D-wave excited $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC}=1^{++}$ and 1^{+-} *

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Abstract: We study the mass spectra of D-wave excited $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC}=1^{++}$ and 1^{+-} in both symmetric $\mathbf{6}_{cs}\otimes\bar{\mathbf{6}}_{\bar{c}\bar{s}}$ and antisymmetric $\bar{\mathbf{3}}_{cs}\otimes\mathbf{3}_{\bar{c}\bar{s}}$ color configurations using the QCD sum rule method. We construct the D-wave diquark-antidiquark type of $cs\bar{c}\bar{s}$ tetraquark interpolating currents in various excitation structures with $(L_\lambda, L_\rho\{l_{\rho_1}, l_{\rho_2}\})=(2,0\{0,0\}), (1,1\{1,0\}), (1,1\{0,1\}), (0,2\{1,1\}), (0,2\{2,0\}), (0,2\{0,2\})$. Our results support the interpretation of the recently observed $X(4685)$ resonance as a D-wave $cs\bar{c}\bar{s}$ tetraquark state with $J^{PC}=1^{++}$ in the $(2,0\{0,0\})$ or $(0,2\{2,0\})$ excitation mode, although some other possible excitation structures cannot be excluded exhaustively within theoretical errors. Moreover, our results provide the mass relations $6_{\rho\rho}<3_{\lambda\lambda}<3_{\lambda\rho}<3_{\rho\rho}$ and $6_{\rho\rho}<3_{\lambda\lambda}<6_{\lambda\lambda}<3_{\rho\rho}$ for the positive and negative $\mathbb{C}$-parity D-wave $cs\bar{c}\bar{s}$ tetraquarks, respectively. We suggest searching for these possible D-wave $cs\bar{c}\bar{s}$ tetraquarks in both the hidden-charm channels $J/\psi\phi$ and $\eta_c\phi$, as well as open-charm channels such as $D_s\bar{D}_s^*$ and $D_s\bar{D}_{s1}^*$.

Keywords: tetraquark states, exotic states, QCD sum rules

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I. INTRODUCTION

The history of multiquarks can be traced to 1964, when Gell-Mann and Zweig proposed such configurations in building the quark model [1, 2]. Although the existence of tetraquarks and pentaquarks has long been speculated, it has rarely, if ever, been proven. The scenario has changed since 2003, owing to the observation of numerous charmoniumlike/bottomoniumlike XYZ states [3], hidden-charm P_c states [4– 7], doubly-charm T_{cc}^+ states [8, 9], and fully-charm tetraquark [10] states, which cannot be well explained within the traditional quark model. They are very good candidates for tetraquark and pentaquark states. Details regarding the experimental as well as theoretical progress can be found in review papers [11–17].

In 2017, the LHCb Collaboration observed four $J/\psi\phi$ structures, i.e., $X(4140)$, $X(4274)$, $X(4500)$ and $X(4700)$, in the $B^+\to J/\psi\phi K^+$ decay process [18, 19], among which $X(4140)$ and $X(4274)$ were confirmed to be consistent with previous measurements performed by the CDF Collaboration [20, 21], CMS Collaboration [22], D0 Collaboration [23], and BABAR Collaboration [24], while $X(4500)$ and $X(4700)$ were new resonances. Inspired by these structures observed in the $J/\psi\phi$ invariant mass spectrum, $X(4140)$ and $X(4274)$ were considered to be the $cs\bar{c}\bar{s}$ tetraquark ground states, whereas $X(4500)$ and $X(4700)$ were interpreted as the $cs\bar{c}\bar{s}$ tetraquark excited states, in various theoretical methods [25–33].

Recently, the LHCb Collaboration performed an improved full amplitude analysis of the $B^+\to J/\psi\phi K^+$ decay using a signal yield 6 times larger than that previously analyzed [34]. They confirmed the four $J/\psi\phi$ states previously reported in Refs. [18, 19]. In addition, a new $X(4685)$ state was observed in the $J/\psi\phi$ final state with 15σ significance, and its spin-parity was determined to be $J^P=1^+$. Considering its observed channel, the quantum numbers of $X(4685)$ should be $J^{PC}=1^{++}$ with positive charge conjugation parity. Its mass and decay width are measured as $m=4684\pm7^{+13}_{-16}$ MeV and $\Gamma=126\pm15^{+37}_{-41}$ MeV. One may wonder if $X(4685)$ and $X(4700)$ are the same resonance, as they were observed in the same final states with very similar masses and decay widths. However, LHCb determined their spin-parity as $J^P=1^+$ for $X(4685)$ and $J^P=0^+$ for $X(4700)$ [34]. They are definitely two different states.

After the observations of the above $J/\psi\phi$ resonances, there have been efforts to understand their underlying

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structures in the diquark-antidiquark picture. If $X(4140)$ and $X(4274)$ can be assigned as the S-wave $cs\bar{c}\bar{s}$ tetraquark ground states with $J^{PC}=1^{++}$, the $X(4685)$ state may be interpreted as the D-wave $cs\bar{c}\bar{s}$ excited tetraquark state. In Ref. [28], the authors calculated the masses of the excited hidden-charm tetraquarks without internal diquark excitation (λ-mode excitation) by using the relativistic quark model. The mass of the D-wave $cs\bar{c}\bar{s}$ tetraquark with $J^{PC}=1^{++}$ was calculated to be approximately 4.8 GeV. The same λ-mode excited 1^{++} D-wave $cs\bar{c}\bar{s}$ tetraquarks were also studied in the color flux-tube model with masses of approximately 4.9 and 5.2 GeV for the $\bar{\mathbf{3}}_{cs}\otimes\mathbf{3}_{\bar{c}\bar{s}}$ and $\mathbf{6}_{cs}\otimes\bar{\mathbf{6}}_{\bar{c}\bar{s}}$ color structures, respectively [35]. In Ref. [29], the D-wave $cs\bar{c}\bar{s}$ tetraquarks were investigated in different excitation modes by considering internal excited diquarks (ρ-mode excitation) in the relativistic quark model. The masses of the ρ-mode D-wave $cs\bar{c}\bar{s}$ tetraquarks with $J^{PC}=1^{++}$ were predicted as 4.6–4.7 GeV, which are far lower than those of the λ-mode tetraquarks. Additionally, the authors of Ref. [36] calculated the mass of the ground state of the 1^{++} S-wave $cs\bar{c}\bar{s}$ tetraquark to be approximately 4.6 GeV according to the QCD sum rules, which is far higher than those obtained in Ref. [25]. In Ref. [37], $X(4685)$ was also considered as the axialvector $2S$ radial excited $cs\bar{c}\bar{s}$ tetraquark state.

According to the above analyses and theoretical investigations, the newly observed $X(4685)$ state may be explained as a ρ-mode excited D-wave $cs\bar{c}\bar{s}$ tetraquark with $J^{PC}=1^{++}$. In this work, we systematically study the mass spectra of the D-wave $cs\bar{c}\bar{s}$ with $J^{PC}=1^{++}$ and 1^{+-} in both color symmetric $\mathbf{6}_{cs}\otimes\bar{\mathbf{6}}_{\bar{c}\bar{s}}$ and antisymmetric $\bar{\mathbf{3}}_{cs}\otimes\mathbf{3}_{\bar{c}\bar{s}}$ configurations within the framework of QCD sum rules [38, 39]. We investigate the D-wave tetraquarks in different excitation structures, including the ρ-mode and λ-mode.

The remainder of this paper is organized as follows. In Sec. II, we construct the nonlocal D-wave interpolating currents for $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC}=1^{++}$ and 1^{+-} in various excitation structures and color configurations. In Sec. III, we introduce the formalism of tetraquark QCD sum rules and calculate the two-point correlation functions and spectral densities for all currents. We perform numerical analyses to extract the full mass spectra of these D-wave $cs\bar{c}\bar{s}$ tetraquark states in Sec. IV. The last section presents a summary.

II. INTERPOLATING CURRENTS FOR D-WAVE $cs\bar{c}\bar{s}$ TETRAQUARKS

In this section, we construct the D-wave $cs\bar{c}\bar{s}$ tetraquark interpolating currents with $J^{PC}=1^{++}$ and 1^{+-}. The $cs\bar{c}\bar{s}$ tetraquark is composed of cs diquark and $\bar{c}\bar{s}$ antidiquark fields. By analogy with the heavy baryon system, the orbital angular momentum of the tetraquark can be decomposed into $\mathbf{L}=\mathbf{L}_\rho+\mathbf{L}_\lambda=\mathbf{l}_{\rho_1}+\mathbf{l}_{\rho_2}+\mathbf{L}_\lambda$, where $\mathbf{l}_{\rho_1}(\mathbf{l}_{\rho_2})$ represents the internal orbital angular momentum for the $cs(\bar{c}\bar{s})$ field, and $\mathbf{L}_\lambda$ represents the orbital angular momentum between the diquark and antidiquark fields. It is convenient to denote the orbital excitation of the tetraquark system as $(L_\lambda,L_\rho\{l_{\rho_1},l_{\rho_2}\})$, as shown in Fig. 1. The D-wave excited $cs\bar{c}\bar{s}$ tetraquarks are the excitations with $L_\rho+L_\lambda=2$. There exist several different excitation structures for the D-wave tetraquarks: $(L_\lambda,L_\rho\{l_{\rho_1},l_{\rho_2}\})=(2,0\{0,0\})$, $(1,1\{1,0\})$, $(1,1\{0,1\})$, $(0,2\{1,1\})$, $(0,2\{2,0\})$, $(0,2\{0,2\})$. We study all these D-wave tetraquarks by constructing the interpolating currents with the same structures and quantum numbers.

The color structure of a diquark-antidiquark tetraquark operator $[cs][\bar{c}\bar{s}]$ can be expressed via $SU(3)$ symmetry:

$$\begin{aligned}&(\mathbf{3}\otimes\mathbf{3})_{[cs]}\otimes(\bar{\mathbf{3}}\otimes\bar{\mathbf{3}})_{[\bar{c}\bar{s}]}\\=&(\mathbf{6}\oplus\bar{\mathbf{3}})_{[cs]}\otimes(\mathbf{3}\oplus\bar{\mathbf{6}})_{[\bar{c}\bar{s}]}\\=&(\mathbf{6}\otimes\bar{\mathbf{6}})\oplus(\bar{\mathbf{3}}\otimes\mathbf{3})\oplus(\mathbf{6}\otimes\mathbf{3})\oplus(\bar{\mathbf{3}}\otimes\bar{\mathbf{6}})\\=&(\mathbf{1}\oplus\mathbf{8}\oplus\mathbf{27})\oplus(\mathbf{1}\oplus\mathbf{8})\oplus(\mathbf{8}\oplus\mathbf{10})\oplus(\mathbf{8}\oplus\overline{\mathbf{10}}),\end{aligned}\tag{1}$$

in which the color singlet structures come from the $\mathbf{6}_{cs}\otimes\bar{\mathbf{6}}_{\bar{c}\bar{s}}$ and $\bar{\mathbf{3}}_{cs}\otimes\mathbf{3}_{\bar{c}\bar{s}}$ terms, which are denoted as the color symmetric and antisymmetric configurations, respectively. In this work, we consider both these color configurations. We use only the S-wave good diquark field $O_S=c_a^T\mathbb{C}\gamma_5 s_b$ with $J^P=0^+$ to compose the D-wave $cs\bar{c}\bar{s}$ tetraquark currents by inserting covariant derivative operators. For example, one can obtain a ρ-mode P-wave diquark field with $J^P=1^-$

$$O_{P,\mu}=c_a^T\mathbb{C}\gamma_5 D_\mu s_b\,,\tag{2}$$

and a ρ-mode D-wave diquark field with $J^P=2^+$

$$O_{D,\mu\nu}=c_a^T\mathbb{C}\gamma_5 D_\mu D_\nu s_b\,,\tag{3}$$

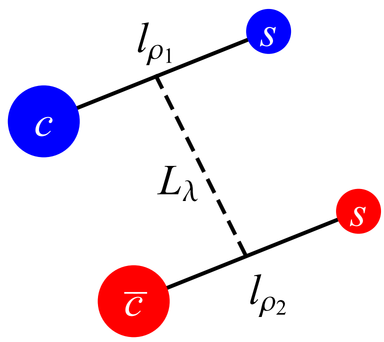

Fig. 1. (color online) Excitation structure of the hidden-charm $cs\bar{c}\bar{s}$ tetraquark system, in which $\mathbf{l}_{\rho_1}(\mathbf{l}_{\rho_2})$ represents the internal orbital angular momentum for the $cs(\bar{c}\bar{s})$ field, and $\mathbf{L}_\lambda$ represents the orbital angular momentum between the diquark and antidiquark fields.

where $D_\mu=\partial_\mu+ig_sA_\mu$ is the covariant derivative, the subscripts a,b are color indices, $\mathbb{C}$ denotes the charge conjugate operator, and T represents the transpose of the quark fields. The corresponding charge conjugate antidiquark fields are

$$\begin{aligned}\bar{O}_S&=\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T,\\ \bar{O}_{P,\mu}&=\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T,\\ \bar{O}_{D,\mu\nu}&=\bar{c}_a\mathbb{C}\gamma_5D_\mu D_\nu\bar{s}_b^T.\end{aligned}\tag{4}$$

To compose the λ-mode excited tetraquark operator, one should insert the covariant derivative operator between the diquark and antidiquark fields.

$$\begin{aligned}L_\lambda=0:&\quad O_S\bar{O}_S,\\ L_\lambda=1:&\quad O_SD_\mu\bar{O}_S,\\ L_\lambda=2:&\quad O_SD_\mu D_\nu\bar{O}_S.\end{aligned}\tag{5}$$

Considering both the symmetric and antisymmetric color configurations, we construct the D-wave $cs\bar{c}\bar{s}$ interpolating tetraquark currents with $J^{PC}=1^{++}$ as

$$\begin{aligned}
J^A_{1,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\{D_\mu,D_\nu\}\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\{D_\mu,D_\nu\}\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{1,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\{D_\mu,D_\nu\}\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\{D_\mu,D_\nu\}\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{2,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu s_b]\,D_\nu\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\,D_\nu\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{2,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu s_b]\,D_\nu\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\,D_\nu\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{3,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\,D_\mu\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\,D_\mu\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^S_{3,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\,D_\mu\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\,D_\mu\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^A_{4,\mu\nu}&=[c_a^T\mathbb{C}D_\mu\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^S_{4,\mu\nu}&=[c_a^T\mathbb{C}D_\mu\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^A_{5,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{5,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{6,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_a]\right),\\
J^S_{6,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_a^T]\right)+[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_a]\right),
\end{aligned}\tag{6}$$

and the D-wave $cs\bar{c}\bar{s}$ interpolating tetraquark currents with $J^{PC}=1^{+-}$ as

$$\begin{aligned}
J^A_{7,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\{D_\mu,D_\nu\}\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\{D_\mu,D_\nu\}\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{7,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\{D_\mu,D_\nu\}\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\{D_\mu,D_\nu\}\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{8,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu s_b]\,D_\nu\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\,D_\nu\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{8,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu s_b]\,D_\nu\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\,D_\nu\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{9,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\,D_\mu\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\,D_\mu\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^S_{9,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\,D_\mu\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\,D_\mu\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^A_{10,\mu\nu}&=[c_a^T\mathbb{C}D_\mu\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^S_{10,\mu\nu}&=[c_a^T\mathbb{C}D_\mu\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\nu s_a]\right),\\
J^A_{11,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5s_b]-[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^S_{11,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]\left([\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5s_b]+[c_b^T\mathbb{C}\gamma_5s_a]\right),\\
J^A_{12,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]-[\bar{c}_b\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]-[c_b^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_a]\right),\\
J^S_{12,\mu\nu}&=[c_a^T\mathbb{C}\gamma_5s_b]\left([\bar{c}_a\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_b^T]+[\bar{c}_b\mathbb{C}\gamma_5D_\mu\,D_\nu\bar{s}_a^T]\right)-[\bar{c}_a\mathbb{C}\gamma_5\bar{s}_b^T]\left([c_a^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_b]+[c_b^T\mathbb{C}\gamma_5D_\mu\,D_\nu s_a]\right),
\end{aligned}\tag{7}$$

where $\{D_\mu, D_\nu\} = D_\mu D_\nu + D_\nu D_\mu$. The interpolating currents with the superscripts "S" and "A" denote the symmetric $[cs]_{\mathbf{6}}[\bar{c}\bar{s}]_{\bar{\mathbf{6}}}$ and antisymmetric $[cs]_{\bar{\mathbf{3}}}[\bar{c}\bar{s}]_{\mathbf{3}}$ color structures, which are abbreviated as **3** and **6**, respectively, hereinafter. The excitation structures $(L_\lambda, L_\rho\{l_{\rho_1}, l_{\rho_2}\})$, color configurations, and J^{PC} quantum numbers for these interpolating currents are presented in Table 1. The abbreviation $\mathbf{3}_{\lambda\lambda}/\mathbf{6}_{\lambda\lambda}$ ($\mathbf{3}_{\rho\rho}/\mathbf{6}_{\rho\rho}$) indicates that the corresponding current contains two λ-orbital (ρ-orbital) momentums with an antisymmetric/symmetric color structure, while $\mathbf{3}_{\lambda\rho}/\mathbf{6}_{\lambda\rho}$ indicates that the current contains one λ-orbital momentum and one ρ-orbital momentum with an antisymmetric/symmetric color structure. In the following, we investigate the mass spectra for the D-wave $cs\bar{c}\bar{s}$ tetraquarks by using these interpolating currents. Among the currents belonging to the $(0,2\{2,0\})$ and $(0,2\{0,2\})$ structures, we only study the $(0,2\{2,0\})$ ones, because the $(0,2\{0,2\})$ currents would yield the same results in our calculations.

Table 1. Excitation structures, color configurations, and J^{PC} quantum numbers for the D-wave $cs\bar{c}\bar{s}$ interpolating currents given by Eqs. (6) and (7).

$(L_\lambda, L_\rho\{l_{\rho_1}, l_{\rho_2}\})$	$[cs]_{\bar{\mathbf{3}}}[\bar{c}\bar{s}]_{\mathbf{3}}$	$[cs]_{\mathbf{6}}[\bar{c}\bar{s}]_{\bar{\mathbf{6}}}$	J^{PC}
$(2,0\{0,0\})$	$J^A_{1,\mu\nu}(\mathbf{3}_{\lambda\lambda})$	$J^S_{1,\mu\nu}(\mathbf{6}_{\lambda\lambda})$	1^{++}
	$J^A_{7,\mu\nu}(\mathbf{3}_{\lambda\lambda})$	$J^S_{7,\mu\nu}(\mathbf{6}_{\lambda\lambda})$	1^{+-}
$(1,1\{1,0\})$	$J^A_{2,\mu\nu}(\mathbf{3}_{\lambda\rho})$	$J^S_{2,\mu\nu}(\mathbf{6}_{\lambda\rho})$	1^{++}
	$J^A_{8,\mu\nu}(\mathbf{3}_{\lambda\rho})$	$J^S_{8,\mu\nu}(\mathbf{6}_{\lambda\rho})$	1^{+-}
$(1,1\{0,1\})$	$J^A_{3,\mu\nu}(\mathbf{3}_{\lambda\rho})$	$J^S_{3,\mu\nu}(\mathbf{6}_{\lambda\rho})$	1^{++}
	$J^A_{9,\mu\nu}(\mathbf{3}_{\lambda\rho})$	$J^S_{9,\mu\nu}(\mathbf{6}_{\lambda\rho})$	1^{+-}
$(0,2\{1,1\})$	$J^A_{4,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{4,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{++}
	$J^A_{10,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{10,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{+-}
$(0,2\{2,0\})$	$J^A_{5,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{5,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{++}
	$J^A_{11,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{11,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{+-}
$(0,2\{0,2\})$	$J^A_{6,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{6,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{++}
	$J^A_{12,\mu\nu}(\mathbf{3}_{\rho\rho})$	$J^S_{12,\mu\nu}(\mathbf{6}_{\rho\rho})$	1^{+-}

III. QCD SUM RULES FOR TETRAQUARK STATES

In this section, we introduce the method of QCD sum rules for the hidden-charm tetraquark states. The two-point correlation functions for the tensor currents can be written as

$$\Pi_{\mu\nu,\rho\sigma}(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x}\langle 0|T\left[J_{\mu\nu}(x)J^\dagger_{\rho\sigma}(0)\right]|0\rangle$$
$$= T^+_{\mu\nu\rho\sigma}\Pi_1(q^2) + \cdots, \tag{8}$$

where

$$T^\pm_{\mu\nu\rho\sigma} = \left(\frac{q_\mu q_\nu}{q^2}\eta_{\nu\sigma} \pm (\mu\leftrightarrow\nu)\right) \pm (\rho\leftrightarrow\sigma),$$
$$\eta_{\mu\nu} = \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}, \tag{9}$$

$\Pi_1\left(q^2\right)$ is the polarization function related to the spin-1 intermediate state, and "$\cdots$" represents other tensor structures relating to different hadron states. The tensor current can couple to the spin-1 physical state X through

$$\langle 0|J_{\mu\nu}(x)|1^{\mathbb{PC}}(\vec{\mathbf{p}},r)\rangle = Z\epsilon^{\mu\nu\alpha\beta}\in_\alpha(\vec{\mathbf{p}},r)p_\beta,$$
$$\langle 0|J_{\mu\nu}(x)|1^{(-\mathbb{P})\mathbb{C}}(\vec{\mathbf{p}},r)\rangle = Z_+(\in^\mu(\vec{\mathbf{p}},r)p^\nu + \in^\nu(\vec{\mathbf{p}},r)p^\mu)$$
$$+ Z_-(\in^\mu(\vec{\mathbf{p}},r)p^\nu - \in^\nu(\vec{\mathbf{p}},r)p^\mu), \tag{10}$$

where Z, Z_+, Z_- are coupling constants, $\epsilon^{\mu\nu\alpha\beta}$ is the antisymmetical tensor, and $\in_\alpha$ is the polarization tensor.

