.

APPENDIX A: THEORETICAL INPUT PARAMETERS

In the numerical calculations, we should input distribution amplitudes and the CKM matrix elements in the Wolfenstein parametrization. For the CKM matrix elements, which are determined from experiments, we use the results in Ref. [43]:

$$\bar{\rho} = 0.117 \pm 0.021, \quad \bar{\eta} = 0.353 \pm 0.013,$$

$$\lambda = 0.225 \pm 0.00061, \quad A = 0.811^{+0.023}_{-0.024}, \tag{A1}$$

where

$$\bar{\rho} = \rho\left(1 - \frac{\lambda^2}{2}\right), \quad \bar{\eta} = \eta\left(1 - \frac{\lambda^2}{2}\right). \quad \text{(A2)}$$

The Wilson coefficients used in our calculations are obtained from Refs. [46–49]. It should be noted that the convention in this work is different from that in Ref. [46] for the effective Hamiltonian (from the expressions of the $Q_{1,2}^p$), so we adopt $c_1 = 1.1502$ and $c_2 = -0.3125$:

$$c_1 = 1.1502, \quad c_2 = -0.3125, \quad c_3 = 0.0174,$$
$$c_4 = -0.0373, \quad c_5 = 0.0104, \quad c_6 = -0.0459,$$
$$c_7 = -1.050 \times 10^{-5}, \quad c_8 = 3.839 \times 10^{-4},$$
$$c_9 = -0.0101, \quad c_{10} = 1.959 \times 10^{-3}. \quad \text{(A3)}$$

For the masses of mesons appeared in *B* decays, we use the following values [43] (in units of GeV):

$$m_{B^-} = 5.279, \quad m_{K^-} = 0.494, \quad m_{f_0(980)} = 0.990,$$
$$m_{a_0^0(980)} = 0.980, \quad m_{\pi^\pm} = 0.14, \quad \text{(A4)}$$

whereas, for the widths we use (in GeV) [43]

$$\Gamma_{f_0(980)} = 0.074, \quad \Gamma_{a_0^0(980)} = 0.092. \quad \text{(A5)}$$

The pole masses of quarks are [43] (in GeV):

$$m_u = m_d = 0.0035, \quad m_b = 4.78,$$
$$m_q = \frac{m_u + m_d}{2}, \quad m_c = 1.67. \quad \text{(A6)}$$

The running masses of quarks are [5, 43] (in GeV):

$$m_s(1\text{GeV}) = 0.119, \quad m_c(m_c) = 1.30,$$
$$m_b(m_b) = 4.20, \quad \frac{m_s(\mu)}{m_{u,d}(\mu)} = 27.5. \quad \text{(A7)}$$

The following numerical values for the decay constants are used [5, 50–52](in GeV):

$$f_{\pi^\pm} = 0.131, \quad f_{B^-} = 0.21 \pm 0.02,$$
$$f_{K^-} = 0.156 \pm 0.007,$$
$$\bar{f}_{f_0(980)} = 0.370 \pm 0.02,$$
$$\bar{f}_{a_0^0(980)} = 0.365 \pm 0.02. \quad \text{(A8)}$$

For the form factors, we use [5]

$$F_0^{B \to K}(0) = 0.35 \pm 0.04,$$
$$F_0^{B \to f_0(980)}(0) = 0.25,$$
$$F_0^{B \to a_0^0(980)}(0) = 0.25. \quad \text{(A9)}$$

The values of Gegenbauer moments at $\mu = 1\text{GeV}$ are obtained from [5]:

$$B_{1,f_0(980)} = -0.78 \pm 0.08, \quad B_{3,f_0(980)} = 0.02 \pm 0.07,$$
$$B_{1,a_0^0(980)} = -0.93 \pm 0.10, \quad B_{3,a_0^0(980)} = 0.14 \pm 0.08. \quad \text{(A10)}$$

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