At the hadron level, the two-point correlation function can be written as

$$\Pi(q^2) = \frac{1}{\pi}\int_{s_<}^{\infty}\frac{\mathrm{Im}\Pi(s)}{s - q^2 - \mathrm{i}\epsilon}\mathrm{d}s, \tag{11}$$

where we use the form of the dispersion relation, and $s_<$ denotes the physical threshold. The imaginary part of the correlation function is defined as the spectral function, which is usually evaluated at the hadron level by inserting intermediate hadron states $\sum_n |n\rangle\langle n|$

$$\rho(s) \equiv \frac{1}{\pi}\mathrm{Im}\Pi(s) = \sum_n \delta(s - M_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle$$
$$= f_X^2\delta(s - m_X^2) + \text{continuum}, \tag{12}$$

where we have adopted the usual parametrization of one-pole dominance for the ground state X and a continuum contribution. Researchers have investigated the excited mesons [40–42], baryons [43], and tetraquarks [44–46] in QCD sum rules by using the non-local interpolating currents under the "pole+continuum" approximation. The spectral density $\rho(s)$ can also be evaluated at the quark-gluon level via the operator product expansion (OPE). To pick out the contribution of the lowest lying resonance in (12), the QCD sum rules are established as

$$\mathcal{L}_k(s_0, M_\mathrm{B}^2) = f_X^2 m_H^{2k}\mathrm{e}^{-m_H^2/M_\mathrm{B}^2} = \int_{4m_c^2}^{s_0}\mathrm{d}s\,\mathrm{e}^{-s/M_\mathrm{B}^2}\rho(s)s^k, \tag{13}$$

where M_B represents the Borel mass introduced by the Borel transformation, and s_0 is the continuum threshold. The mass of the lowest-lying hadron can be thus extracted as

$$m_X(s_0,M_B^2)=\sqrt{\frac{\mathcal{L}_1(s_0,M_B^2)}{\mathcal{L}_0(s_0,M_B^2)}},\tag{14}$$

which is the function of two parameters M_B^2 and s_0. We discuss the details of obtaining suitable parameter working regions in QCD sum rule analyses in next section. Using the operator production expansion method, the two-point function can also be evaluated at the quark-gluonic level as a function of various QCD parameters, such as QCD condensates, quark masses, and the strong coupling constant α_s. To evaluate the Wilson coefficients, we adopt the heavy quark propagator in the momentum space and the strange quark propagator in the coordinate space:

$$\begin{aligned}
iS_c^{ab}(p)=&\frac{i\delta^{ab}}{\hat{p}-m_c}+\frac{i}{4}g_s\frac{\lambda^n_{ab}}{2}G^n_{\mu\nu}\frac{\sigma^{\mu\nu}(\hat{p}+m_c)+(\hat{p}+m_c)\sigma^{\mu\nu}}{12}\\
&+\frac{i\delta^{ab}}{12}\langle g_s^2GG\rangle m_c\frac{p^2+m_c\hat{p}}{(p^2-m_c^2)^4},\\
iS_s^{ab}(x)=&\frac{i\delta^{ab}}{2\pi^2x^4}\hat{x}-\frac{\delta^{ab}}{12}\langle\bar{s}s\rangle+\frac{i}{32\pi^2}\frac{\lambda^n_{ab}}{2}g_sG^n_{\mu\nu}\frac{1}{x^2}(\sigma^{\mu\nu}\hat{x}+\hat{x}\sigma^{\mu\nu})\\
&+\frac{\delta^{ab}x^2}{192}\langle\bar{s}g_s\sigma\cdot Gs\rangle-\frac{m_s\delta^{ab}}{4\pi^2x^2}+\frac{i\delta^{ab}m_s\langle\bar{s}s\rangle}{48}\hat{x}\\
&-\frac{im_s\langle\bar{s}g_s\sigma\cdot Gs\rangle\delta^{ab}x^2\hat{x}}{1152},
\end{aligned}\tag{15}$$

where $\hat{p}=p^\mu\gamma_\mu$ and $\hat{x}=x^\mu\gamma_\mu$. In this work, we evaluate the Wilson coefficients of the correlation function up to dimension ten condensates at the leading order of α_s. We find that the calculations are highly complex owing to the existence of the covariant derivative operators. The results of spectral functions are too lengthy to present here; thus, they are provided in the Appendix.

IV. MASS SUM RULE ANALYSES

In this section, we perform the QCD sum rule analyses for the $cs\bar{c}\bar{s}$ tetraquark systems. We use the following values of the quark masses and various QCD condensates [3, 47–55]:

$$\begin{aligned}
&m_c(m_c)=1.27\pm0.02\ \text{GeV},\\
&m_c/m_s=11.76^{+0.05}_{-0.10},\\
&\langle\bar{q}q\rangle=-(0.24\pm0.03)^3\ \text{GeV}^3,\\
&\langle\bar{q}g_s\sigma\cdot Gq\rangle=-M_0^2\langle\bar{q}q\rangle,\\
&\langle\bar{q}q\bar{q}q\rangle=\langle\bar{q}q\rangle^2,\\
&M_0^2=(0.8\pm0.2)\ \text{GeV}^2,\\
&\langle\bar{s}s\rangle/\langle\bar{q}q\rangle=0.8\pm0.1,\\
&\langle g_s^2GG\rangle=(0.48\pm0.14)\ \text{GeV}^4,
\end{aligned}\tag{16}$$

where the charm quark mass m_c is the "running" mass in the $\overline{\text{MS}}$ scheme. To ensure the unified renormalization scale in our analyses, we use the renormalization scheme and scale independent m_c/m_s mass ratio from PDG [3] to obtain the strange quark mass m_s.

To establish a stable mass sum rule, one should initially find the appropriate parameter working regions, i.e, for the continuum threshold s_0 and the Borel mass M_B^2. The threshold s_0 can be determined via the minimized variation of the hadronic mass m_X with respect to the Borel mass M_B^2. The lower bound on the Borel mass M_B^2 can be fixed by requiring a reasonable OPE convergence, while its upper bound is determined through a sufficient pole contribution. The pole contribution is defined as

$$\text{PC}(s_0,M_B^2)=\frac{\mathcal{L}_0(s_0,M_B^2)}{\mathcal{L}_0(\infty,M_B^2)},\tag{17}$$

where $\mathcal{L}_0$ is defined in Eq. (13).

As an example, we use the color antisymmetric current $J^A_{5,\mu\nu}(x)$ with $J^{PC}=1^{++}$ in the $(0,2\{2,0\})$ excitation mode to show the details of the numerical analysis. For this current, the dominant non-perturbative contribution to the correlation function comes from the quark condensate, which is proportional to the charm quark mass m_c. Figure 2 shows the contributions of the perturbative term and various condensate terms to the correlation function with respect to M_B^2 when s_0 tends to infinity. It is clear that the Borel mass M_B^2 should be large enough to ensure the convergence of the OPE series. In this work, we require that the perturbative term be two times larger than the quark condensate term, providing the lower bound of the Borel mass $M_B^2\geq2.82\ \text{GeV}^2$. The other QCD condensates are far smaller than the quark condensate in this region of M_B^2. Studying the pole contribution defined in Eq. (17) reveals that the PC is very small for such D-wave $cs\bar{c}\bar{s}$ tetraquark systems owing to the high dimension of the interpolating current. To find an upper

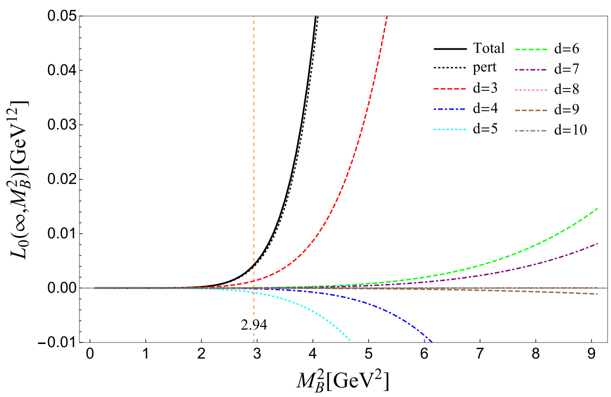

Fig. 2. (color online) Contributions of various OPE terms to the correlation function for the current $J^A_{5,\mu\nu}(x)$ as a function of M_B^2 when $s_0\to\infty$.

bound on the Borel mass, we require the pole contribution to be larger than 20%. As a result, the reasonable Borel window for the current $J^A_{5,\mu\nu}(x)$ is obtained as $2.94\ \text{GeV}^2 \le M_\text{B}^2 \le 3.90\ \text{GeV}^2$.

As mentioned previously, the variation of the extracted hadron mass m_X with respect to M_B^2 should be minimized to obtain the optimal value of the continuum threshold s_0. We show the variation of m_X with s_0 in the left panel of Fig. 3, from which the optimized value of the continuum threshold can be chosen as $s_0 \approx (30.0 \pm 1.5)$ GeV^2. In the right panel of Fig. 3, the mass sum rules are established to be very stable in the above parameter regions of s_0 and M_B^2. The hadron mass for this D-wave $cs\bar{c}\bar{s}$ tetraquark with $J^{PC} = 1^{++}$ can be obtained as

$$m_{J^A_5} = 5.16^{+0.12}_{-0.13}\ \text{GeV}, \qquad (18)$$

where the errors come from the uncertainties of the threshold s_0, Borel mass M_B^2, quark masses, and various QCD condensates in Eq. (16). Performing the same numerical analyses for all the interpolating currents in Eqs. (6)–(7), we find that only the currents $J^S_{5,\mu\nu}(x)$, $J^{A(S)}_{11,\mu\nu}(x)$, and $J^{A(S)}_{4,\mu\nu}(x)$ with $J^{PC} = 1^{++}$ exhibit the same mass sum rule behaviors as $J^A_{5,\mu\nu}(x)$. We present the numerical results in Table 2.

Except for $J^{A(S)}_{5,\mu\nu}(x)$, $J^{A(S)}_{11,\mu\nu}(x)$, and $J^{A(S)}_{4,\mu\nu}(x)$, the interpolating currents exhibit very different mass sum rule behaviors. As shown in the left panel for $J^S_{1,\mu\nu}(x)$, the extracted hadron mass increases monotonically with the continuum threshold s_0. Thus, one is not able to find an optimal value of s_0 to minimize the variation of the hadron mass with respect to M_B^2. For such a situation, we define

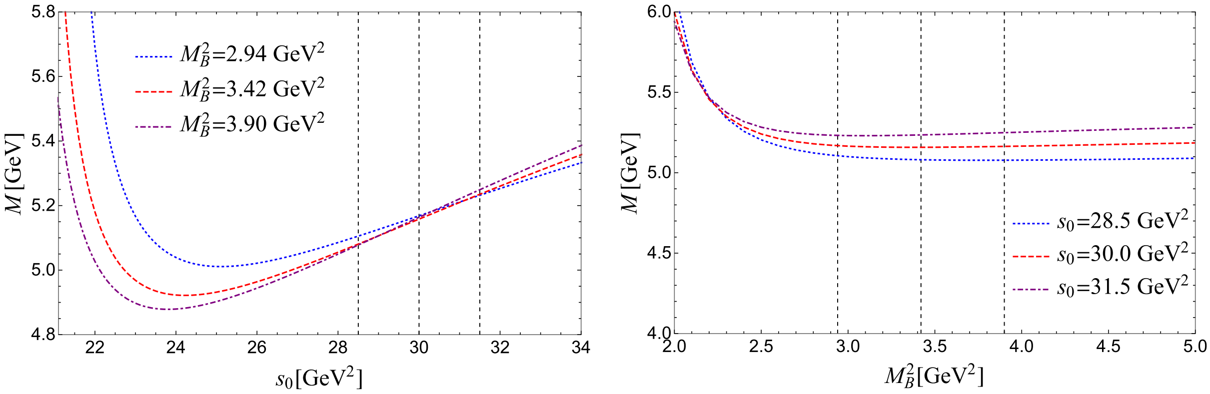

Fig. 3. (color online) Mass curves for the interpolating current $J^A_{5,\mu\nu}(x)$ with $J^{PC} = 1^{++}$.

Table 2. Hadron masses of the $cs\bar{c}\bar{s}$ tetraquark states with different J^{PC} quantum numbers and $(L_\lambda, L_\rho\{l_{\rho_1}, l_{\rho_2}\})$ excitation structures. The subscripts "A" and "S" denote the numerical results for the color antisymmetric and symmetric currents, respectively.

$(L_\lambda, L_\rho\{l_{\rho_1}, l_{\rho_2}\})$	Current	J^{PC}	m_A/GeV	$s_{0,A}$/GeV^2	$M^2_{\text{B},A}$/GeV^2	PC_A(%)	m_S/GeV	$s_{0,S}$/GeV^2	$M^2_{\text{B},S}$/GeV^2	PC_S(%)
(2,0{0,0})	$J^{A(S)}_{1,\mu\nu}$	1^{++}	$4.70^{+0.12}_{-0.11}$	27(±5%)	3.27 ~ 3.92	27.3	$4.91^{+0.11}_{-0.12}$	28(±5%)	3.56 ~ 4.20	26.5
(2,0{0,0})	$J^{A(S)}_{7,\mu\nu}$	1^{+-}	$4.78^{+0.12}_{-0.11}$	27(±5%)	3.58 ~ 4.16	25.4	$4.89^{+0.10}_{-0.11}$	28(±5%)	3.60 ~ 4.50	28.5
(1,1{1,0})	$J^{A(S)}_{2,\mu\nu}$	1^{++}	$4.80^{+0.12}_{-0.16}$	28(±5%)	3.15 ~ 3.94	39.6	$4.84^{+0.12}_{-0.16}$	29(±5%)	2.63 ~ 4.13	37.9
(1,1{1,0})	$J^{A(S)}_{8,\mu\nu}$	1^{+-}	4.81 ± 0.10	27(±5%)	3.71 ~ 4.51	26.3	$4.85^{+0.11}_{-0.10}$	28(±5%)	4.69 ~ 5.16	28.2
(1,1{0,1})	$J^{A(S)}_{3,\mu\nu}$	1^{++}	$4.80^{+0.11}_{-0.10}$	26(±5%)	2.75 ~ 3.31	26.1	$4.82^{+0.12}_{-0.11}$	27(±5%)	3.37 ~ 4.11	47.0
(1,1{0,1})	$J^{A(S)}_{9,\mu\nu}$	1^{+-}	$4.98^{+0.13}_{-0.23}$	26(±5%)	2.73 ~ 3.14	24.0	$4.92^{+0.11}_{-0.10}$	28(±5%)	3.55 ~ 3.91	23.4
(0,2{1,1})	$J^{A(S)}_{4,\mu\nu}$	1^{++}	$4.80^{+0.10}_{-0.11}$	26(±5%)	2.51 ~ 3.14	27.5	$4.80^{+0.10}_{-0.11}$	26(±5%)	2.52 ~ 3.15	27.4
(0,2{1,1})	$J^{A(S)}_{10,\mu\nu}$	1^{+-}	$4.83^{+0.10}_{-0.11}$	28(±5%)	3.06 ~ 3.82	28.6	$4.83^{+0.10}_{-0.12}$	28(±5%)	3.08 ~ 3.82	28.3
(0,2{2,0})	$J^{A(S)}_{5,\mu\nu}$	1^{++}	$5.16^{+0.12}_{-0.13}$	30(±5%)	2.94 ~ 3.90	41.4	4.69 ± 0.09	24(±5%)	2.22 ~ 2.82	27.5
(0,2{2,0})	$J^{A(S)}_{11,\mu\nu}$	1^{+-}	$5.19^{+0.12}_{-0.13}$	30(±5%)	3.55 ~ 3.92	43.4	4.67 ± 0.09	23(±5%)	2.69 ~ 2.87	21.6
$(1,1)_\text{mix}$	$J^{A(S)}_{2,\mu\nu} + J^{A(S)}_{3,\mu\nu}$	1^{++}	4.80 ± 0.10	27(±5%)	3.01 ~ 3.76	24.1	$4.93^{+0.09}_{-0.10}$	29(±5%)	3.22 ~ 4.02	38.4
$(1,1)_\text{mix}$	$J^{A(S)}_{8,\mu\nu} + J^{A(S)}_{9,\mu\nu}$	1^{+-}	$4.80^{+0.11}_{-0.13}$	26(±5%)	2.71 ~ 3.13	30.2	4.94 ± 0.10	29(±5%)	3.37 ~ 4.21	38.2

the following hadron mass $\bar{m}_X$ and quantity $\chi^2(s_0)$ to study the stability of the mass sum rules:

$$\bar{m}_X(s_0)=\sum_{i=1}^{N}\frac{m_X(s_0,M_{\mathrm{B},i}^2)}{N}, \tag{19}$$

$$\chi^2(s_0)=\sum_{i=1}^{N}\left[\frac{m_X(s_0,M_{\mathrm{B},i}^2)}{\bar{m}_X(s_0)}-1\right]^2, \tag{20}$$

where $M_{\mathrm{B},i}^2(i=1,2,\ldots,N)$ represents N definite values for the Borel parameter M_{B}^2 in the Borel window. According to the above definition, the optimal choice for the continuum threshold s_0 in the QCD sum rule analysis can be obtained by minimizing the quantity $\chi^2(s_0)$, which is a function of only s_0. This relation is shown in the right panel of Fig. 4, in which there is a minimum point at approximately $s_0\approx 28.0\,\mathrm{GeV}^2$. We can thus determine the working range for the continuum threshold to be $s_0=(28.0\pm1.4)\,\mathrm{GeV}^2$, as shown in the left panel of Fig. 4. The hadron mass is thus obtained as

$$m_{J_1^S}=4.91^{+0.11}_{-0.12}\,\mathrm{GeV}. \tag{21}$$

In these analyses, we find that the OPE series for the $J^{A(S)}_{4,\mu\nu}(x)$ and $J^{A(S)}_{10,\mu\nu}(x)$ belonging to the $(0,2\{1,1\})$ structure differ significantly from those of other interpolating currents. As shown in the Appendix, the quark condensate does not contribute to the correlation function for any of the $(0,2\{1,1\})$ currents.

By performing similar analyses, we obtain the numerical results for all the other interpolating currents in Eqs. (6) and (7), and they are presented in Table 2. The extracted hadron masses from $J^{A}_{1,\mu\nu}(x)$ and $J^{S}_{5,\mu\nu}(x)$ with $J^{PC}=1^{++}$ agree well with the mass of the newly observed resonance $X(4685)$, implying that $X(4685)$ can be interpreted as a D-wave $cs\bar{c}\bar{s}$ tetraquark state with $J^{PC}=1^{++}$ in the excitation mode of $(2,0\{0,0\})$ or $(0,2\{2,0\})$.

Considering the same physical picture for the $(1,1\{1,0\})$ and $(1,1\{0,1\})$ excitation structures, the interpolating currents $J^{A(S)}_{2,\mu\nu}(x)$ and $J^{A(S)}_{3,\mu\nu}(x)$ exhibit similar mass sum rules. The currents $J^{A}_{2,\mu\nu}(x)$ and $J^{A}_{3,\mu\nu}(x)$ give almost degenerate hadron masses, as shown in Table 2. To study their mixing effects, we also perform analyses for the mixed currents $J^{A(S)}_{2,\mu\nu}+J^{A(S)}_{3,\mu\nu}$. Our calculations show that the off-diagonal correlator $\Pi^{A(S)}_{23}(q^2)$ is nonzero, implying that the currents $J^{A}_{2,\mu\nu}(x)$ and $J^{A}_{3,\mu\nu}(x)$ may couple to the same hadron state. The same situation arises for the interpolating currents $J^{A(S)}_{8,\mu\nu}(x)$ and $J^{A(S)}_{9,\mu\nu}(x)$, which couple to the same tetraquark state.

V. CONCLUSION AND DISCUSSION

We investigated the mass spectra for the D-wave $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC}=1^{++}$ and 1^{+-} in the framework of QCD sum rules. We constructed the D-wave non-local interpolating tetraquark currents with covariant derivative operators in the $(L_\lambda,L_\rho\{l_{\rho_1},l_{\rho_2}\})=(2,0\{0,0\})$, $(1,1\{1,0\}),(1,1\{0,1\}),(0,2\{1,1\}),(0,2\{2,0\}),(0,2\{0,2\})$ excitation structures. The two-point correlation functions were calculated up to dimension ten condensates in the leading order of α_s. We established reliable mass sum rules for all these currents and obtained the mass spectra of D-wave $cs\bar{c}\bar{s}$ tetraquarks, as shown in Table 2. Our results support the interpretation of the recently observed $X(4685)$ structure as a D-wave $cs\bar{c}\bar{s}$ tetraquark state with $J^{PC}=1^{++}$ in the $(2,0\{0,0\})$ or $(0,2\{2,0\})$ excitation mode. However, some other possibilities of the excitation modes cannot be excluded by our results within errors.

The mass spectra of $cs\bar{c}\bar{s}$ tetraquark states in different color configurations were studied in Ref. [35], and the results indicated that the masses of color symmetric tetraquarks are lower than those of color antisymmetric tetraquarks in the ground state ($L=0$). Similar results were obtained for the fully heavy tetraquark states [56–58]. However, the situation is different for the excited $cs\bar{c}\bar{s}$ tetraquarks: the masses of color antisymmetric tetraquarks are lower than those of color symmetric tetra-

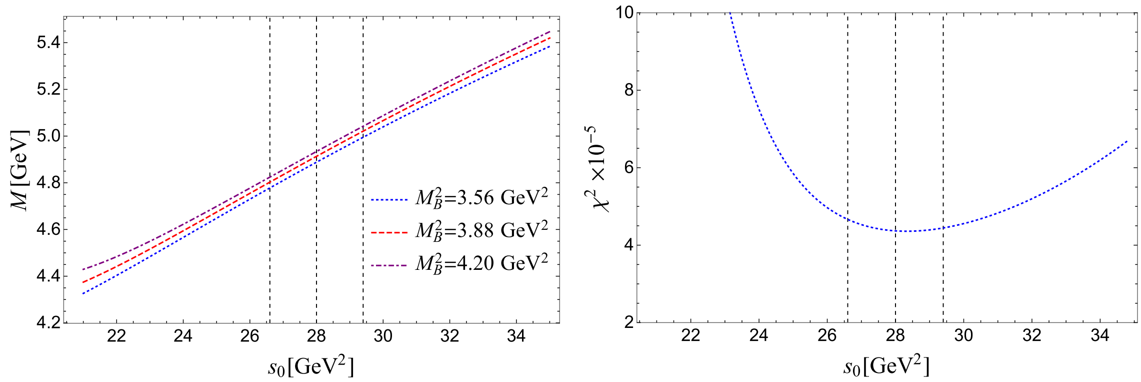

Fig. 4. (color online) Mass curves (left) and χ^2 curve (right) for the current $J^{S}_{1,\mu\nu}(x)$ with $J^{PC}=1^{++}$.

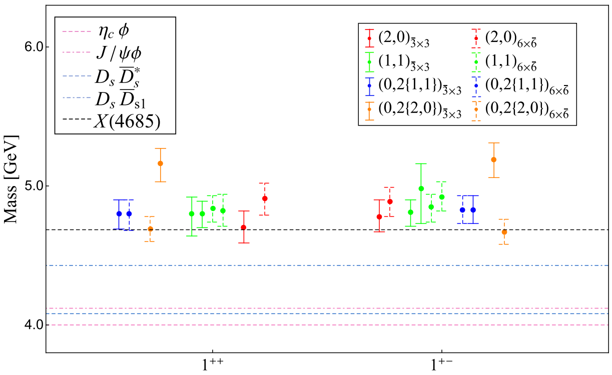

Fig. 5. (color online) Mass spectra for the D-wave $cs\bar{c}\bar{s}$ tetraquark with $J^{PC}=1^{++}$ and 1^{+-}, compared with the corresponding two-meson mass thresholds.

quarks. Such behavior is consistent with our results in Table 2 for the D-wave $cs\bar{c}\bar{s}$ tetraquarks, except for those in the $(0,2\{2,0\})$ structures with two ρ-mode excitations. In Table 2, the masses for the positive $\mathbb{C}$-parity tetraquarks follow the relation $6_{\rho\rho}<3_{\lambda\lambda}<3_{\lambda\rho}<3_{\rho\rho}$, and those for the negative $\mathbb{C}$-parity tetraquarks exhibit the relation $6_{\rho\rho}<3_{\lambda\lambda}<6_{\lambda\lambda}<3_{\rho\rho}$, which is consistent with the conclusion for P-wave $cc\bar{c}\bar{c}$ systems [57].

We present the mass spectra of these $cs\bar{c}\bar{s}$ tetraquarks in comparison with the corresponding two-meson open-charm mass thresholds in Fig. 5. Clearly, these D-wave $cs\bar{c}\bar{s}$ tetraquarks with $J^{PC}=1^{++}$ and 1^{+-} lie above the mass thresholds of $D_s\bar{D}_s^*$, $J/\psi\phi$, and $\eta_c\phi$. Accordingly, we present their possible decay channels in both the S-wave and P-wave in Table 3. We suggest searching for these D-wave $cs\bar{c}\bar{s}$ tetraquarks in both the hidden-charm channels $J/\psi\phi$ and $\eta_c\phi$, as well as open-charm channels such as $D_s\bar{D}_s^*$ and $D_s\bar{D}_{s1}^*$.

Table 3. Possible decay channels of the D-wave $cs\bar{c}\bar{s}$ tetraquark states with $J^{PC}=1^{++}$ and 1^{+-}.

J^{PC}	S-wave	P-wave
1^{++}	$D_{s0}^*\bar{D}_{s1}, D_s\bar{D}_s^*, D_s\bar{D}_{s1}^*,$	$D_s\bar{D}_{s1}, D_{s0}^*\bar{D}_s^*, D_{s0}^*\bar{D}_{s1}^*,$
	$D_{s1}\bar{D}_{s1}, D_{s1}\bar{D}_{s2}^*,$	$D_s^*\bar{D}_{s1}, D_{s1}^*\bar{D}_{s1}, D_s^*\bar{D}_{s2}^*,$
	$J/\psi\phi$	$D_{s1}^*\bar{D}_{s2}^*, h_c(1P)\phi$
1^{+-}	$D_{s0}^*\bar{D}_{s1}, D_s\bar{D}_s^*, D_s\bar{D}_{s1}^*,$	$D_s\bar{D}_{s1}, D_{s0}^*\bar{D}_s^*, D_{s0}^*\bar{D}_{s1}^*,$
	$D_{s1}\bar{D}_{s1}, D_{s1}\bar{D}_{s2}^*,$	$D_s^*\bar{D}_{s1}, D_{s1}^*\bar{D}_{s1}, D_s^*\bar{D}_{s2}^*,$
	$\eta_c\phi$	$\chi_{c0}(1P)\phi, \chi_{c1}(1P)\phi$

APPENDIX: SPECTRAL FUNCTIONS FOR *D*-WAVE INTERPOLATING CURRENT

The spectral functions for the D-wave interpolating current $J_i^{A(S)}$ can be written as

$$\begin{aligned}\rho_{i;A(S)}(s)=&\rho^{\rm pert}_{i;A(S)}(s)+\langle\bar{s}s\rangle\rho^{\langle\bar{s}s\rangle}_{i;A(S)}(s)+m_s\langle\bar{s}s\rangle\rho^{m_s\langle\bar{s}s\rangle}_{i;A(S)}(s)+\langle g_s^2G^2\rangle\rho^{\langle g_s^2G^2\rangle}_{i;A(S)}(s)+\langle\bar{s}\sigma\cdot Gs\rangle\rho^{\langle\bar{s}\sigma\cdot Gs\rangle}_{i;A(S)}(s)+m_s\langle\bar{s}\sigma\cdot Gs\rangle\rho^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}_{i;A(S)}(s)\\&+\langle\bar{s}s\bar{s}s\rangle\rho^{\langle\bar{s}s\bar{s}s\rangle}_{i;A(S)}(s)+\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle\rho^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{i;A(S)}(s)+\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\rho^{\langle g_s^2G^2\rangle\langle\bar{s}s\rangle}_{i;A(S)}(s)+m_s\langle g_s^2G^2\rangle\langle\bar{s}s\rangle\rho^{m_s\langle g_s^2G^2\rangle\langle\bar{s}s\rangle}_{i;A(S)}(s)\\&+\langle g_s^2G^2\rangle^2\rho^{\langle g_s^2G^2\rangle^2}_{i;A(S)}(s)+\langle g_s^2G^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle\rho^{\langle g_s^2G^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{i;A(S)}(s)+m_s\langle g_s^2G^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle\rho^{m_s\langle g_s^2G^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{i;A(S)}(s).\end{aligned}\tag{22}$$

The spectral functions for the $(2,0\{0,0\})$ structure are given as follows:

$$\begin{aligned}\rho^{\rm pert}_{1,7;A(S)}(s)=&-\int_0^{x_{\max}}{\rm d}x\int_{y_{\min}}^{y_{\max}}{\rm d}y\frac{x}{1612800\pi^5(y-1)^5}F(s,x,y)^3c_1\,(2(x-1)\,(10((x(5(13x-42)x+273)-140)x\\&+35)y^2-28\left(39x^2-45x+20\right)y-21x(2x+5)+\left(10((x(2(x-35)x+189)-140)x+35)y^2+28(x((15x\right.\\&\left.-74)x+70)-20)y+21((23x-30)x+10))\,c_p+210\right)x^2yF(s,x,y)^3+42x((x-1)(y-1)\,(10((x(5(13x\\&-42)x+273)-140)x+35)y^4-((x(50x+923)-1165)x+590)y^3+2((32x-165)x+190)y^2+60(x)\\&-2)y+10sx+\left(-4((x((59x-184)x+195)-90)x+15)xy^3+((x(58(x-3)x+195)-120)x+30)y^2\right.\\&+2(3(16x-45)x+110)xy-60y+5(3x-8)x+30)\,m_cm_s+c_p\,((x-1)(y-1)\,(10((x(2(x-35)x+189)\\&\left.-140)x+35)y^2+((450x-2183)x+2065)xy-590y+24(23x-30)x+240\right)sxy^2+(-4(x(4(x-5)x\\&+35)-15)(x-1)xy^3+(((4(64-13x)x-245)x+20)x+30)y^2-2\left(37x^3-60x+30\right)y+55x^2-80x\\&+30)\,m_cm_s))\,F(s,x,y)^2+15(y-1)\left((x-1)(y-1)\left(50((x(5(13x-42)x+273)-140)x+35)y^4\right.\right.\\&\left.-14((x(10x+361)-455)x+230)y^3+7((79x-310)x+330)y^2+420(x-2)y+70\right)s^2yx^2+14m_cm_s\\&((-11((x((59x-184)x+195)\;-90)x+15)xy^4+((x(2(96x-325)x+765)-440)x+105)y^3+(x((153x\\&-485)x+430)-150)y^2+5((7x-12)x+12)y+5(x-3)\Big)\,sx+\Big(6(4(2x((2x-5)x+5)-5)x+5)y^2\end{aligned}$$

$$
\begin{aligned}
&+5(7(x-4)x+30)xy-60y+10(x-3)x+30)m_cm_s)+c_p((x-1)(y-1)(50((x(2(x-35)x+189)\\
&-140)x+35)y^2+14(x((180x-851)x+805)-230)y+147((23x-30)x+10)\Big)s^2x^2y^3+7m_cm_s((-22\\
&(x(4(x-5)x+35)-15)(x-1)xy^3-2(x(((143x-680)x+595)x+20)-105)y^2-((x(359x+240)\\
&-930)x+420)y+35((11x-16)x+6))sxy+2\Big(2(4(2x((x-5)x+10)-15)x+15)y^2+5(11(x-4)x\\
&+42)xy-60y+10(7x-9)x+30)m_cm_s)))F(s,x,y)+60(y-1)^2s((x-1)(y-1)(4((x(5(13x-42)x\\
&+273)-140)x+35)y^3-56(2(4x-5)x+5)y^2+21((3x-10)x+10)y+35(x-2)\Big)s^2x^2y^3+2((x-1)\\
&(y-1)\Big(2((x(2(x-35)x+189)-140)x+35)y^2+14(x((8x-37)x+35)-10)y+7(23x-30)x+70\Big)\\
&s^2x^2y^3+7\Big(2\Big(2(4(2x((x-5)x+10)-15)x+15)y^2+5(11(x-4)x+42)xy-60y+10(7x-9)x+30\Big)\\
&m_cm_s-sxy(xy-1)\Big(55x^2-80x+2(x(4(x-5)x+35)-15)(x-1)y^2+2(x((17x-80)x+90)-30)y\\
&+30))m_cm_s)c_py+7m_cm_s((-4((x((59x-184)x+195)-90)x+15)xy^4+4((x-2)((17x-25)x+25)x\\
&+15)y^3+(x((53x-210)x+250)-120)y^2+5((7x-20)x+18)y+20x-30\Big)sxy+2(12(4(2x((2x-5)\\
&x+5)-5)x+5)y^3+10(x(5(x-4)x+24)-12)y^2+30((x-3)x+3)y+15(x-2)\Big)m_cm_s\Big)\Big)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\rho^{\langle\bar{s}s\rangle}_{1,7;A(S)}(s)=&-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1xm_c}{96\pi^3(y-1)^4}F(s,x,y)^2\Big(2s(y-1)\Big((x-1)y^2\Big((x(26x-1)-14)c_p-22x^2+50x\\
&-10\Big)+11xy^4\Big(x^3\Big(c_p-23\Big)-2x\Big(c_p+5\Big)+c_p+7x^4+24x^2+1\Big)-y^3((x(x(11x+37)-19)-7)(x-1)\\
&c_p-2((21x-40)x+28)x+7\Big)-(x-1)y\Big(7(x-1)c_p+10x-4\Big)-x+1\Big)F(s,x,y)+\Big((x-1)(y-1)c_p\\
&\Big(2\Big(x^2+x-1\Big)xy^2+x(2-5x)y+x+y-1\Big)+2((x((7x-23)x+24)-10)x+1)xy^3+(x((5x-9)x\\
&+7)-1)y^2-(x-1)((5x-9)x+2)y-(x-1)^2\Big)F(s,x,y)^2+6s^2(y-1)^2y\Big(-2y^3\Big((x(x+2)-4)x^2c_p\\
&+c_p-2(2(x-2)x+3)x+1\Big)+(x-1)y^2\Big(2\Big(2x^2+x-2\Big)c_p+x(11-4x)-4\Big)+2xy^4\Big(x^3\Big(c_p-23\Big)\\
&-2x\Big(c_p+5\Big)+c_p+7x^4+24x^2+1\Big)-(x-1)^2y\Big(2c_p+3\Big)-x+1\Big)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\rho^{\langle m_s\bar{s}s\rangle}_{1,7;A(S)}(s)=&-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1}{96\pi^3(y-1)^3}F(s,x,y)\Big(-2F(s,x,y)^2\Big(m_c^2\Big(5x^2c_p+y^2\Big(x^4\Big(c_p+9\Big)-4x^3\Big(c_p+5\Big)\\
&+2x^2\Big(5c_p+9\Big)-8x\Big(c_p+1\Big)+2\Big(c_p+1\Big)\Big)+2y\Big((x((x-7)x+7)-2)c_p+((x-5)x+5)x-2\Big)-6xc_p\\
&+2c_p+x^2-2x+2\Big)-s(x-1)x(y-1)\Big(35y^4\Big(((x((x-4)x+10)-8)x+2)c_p+(3x((x-4)x+6)-8)x\\
&+2\Big)+y^3\Big(59(x((x-7)x+7)-2)c_p+(233-x(81x+233))x-118\Big)+2y^2\Big(12((5x-6)x+2)c_p+(14x\\
&-33)x+38)+12(x-2)y+2))-3s(y-1)F(s,x,y)\Big(2m_c^2\Big(2y^3\Big(x^4\Big(c_p+9\Big)-4x^3\Big(c_p+5\Big)+2x^2\Big(5c_p\\
&+9\Big)-8x\Big(c_p+1\Big)+2\Big(c_p+1\Big)\Big)+4y^2\Big((x((x-7)x+7)-2)c_p+x(x-2)^2-2\Big)+xy\Big(2(5x-6)c_p+3(x\\
&-2)\Big)+y\Big(4c_p+6\Big)+x-2\Big)-s(x-1)x(y-1)y\Big(25y^4\Big(((x((x-4)x+10)-8)x+2)c_p+(3x((x-4)x\\
&+6)-8)x+2\Big)+2y^3\Big(23(x((x-7)x+7)-2)c_p+(91-x(27x+91))x-46\Big)+y^2\Big(21((5x-6)x+2)c_p\\
&+(29x-62)x+66\Big)+12(x-2)y+2\Big)\Big)+(x-1)xy\Big(5y^2\Big(((x((x-4)x+10)-8)x+2)c_p+(3x((x-4)x\\
&+6)-8)x+2\Big)+4y\Big(2(x((x-7)x+7)-2)c_p-3(x(x+3)-3)x-4\Big)+3((5x-6)x+2)c_p-3x+6\Big)\\
&F(s,x,y)^3+6s^3(x-1)x(y-1)^3y^3\Big(2y^3\Big(((x((x-4)x+10)-8)x+2)c_p+(3x((x-4)x+6)-8)x+2\Big)\\
&+4y^2((x((x-7)x+7)-2)c_p-(x(x+4)-4)x-2\Big)+xy\Big(2(5x-6)c_p+3(x-2)\Big)+y\Big(4c_p+6\Big)+x-2\Big)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\rho^{\langle g_sG^2\rangle}_{1,7;A(S)}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{x^2c_1m_c}{1935360\pi^5(x-1)^3(y-1)^5}\Big(-2(x-1)\Big(2(y((x((x-3)x+3)y-3)y+3)-1)\Big(10\Big(\Big(2c_p\\
&+65\Big)x^4-70\Big(c_p+3\Big)x^3+21\Big(9c_p+13\Big)x^2-140\Big(c_p+1\Big)x+35\Big(c_p+1\Big)\Big)y^2+28\Big(-39x^2+45x+(x\\
&((15x-74)x+70)-20)c_p-20\Big)y+21\Big((2x+5)(-x)+((23x-30)x+10)c_p+10\Big)\Big)xym_c+21(-4
\end{aligned}
$$

$$
\Big(\Big(\Big(x\Big(4x^2\ -34x+105\Big)-125\Big)x+45\Big)c_p(x-1)^2+(x((x((59x-327)x+762)-949)x+660)-255)x
$$

$$
+45\Big)xy^5+(3(x(2((-7(x-8)x-116)x+61)x+65)-120)x+(150-x((4(x(23(x-8)x+494)-554)x
$$

$$
+805)x+220))c_p+150\Big)y^4-2((3x((-5(x-13)x-218)x+300)-650)x+((x((5(17x-59)x+141)x
$$

$$
+535)-660)x+210)c_p+210\Big)y^3+2((x((50x-279)x+570)-540)x+((x(4(5x-61)x+725)-690)
$$

$$
x+210)c_p+210\Big)y^2+6\Big(((11x-45)x+60)x+2(x(8(x-5)x+45)-15)c_p-30\Big)y+5((3x-8)x
$$

$$
+((11x-16)x+6)c_p+6)\Big)m_s\Big)F(s,x,y)^3+21(2(y((x((x-3)x+3)y-3)y+3)-1)(4((x((59x-184)
$$

$$
x+195)-90)x+(x(4(x-5)x+35)-15)(x-1)c_p+15\Big)xy^3+(((-58(x-3)x-195)x+120)x+(x
$$

$$
(x(4(13x-64)x+245)-20)-30)c_p-30\Big)y^2+2\Big((3(45-16x)x-110)x+\Big(37x^3-60x+30\Big)c_p+30\Big)
$$

$$
y-30\Big(c_p+1\Big)-5x^2\Big(11c_p\ +3\Big)+40\Big(2c_p+1\Big)x\Big)m_sm_c^2-2s(x-1)x(y-1)(y((x((x-3)x+3)y-3)y
$$

$$
+3)-1)\Big(10\Big(\Big(2c_p+65\Big)x^4-70\ \Big(c_p+3\Big)x^3+21\Big(9c_p+13\Big)x^2-140\Big(c_p+1\Big)x+35\Big(c_p+1\Big)\Big)y^4
$$

$$
+\Big(50\Big(9c_p-1\Big)x^3-\Big(2183c_p+923\Big)x^2\ +5\Big(413c_p+233\Big)x-590\Big(c_p+1\Big)\Big)y^3+\Big(8\Big(69c_p+8\Big)x^2
$$

$$
-30\Big(24c_p+11\Big)x+240c_p+380\Big)y^2+60(x-2)y+10\Big)m_c+3(x-1)(y-1)\Big(22\Big(4c_p+59\Big)y^6x^7
$$

$$
+2\Big(-33\Big(14c_p+109\Big)y+253c_p+83\Big)y^5x^6+\Big(66\Big(59c_p\ +254\Big)y^2-16\Big(253c_p+83\Big)y+935c_p+10\Big)y^4
$$

$$
x^5-y^3\Big(2(11y(949y-109)-670)y+11\Big(\Big(738y^2-988y\ \ +295\Big)y+20\Big)c_p+470\Big)x^4+(6((20y(121y
$$

$$
-2)-913)y+408)y+(y((20(440y-607)y+1407)y+2828)-576)c_p-286\Big)y^2x^3-5y(2(y((y(561y
$$

$$
+205)-794)y+513)-147)y+((y(2(y(473y-427)-644)y+1739)-609)y+77)c_p+30\Big)x^2+10
$$

$$
(y((y((11y(9y+20)-630)y+560)-251)y+(y((y(99y+235)\ -548)y+292)-56)(y-1)c_p+52\Big)
$$

$$
-4)x-30(y-1)\Big(\Big(((29y-50)y+39)y+(y-1)((29y-25)y+7)c_p\ -14\Big)y+2\Big)\Big)sm_s\Big)F(s,x,y)^2
$$

$$
+6(y-1)s\Big(7(y((x((x-3)x+3)y-3)y+3)-1)\Big(22\Big(4c_p+59\Big)y^4x^5\ -2y^3\Big(88\Big(3c_p+23\Big)y-143c_p
$$

$$
+192\Big)x^4+\Big(110\Big(11c_p+39\Big)y^2+20\Big(65-68c_p\Big)y+359c_p-306\Big)y^2x^3-5y\Big(44\Big(5c_p+9\Big)y^3+34
$$

$$
\Big(9-7c_p\Big)y^2-2\Big(24c_p+97\Big)y+77c_p+14\Big)x^2+10\Big(y\Big(33\Big(c_p+1\Big)y^3+4\Big(c_p+22\Big)y^2-\Big(93c_p+86\Big)y
$$

$$
+56c_p+12\Big)-1\Big)x-30(y-1)\Big(\Big(7\Big(c_p+1\Big)y-7c_p-3\Big)y+1\Big)\Big)m_sm_c^2-s(x-1)x(y-1)y(y((x((x-3)x
$$

$$
+3)y-3)y+3)-1)\Big(\Big(50\Big(2c_p+65\Big)y^3x^4-140y^2\Big(25\Big(c_p+3\Big)y-18c_p+1\Big)x^3\ \ +7\Big(150\Big(9c_p+13\Big)y^2
$$

$$
-2\Big(851c_p+361\Big)y+483c_p+79\Big)yx^2+70\Big(\Big(-100\Big(c_p+1\Big)y^2+7\Big(23c_p+13\Big)y\ -63c_p-31\Big)y+6\Big)x
$$

$$
+70(y-1)\Big((25y-21)\Big(c_p+1\Big)y+12\Big)\Big)y+70\Big)m_c+21(x-1)(y-1)\Big(4\Big(4c_p+59\Big)y^6x^7+4\Big(-327y
$$

$$
+(23-42y)c_p+8\Big)y^5x^6+\Big(8(381y-32)y+2\Big(354y^2-368y+85\Big)c_p+15\Big)y^4x^5-y^3\Big((4y(949y
$$

$$
-119)-195)y+2\Big(\Big(738y^2-988y+295\Big)y+20\Big)c_p+90\Big)x^4+(3((4y(220y-7)-323)y+168)y
$$

$$
+2(y((4(200y-273)y+93)y+292)-64)c_p-68\Big)y^2x^3-5y((y(4(y(51y+19)-78)y+231)-75)y
$$

$$
+2((y(2(y(43y-35)-72)y+193)-75)y+11)c_p+9\Big)x^2+10(y((y((2y(9y+22)-135)y+136)
$$

$$
-70)y+2(y((y(9y+25)-60)y+36)-8)(y-1)c_p+18\Big)-2\Big)x-30(y-1)(3(y-1)y+1)(2(y-1)
$$

$$
\Big(c_p+1\Big)y+1\Big)\Big)sym_s\Big)F(s,x,y)+6(y((x((x-3)x+3)y-3)y+3)-1)(y-1)^2\Big(7\Big(4\Big(4c_p+59\Big)y^4x^5-4
$$

$$
y^3\Big(8\Big(3c_p+23\Big)y-13c_p+17\Big)x^4+\Big(4(195y+59)y+(4(55y-56)y+42)c_p-53\Big)y^2x^3-5y(6(2y(6y
$$

$$
+5)-7)y+(4(y(10y-7)-8)y+22)c_p+7\Big)x^2+10\Big(6\Big(c_p+1\Big)y^4+4\Big(2c_p+5\Big)y^3-5\Big(6c_p+5\Big)y^2+2
$$

$$
\Big(8c_p+5\Big)y-2\Big)x-30(y-1)\Big(2(y-1)\Big(c_p+1\Big)y+1\Big)\Big)m_cm_s-s(x-1)x(y-1)y^2\Big(4\Big(2c_p+65\Big)y^3x^4
$$

$$
-56y^2\Big(5y\Big(c_p+3\Big)-4c_p\Big)x^3+7\Big(12\Big(9c_p+13\Big)y^2-4\Big(37c_p+16\Big)y+46c_p+9\Big)yx^2+35(1-2y
$$

$$\begin{aligned}
&(8(y-1)y+2(4y-3)(y-1)c_p+3)\Big)x+70(y-1)\Big(2(y-1)\Big(c_p+1\Big)y+1\Big)\Big)\Big)s^2ym_c\Big),\\
\rho^{\langle\bar{s}\sigma\cdot Gs\rangle}_{1,7;A(S)}(s)=&-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1m_c}{192\pi^3(y-1)^3}F(s,x,y)\Big(3s(y-1)\Big(y\Big(x^2\Big(7c_p-38\Big)-7c_p+44x-4\Big)+11xy^4\Big(9x^3\\
&\Big(c_p-7\Big)-12x^2\Big(c_p-5\Big)+2x\Big(c_p-11\Big)+c_p+19x^4+1\Big)+y^3\Big(-(x-1)(x(11x(9x+5)-59)-7)c_p\\
&+2((12x(x+4)-97)x+68)x-7)+(x-1)y^2\Big((11x(8x-5)-14)c_p-2(6(5x-12)x+5)\Big)-3x+1\Big)\\
&F(s,x,y)+2\Big((x-1)(y-1)c_p(x(y(2x((9x-3)y-8)-2y+9)-1)+y-1)+y(x(2((x((19x-63)x\\
&+60)-22)x+1)y^2+((2x(x+7)-21)x+16)y-10(x-3)x-22\Big)-y\Big)+x(4-3x)+2y-1\Big)F(s,x,y)^2\\
&+6s^2(y-1)^2y\Big(2xy^4\Big(9x^3\Big(c_p-7\Big)-12x^2\Big(c_p-5\Big)+2x\Big(c_p-11\Big)+c_p+19x^4+1\Big)-2y^3((x(9x-4)-1)\\
&(x-1)(x+1)c_p-2((x(x+5)-10)x+7)x+1\Big)+(x-1)y^2\Big(2(x(8x-5)-2)c_p+x(31-12x)-4\Big)\\
&+2xy\Big(x\Big(c_p-5\Big)+7\Big)-y\Big(2c_p+3\Big)-2x+1\Big)\Big)+\frac{c_2xm_c}{128\pi^3(x-1)(y-1)^4}F(s,x,y)\Big(3s(y-1)\Big(-2y^4\Big(\Big(55x^2\\
&-75x+31)(x-1)^2c_p+(x((10x-83)x+134)-103)x+31\Big)+11y^5\Big(((5x-6)x+2)(x-1)^2c_p\\
&+(x((x((7x-30)x+51)-48)x+27)-10)x+2)+y^3\Big(((55x-102)x+65)(x-1)^2c_p-2x((2x(x+17)\\
&-95)x+102)+75)+2(x-1)y^2\Big((9x-16)(x-1)c_p+(9x-32)x+24\Big)+xy\Big(7(x-2)c_p+11x-26\Big)\\
&+y\Big(7c_p+15\Big)+2x-2\Big)F(s,x,y)+2\Big((x-1)^2(y-1)^2c_p(y(x(2(5x-6)y+3)+4y-3)+1)+2((x((x\\
&((7x-30)x+51)-48)x+27)-10)x+2)y^4+(x(((37-6x)x-55)x+39)-11)y^3+(11-x((2x(x+4)\\
&-27)x+30))y^2+(3x-5)(x-1)^2y+(x-1)^2\Big)F(s,x,y)^2+6s^2(y-1)^2y\Big(2y^5\Big(((5x-6)x+2)(x-1)^2c_p\\
&+(x((x((7x-30)x+51)-48)x+27)-10)x+2)-4y^4\Big(((5x-7)x+3)(x-1)^2c_p+(x((x-8)x+13)\\
&-10)x+3\Big)+2y^3\Big((5(x-2)x+7)(x-1)^2c_p+((21-8x)x-22)x+8\Big)+(x-1)y^2\Big(4(x-2)(x-1)c_p\\
&+(4x-15)x+12)+xy\Big(2(x-2)c_p+3x-8\Big)+y\Big(2c_p+5\Big)+x-1\Big)\Big)\Bigg\},
\end{aligned}$$

$$\begin{aligned}
\rho^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}_{1,7;A(S)}(s)=&-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1}{288\pi^3(y-1)^2}\Big(3s(y-1)yF(s,x,y)\Big(s(x-1)(y-1)\Big(25y^4\Big(x^4\Big(5c_p-1\Big)-4x^3\\
&\Big(c_p+3\Big)+6x^2\Big(c_p+5\Big)-8x\Big(c_p+1\Big)+2\Big(c_p+1\Big)\Big)-2y^3\Big(((x(42x+115)-161)x+46)c_p+(x(152x\\
&+185)-91)x+46\Big)+y^2\Big(21((7x-6)x+2)c_p+(95x-62)x+66\Big)+12(x-2)y+2\Big)-6xm_c^2\Big(2c_p(y(x(4\\
&y-5)-2y+2)+1)+2y(4(2x-3)xy+x+6y-4)+3))+3F(s,x,y)^2\Big(s(x-1)(y-1)\Big(35y^4\Big(x^4\Big(5c_p\\
&-1\Big)-4x^3\Big(c_p+3\Big)+6x^2\Big(c_p+5\Big)-8x\Big(c_p+1\Big)+2\Big(c_p+1\Big)\Big)-y^3\Big(((x(96x+295)-413)x+118)c_p+(x\\
&(436x+475)-233)x+118)+2y^2\Big(12((7x-6)x+2)c_p+(52x-33)x+38\Big)+12(x-2)y+2\Big)-3xm_c^2\\
&\Big(2y^2\Big(2x\Big(c_p-3\Big)-c_p+4x^2+3\Big)-y\Big((5x-2)c_p+x+2\Big)+c_p+1\Big)\Big)+2(x-1)y(-4y((x(x+4)-2)\\
&(3x-2)c_p+3(6x+5)x^2-9x+4\Big)+5y^2\Big(x^4\Big(5c_p-1\Big)-4x^3\Big(c_p+3\Big)+6x^2\Big(c_p+5\Big)-8x\Big(c_p+1\Big)+2\\
&\Big(c_p+1\Big)\Big)+3((7x-6)x+2)c_p+6x^2-3x+6\Big)F(s,x,y)^3+3s^3(x-1)(y-1)^3y^3\Big(2x^4y^3\Big(5c_p-1\Big)-8x^3\\
&y^2(y+1)\Big(c_p+3\Big)+x^2y\Big(12y^2\Big(c_p+5\Big)-4y\Big(5c_p+8\Big)+14c_p+9\Big)+2xy\Big(-8y^2\Big(c_p+1\Big)+2y\Big(7c_p+4\Big)\\
&-6c_p-3\Big)+2(y-1)\Big(2(y-1)y\Big(c_p+1\Big)+1\Big)+x\Big)\Big)+\frac{c_2}{128\pi^3(y-1)^3}\Big(-3F(s,x,y)^2\Big(m_c^2\Big((y-1)c_p(xy-1)\\
&\Big(\Big(x^2+x-1\Big)y-x+1\Big)+y\Big(x\Big(\Big(-x^2+x-3\Big)y+(x((8x-15)x+9)-1)y^2-2x+5\Big)+y\Big)-x-2y+1\Big)\\
&-sx(y-1)\Big(y\Big(35xy^4\ \Big(\Big(x^3-2x+1\Big)c_p+(3x((x-5)x+8)-10)x+1\Big)-y^3((x(x(35x+72)-13)-35)\\
&(x-1)c_p+(x((11x+115)x+275)-202)x+35\Big)+y^2\Big((x(37x+46)-59)(x-1)c_p+5(x(12x+47)\\
&-45)x+59\Big)-2(x-1)^2y\Big(12c_p+19\Big)-11x+12\Big)-1\Big)\Big)+3s(y-1)F(s,x,y)\Big(sx(y-1)y\Big(y\Big(25xy^4
\end{aligned}$$

$$((x^3-2x+1)c_p+(3x((x-5)x+8)-10)x+1)-y^3((x(x(25x+63)-17)-25)(x-1)c_p+2(x(x$$
$$+20)(2x+5)-74)x+25)+y^2((x(38x+29)-46)(x-1)c_p+(x(46x+179)-175)x+46)-3(x-1)^2$$
$$y(7c_p+11)-11x+12)-1)-2m_c^2(y(2(y-1)c_p(xy-1)((x^2+x-1)y-x+1)+y(x(2(x((8x-15)$$
$$x+9)-1)y^2-2((x-3)x+5)y-4x+11)+2y)-3x-4y+3)-1))+2xy((x-1)(y-1)c_p(x(5(x^2$$
$$+x-1)y^2-4(x+1)y+3)+5y-3)+5((3x((x-5)x+8)-10)x+1)xy^3-(2(x((x+10)x+15)-12)x$$
$$+5)y^2+((x(7x+33)-30)x+8)y-3(x-1)^2)F(s,x,y)^3+3s^3x(y-1)^3y^3(2xy^4((x^3-2x+1)c_p+(3x$$
$$((x-5)x+8)-10)x+1)-2y^3((x(x+2)-4)x^2c_p+c_p+2(2x(x+2)-3)x+1)+y^2(2(2x^2+x-2)$$
$$(x-1)c_p+(x(4x+15)-15)x+4)-(x-1)^2y(2c_p+3)-x+1))\Big\},$$

$$\rho_{1,7;A(S)}^{\langle\bar{s}s\bar{s}s\rangle}(s)=-\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\frac{c_1}{24\pi}(m_cm_s(s(z-1)(z(-14c_p+22z-5)+2)G(s,z)+(-2c_p+4z-3)G(s,z)^2+2s^2(z-1)^2z$$
$$(z(-2c_p+2z-1)+1))+m_s^2(-2s(z(z((35z-24)(z-1)c_p+(35z-59)z+38)-12)+1)G(s,z)$$
$$-2z(5z-3)(c_p+1)G(s,z)^2+s^2(-(z-1))z((z-1)z((25z-21)z(c_p+1)+12)+1))+2m_c^2G(s,z)$$
$$(c_pG(s,z)+G(s,z)+4s(z-1)z(c_p+1)+2s))+\int_0^1\mathrm{d}z\frac{c_1}{24\pi}s^3(z-1)^3z^3m_s^2(2(z-1)z(c_p+1)+1),$$

$$\rho_{1,7;A(S)}^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=-\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\Big\{\frac{c_1}{288\pi}(12m_c^2(((8z^2-4z-2)c_p+8z^2-4z+2)G(s,z)+2s(z(11z-7)-2)(z-1)zc_p$$
$$+2s(z((11z-18)z+14)-5)z+s)+m_cm_s(s(-4(z-1)(10z(12z-7)-21)zc_p+((5((10z-59)z+9)z$$
$$+51)z+45)z-12)-8((z(15z-7)-3)c_p+3(5z-2)z+3)G(s,z))-4zm_s^2(2(2(2(5(3z-4)z+4)z$$
$$+3)c_p+4(5(3z-4)z+9)z-9)G(s,z)+s(z-1)(((2(z(130z-77)-115)z+127)z+8)c_p+2(z((130z$$
$$-77)z+11)-10)z)))+\frac{c_2}{64\pi}(4m_c^2((c_p+1)G(s,z)+2s(z-1)zc_p+2s(z-1)z+s)+m_cm_s((-4c_p$$
$$+8z-6)G(s,z)+s(z-1)(-14zc_p+(22z-5)z+2))-2m_s^2(2(5z-3)z(c_p+1)G(s,z)+sz(z((35z$$
$$-24)(z-1)c_p+(35z-59)z+38)-12)+s))\Big\}+\int_0^1\mathrm{d}z\Big\{\frac{c_1s^2(z-1)z}{144\pi}(m_cm_s(z(3(z(35z-27)-4)$$
$$+(z-1)c_p(60z^2-58z+15)z-4)-2)+2(z-1)zm_s^2((((2z(73z-77)-39)z+37)z+12)c_p$$
$$+(z(2(73z-77)z+79)-26)z-4)-12(z-1)zm_c^2(2(z-1)z(c_p+1)+1))-\frac{\mathrm{i}c_2s^2(z-1)^2z}{32\pi}m_cm_s$$
$$(z(-2c_p+2z-1)+1)\Big\},$$

$$\rho_{1,7;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{xc_1}{576\pi^3(x-1)^3(y-1)^4}m_c(-3(x-1)(2((x((x((7x-37)x+81)-95)x+61)-21)x$$
$$+3)xy^5-((x((6(x-4)x+13)x+9)-15)x+5)y^4+((x((-2(x-4)x-43)x+66)-49)x+14)y^3$$
$$+(x((17x-42)x+41)-14)y^2-3(x-2)(x-1)^2y-(x-1)^2+(y((y(-2x+2(4x-3)(x-1)y+7)-3)x$$
$$-5y+4)-1)(x-1)^2(y-1)^2c_p)F(s,x,y)^2+(-2(y((x((x-3)x+3)y-3)y+3)-1)(2((x((7x-23)x$$
$$+24)-10)x+1)xy^3+(x((5x-9)x+7)-1)y^2-(x-1)((5x-9)x+2)y-(x-1)^2+(x-1)$$
$$(2(x^2+x-1)xy^2+x(2-5x)y+y+x-1)(y-1)c_p)m_c^2-3s(x-1)(y-1)(11((4x-3)c_p(x-1)^3$$
$$+(x((x((7x-37)x+81)-95)x+61)-21)x+3)xy^6+(-(11x(8x-5)-29)c_p(x-1)^3+2((2x(2$$
$$-5(x-4)x)-53)x+50)x-29)y^5+(((11x(4x-5)-65)x+83)c_p(x-1)^2-4x((x((x-4)x+48)$$
$$-83)x+66)+79)y^4+(-((11x-76)x+86)c_p(x-1)^2+2(9(5x-13)x+122)x-89)y^3-(x-1)$$

$$\Big((17x-64)x+3(6x-13)(x-1)c_p+53\Big)y^2-\Big(7c_p+16\Big)y+\Big(-10x-7(x-2)c_p+26\Big)xy-2x+2\Big)\Big)$$
$$F(s,x,y)-s(y-1)\Big(\big(y((x((x-3)x+3)y-3)y+3)-1\big)\Big(11\Big(7x^4+\Big(c_p-23\Big)x^3+24x^2-2\Big(c_p+5\Big)x$$
$$+c_p+1\Big)xy^4-\Big(-2((21x-40)x+28)x+(x(x(11x+37)-19)-7)(x-1)c_p+7\Big)y^3+\Big(-22x^2+50x$$
$$+(x(26x-1)-14)c_p-10\Big)(x-1)y^2-(x-1)\Big(10x+7(x-1)c_p-4\Big)y-x+1\Big)m_c^2+3(x-1)(y-1)$$
$$\Big(2\Big((4x-3)c_p(x-1)^3+(x(x(x((7x-37)x+81)-95)x+61)-21)x+3\Big)xy^6-2\Big((8x+3)c_p(x-1)^4$$
$$+2\Big(x^4-4x^3+5x-5\Big)x+3\Big)y^5+\Big(2((x(4x-5)-7)x+9)c_p(x-1)^2-3x(11(x-2)x+19)+18\Big)y^4$$
$$+\Big(-2((x-8)x+10)c_p(x-1)^2+((19x-54)x+60)x-23\Big)y^3-(x-1)((4x-17)x+2(2x-5)(x-1)$$
$$c_p+16\Big)y^2-2\Big(c_p+3\Big)y+\Big(-3x-2(x-2)c_p+9\Big)xy-x+1\Big)sy\Big)\Big)\Bigg\}$$
$$-\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1s^2xy}{576\pi^3(x-1)^3(y-1)^2}m_c^3(y(y(x((x-3)x+3)y-3)+3)-1)\Big(-2y^3\Big((x(x+2)-4)x^2c_p$$
$$+c_p-2(2(x-2)x+3)x+1\Big)+(x-1)y^2\Big(2\Big(2x^2+x-2\Big)c_p+x(11-4x)-4\Big)+2xy^4\Big(x^3\Big(c_p-23\Big)$$
$$-2x\Big(c_p+5\Big)+c_p+7x^4+24x^2+1\Big)-(x-1)^2y\Big(2c_p+3\Big)-x+1\Big),$$

$$\rho_{1,7;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1}{576\pi^3(x-1)^3(y-1)^3}m_c^2\Big(-(y((x((x-3)x+3)y-3)y+3)-1)\Big(5c_px^2+x^2-6c_p$$
$$x-2x+\Big(\Big(c_p+9\Big)x^4-4\Big(c_p+5\Big)x^3+2\Big(5c_p+9\Big)x^2-8\Big(c_p+1\Big)x+2\Big(c_p+1\Big)\Big)y^2+2\,(((x-5)x+5)x$$
$$+(x((x-7)x+7)-2)c_p-2\Big)y+2c_p+2\Big)m_c^2+(x-1)(y-1)\,(35\,((3x((x-4)x+6)-8)x+((x((x-4)$$
$$x+10)-8)x+2)c_p+2\Big)((x-3)x+3)x^2y^7+(((x(x(x(10-81x)+374)-256)-837)x+486)x$$
$$+(x(((x(59(x-10)x+1724)-2176)x+543)x+486)-210)c_p-210\Big)xy^6+(((x(((393-26x)x$$
$$-1115)x+2667)-1635)x+684)x+3(x((x(((38x-121)x+41)x+643)-721)x+228)-8)c_p-24\Big)$$
$$y^5+\Big((x(1537-3x((35x-99)x+571))-892)x+((x(((437-47x)x-2273)x+2431)-880)x+72)c_p$$
$$+72\Big)y^4+\Big(((x(20x+491)-737)x+634)x+(x(((917-95x)x-1217)x+574)-84)c_p-96\Big)y^3$$
$$+\Big(((180-43x)x-256)x-12(x-1)((11x-13)x+4)c_p+72\Big)y^2-6\Big(2c_p+5\Big)y+3\Big(-7x+2(6-5x)$$
$$c_p+18\Big)xy-5x+6\Big)s+(x-1)\Big(10\Big((3x((x-4)x+6)-8)x+((x((x-4)x+10)-8)x+2)c_p+2\Big)$$
$$((x-3)x+3)x^2y^6+2\Big(\Big(72-x\Big(\Big(12x^3-63x+52\Big)x+114\Big)\Big)x+(x(((x(8(x-10)x+233)-292)x+66)$$
$$x+72)-30)c_p-30\Big)xy^5+3\,(((3x(((22-3x)x-54)x+106)-194)x+72)x+(x((x((x(9x-26)-6)x$$
$$+200)-218)x+72)-4)c_p-4\Big)y^4+2\,((x(277-3x(6(x-4)x+103))-142)x+2((x(((37-4x)x$$
$$-172)x+191)-77)x+9)c_p+18\Big)y^3-2\,((3x((x-26)x+38)-88)x+((x((17x-137)x+203)-112)$$
$$x+21)c_p+21\Big)y^2-6\Big(((x-7)x+9)x+(3x(2(x-3)x+5)-4)c_p-4\Big)y-3\,((x-2)x+((5x-6)x+2)$$
$$c_p+2)\Big)F(s,x,y)\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1s}{1152\pi^3(x-1)^3(y-1)^2}m_c^2(y(y(x((x-3)x+3)y-3)+3)-1)$$
$$\Big(2m_c^2\Big(2y^3\Big(x^4\Big(c_p+9\Big)-4x^3\Big(c_p+5\Big)+2x^2\Big(5c_p+9\Big)-8x\Big(c_p+1\Big)+2\Big(c_p+1\Big)\Big)+4y^2\,((x((x-7)x+7)$$
$$-2)c_p+x(x-2)^2-2\Big)+xy\Big(2(5x-6)c_p+3(x-2)\Big)+y\Big(4c_p+6\Big)+x-2\Big)-s(x-1)x(y-1)y\Big(25y^4$$
$$\Big(((x((x-4)x+10)-8)x+2)c_p+(3x((x-4)x+6)-8)x+2\Big)+2y^3\Big(23(x((x-7)x+7)-2)c_p+(91$$
$$-x(27x+91))x-46\Big)+y^2\Big(21((5x-6)x+2)c_p+(29x-62)x+66\Big)+12(x-2)y+2\Big)\Big)\Bigg\},$$

$$\rho_{1,7;A(S)}^{\langle g_sG^2\rangle^2}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^3y^4}{5806080\pi^5(x-1)^2(y-1)^2}m_c^4\Big(28y\Big((x((15x-74)x+70)-20)c_p-39x^2+45x$$

$$-20\Big)+10y^2\Big(x^4\Big(2c_p+65\Big)-70x^3\Big(c_p+3\Big)+21x^2\Big(9c_p+13\Big)-140x\Big(c_p+1\Big)+35\Big(c_p+1\Big)\Big)+21\Big(((23x -30)x+10)c_p-(2x+5)x+10)\Big)\Big)-\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1sx^3y^3}{1658880\pi^5(x-1)^2(y-1)}m_c^4\Big(y^2\Big(8x^2\Big(69c_p+8\Big) -30x\Big(24c_p+11\Big)+240c_p+380\Big)+y^3\Big(50x^3\Big(9c_p-1\Big)-x^2\Big(2183c_p+923\Big)+5x\Big(413c_p+233\Big) -590\Big(c_p+1\Big)\Big)+10y^4\Big(x^4\Big(2c_p+65\Big)-70x^3\Big(c_p+3\Big)+21x^2\Big(9c_p+13\Big)-140x\Big(c_p+1\Big)+35\Big(c_p+1\Big)\Big) +60(x-2)y+10\Big),$$

$$\rho_{1,7;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1}{2304\pi^3(x-1)^3(y-1)^3}m_c\Big(6(x-1)\Big(y^4\Big(\Big(2\Big(8x^3-15x+6\Big)x+5\Big)(x-1)^2c_p +(x(2(3x((x-2)x-4)-7)x+33)-24)x+5\Big)-2xy^5\Big(\Big(8x^2-3\Big)(x-1)^3c_p+(x((x((19x-97)x+201) -219)x+121)-33)x+3\Big)+2y^3\Big(-(x-1)(x(((8x-19)x+12)x+5)-7)c_p+\Big(\Big(2x\Big(x^2+x+19\Big)-53\Big) x+33\Big)x-7\Big)+2y^2\Big((x((x-3)x+6)-7)(x-1)c_p-((x(4x+13)-33)x+28)x+7\Big)+6(x-1)y\Big(c_p +(x-3)x+1\Big)+(1-x)\Big(x\Big(c_p-3\Big)+c_p+1\Big)\Big)F(s,x,y)-2m_c^2(y(y(x((x-3)x+3)y-3)+3)-1)\Big((x -1)(y-1)c_p(x(y(2x((9x-3)y-8)-2y+9)-1)+y-1)+y\Big(x\Big(2((x((19x-63)x+60)-22)x+1)y^2 +((2x(x+7)-21)x+16)y-10(x-3)x-22)-y\Big)+x(4-3x)+2y-1\Big)+3s(x-1)(y-1)\Big(y^5\Big(\Big(11 \Big(8x^3-15x+6\Big)x+29\Big)(x-1)^2c_p+2(x((x(10(x-2)x-53)-100)x+156)-90)x+29\Big)-11xy^6 \Big(\Big(8x^2-3\Big)(x-1)^3c_p+(x((x((19x-97)x+201)-219)x+121)-33)x+3\Big)+y^4\Big(-(x-1)(x(11((8x -19)x+12)x+58)-83)c_p+2((x(4(x+8)x+213)-340)x+213)x-79\Big)+y^3\Big((11x((x-3)x+6) -86)(x-1)c_p-2(9(2x(x+6)-29)x+208)x+89\Big)+y^2\Big(3(x-1)(x+13)c_p+((51x-209)x+217)x -53\Big)+xy\Big(x\Big(30-7c_p\Big)-52\Big)+y\Big(7c_p+16\Big)+4x-2\Big)\Big)-\frac{c_2xy^2}{768\pi^3(x-1)^2(y-1)^2}m_c^3\Big(y\Big(((x(x+21) -21)x+5)c_p+\Big(x^2+x-1\Big)x+5\Big)-2\Big((1-2x)^2c_p-x^2+x+1\Big)+2xy^3\Big((x-1)((x-4)x+6)xc_p+c_p +(3(x-2)x+2)x+1)+y^2\Big(3(x(((x-4)x-2)x+4)-1)c_p+(((20-11x)x-12)x+2)x-3\Big)\Big)\Bigg\}$$

$$+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1s}{2304\pi^3(x-1)^3(y-1)^2}m_c\Big(m_c^2(-(y(y(x((x-3)x+3)y-3)+3)-1))\Big(y\Big(x^2\Big(7c_p-38\Big) -7c_p+44x-4\Big)+11xy^4\Big(9x^3\Big(c_p-7\Big)-12x^2\Big(c_p-5\Big)+2x\Big(c_p-11\Big)+c_p+19x^4+1\Big)+y^3\Big(-(x-1) (x(11x(9x+5)-59)-7)c_p+2((12x(x+4)-97)x+68)x-7\Big)+(x-1)y^2\Big((11x(8x-5)-14)c_p -2(6(5x-12)x+5))-3x+1\Big)-3s(x-1)(y-1)y\Big(2y^5\Big(-\Big(\Big(8x^3-15x+6\Big)x+3\Big)(x-1)^2c_p+2((x(x (5-(x-2)x)+9)-15)x+9)x-3\Big)+2xy^6\Big(\Big(8x^2-3\Big)(x-1)^3c_p+(x((x((19x-97)x+201)-219)x +121)-33)x+3\Big)+y^4\Big(2(x(((8x-19)x+12)x+6)-9)(x-1)c_p-3((x(6x+25)-46)x+31)x+18\Big) +y^3\Big(-2(x-1)(x((x-3)x+6)-10)c_p+((x(7x+46)-123)x+104)x-23\Big)+y^2\Big(-2(x-1)(x+5)c_p -(x-2)(12x-31)x+16\Big)+xy\Big(x\Big(2c_p-9\Big)+18\Big)-2y\Big(c_p+3\Big)-2x+1\Big)\Big)-\frac{c_2sxy}{1536\pi^3(x-1)^2(y-1)} m_c^3\Big(y\Big(-y\Big(11(1-2x)^2c_p-6x^2+8x+24\Big)+11xy^4\Big((x-1)((x-4)x+6)xc_p+c_p+(3(x-2)x+2)x+1\Big) +y^3\Big(3(2x((3(x-4)x-4)x+10)-5)c_p-(((59x-104)x+44)x+8)x-15\Big)+y^2\Big(((x(7x+111)-111) x+26)c_p+2((5x-7)x+7)x+30\Big)+9\Big)-1\Big)\Bigg\},$$

$$\rho_{1,7;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1}{3456\pi^3(x-1)^2(y-1)^2}m_c^2\Big(10((x-3)x+3)xy^6\Big(x^4\Big(5c_p-1\Big)-4x^3\Big(c_p+3\Big)+6x^2$$

$$
\begin{aligned}
&(c_p+5)-8x(c_p+1)+2(c_p+1))-2y^5(12x^6(c_p+6)+4x^5(c_p-39)-5x^4(13c_p+3)+4x^3(61c_p \\
&+31)+42x^2(7-3c_p)-72x(c_p+1)+30(c_p+1))+6y^4(((x(((7x-8)x+48)x+28)-84)x+26)c_p \\
&+(x(255-2x(5(x-3)x+23))-88)x+26)-y^3(((x(5(x+28)x+228)-452)x+152)c_p+(x(x(168 \\
&-19x)+930)-458)x+152)+y^2(((x(15x+98)-184)x+68)c_p+3(x(21x+88)-66)x+68) \\
&+3y((x(x+12)-4)c_p-(x-14)x-4)-9x(c_p+1))+\frac{c_2 y}{1536\pi^3(x-1)^2(y-1)^2}m_c^2(3y((x((7x-17)x \\
&+11)-2)c_p+(-3x^2+x+3)x-2)+10xy^5(((x((x((x-6)x+15)-20)x+19)-10)x+2)c_p \\
&+(x((9x-19)x+8)-2)(x-1))+4xy^4((4x((x((x-5)x+10)-19)x+14)-13)c_p+(46-x((x(5x \\
&+17)-90)x+106))x-13)+xy^3(3((x((x-3)x+66)-66)x+17)c_p+(x((33x-145)x+216)-112) \\
&x+51)+y^2((3-2x((x(3x+44)-56)x+20))c_p+2(x((7x-18)x+12)-14)x+3) \\
&+3((1-2x)^2c_p+1))\Big\}+\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\Big\{-\frac{c_1}{3456\pi^3(x-1)^3(y-1)^2}m_c^2(s(x-1)(y-1)(y(35((x-3)x \\
&+3)xy^6(x^4(5c_p-1)-4x^3(c_p+3)+6x^2(c_p+5)-8x(c_p+1)+2(c_p+1))-y^5(4x^6(24c_p+109) \\
&+7x^5(c_p-119)-5x^4(97c_p+91)+2x^3(911c_p+491)+9x^2(233-107c_p)-486x(c_p+1)+210 \\
&(c_p+1))+y^4(3((x(((56x-65)x+344)x+241)-621)x+188)c_p+(x(((21-40x)x-194)x+4797) \\
&-1527)x+564)-y^3(((x((85x+364)x+1203)-1807)x+568)c_p+(x(x(552-29x)+3291)-1465) \\
&x+652)+y^2(((x(78x+583)-773)x+262)c_p+(x(267x+1087)-767)x+418)-2y(3((13x-24) \\
&x+8)c_p+(61x-114)x+77)-3x(6c_p+13)+30)-2)-3xm_c^2(y(y(x((x-3)x+3)y-3)+3)-1) \\
&(2y^2(2x(c_p-3)-c_p+4x^2+3)-y((5x-2)c_p+x+2)+c_p+1))+\frac{c_2 y}{1536\pi^3(x-1)^3(y-1)^2}m_c^2 \\
&(m_c^2(4x^2c_p+y^4(((x((x((x-6)x+15)-20)x+19)-10)x+2)c_p+(x((x((x-6)x+23)-36)x+27) \\
&-10)x+2)+2y^3((x((x((x-5)x+10)-18)x+13)-3)c_p+((x((x-13)x+26)-24)x+11)x-3) \\
&+y^2(((x((x-4)x+30)-28)x+7)c_p+(x((9x-20)x+26)-16)x+7)-4y((1-2x)^2c_p+(x-1)x \\
&+1)-4xc_p+c_p+1)-s(x-1)(y-1)(35xy^6(((x((x((x-6)x+15)-20)x+19)-10)x+2)c_p \\
&+(x((9x-19)x+8)-2)(x-1))+xy^5((x((59x((x-5)x+10)-1106)x+811)-188)c_p+(631 \\
&-x((x(70x+247)-1290)x+1486))x-188)+xy^4(3(2(x((3x-11)x+132)-128)x+65)c_p \\
&+(x((137x-568)x+786)-406)x+223)+y^3((6-x(4(x(3x+95)-107)x+131))c_p+(x((37x \\
&-120)x+84)-147)x+6)+y^2(6(3x-2)(5(x-1)x+1)c_p+(x(23-18x)+49)x-12) \\
&+xy(24(x-1)\,c_p-3x-8)+y(6c_p+9)+x-3))\Big\},
\end{aligned}
\tag{23}
$$

where $F(s,x,y)=\frac{m_c^2(1-xy)}{1-x}-s(1-y)y$, $G(s,z)=m_c^2-s(1-z)z$, $y_{\max}=\frac{1}{2}+\frac{\sqrt{4m_c^2 s(x-1)+\left(s(x-1)-m_c^2 x\right)^2}+m_c^2 x}{2s(1-x)}$, $y_{\min}=\frac{1}{2}-\frac{\sqrt{4m_c^2 s(x-1)+\left(s(x-1)-m_c^2 x\right)^2}-m_c^2 x}{2s(1-x)}$, $x_{\max}=\left(1-2\sqrt{m_c^2/s}\right)/\left(\sqrt{m_c^2/s}-1\right)^2$, $z_{\max}=\frac{1}{2}\left(1+\sqrt{1-4m_c^2/s}\right)$, $z_{\min}=\frac{1}{2}\left(1-\sqrt{1-4m_c^2/s}\right)$, coefficient $c_p=1$ for current $J^{A(S)}_{1,\mu\nu}$ while $c_p=-1$ for current $J^{A(S)}_{7,\mu\nu}$, and $c_1=12, c_2=-8, c_3=4$ for color antisymmetric current $J^A_{i,\mu\nu}$ while $c_1=24, c_2=8, c_3=20$ for color symmetric current $J^S_{i,\mu\nu}$. The spectral functions for $(1,1\{1,0\})$ structure are shown as

$$\rho^{\rm pert}_{2,8;A(S)}(s)=\int_0^{x_{\max}}{\rm d}x\int_{y_{\min}}^{y_{\max}}{\rm d}y\frac{x^2}{51609600\pi^5(y-1)^5}F(s,x,y)^3c_1\,((x-1)x((350(4y-3)y+(231-8y(-266x+5((x-14)x+42)y+147))xy+35)x+(x(140(4y-1)y+(8(5x(20xy+21)-28(5y+4))y+483)xy-105)-70(y-1))c_p\Big)F(s,x,y)^3+42x\Big((x-1)(y-1)s\Big(\Big(\Big(\Big(-20((x-14)x+42)y^2+6(141x-32)y-29\Big)x+500y-410\Big)y+105\Big)xy+\Big(-20(y-1)y+2\Big(55(2y-1)y+\Big(\Big(115x+40\Big(5x^2-7\Big)y-96\Big)y+143\Big)xy-15\Big)xy-5\Big)c_p\Big)+(50(y-1)+(-10(3y+10)y+((8(7x-8)yx-183x+166)y+140)xy+2(y(x(y(11x+8((x-9)x+5)y+68)-20)-40y)+5)c_p+15\Big)x\Big)m_cm_s\Big)F(s,x,y)^2+30(y-1)\Big((x-1)(y-1)\Big(50\Big(-(x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^3+14\Big((131x+3)x+4((5x+2)x+5)c_p+75\Big)y^2+7\,((82x-30)c_p-35(x+4)\Big)y+385\Big)s^2x^2y^2+14\Big(30(y-1)+\Big(\Big(-2\Big(7c_p+17\Big)yx^2+5\Big(8c_py+12y-2c_p+3\Big)x-20y\Big(c_p+2\Big)-10\Big)y+5\Big)x\Big)m_c^2m_s^2+7\,(10(y+1)y+(-30(y+4)y+((22(7x-8)yx-419x+330)y+346)xy-125)xy+2(y((((2y(34x+11((x-9)x+5)y+75)-107)x-110y+30)y+30)x+10y)-5)c_p+20\Big)sx\,m_cm_s\Big)F(s,x,y)+120(y-1)^2s\Big((x-1)(y-1)\Big(\Big(4\Big(-(x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^2+14\Big(\Big(9x+4c_p+3\Big)x+5\Big)y+7\Big(4c_p-5\Big)x-70\Big)y+35\Big)s^2x^2y^3+7\,(10(y-1)y+(y(x((4(7x-8)yx-77x+82)y+44)-30y)+2(2y(9x+2((x-9)x+5)y-5)-5)(xy-1)c_p-25\Big)xy+10\Big)sxm_cm_sy+14\,((-4\,((17x-30)x+((7x-20)x+10)c_p+20\Big)xy^2+5\Big(\Big(x+(4-8x)c_p+6\Big)x+6\Big)y+10\Big(c_p-2\Big)x-30\Big)y+15\Big)m_c^2m_s^2\Big)\Big),$$

$$\rho^{\langle\bar{s}s\rangle}_{2,8;A(S)}(s)=-\int_0^{x_{\max}}{\rm d}x\int_{y_{\min}}^{y_{\max}}{\rm d}y\frac{c_1(x-1)x^2}{3072\pi^3(y-1)^3}m_cF(s,x,y)^2\Big(4s(y-1)y(11xy-5)(2xy-1)F(s,x,y)+(xy-1)(8xy+1)F(s,x,y)^2+12s^2(y-1)^2y^2(xy(4xy-3)+1)\Big),$$

$$\rho^{\langle m_s\bar{s}s\rangle}_{2,8;A(S)}(s)=-\int_0^{x_{\max}}{\rm d}x\int_{y_{\min}}^{y_{\max}}{\rm d}y\frac{c_1x}{6144\pi^3(y-1)^3}F(s,x,y)\Big(12s(y-1)F(s,x,y)\Big(2m_c^2\Big(xy\Big(-4((x-2)x+2)y^2+6y-3\Big)+2(y-1)y+1\Big)+s(x-1)x(y-1)y^2\Big(y\Big(50((x-2)x+2)xy^2-2(x+15)(2x+1)y+29x+28\Big)-11\Big)\Big)+4F(s,x,y)^2\Big(4m_c^2(y(-((x-2)x+2)xy+x+1)-1)+s(x-1)x(y-1)y(y(x(2y(70((x-2)x+2)y-11x-68)+57)-100y+82)-21))+(x-1)c_p\Big((x(y(-8((x-4)x+2)y-15x+4)+3)+2(y-1))F(s,x,y)^2-4s(y-1)(2y(x(y(11((x-4)x+2)y+25x-11)-3)-2y+2)-1)F(s,x,y)-24s^2x(y-1)^2y^3(2((x-4)x+2)y+5x-3))\,F(s,x,y)+(x-1)x\Big(40((x-2)x+2)xy^3-8((x+3)x+5)y^2+3(x+10)y-1\Big)F(s,x,y)^3+24s^3(x-1)x(y-1)^3y^3\Big(y\Big(x\Big(4((x-2)x+2)y^2-6y+3\Big)-2y+2\Big)-1\Big)\Big),$$

$$\rho^{\langle g_sG^2\rangle}_{2,8;A(S)}(s)=\int_0^{x_{\max}}{\rm d}x\int_{y_{\min}}^{y_{\max}}{\rm d}y\Big\{\frac{x^2c_1}{30965760\pi^5(x-1)^3(y-1)^5}m_c\,((x-1)x((-350(4y-3)y+(8y(-266x+5((x-14)x+42)y+147)-231)xy-35)(xy-1)((x+((x-3)x+3)y-3)y+1)(-x)m_c+21\,(8((x((7x-41)x+101)-108)x+40)x^2y^5+(x(((23-31x)x-469)x+1056)-610)xy^4+(((x(25x+541)-1020)x+320)x+310)y^3+(13x(x(5-14x)+35)-600)y^2+5((23x-65)x+74)y+45x-80\Big)m_s+c_p\,((xy-1)((x+((x-3)x+3)y-3)y+1)(x(140(4y-1)y+(8(5x(20xy+21)-28(5y+4))y+483)xy-105)-70(y-1))m_c+42\Big(8((x((x-13)x+38)-44)x+15)x^2y^5+(x((x(27x-71)-7)x+278)-140)xy^4+((((103-15x)x-330)x+100)x+20)y^3+(x(x(95-16x)+35)-30)y^2-5(x(x+7)-2)y+5x\Big)m_s\Big)\Big)F(s,x,y)^3+21\,((y((x((x-3)x+3)y-3)y+3)-1)\,(50(y-1)+(-10(3y+10)y+((8(7x-8)yx-183x+166)y+140)xy+2(y(x(y(11x+8((x-9)x+5)y+68)-20)-40y)+5)c_p+15\Big)x\Big)xm_sm_c^2+(x-1)(y-1)\Big(sx(y((x((x-3)x+3)y-3)y+3)-1)\Big(\Big(\Big(\Big(\Big(-20((x-14)x+42)y^2+6(141x-32)y-29\Big)x+500y-410\Big)y+105\Big)xy+\Big(-20(y-1)y+2\Big(55(2y-1)y+\Big(\Big(115x+40\Big(5x^2-7\Big)y-96\Big)y+143\Big)xy-15\Big)xy-5\Big)c_p\Big)$$

$$-6((((x-2)x+2)y-2)y+1)\Big(x\Big(y\Big(2\Big(7c_p+17\Big)yx^2-5\Big(8c_py+12y-2c_p+3\Big)x+20\Big(c_p+2\Big)y+10\Big)$$
$$-5\Big)-30(y-1)\Big)m_s^2\Big)m_c+3(x-1)(y-1)(22((x((7x-41)x+101)-108)x+2((x((x-13)x+38)-44)$$
$$x+15)c_p+40\Big)x^2y^6+\Big((x((x+367)x+377)-2040)(-x)+4(x(2(28x-61)(x-1)x+305)-185)c_p$$
$$-1370)xy^5+\Big(3((11x(5x+33)-864)x+440)x+(2(((287-45x)x-1041)x+400)x+100)c_p+470\Big)y^4$$
$$-\Big(((558x-757)x+5)x+2(((49x-421)x+20)x+80)c_p+900\Big)y^3+\Big((181x-315)x-2(62x+105)c_px$$
$$+70c_p+620\Big)y^2+5\Big(3x+2(7x+1)c_p\Big)y-170y-10c_p+20\Big)sxm_s\Big)F(s,x,y)^2-6(y-1)(14(y((x((x$$
$$-3)x+3)y-3)y+3)-1)\Big(x\Big(y\Big(2\Big(7c_p+17\Big)yx^2-5\Big(8c_py+12y-2c_p+3\Big)x+20\Big(c_p+2\Big)y+10\Big)-5\Big)$$
$$-30(y-1))m_s^2m_c^3-7sx(y((x((x-3)x+3)y-3)y+3)-1)(10(y+1)y+(-30(y+4)y+((22(7x-8)yx$$
$$-419x+330)y+346)xy-125)xy+2(y((((2y(34x+11((x-9)x+5)y+75)-107)x-110y+30)y$$
$$+30)x+10y)-5)c_p+20\Big)m_sm_c^2-s(x-1)(y-1)\Big(sx^2y^2(y((x((x-3)x+3)y-3)y+3)-1)(50(-(x$$
$$-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^3+14\Big((131x+3)x+4((5x+2)x+5)c_p+75\Big)y^2+7\Big((82x-30)c_p$$
$$-35(x+4))y+385)-42((((x-2)x+2)y-2)y+1)\Big(y\Big(4\Big((17x-30)x+((7x-20)x+10)c_p+20\Big)xy^2$$
$$+5\Big(x\Big(-x+(8x-4)c_p-6\Big)-6\Big)y-10x\Big(c_p-2\Big)+30\Big)-15\Big)m_s^2\Big)m_c-21s^2(x-1)x(y-1)y((4((x((7x$$
$$-41)x+101)-108)x+2((x((x-13)x+38)-44)x+15)c_p+40\Big)x^2y^5+((x((x+67)x+51)-352)$$
$$(-x)+4(x(((11x-38)x+36)x+43)-30)c_p-250\Big)xy^4+(((x(35x+163)-468)x+2(((39-5x)x$$
$$-189)x+90)c_p+300\Big)x+70\Big)y^3-\Big(\Big(16\Big(c_p+6\Big)x^2-\Big(188c_p+193\Big)x+70c_p+125\Big)x+140\Big)y^2$$
$$+\Big(\Big(9x-2(18x+5)c_p+5\Big)x+120\Big)y+5\Big(2c_p-1\Big)x-50\Big)y+10\Big)m_s\Big)F(s,x,y)+6(y((x((x-3)x+3)y$$
$$-3)y+3)-1)(y-1)^2sm_c\Big((x-1)(y-1)\Big(\Big(4\Big(-(x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^2+14\Big(\Big(9x+4c_p+3\Big)$$
$$x+5)y+7\Big(4c_p-5\Big)x-70\Big)y+35\Big)s^2x^2y^3+7(10(y-1)y+(y(x((4(7x-8)yx-77x+82)y+44)$$
$$-30y)+2(2y(9x+2((x-9)x+5)y-5)-5)(xy-1)c_p-25\Big)xy+10\Big)sxm_cm_sy+14((-4((17x-30)x$$
$$+((7x-20)x+10)c_p+20\Big)xy^2+5\Big(\Big(x+(4-8x)c_p+6\Big)x+6\Big)y+10\Big(c_p-2\Big)x-30\Big)y+15\Big)m_c^2m_s^2\Big)\Big)$$
$$+\frac{c_3}{23592960\pi^5(y-1)^3}(x-1)x^2\Big(c_p+1\Big)F(s,x,y)^2\Big(8s(y-1)y((393x-200)y+73)F(s,x,y)+((477x$$
$$-230)y+43)F(s,x,y)^2+24s^2(y-1)^2y^2((64x-30)y+15)\Big)\Big\},$$

$$\rho_{2,8;A(S)}^{\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1x}{3072\pi^3(y-1)^3}m_cF(s,x,y)\Big(3s(y-1)\Big(yc_p\Big(x\Big(2y\Big(22(x-1)xy^2-4(x-1)(3x+5)y$$
$$+13x-9)-7)-2y)+c_p+y(x(y(x(y(66(x-1)y-41x-23)+42)+69y-28)-15)-21y+19)-2\Big)$$
$$F(s,x,y)+\Big(x\Big(y\Big(2(x-1)c_p(y(8xy-2x-7)+1)+x(y(24(x-1)y-9x-13)+7)+25y-12\Big)+2\Big)-y$$
$$+1\Big)F(s,x,y)^2+6s^2(y-1)^2y\Big(y\Big(x\Big(2c_p(xy-1)(2(x-1)(2y-1)y+1)+y(x(y(12(x-1)y-7x-3)+6)$$
$$+11y-2)-4)-5y+5)-1)\Big)-\frac{c_2x^2}{2048\pi^3(y-1)^3}m_cF(s,x,y)\Big(3s(y-1)y\Big(xy\Big(44(x-1)y^2+(26-39x)y$$
$$+11)+18(y-1)y+2)F(s,x,y)+(xy(y(16(x-1)y-13x+6)+4)+10(y-1)y+3)F(s,x,y)^2+6s^2$$
$$(y-1)^2y^3\Big(8(x-1)xy^2+(x(6-7x)+2)y+x-2\Big)\Big)\Big\},$$

$$\rho_{2,8;A(S)}^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1}{9216\pi^3(y-1)^2}\Big(c_p\Big(12sx(y-1)yF(s,x,y)\Big(6m_c^2(2y(((x-4)x+2)y+2x-1)-1)$$
$$+s(x-1)(y-1)y^2\Big(x\Big(50\Big(x^2-2\Big)y^2+8(x+7)y+11\Big)-4y+3\Big)\Big)+3F(s,x,y)^2\Big(12xym_c^2(((x-4)x+2)y$$
$$+x)+s(x-1)(y-1)\Big(2xy\Big(y\Big(7x\Big(y\Big(20\Big(x^2-2\Big)y+7x+24\Big)-1\Big)-22y+11\Big)+3\Big)+4(y-1)y+1\Big)\Big)$$

$$
\begin{aligned}
&+(x-1)\Big(x\Big(y\Big(x\Big(8y\Big(10\Big(x^2-2\Big)y+9x+8\Big)-21\Big)-16y+4\Big)+3\Big)+2(y-1)\Big)F(s,x,y)^3+24s^3(x-1)x^2\\
&(y-1)^3y^4\Big(2y\Big(\Big(x^2-2\Big)y+1\Big)+1\Big)\Big)+x\Big(-6s(y-1)yF(s,x,y)\Big(6m_c^2(y(x+8y-12)+5)-s(x-1)(y-1)\\
&y(y(x(2y(25(5(x-2)x+6)y+119x-183)+139)-30y+28)-11))+3F(s,x,y)^2\,(s(x-1)(y-1)y(y\\
&(x(14y(x(50(x-2)y+51)+60y-72)+313)-100y+82)-21)-6m_c^2(y(3x+4y-8)+1)\Big)+(x-1)\\
&(y(x(8y(5(5(x-2)x+6)y+32x-39)+51)-40y+30)-1)F(s,x,y)^3+6s^3(x-1)(y-1)^3y^3\Big(y\Big(x\Big(4(5\\
&(x-2)x+6)y^2+6(3x-5)y+13\Big)-2y+2\Big)-1\Big)\Big)\Big)-\frac{c_2}{8192\pi^3(y-1)^3}x\Big(2\Big(6s(y-1)^2yF(s,x,y)\Big(m_c^2(2xy\\
&(3-4xy)-2)+sxy^2(x(xy(5y(10(x-1)y-10x+3)+36)+7(5y-4)y-8)-9y+9)\Big)+3(y-1)\\
&F(s,x,y)^2\Big(sxy(y(x(5xy(2y(14(x-1)y-14x+5)+19)+90(y-1)y-7)-10y+14)-2)-2m_c^2(xy-1)\\
&(2xy+1))+x(y(x(y(x(20y(2x(y-1)-2y+1)+23)+20y-34)+8)+8y-2)-3)F(s,x,y)^3+6s^3\\
&(x-1)x(y-1)^4y^4(xy(4xy-3)+1)\Big)+c_p\Big(3s(y-1)\Big(y\Big(x\Big(2y\Big(22(x-1)xy^2-6(x-1)(2x+5)y+23x\\
&-24\Big)+3\Big)-4y+4\Big)-1\Big)F(s,x,y)+2(x-1)(xy(y(8xy-2x-13)+7)+y-1)F(s,x,y)^2+12s^2x(y-1)^2\\
&y^2\Big(y\Big(4(x-1)xy^2-2(x-1)(x+3)y+5x-6\Big)+1\Big)\Big)F(s,x,y)\Big)\Big\},
\end{aligned}
$$

$$
\rho_{2,8;A(S)}^{\langle\bar{s}s\bar{s}s\rangle}(s)=\int_{z_{min}}^{z_{max}}\mathrm{d}z\frac{c_1c_p}{768\pi}m_s^2G(s,z)\Big(G(s,z)+s(1-2z)^2\Big),
$$

$$
\begin{aligned}
\rho_{2,8;A(S)}^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\Big\{-\frac{c_1}{2304\pi}\Big(m_cm_s\Big(s\Big(z^2\Big(-\Big(6c_p+23\Big)\Big)+3\Big(c_p-2\Big)+17z\Big)-11G(s,z)\Big)+m_s^2\Big(\Big(-8z^2\Big(3c_p+5\Big)\\
&+2z\Big(7c_p+15\Big)+c_p-1\Big)G(s,z)-s(z-1)z\Big(6(11z-8)zc_p+c_p+2(50z-41)z+21\Big)\Big)+6m_c^2(2G(s,z)\\
&+2s(z-1)z+s))+\frac{c_2c_p}{2048\pi}m_s^2\Big(2G(s,z)+s(1-2z)^2\Big)\Big\}+\int_0^1\mathrm{d}z\Big\{\frac{c_1s^2(z-1)z}{2304\pi}m_s((z-1)zm_s((2(6z-5)z\\
&+1)c_p+30z^2-28z+11\Big)+(7(z-1)z+3)m_c\Big)\Big\},
\end{aligned}
$$

$$
\begin{aligned}
\rho_{2,8;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=&\int_0^{x_{max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1x^2}{18432\pi^3(x-1)^2(y-1)^3}m_c\Big(2F(s,x,y)\Big(m_c^2(xy-1)^2(y(((x-3)x+3)y+x-3)+1)\\
&(8xy+1)+3s(x-1)(y-1)y(xy(y(x(y(22(x-1)y-17x-19)+29)+41y-28)-4)-13(y-1)y-2))\\
&+3(x-1)(xy-1)(x(8y-5)y((x-1)y-1)+11(y-1)y+3)F(s,x,y)^2+2s(y-1)y\Big(m_c^2(11xy-5)(xy-1)\\
&(2xy-1)(y(((x-3)x+3)y+x-3)+1)+3s(x-1)(y-1)y^2\Big(y\Big(x\Big(x\Big(4(x-1)y^2-3(x+1)y+5\Big)+7y\\
&-6\Big)-1\Big)+1\Big)\Big)\Big)\Big\}-\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1s^2x^2y^2}{9216\pi^3(x-1)^2(y-1)}m_c^3(xy-1)(xy(4xy-3)+1)(y(((x-3)x+3)y\\
&+x-3)+1),
\end{aligned}
$$

$$
\begin{aligned}
\rho_{2,8;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x}{18432\pi^3(x-1)^3(y-1)^3}m_c^2\Big((x-1)\Big(c_p(xy-1)(y(((x-3)x+3)y+x-3)+1)(x(y(8\\
&((x-4)x+2)y+15x-4)-3)-2y+2)-40x^3((x-3)x+3)((x-2)x+2)y^6+8\Big(\Big(x^3+14x-36\Big)x\\
&+45\Big)x^2y^5+3(x(x(3(x-21)x+131)-166)-24)xy^4+(x(((41x-95)x+224)x+162)-24)y^3+((x(4x\\
&-45)-109)x+48)y^2+3(x(x+7)-12)y-x+12\Big)F(s,x,y)+4m_c^2(xy-1)(y(((x-3)x+3)y+x-3)\\
&+1)\Big(((x-2)x+2)xy^2-(x+1)y+1\Big)+s(x-1)(y-1)\Big(c_p(2y(x(y(11((x-4)x+2)y+25x-11)-3)\\
&-2y+2)-1)(xy-1)(y(((x-3)x+3)y+x-3)+1)+y\Big(-140x^3((x-3)x+3)((x-2)x+2)y^6+2((x\\
&((11x+35)x+89)-366)x+570)x^2y^5-x(((x(33x+427)-1041)x+1686)x+204)y^4+(x(((161x
\end{aligned}
$$

$$\begin{aligned}
&-361)x+1078)x+402)-24)y^3+((x(20x-379)-277)x+48)y^2+17(3x+5)xy-3x-48y+24)-6))\Big\}\\
&+\int_0^1 \mathrm{d}x\int_0^1 \mathrm{d}y\Big\{\frac{c_1 s x}{18432\pi^3(x-1)^3(y-1)^2}m_c^2(y(y(x((x-3)x+3)y-3)+3)-1)\Big(2m_c^2(y(x(4((x-2)x\\
&+2)y^2-6y+3)-2y+2)-1)-s(x-1)x(y-1)y^2\Big(y\Big(-2c_p(2((x-4)x+2)y+5x-3)+50((x-2)x\\
&+2)xy^2-2(x+15)(2x+1)y+29x+28\Big)-11\Big)\Big)\Big\},
\end{aligned}$$

$$\begin{aligned}
\rho_{2,8;A(S)}^{\langle g_sG^2\rangle^2}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x^3y^3}{371589120\pi^5(x-1)^2(y-1)^2}m_c^4\Big(c_p(x(xy(8y(5x(20xy+21)-28(5y+4))+483)\\
&+140(4y-1)y-105)-70(y-1))+x(xy(8y(7(38x-21)-5((x-14)x+42)y))+231)+350(4y-3)y\\
&+35)\Big)\Big\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Big\{\frac{c_1sx^3y^3}{53084160\pi^5(x-1)^2(y-1)}m_c^4\Big(c_p\Big(2xy\Big(xy\Big(y\Big(40\Big(5x^2-7\Big)y+115x-96\Big)+143\Big)\\
&+55(2y-1)y-15)-20(y-1)y-5)+xy\Big(y\Big(x\Big(-20((x-14)x+42)y^2+6(141x-32)y-29\Big)+500y\\
&-410\Big)+105\Big)\Big)\Big\},
\end{aligned}$$

$$\begin{aligned}
\rho_{2,8;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&\int_0^{x_{max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1x}{36864\pi^3(x-1)^3(y-1)^3}m_c\Big(3(x-1)\Big(8(2x-3)(x-1)x^2y^5\Big(2(x-1)c_p+x+1\Big)+y^3\\
&\Big(2(x(2x-27)-2)(x-1)^2c_p+\Big(\Big(22x^2-30x-111\Big)x+94\Big)x+13\Big)-xy^4\Big(10(2(x-1)x-5)(x-1)^2c_p\\
&+((x(x+58)-149)x+50)x+37)+y^2\Big(2(8x+3)(x-1)^2c_p+((92-13x)x-33)x-28\Big)-y\Big(2(x-1)^2c_p\\
&+(15x+16)x-19\Big)+7x-4\Big)F(s,x,y)+m_c^2(y(y(x((x-3)x+3)y-3)+3)-1)\Big(x\Big(y\Big(2(x-1)c_p(y(x(8y\\
&-2)-7)+1)+x(y(24(x-1)y-9x-13)+7)+25y-12)+2)-y+1)+3s(x-1)(y-1)\Big(22(2x-3)\\
&(x-1)x^2y^6\Big(2(x-1)c_p+x+1\Big)-xy^5\Big(4(x(17x-4)-17)(x-2)(x-1)c_p+((x(19x+98)-329)x\\
&+118)x+89)+y^4\Big(2(x(x((7x-89)x+177)-87)-5)c_p+((x(67x-104)-231)x+246)x+5\Big)+y^3\\
&\Big((((45x-127)x+47)x+16)c_p+((205-36x)x-132)x-14\Big)+y^2\Big((4x(3x+4)-7)c_p+5x(2-5x)-2\Big)\\
&-y\Big(8xc_p+c_p+x-9\Big)+c_p-2\Big)\Big)-\frac{c_2x^2y}{24576\pi^3(x-1)^2(y-1)^2}m_c^3(xy-1)\Big(y(x(y(8((x-3)x+3)y+6x\\
&-15)+3)-9y+9)-2\Big)\Big\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Big\{-\frac{c_1sx}{36864\pi^3(x-1)^3(y-1)^2}m_c\Big(m_c^2(y(y(x((x-3)x+3)y-3)\\
&+3)-1)\Big(yc_p\Big(x\Big(2y\Big(22(x-1)xy^2-4(x-1)(3x+5)y+13x-9\Big)-7\Big)-2y\Big)+c_p+y(x(y(x(y(66(x-1)y\\
&-41x-23)+42)+69y-28)-15)-21y+19)-2)+3s(x-1)(y-1)y\Big(y\Big(4(2x-3)(x-1)x^2y^5\Big(2(x\\
&-1)c_p+x+1\Big)-xy^4\Big(4(x(3x-1)-3)(x-2)(x-1)c_p+(3x(x+8)-11)(x-2)x+15\Big)+y^3(2(x((x-15)\\
&x+33)-18)xc_p+(x(11x-6)-48)(x-1)x-3\Big)+y^2\Big(x\Big(2(2(2x-7)x+7)c_p+x(37-6x)-33\Big)+6\Big)\\
&y\Big((4x+2)c_p-5x+9\Big)-2x\Big(c_p+1\Big)-8y+5\Big)-1\Big)\Big)-\frac{c_2sx^2y^2}{24576\pi^3(x-1)^2(y-1)}m_c^3\Big(xy(y(x(y(22((x\\
&-3)x+3)y-4x+17)-2)-83y+60)-17)+23(y-1)y+7\Big)\Big\},
\end{aligned}$$

$$\begin{aligned}
\rho_{2,8;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1}{110592\pi^3(x-1)^2(y-1)^2}m_c^2\Big(8x^2y^5\Big((((x(9x-19)-29)x+30)x+54)c_p+(((32x\\
&-135)x+133)x+48)x-105)+40((x-3)x+3)x^3y^6\Big(2\Big(x^2-2\Big)c_p+5(x-2)x+6\Big)+xy^4((x((15x+91)\\
&x+143)-1098)xc_p+198c_p+3((x(17x+159)-659)x+630)x-24\Big)+y^3((x(x(x(61-41x)+790)\\
&-210)-6)c_p+(((1423-255x)x-1728)x+222)x\Big)+y^2\Big(((91-x(36x+343))x+12)c_p+((609-274x)
\end{aligned}$$

$$x-263)x)+y\Big(((x(57x+5)-8)c_p+21x(7-5x)\Big)-3xc_p+2c_p-17x\Big)+\frac{c_2xy}{49152\pi^3(x-1)^2(y-1)^2}m_c^2\Big(y^2\Big(x^3\Big(-\Big(c_p+3\Big)\Big)+x^2\Big(94-33c_p\Big)+3x\Big(c_p+7\Big)+4c_p\Big)+4xy^4\Big(2((x((x-4)x+6)-6)x+2)c_p+(((2x-11)x+21)x+16)x+10\Big)+y^3\Big((x(x((5x-14)x+54)-18)-2)c_p+(3x(x+1)(8x-31)-70)x\Big)+y((2x(5x+2)-3)c_p+7x(1-3x)+6\Big)-2xc_p+c_p-40x^2((x-2)((x-2)x+2)x+2)y^5-2x-6\Big)\Big\}$$
$$+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Big\{\frac{c_1}{110592\pi^3(x-1)^3(y-1)^2}m_c^2\Big(c_p\Big(-12xym_c^2(xy-1)(((x-4)x+2)y+x)(y(((x-3)x+3)y+x-3)+1)-s(x-1)(y-1)\Big(y\Big(2(((7x(7x+3)-799)x+570)x+774)x^2y^5+280\Big(x^2-2\Big)((x-3)x+3)x^3y^6+2((x((29x+236)x+233)-1749)x+216)xy^4-2(((x(65x+116)-1324)x+321)x+6)y^3+((311-x(61x+885))x+24)y^2+(2x(79x-2)-19)y-42x+7\Big)-1\Big)\Big)+x\Big(6m_c^2(xy-1)(y(3x+4y-8)+1)(y(((x-3)x+3)y+x-3)+1)-s(x-1)(y-1)y\Big(y\Big(140((x-3)x+3)(5(x-2)x+6)x^2y^5+2(x((21(17x-75)x+1483)x+738)-1410)xy^4+(((x(313x+1243)-5793)x+6078)x+12)y^3+(((3857-739x)x-5334)x+174)y^2+(3(721-268x)x-347)y-331x+251\Big)-69\Big)\Big)\Big)-\frac{c_2xy}{98304\pi^3(x-1)^3(y-1)^2}m_c^2\Big(s(x-1)(y-1)\Big(yc_p\Big(-44x((x((x-4)x+6)-6)x+2)y^4+4((((23-8x)x-82)x+31)x+2)y^3-2(((x-112)x+28)x+8)y^2+(14-x(67x+14))y+14x-6\Big)+c_p+2\Big(xy\Big(y\Big(140((x-2)((x-2)x+2)x+2)xy^4-2((((11x-58)x+108)x+158)x+50)y^3+((5x(23-8x)+519)x+182)y^2+(2(x-182)x-135)y+73x+57\Big)-5\Big)-18(y-1)y-6\Big)\Big)-4m_c^2(2((x-2)((x-2)x+2)x+2)xy^4+((x-4)x(x+1)-2)y^3+(x(6-(x-3)x)+4)y^2-(5x+3)y+x+1)\Big)\Big\},\tag{24}$$

where the coefficient $c_p=1$ for current $J^{A(S)}_{2,\mu\nu}$ and $c_p=-1$ for current $J^{A(S)}_{8,\mu\nu}$. The spectral functions for the $(1,1\{0,1\})$ structure are given as

$$\rho^{pert}_{3,9;A(S)}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{\mathrm{i}x^2}{51609600\pi^5(y-1)^5}F(s,x,y)^3c_1\Big((x-1)x\Big(-40\Big((27x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)x^2y^3-56x\Big(7(12x-7)x+((3x-4)x+10)c_p+25\Big)y^2+7\Big(x(140-57x)+(x(50-99x)+10)c_p\Big)y+35\Big(x+(3x-2)c_p\Big)\Big)F(s,x,y)^3+42x\Big(\Big(-20c_p(y-1)-110(y-1)-75x+((y(443x-8((27x-88)x+40)y-1026)-120)x+470y+140)xy+2(y((50-y(41x+8((x-9)x+5)y+38))x+40y-20)-15)xc_p\Big)m_cm_s-s(x-1)(y-1)y((100(5y-4)y+(2(909x+10(x(27x-14)-42)y-364)y+127)xy+95)x+2\Big(x\Big(y\Big(\Big(\Big(3x+40\Big(5x^2-7\Big)y+16\Big)y+144\Big)x+110y-80\Big)-15\Big)-10(y-1)\Big)c_p\Big)\Big)F(s,x,y)^2+30(y-1)\Big((x-1)(y-1)\Big(50\Big((27x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^3-14\Big(-303x^2+93x+4(2x-5)(3x+1)c_p-75\Big)y^2+7\Big(37x+20(3x-2)c_p-130\Big)y+315\Big)\Big(-s^2\Big)x^2y^2+14\big(x\big(y\Big(-75x+2\big((37x-50)x+((7x-20)x+10)c_p+20\Big)y+5(6x-4)c_p+70\Big)-5\Big)-30(y-1)\Big)m_c^2m_s^2-7sx(((-10(115y+12)y+((-719x+22((27x-88)x+40)y+2190)y+26)xy+95)x+350y+4(10(y-1)+(((y(59x+11((x-9)x+5)y+50)-71)x-55y+35)y+15)x)c_p-270\Big)y+60\Big)m_cm_s\Big)F(s,x,y)-120s(y-1)^2\Big((x-1)(y-1)\Big(70(y-1)y+\Big(4\Big((27x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)y^2-14\Big(-23x+4(x-2)c_p+5\Big)y+21\Big)xy+35\Big)s^2x^2y^3+7\big(50(y-1)y+\Big(\Big((y(-97x+4((27x-88)x+40)y+342)-16)x-190y+8(7x+((x-9)x+5)y-5)(xy-1)c_p+20\Big)y+5\Big)xy+20\Big)sxm_cm_sy+14\Big(30(y-1)y+\Big(-4\Big(\Big(7c_p+37\Big)x^2-10\Big(2c_p+5\Big)x+10\Big(c_p+2\Big)\Big)y^2-5(3x-2)\Big(4c_p-3\Big)y+10\Big)xy+15\Big)m_c^2m_s^2\Big)\Big),$$

$$
\begin{aligned}
\rho^{\langle\bar{s}s\rangle}_{3,9;A(S)}(s) =& -\int_0^{x_{\max}} \mathrm{d}x \int_{y_{\min}}^{y_{\max}} \mathrm{d}y \frac{c_1 x^2}{3072\pi^3(y-1)^4} m_c F(s,x,y)^2 (4s(y-1)(y(x(y(x(y(22(x-3)(2x-1)y-34x+153) \\
& -12)-75y-4)+9)+19y-15)+3)F(s,x,y)+(xy-1)(x(y(8(x-3)(2x-1)y-8x+13)-5)-5y+5) \\
& F(s,x,y)^2+12s^2(y-1)^2y(y(xy(x(y(4(x-3)(2x-1)y-4x+25)-4)-13y+2)+x+3(y-1))+1)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\rho^{\langle m_s\bar{s}s\rangle}_{3,9;A(S)}(s) =& \int_0^{x_{\max}} \mathrm{d}x \int_{y_{\min}}^{y_{\max}} \mathrm{d}y \frac{c_1 x}{6144\pi^3(y-1)^3} F(s,x,y)\Big(12s(y-1)F(s,x,y)\Big(2m_c^2(y(x(2y(2((5x-6)x+2)y-2x+1) \\
& -1)-2y+2)-1)+s(x-1)x(y-1)y^2(y(x(2y(25(x(3x-2)-2)y+130x-21)+11)+30y-26)+9)\Big) \\
& +4F(s,x,y)^2\Big(4m_c^2(y(x(((5x-6)x+2)y-4x+3)-1)+1)+s(x-1)x(y-1)y(xy(2y(70(x(3x-2)-2)y \\
& +385x-88)+35)+20(5y-4)y+19))+(x-1)c_p(8s(y-1)y(x(y(11((x-4)x+2)y+30x-16)-3) \\
& -2y+2)F(s,x,y)+(x(y(8((x-4)x+2)y+21x-10)-3)-2y+2)F(s,x,y)^2+48s^2x(y-1)^2y^3(((x-4) \\
& x+2)y+3x-2))F(s,x,y)+(x-1)x(y(x(8y(5(x(3x-2)-2)y+35x-13)+13)+40y-28)-1) \\
& F(s,x,y)^3+24s^3(x-1)x(y-1)^3y^3\Big(y\Big(4(x(3x-2)-2)xy^2+2((10x-1)x+1)y+x-2\Big)+1\Big)\Big),
\end{aligned}
$$

$$
\begin{aligned}
\rho^{\langle g_sG^2\rangle}_{3,9;A(S)}(s) =& \int_0^{x_{\max}} \mathrm{d}x \int_{y_{\min}}^{y_{\max}} \mathrm{d}y \Big\{ \frac{x^2 c_1}{30965760\pi^5(x-1)^3(y-1)^5} m_c\Big((x-1)x\Big(\Big(y\Big(40(x(27x-14)-42)xy^2+56(7(12x-7) \\
& x+25)y+399x-980)-35)(xy-1)((x+((x-3)x+3)y-3)y+1)xm_c+21(8((x(3(9x-47)x+281) \\
& -268)x+80)x^2y^5+(x(((1743-451x)x-2629)x+2836)-1050)xy^4+((((121-15x)x-900)x+120)x \\
& +370)y^3+(x((38x-175)x+695)-660)y^2+5(3(9x-23)x+74)y+45x-80\Big)m_s+c_p\Big((xy-1) \\
& ((x+((x-3)x+3)y-3)y+1)\Big(x\Big(70(8y-5)y+\Big(8y\Big(7(3x-4)+20\Big(5x^2-7\Big)y\Big)+693\Big)xy-105\Big)-70 \\
& (y-1)\Big)m_c+42\Big(8((x((x-13)x+38)-44)x+15)x^2y^5+(x(((57x-191)x+173)x+188)-140)xy^4 \\
& +((((103-15x)x-420)x+180)x+30)y^3-(((16x-185)x+25)x+60)y^2-35x(x+1)y+40y+15x \\
& -10\Big)m_s\Big)\Big)F(s,x,y)^3+21((y((x((x-3)x+3)y-3)y+3)-1)x(110(y-1)+(-10(47y+14)y+((-443x \\
& +8((27x-88)x+40)y+1026)y+120)xy+75)x+2(10(y-1)+(((y(41x+8((x-9)x+5)y+38)-50) \\
& x-40y+20)y+15)x)c_p\Big)m_sm_c^2+(x-1)(y-1)(sxy(y((x((x-3)x+3)y-3)y+3)-1)((100(5y-4)y \\
& +(2(909x+10(x(27x-14)-42)y-364)y+127)xy+95)x+2\Big(x\Big(y\Big(\Big(\Big(3x+40\Big(5x^2-7\Big)y+16\Big)y+144\Big) \\
& x+110y-80)-15)-10(y-1))c_p\Big)-6((((x-2)x+2)y-2)y+1)(x(y(-75x+2((37x-50)x+((7x \\
& -20)x+10)c_p+20\Big)y+5(6x-4)c_p+70\Big)-5\Big)-30(y-1)\Big)m_s^2\Big)m_c+3(x-1)(y-1)\Big(22((x(3(9x-47) \\
& x+281)-268)x+2((x((x-13)x+38)-44)x+15)c_p+80\Big)x^2y^6+((((2833-741x)x-4217)x+5860) \\
& x+4(x(((81x-278)x+272)x+230)-185)c_p-2490\Big)xy^5+((((109-15x)x-2952)x+1200)x+4 \\
& ((((121-15x)x-573)x+270)x+30)c_p+810\Big)y^4-\Big((x(18x-877)-235)x+4\Big(\Big(22x^2-278x+85\Big)x \\
& +60\Big)c_p+1460\Big)y^3+\Big((41x-535)x-16(x(14x+5)-10)c_p+1160\Big)y^2+5\Big(27x+4(3x-2)c_p-86\Big)y \\
& +60\Big)sxm_s\Big)F(s,x,y)^2-6(y-1)(14(y((x((x-3)x+3)y-3)y+3)-1)(x(y(-75x+2((37x-50)x \\
& +((7x-20)x+10)c_p+20\Big)y+5(6x-4)c_p+70\Big)-5\Big)-30(y-1)\Big)m_s^2m_c^3-7sx(y((x((x-3)x+3)y-3) \\
& y+3)-1)(((-10(115y+12)y+((-719x+22((27x-88)x+40)y+2190)y+26)xy+95)x+350y+4 \\
& (10(y-1)+(((y(59x+11((x-9)x+5)y+50)-71)x-55y+35)y+15)x)c_p-270\Big)y+60\Big)m_sm_c^2 \\
& -s(x-1)(y-1)\Big(sx^2y^2(y((x((x-3)x+3)y-3)y+3)-1)\Big(50\Big((27x-14)x+4\Big(5x^2-7\Big)c_p-42\Big)xy^3 \\
& -14\Big(-303x^2+93x+4(2x-5)(3x+1)c_p-75\Big)y^2+7\Big(37x+20(3x-2)c_p-130\Big)y+315\Big)-42((((x-2) \\
& x+2)y-2)y+1)\Big(4\Big(\Big(7c_p+37\Big)x^2-10\Big(2c_p+5\Big)x+10\Big(c_p+2\Big)\Big)xy^3+5\Big(x(3x-2)\Big(4c_p-3\Big)-6\Big)y^2
\end{aligned}
$$

$$\begin{aligned}
&-10(x-3)y-15)m_s^2\Big)m_c-21s^2(x-1)x(y-1)y((4(((x(3(9x-47)x+281)-268)x+2((x((x-13)x\\
&+38)-44)x+15)c_p+80\Big)x^2y^5+\Big((((373-101x)x-531)x+892)x+8(x(((8x-29)x+33)x+14)-15)\\
&c_p-410\Big)xy^4+\Big(\Big(((23-5x)x-588)x+24((2x-17)x+10)c_p+320\Big)x+110\Big)y^3-(((6x-223)x+8((2x\\
&-31)x+20)c_p+95\Big)x+220\Big)y^2-x\Big(11x+8(7x-5)c_p+35\Big)y+210y+15x-100\Big)y+20\Big)m_s\Big)F(s,x,y)\\
&-6s(y-1)^2(y((x((x-3)x+3)y-3)y+3)-1)m_c\Big((x-1)(y-1)\Big(70(y-1)y+\Big(4\Big((27x-14)x+4\Big(5x^2-7\Big)\\
&c_p-42\Big)y^2-14\Big(-23x+4(x-2)c_p+5\Big)y+21\Big)xy+35\Big)\Big(-s^2\Big)x^2y^3-7sx(50(y-1)y+(((y(-97x+4((27\\
&x-88)x+40)y+342)-16)x-190y+8(7x+((x-9)x+5)y-5)(xy-1)c_p+20\Big)y+5\Big)xy+20\Big)m_cm_s\\
&y+14\Big(4\Big((37x-50)x+((7x-20)x+10)c_p+20\Big)xy^3+5\Big(x(3x-2)\Big(4c_p-3\Big)-6\Big)y^2-10(x-3)y-15\Big)\\
&m_c^2m_s^2\Big)\Big)\Big\},
\end{aligned}$$

$$\begin{aligned}
\rho^{\langle\bar{s}\sigma\cdot Gs\rangle}_{3,9;A(S)}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1x}{3072\pi^3(y-1)^3}m_cF(s,x,y)\Big(3s(y-1)\Big(y\Big(4(x-1)(y-1)c_p(xy(11xy-10)+1)\\
&+x(xy(y(22((6x-25)x+9)y-73x+517)-51)-7(29y+2)y+30)+41y-33)+6)F(s,x,y)\\
&+\Big(2(x-1)(y-1)c_p(xy(8xy-7)+1)+xy(x(y(8((6x-25)x+9)y-65x+223)+11)-75y-26)+12x\\
&+9y-9)F(s,x,y)^2+6s^2(y-1)^2y\Big(xy\Big(y\Big(8(x-1)(y-1)c_p(xy-1)+x(y(4((6x-25)x+9)y-7x+89)\\
&-17)-37y+6\Big)+4\Big)+7(y-1)y+2\Big)\Big)-\frac{c_2x^2}{2048\pi^3(x-1)(y-1)^4}m_cF(s,x,y)(3s(y-1)(y(44((x-3)x+1)\\
&((x-1)x+1)xy^4+(x(((197-56x)x-47)x+12)-18)y^3+((15-22x(x+3))x+32)y^2+3(4x+5)xy\\
&-9x-36y+18)-3)F(s,x,y)+\Big(16((x-3)x+1)((x-1)x+1)xy^4+(x(((103-32x)x-53)x+24)\\
&-10)y^3-(x((7x+11)x+14)-16)y^2+9(x-1)y-3x+3\Big)F(s,x,y)^2+6s^2(y-1)^2y\Big(y\Big(\Big(x\Big(-8x^3+29x^2\\
&+x-4)-2)y^3+8((x-3)x+1)((x-1)x+1)xy^4+((9-4x(x+4))x+4)y^2+(x(4x-1)-6)y-x+4\Big)\\
&-1)\Big)\Big\},
\end{aligned}$$

$$\begin{aligned}
\rho^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}_{3,9;A(S)}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{-\frac{c_1}{9216\pi^3(y-1)^2}\Big(c_p\Big(24sx(y-1)y^2F(s,x,y)\Big(6m_c^2(((x-4)x+2)y+3x-2)+s\\
&(x-1)(y-1)y\Big(x\Big(25\Big(x^2-2\Big)y^2+2(25-9x)y-3\Big)-2y+2\Big)\Big)+6yF(s,x,y)^2\Big(6xm_c^2(((x-4)x+2)y+3x\\
&-2)+s(x-1)(y-1)\Big(x\Big(y\Big(x\Big(7y\Big(20\Big(x^2-2\Big)y-9x+40\Big)-36\Big)-22y+16\Big)+3\Big)+2(y-1)\Big)\Big)+(x-1)\\
&\Big(x\Big(y\Big(x\Big(8y\Big(10\Big(x^2-2\Big)y-3x+20\Big)-27\Big)-16y+10\Big)+3\Big)+2(y-1)\Big)F(s,x,y)^3+48s^3(x-1)x^2(y-1)^3\\
&y^5\Big(\Big(x^2-2\Big)y-x+2\Big)\Big)-x\Big(6s(y-1)yF(s,x,y)\Big(6m_c^2\Big(8(1-2x)^2y^2-3xy-1\Big)+s(x-1)(y-1)y\Big(y\Big(x\Big(50\\
&(5x(3x-2)-6)y^2+(954x-66)y+29\Big)+30y-26\Big)+9\Big)\Big)+3F(s,x,y)^2\Big(6m_c^2\Big(y\Big(4(1-2x)^2y-9x+4\Big)\\
&+1\Big)+s(x-1)(y-1)y(xy(2y(70(5x(3x-2)-6)y+1401x-164)+83)+20(5y-4)y+19)\Big)+(x-1)\\
&(y(x(8y(5(5x(3x-2)-6)y+126x-29)+21)+40y-28)-1)F(s,x,y)^3+6s^3(x-1)(y-1)^3y^3(xy(2y\\
&(2(5x(3x-2)-6)y+37x-1)+3)+2(y-1)y+1)))+\frac{c_2x}{4096\pi^3(x-1)(y-1)^3}(6s(y-1)F(s,x,y)\\
&\Big(2m_c^2(y(xy(x(y(4(4x-3)(x-1)y-12x+25)-4)-13y+2)+x+3(y-1))+1)+s(x-1)x(y-1)y^2(y(x\\
&(xy(y(50(2x-3)(x+1)y+62x+65)-148)+5(33y-4)y-11)-37y+33)-9))+3(x-1)F(s,x,y)^2\\
&\Big(2m_c^2(y(2(4x-3)xy-2x+1)-1)(xy-1)+sx(y-1)y(y(x(y(x(2y(70(2x-3)(x+1)y+108x+55)-415)
\end{aligned}$$

$$+470y-26)-38)-108y+92)-21))-(x-1)^2(y-1)c_pF(s,x,y)(6s(y-1)y(xy(11xy-10)+1)F(s,x,y)$$
$$+(xy(8xy-7)+1)F(s,x,y)^2+24s^2x(y-1)^2y^3(xy-1)\Big)+(x-1)x(y(x(y(x(4y(10(2x-3)(x+1)y+26x$$
$$-5)-139)+140y+22)-20)-30y+26)-3)F(s,x,y)^3+6s^3(x-1)x(y-1)^3y^3(y(x(y(x(y(4(2x-3)$$
$$(x+1)y+4x+7)-12)+13y-2)-1)-3y+3)-1))\Bigg\},$$

$$\rho_{3,9;A(S)}^{\langle\bar{s}s\bar{s}s\rangle}(s)=\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\frac{c_1c_p}{768\pi}m_s^2G(s,z)(G(s,z)+4s(z-1)z),$$
$$\rho_{3,9;A(S)}^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_{z_{\min}}^{z_{\max}}\mathrm{d}z\Bigg\{-\frac{c_1}{2304\pi}\Big(m_s^2\Big(\Big((4z(5-6z)+1)c_p+4z(7-10z)+1\Big)G(s,z)-s(z-1)z\Big(\Big(66z^2-58z-2\Big)c_p$$
$$+20(5z-4)z+19)\Big)-m_cm_s\Big(\Big(6c_p+13\Big)G(s,z)+s\Big(z\Big(12(z-1)c_p+29z-21\Big)+6\Big)\Big)+6m_c^2(2G(s,z)$$
$$+2s(z-1)z+s)\Big)+\frac{c_2c_p}{1024\pi}m_s^2(G(s,z)+2s(z-1)z)\Bigg\}+\int_0^1dz\Bigg\{\frac{c_1}{2304\pi}s^2(z-1)zm_s\Big((z-1)zm_s\Big(12(z-1)z$$
$$c_p+30z^2-26z+9\Big)+(3(z-1)z+2)m_c\Big)\Bigg\},$$

$$\rho_{3,9;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x^2}{18432\pi^3(x-1)^3(y-1)^4}m_c\Big(2F(s,x,y)\Big(m_c^2(xy-1)^2(y(((x-3)x+3)y+x-3)+1)$$
$$(x(y(8(x-3)(2x-1)y-8x+13)-5)-5y+5)+3s(x-1)(y-1)\big(y(22((x(2(x-5)x+19)-18)x+5)$$
$$x^2y^5+(x(((209-56x)x-234)x+350)-137)xy^4+(37-2x(x(32x+87)-36))y^3+2(x(x(11x+40)$$
$$-2)-33)y^2-6(2x(x+2)-9)y+9x-21\big)+3\Big)\Big)+3(x-1)\Big(8((x(2(x-5)x+19)-18)x+5)x^2y^5$$
$$+(x(((119-32x)x-146)x+162)-55)xy^4+((x+2)((x-26)x+4)x+15)y^3+2(x((5x+2)x+14)-13)$$
$$y^2+2(x-7)(x-1)y+3(x-1)\Big)F(s,x,y)^2+2s(y-1)\Big(m_c^2(xy-1)(y(((x-3)x+3)y+x-3)+1)(y(x(y$$
$$(x(y(22(x-3)(2x-1)y-34x+153)-12)-75y-4)+9)+19y-15)+3)+3s(x-1)(y-1)y\Big(y\Big(4((x$$
$$(2(x-5)x+19)-18)x+5)x^2y^5+2\Big(5-2x^2\Big)y+(x(((29-8x)x-28)x+54)-23)xy^4+(5-4x(3x(x$$
$$+3)-5))y^3+2(x(2x(x+5)-5)-5)y^2+x-5\Big)+1\Big)\Big)\Big)\Bigg\}-\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1s^2x^2y}{9216\pi^3(x-1)^3(y-1)^2}m_c^3$$
$$(xy-1)(y(((x-3)x+3)y+x-3)+1)(y(xy(x(y(4(x-3)(2x-1)y-4x+25)-4)-13y+2)+x$$
$$+3(y-1))+1),$$

$$\rho_{3,9;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x}{18432\pi^3(x-1)^3(y-1)^3}m_c^2\Big((x-1)\Big(c_p(xy-1)(y(((x-3)x+3)y+x-3)+1)(x$$
$$(y(8((x-4)x+2)y+21x-10)-3)-2y+2)+8((x((35x-118)x+104)-24)x+45)x^2y^5$$
$$+40\Big(x^2((3x-11)x+13)-6\Big)x^3y^6+(x(x(x(73x+101)-669)-204)-72)xy^4+(x(((935-169x)x$$
$$-310)x+252)-24)y^3+(((179-220x)x-193)x+48)y^2+(-61(x-1)x-36)y+x+12\Big)F(s,x,y)$$
$$+4m_c^2(xy-1)(y(((x-3)x+3)y+x-3)+1)(y(x(((5x-6)x+2)y-4x+3)-1)+1)+s(x-1)(y-1)$$
$$\Big(y\Big(2c_p(xy-1)(y(((x-3)x+3)y+x-3)+1)(x(y(11((x-4)x+2)y+30x-16)-3)-2y+2)+2(((11$$
$$(35x-113)x+839)x+6)x+570)x^2y^5+140\Big(x^2((3x-11)x+13)-6\Big)x^3y^6+(x(x(x(155x+691)$$
$$-2229)-936)-204)xy^4+(x(((2353-425x)x-92)x+492)-24)y^3+(((209-656x)x-409)x+48)y^2$$
$$+((173-65x)x-48)y-25x+24)-6)\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{-\frac{\mathrm{i}c_1sx}{18432\pi^3(x-1)^3(y-1)^2}m_c^2(y(y(x((x-3)$$

$$x+3)y-3)+3)-1)\Big(2m_c^2\Big(y\Big(-4x((5x-6)x+2)y^2+2((2x-1)x+1)y+x-2\Big)+1\Big)-s(x-1)x(y-1)y^2$$
$$\Big(y\Big(4c_p(((x-4)x+2)y+3x-2)+x(2y(25(x(3x-2)-2)y+130x-21)+11)+30y-26\Big)+9\Big)\Big)\Bigg\},$$

$$\rho_{3,9;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x}{36864\pi^3(x-1)^3(y-1)^3}m_c\Big(3(x-1)\Big(2(x-1)(y-1)c_p(xy(y(x(y(8(2x-3)(x-1)y$$
$$-2x+21)-8)-25y+18)-5)+3(y-1)y+1)+8\Big(\Big(x\Big(6x^2-32x+63\Big)-62\Big)x+15\Big)x^2y^5+(x(((374$$
$$-105x)x-379)x+490)-137)xy^4+(((2x(3x-68)-165)x+20)x+23)y^3+(x((45x+34)x+57)-38)$$
$$y^2+((5x-36)x+19)y+7x-4\Big)F(s,x,y)+m_c^2(y(y(x((x-3)x+3)y-3)+3)-1)\Big(2(x-1)(y-1)c_p(xy$$
$$(8xy-7)+1)+xy(x(y(8((6x-25)x+9)y-65x+223)+11)-75y-26)+12x+9y-9)+3s(x-1)$$
$$(y-1)\Big(y\Big(4(x-1)(y-1)c_p(xy(y(x(y(11(2x-3)(x-1)y-2x+27)-11)-34y+27)-8)+3(y-1)y$$
$$+1)+22\Big(\Big(x\Big(6x^2-32x+63\Big)-62\Big)x+15\Big)x^2y^5+(x((3(218-61x)x-547)x+1118)-361)xy^4$$
$$+(((x(2x-341)-666)x+212)x+67)y^3+(x(19x(6x+17)-33)-118)y^2+(100-x(53x+59))y+26x$$
$$-41\Big)+6\Big)\Big)+\frac{c_2x^2y}{24576\pi^3(x-1)^2(y-1)^2}m_c^3(xy-1)(y(x(y(8((x-1)x+1)y+26x-17)-3)+5y-1)-2)\Bigg\}$$
$$+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1sx}{36864\pi^3(x-1)^3(y-1)^2}m_c\Big(m_c^2(-(y(y(x((x-3)x+3)y-3)+3)-1))(y(4(x-1)(y-1)$$
$$c_p(xy(11xy-10)+1)+x(xy(y(22((6x-25)x+9)y-73x+517)-51)-7(29y+2)y+30)+41y-33\Big)$$
$$+6\Big)-3s(x-1)(y-1)y\Big(y\Big(4x^2y^5\Big(2(2x-3)(x-1)^2c_p+\Big(x\Big(6x^2-32x+63\Big)-62\Big)x+15\Big)-xy^4(8(2x-3)$$
$$(x-1)((x-1)x-1)c_p+(x((27x-94)x+61)-182)x+63\Big)+y^3\Big(9-x\Big(24(x-2)(x-1)c_p+(62x$$
$$+141)x-60)\Big)+y^2\Big(x\Big(8(x-4)(x-1)c_p+(21x+80)x-33\Big)-18\Big)+y\Big(x\Big(8(x-1)c_p-17x+2\Big)+19\Big)$$
$$+3x-10\Big)+2\Big)\Big)+\frac{c_2sx^2y^3}{24576\pi^3(x-1)^2(y-1)}m_c^3(x(y(x(y(22((x-1)x+1)y+32x-17)-47)-13y+20)$$
$$+3)+y-1)\Bigg\},$$

$$\rho_{4;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1}{110592\pi^3(x-1)^2(y-1)^2}m_c^2\Big(-8x^2y^5\Big((x(x((3x-29)x+101)-66)-54)c_p$$
$$+(((126x-407)x+245)x+48)x+105)+40((x-3)x+3)x^3y^6\Big(2\Big(x^2-2\Big)c_p+5x(2-3x)+6\Big)$$
$$+xy^4\Big(((x((9x+115)x+395)-1368)x+198)c_p+((2853-x(309x+845))x+828)x-24\Big)+y^3((x(x((31$$
$$x-443)x+1384)-372)-6)c_p+(x((763x-3247)x+414)-204)x\Big)+y^2((((60x-601)x+253)x+12)$$
$$c_p+((702x-295)x+157)x\Big)+y\Big((x(135x-73)-8)c_p+61(3x-1)x\Big)-3xc_p+2c_p-19x\Big)$$
$$+\frac{c_2xy}{49152\pi^3(x-1)^2(y-1)^2}m_c^2\Big(c_p\Big(8((x((x-4)x+6)-6)x+2)xy^4+(x(x((5x-14)x+60)-24)-2)y^3$$
$$-(x(x(x+45)-15)-4)y^2+(2x-1)(8x+3)y-2x+1\Big)+xy\Big(40\Big(x^2((x-4)x+6)-2\Big)xy^4+4((x((34x$$
$$-111)x+81)-16)x+10)y^3+(3x((36x-109)x+61)-68)y^2+((99x-86)x+43)y+39x-13\Big)-10x$$
$$+6y-6\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{-\frac{c_1}{110592\pi^3(x-1)^3(y-1)^2}m_c^2\Big(6xm_c^2(y(y(x((x-3)x+3)y-3)+3)-1)\Big(y$$
$$\Big(2c_p(((x-4)x+2)y+3x-2)-4(1-2x)^2y+9x-4\Big)-1\Big)+s(x-1)(y-1)y\Big(-2x^2y^5((x(x(7(9x-67)x$$
$$+1471)-906)-774)c_p+(((1401x-4367)x+1595)x+1458)x+1410\Big)+140((x-3)x+3)x^3y^6\Big(2\Big(x^2$$
$$-2\Big)c_p+5x(2-3x)+6\Big)+xy^4\Big((2(x(328x+467)-2070)x+432)c_p+((9669-x(659x+4243))x+3216)$$

$$x+12\Big)-y^3\Big(2(((x(29x+560)-1891)x+408)x+6)c_p+((5x(1741-427x)+708)x+252)x\Big)+y^2(2 ((11(9x-68)x+277)x+12)c_p+((2244x-145)x+289)x\Big)+y\Big(2(x(144x-79)-8)c_p+(137x-173)x\Big) +(4-6x)c_p+37x\Big)\Big)+\frac{c_2xy}{49152\pi^3(x-1)^3(y-1)^2}m_c^2\Big(2m_c^2\Big(2((x((x-4)x+10)-8)x+2)xy^4+(x(5x(1-3x). +4)-2)y^3+((x(7x+11)-10)x+4)y^2+((3-8x)x-3)y+x+1\Big)+s(x-1)(y-1)y\Big(2c_p(11((x(((x-4)x +6)-6)x+2)xy^4+(x(x((8x-23)x+87)-36)-2)y^3-(x(x(x+63)-24)-4)y^2+(2x-1)(11x+3)y -2x+1)+x\Big(y\Big(140\Big(x^2((x-4)x+6)-2\Big)xy^4+2((((207x-668)x+358)x+2)x+50)y^3+(x((340x -829)x+465)-180)y^2+((305x-262)x+109)y+41x-19\Big)-8\Big)-6y+6\Big)\Big)\Bigg\}, \tag{25}$$

where the coefficient $c_p=1$ for current $J^{A(S)}_{3,\mu\nu}$ and $c_p=-1$ for current $J^{A(S)}_{9,\mu\nu}$. The spectral functions for the $(0,2\{1,1\})$ structure with $\mathbb{C}=+1$ are given as

$$\rho^{pert}_{4;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^3}{12902400\pi^5(y-1)^5}\Big(c_p+1\Big)F(s,x,y)^3(-15(y-1)F(s,x,y)(7sxym_cm_s(11xy(2xy+3) -10)(xy-1)+14m_c^2m_s^2(xy(2xy-5)+5)-6s^2(x-1)x^2(y-1)y^3(5xy(5xy+7)-7)\Big)+21xF(s,x,y)^2\Big(20s (x-1)x(y-1)y^2(xy+2)(3xy-1)-m_cm_s(xy-1)(xy+3)(8xy-5)\Big)+2(x-1)x^2y(10xy(3xy+7)-49) F(s,x,y)^3-60sx(y-1)^2y^2\Big(7sxym_cm_s(xy-1)(4xy+5)+14m_c^2m_s^2(4xy-5)-2s^2(x-1)x^2(y-1)y^3(6xy +7)\Big)\Big),$$

$$\rho^{\langle\bar{s}s\rangle}_{4;A(S)}(s)=0,$$

$$\rho^{\langle m_s\bar{s}s\rangle}_{4;A(S)}(s)=0,$$

$$\rho^{\langle g_sG^2\rangle}_{4;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x^3}{15482880\pi^5(x-1)^3(y-1)^5}m_c\Big(c_p+1\Big)\Big(6(y-1)F(s,x,y)\Big(6s(x-1)x(y-1)y^2m_c\Big(sxy (xy-1)(5xy(5xy+7)-7)(y(((x-3)x+3)y+x-3)+1)-7m_s^2(4xy-5)(y(((x-2)x+2)y-2)+1)\Big) -7sxym_c^2m_s(xy-1)^2(11xy(2xy+3)-10)(y(((x-3)x+3)y+x-3)+1)-14m_c^3m_s^2(xy-1)(xy(2xy-5) +5)(y(((x-3)x+3)y+x-3)+1)-21s^2(x-1)x^2(y-1)y^3m_s(xy-1)(4xy+5)(y(((x-3)x+3)y+x-3) +1)\Big)-21F(s,x,y)^2\Big(2(x-1)(y-1)m_c\Big(3m_s^2(xy(2xy-5)+5)(y(((x-2)x+2)y-2)+1)-10sx^2y^2(xy-1) (xy+2)(3xy-1)(y(((x-3)x+3)y+x-3)+1)\Big)+xm_c^2m_s(8xy-5)(xy-1)^2(y(((x-3)x+3)y+x-3) +1)(xy+3)+3s(x-1)x(y-1)ym_s(11xy(2xy+3)-10)(xy-1)(y(((x-3)x+3)y+x-3)+1)\Big)+(x-1) x(y(y(x((x-3)x+3)y-3)+3)-1)F(s,x,y)^3(4xym_c(10xy(3xy+7)-49)-21m_s(xy+3)(8xy-5)) -6sx(y-1)^2y^2m_c(y(y(x((x-3)x+3)y-3)+3)-1)\Big(7sxym_cm_s(xy-1)(4xy+5)+14m_c^2m_s^2(4xy-5) -2s^2(x-1)x^2(y-1)y^3(6xy+7)\Big)\Big)+\frac{c_3}{11796480\pi^5(y-1)^3}(x-1)x^2\Big(c_p+1\Big)F(s,x,y)^2(4s(y-1)y(293xy -27)F(s,x,y)+(181xy-36)F(s,x,y)^2+588s^2x(y-1)^2y^3\Big)\Bigg\},$$

$$\rho^{\langle\bar{s}\sigma\cdot Gs\rangle}_{4;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x^2}{512\pi^3(y-1)^3}m_c\Big(c_p+1\Big)(xy-1)F(s,x,y)(s(y-1)y(5xy-2)F(s,x,y)+(xy-1) F(s,x,y)^2+2s^2x(y-1)^2y^3)\Bigg\},$$

$$\rho^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}_{4;A(S)}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x}{2304\pi^3(y-1)^2}\Big(c_p+1\Big)\Big(18sx(y-1)y^2F(s,x,y)\Big(s(x-1)(y-1)y(5xy-1)-m_c^2\Big)$$

$$
\begin{aligned}
&+3F(s,x,y)^2\Big(m_c^2(3-3xy)+10s(x-1)x(y-1)y^2(5xy-2)\Big)+2(x-1)xy(10xy-7)F(s,x,y)^3+6s^3(x-1)\\
&x^2(y-1)^3y^5\Big)\Bigg\},\\
\rho_{4;A(S)}^{\langle\bar{s}s\bar{s}s\rangle}(s)=&0,\\
\rho_{4;A(S)}^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&0,\\
\rho_{4;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=&0,\\
\rho_{4;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=&0,\\
\rho_4^{\langle g_sG^2\rangle^2\,A(S)}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^5y^4m_c^4\Big(c_p+1\Big)(10xy(3xy+7)-49)}{46448640\pi^5(x-1)^2(y-1)^2}\\
&+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1sx^5y^5m_c^4\Big(c_p+1\Big)(xy+2)(3xy-1)}{1327104\pi^5(x-1)^2(y-1)},\\
\rho_{4;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x^2}{6144\pi^3(x-1)^3(y-1)^3}m_c\Big(c_p+1\Big)(y(y(x((x-3)x+3)y-3)+3)-1)\Big(3(x-1)\\
&(xy-1)F(s,x,y)+m_c^2(xy-1)^2+s(x-1)(y-1)y(5xy-2)\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{-\frac{c_1sx^2y}{18432\pi^3(x-1)^3(y-1)^2}\\
&m_c\Big(c_p+1\Big)(y(y(x((x-3)x+3)y-3)+3)-1)\Big(m_c^2(5xy-2)(xy-1)+3s(x-1)x(y-1)y^2\Big)\Bigg\},\\
\rho_{10;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x}{27648\pi^3(x-1)^2(y-1)^2}m_c^2\Big(c_p+1\Big)(xy-1)^2\Big(xy\Big(20((x-3)x+3)y^2+26(x-3)y\\
&+23\Big)+18(y-1)y+9\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{-\frac{c_1x}{27648\pi^3(x-1)^3(y-1)^2}m_c^2\Big(c_p+1\Big)\Big(s(x-1)x(y-1)y^2(xy(x\\
&y(10y(5((x-3)x+3)y-2x+6)-9)+42(4-5y)y-50)+42(y-1)y+11)-3m_c^2(xy-1)^2(y(((x-3)x\\
&+3)y+x-3)+1))\Bigg\},
\end{aligned}
\tag{26}
$$

where the coefficient $c_p=1$. The spectral functions for the $(0,2\{1,1\})$ structure with $\mathbb{C}=-1$ are given as

$$
\begin{aligned}
\rho_{10;A(S)}^{pert}(s)=&-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^3}{614400\pi^5(y-1)^5}\Big(c_p-1\Big)F(s,x,y)^3\Big(5(y-1)F(s,x,y)\Big(2s^2(x-1)x^2(y-1)y^3(11xy\\
&-3)-5m_cm_s(xy-1)(sxy(5xy-2)-2m_cm_s))+xF(s,x,y)^2\Big(8s(x-1)x(y-1)y^2(7xy-4)-15m_cm_s\\
&(xy-1)^2\Big)+4(x-1)x^2y(xy-1)F(s,x,y)^3+20sx(y-1)^2y^2\Big(5m_cm_s(2m_cm_s+sxy(1-xy))+2s^2(x-1)\\
&x^2(y-1)y^3\Big)\Big),\\
\rho_{10;A(S)}^{\langle\bar{s}s\rangle}(s)=&0,\\
\rho_{10;A(S)}^{\langle m_s\bar{s}s\rangle}(s)=&0,\\
\rho_{10;A(S)}^{\langle g_sG^2\rangle}(s)=&\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{-\frac{c_1x^3}{737280\pi^5(x-1)^3(y-1)^5}m_c\Big(c_p-1\Big)\Big(2(y-1)F(s,x,y)\Big(2s(x-1)x(y-1)y^2m_c\\
&\Big(15m_s^2(y(((x-2)x+2)y-2)+1)+sxy(11xy-3)(xy-1)(y(((x-3)x+3)y+x-3)+1)\Big)-5sxym_c^2m_s\\
&(xy-1)^2(5xy-2)(y(((x-3)x+3)y+x-3)+1)+10m_c^3m_s^2(xy-1)^2(y(((x-3)x+3)y+x-3)+1)\\
&-15s^2(x-1)x^2(y-1)y^3m_s(y(y(x((x-3)x+3)y-3)+3)-1)\Big)+(xy-1)F(s,x,y)^2\,(2(x-1)(y-1)m_c\\
&\Big(15m_s^2(y(((x-2)x+2)y-2)+1)+4sx^2y^2(7xy-4)(y(((x-3)x+3)y+x-3)+1)\Big)-15xm_c^2m_s(xy-1)^2
\end{aligned}
$$

$$(y(((x-3)x+3)y+x-3)+1)-15s(x-1)x(y-1)ym_s(5xy-2)(y(((x-3)x+3)y+x-3)+1))$$
$$+(x-1)x(xy-1)^2(y(((x-3)x+3)y+x-3)+1)F(s,x,y)^3\,(8xym_c-15m_s)+2sx(y-1)^2y^2m_c(y(y(x((x$$
$$-3)x+3)y-3)+3)-1)\left(-5sxym_cm_s(xy-1)+10m_c^2m_s^2+2s^2(x-1)x^2(y-1)y^3\right)\Big)+\frac{c_3}{2359296\pi^5(y-1)^3}$$
$$(x-1)x^2\left(c_p-1\right)F(s,x,y)^2\left(4s(y-1)y(15xy-2)F(s,x,y)+(9xy-3)F(s,x,y)^2+36s^2x(y-1)^2y^3\right)\Big\},$$

$$\rho_{10;A(S)}^{\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x^2}{512\pi^3(y-1)^3}m_c\left(c_p-1\right)(xy-1)F(s,x,y)\,(s(y-1)y(5xy-2)F(s,x,y)+(xy-1)$$
$$F(s,x,y)^2+2s^2x(y-1)^2y^3)\Big\},$$

$$\rho_{10;A(S)}^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x}{768\pi^3(y-1)^2}\left(c_p-1\right)\Big(2sx(y-1)y^2F(s,x,y)\left(s(x-1)(y-1)y(11xy-3)-3m_c^2\right)$$
$$+F(s,x,y)^2\left(m_c^2(3-3xy)+4s(x-1)x(y-1)y^2(7xy-4)\right)+4(x-1)xy(xy-1)F(s,x,y)^3+2s^3(x-1)x^2$$
$$(y-1)^3y^5\Big)\Big\},$$

$$\rho_{10;A(S)}^{\langle\bar{s}s\bar{s}s\rangle}(s)=0,$$
$$\rho_{10;A(S)}^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=0,$$
$$\rho_{10;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=0,$$
$$\rho_{10;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}(s)=0,$$

$$\rho_{10;A(S)}^{\langle g_sG^2\rangle^2}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^5y^4m_c^4\left(c_p-1\right)(xy-1)}{1105920\pi^5(x-1)^2(y-1)^2}-\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{ic_1sx^5y^5m_c^4\left(c_p-1\right)(7xy-4)}{3317760\pi^5(x-1)^2(y-1)},$$

$$\rho_{10;A(S)}^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x^2}{6144\pi^3(x-1)^3(y-1)^3}m_c\left(c_p-1\right)(y(y(x((x-3)x+3)y-3)+3)-1)\,(3(x-1)(xy$$
$$-1)F(s,x,y)+m_c^2(xy-1)^2+s(x-1)(y-1)y(5xy-2))\Big\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Big\{\frac{c_1sx^2y}{18432\pi^3(x-1)^3(y-1)^2}m_c$$
$$\left(c_p-1\right)(y(y(x((x-3)x+3)y-3)+3)-1)\left(m_c^2(5xy-2)(xy-1)+3s(x-1)x(y-1)y^2\right)\Big\},$$

$$\rho_{5,11;A(S)}^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Big\{\frac{c_1x}{9216\pi^3(x-1)^2(y-1)^2}m_c^2\left(c_p-1\right)(xy-1)\Big(y\Big(x\Big(y\Big(x\left(4((x-3)x+3)y^2-3\right)-12y$$
$$+18\Big)-4\Big)-6y+6\Big)-3\Big)\Big\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Big\{\frac{c_1x}{27648\pi^3(x-1)^3(y-1)^2}m_c^2\left(c_p-1\right)\Big(s(x-1)x(y-1)y^2$$
$$\Big(xy\Big(y\left(28((x-3)x+3)xy^2-4(4(x-3)x+33)y-9x+102\right)-28\Big)+30(y-1)y+7\Big)-3m_c^2(xy-1)^2$$
$$(y(((x-3)x+3)y+x-3)+1)\Big)\Big\}, \qquad (27)$$

where the coefficient $c_p=-1$. The spectral functions for the $(0,2\{2,0\})$ structure are given as

$$\rho_{5,11;A(S)}^{pert}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^3}{6451200\pi^5(y-1)^5}F(s,x,y)^3\Big(15(y-1)F(s,x,y)\Big(7sxym_cm_s(xy-1)\Big(xy\Big(11xy\left(c_p+6\right)$$
$$+4c_p+99\Big)-25\Big)+14m_c^2m_s^2\Big(xy\Big(xy\left(c_p+6\right)-15\Big)+10\Big)-s^2(x-1)x^2(y-1)y^3\Big(xy\Big(25xy\left(3c_p+10\right)+28$$
$$\left(c_p+13\right)\Big)-63\Big)\Big)+21xF(s,x,y)^2\Big(m_cm_s(xy-1)\Big(xy\left(4xy\left(c_p+6\right)+2c_p+57\right)-35\Big)-2s(x-1)x(y-1)y^2$$
$$\Big(5x^2y^2\left(3c_p+10\right)+xy\left(11c_p+78\right)-2\left(c_p+12\right)\Big)\Big)-(x-1)x^2y\Big(10x^2y^2\left(3c_p+10\right)+28xy\left(c_p+8\right)$$

$$-7\left(c_p+18\right)\big)F(s,x,y)^3+60sx(y-1)^2y^2\left(7sxym_cm_s(xy-1)\left(2xy\left(c_p+6\right)+15\right)+14m_c^2m_s^2\left(2xy\left(c_p+6\right)-15\right)-2s^2(x-1)x^2(y-1)y^3\left(xy\left(3c_p+10\right)+14\right)\right)\Big),$$

$$\rho^{\langle\bar{s}s\rangle}_{5,11;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^3}{384\pi^3(y-1)^4}m_c(xy-1)F(s,x,y)^2\Big(s(y-1)y(xy(11xy+14)-3)F(s,x,y)+(xy(xy+2)-1)F(s,x,y)^2+6s^2x(y-1)^2y^3(xy+1)\Big),$$

$$\rho^{\langle m_s\bar{s}s\rangle}_{5,11;A(S)}(s)=-\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\frac{c_1x^2}{384\pi^3(y-1)^3}F(s,x,y)\Big(-3sx(y-1)y^2F(s,x,y)\Big(4m_c^2(xy-1)+s(x-1)(y-1)y(xy(25xy+26)-3)\Big)-2F(s,x,y)^2\Big(m_c^2(xy-1)^2+s(x-1)x(y-1)y^2(xy(35xy+39)-8)\Big)-(x-1)xy(xy(5xy+8)-3)F(s,x,y)^3-12s^3(x-1)x^2(y-1)^3y^5(xy+1)\Big),$$

$$\rho^{\langle g_sG^2\rangle}_{5,11;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\bigg\{-\frac{c_1x^3}{7741440\pi^5(x-1)^3(y-1)^5}m_c\Big(-6(y-1)F(s,x,y)\Big(-21s^2(x-1)x^2(y-1)y^3m_s(y(y(x((x-3)x+3)y-3)+3)-1)\left(2xy\left(c_p+6\right)+15\right)+s(x-1)x(y-1)y^2m_c(sxy(y(y(x((x-3)x+3)y-3)+3)-1)\left(xy\left(25xy\left(3c_p+10\right)+28\left(c_p+13\right)\right)-63\right)-42m_s^2(y(((x-2)x+2)y-2)+1)\left(2xy\left(c_p+6\right)-15\right)\Big)-7sxym_c^2m_s(xy-1)^2(y(((x-3)x+3)y+x-3)+1)\left(xy\left(11xy\left(c_p+6\right)+4c_p+99\right)-25\right)-14m_c^3m_s^2(y(y(x((x-3)x+3)y-3)+3)-1)\left(xy\left(xy\left(c_p+6\right)-15\right)+10\right)\Big)+(x-1)x(y(y(x((x-3)x+3)y-3)+3)-1)F(s,x,y)^3\Big(21m_s\left(xy\left(4xy\left(c_p+6\right)+2c_p+57\right)-35\right)+2xym_c\left(-10x^2y^2\left(3c_p+10\right)-28xy\left(c_p+8\right)+7\left(c_p+18\right)\right)\Big)+21F(s,x,y)^2\Big(2(x-1)(y-1)m_c\Big(3m_s^2(y(((x-2)x+2)y-2)+1)\left(xy\left(xy\left(c_p+6\right)-15\right)+10\right)-sx^2y^2(y(y(x((x-3)x+3)y-3)+3)-1)\left(5x^2y^2\left(3c_p+10\right)+xy\left(11c_p+78\right)-2\left(c_p+12\right)\right)\Big)+xm_c^2m_s(xy-1)^2(y(((x-3)x+3)y+x-3)+1)\left(xy\left(4xy\left(c_p+6\right)+2c_p+57\right)-35\right)+3s(x-1)x(y-1)ym_s(y(y(x((x-3)x+3)y-3)+3)-1)\left(xy\left(11xy\left(c_p+6\right)+4c_p+99\right)-25\right)\Big)+6sx(y-1)^2y^2m_c(y(y(x((x-3)x+3)y-3)+3)-1)\Big(7sxym_cm_s(xy-1)\left(2xy\left(c_p+6\right)+15\right)+14m_c^2m_s^2\left(2xy\left(c_p+6\right)-15\right)-2s^2(x-1)x^2(y-1)y^3\left(xy\left(3c_p+10\right)+14\right)\Big)\Big)\bigg\},$$

$$\rho^{\langle\bar{s}\sigma\cdot Gs\rangle}_{5,11;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\bigg\{\frac{c_1x^2}{1536\pi^3(y-1)^3}m_c(xy-1)F(s,x,y)\Big(3s(y-1)y(xy(55xy+56)-9)F(s,x,y)+4(xy(5xy+8)-3)F(s,x,y)^2+12s^2x(y-1)^2y^3(5xy+4)\Big)+\frac{c_2x^3}{1024\pi^3(x-1)(y-1)^4}m_c(y(((x-2)x+2)y-2)+1)F(s,x,y)\Big(3s(y-1)y(xy(11xy+14)-3)F(s,x,y)+4(xy(xy+2)-1)F(s,x,y)^2+12s^2x(y-1)^2y^3(xy+1)\Big)\bigg\},$$

$$\rho^{m_s\langle\bar{s}\sigma\cdot Gs\rangle}_{5,11;A(S)}(s)=\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\bigg\{-\frac{c_1x}{1152\pi^3(y-1)^2}\Big(-3sx(y-1)y^2F(s,x,y)\Big(6m_c^2(4xy-3)+s(x-1)(y-1)y\left(xy\left(-4c_p+125xy+104\right)-9\right)\Big)-3F(s,x,y)^2\Big(3m_c^2(xy-1)(2xy-1)+s(x-1)x(y-1)y^2\left(c_p(2-11xy)+xy(175xy+156)-24\right)\Big)-(x-1)xy\left(-4xyc_p+c_p+2xy(25xy+32)-18\right)F(s,x,y)^3-6s^3(x-1)x^2(y-1)^3y^5(5xy+4)\Big)+\frac{c_2x^2(xy-1)}{1024\pi^3(x-1)(y-1)^3}\Big(3sx(y-1)y^2F(s,x,y)\Big(4m_c^2(xy-1)+s(x-1)(y-1)y(xy(25xy+26)-3)\Big)+3F(s,x,y)^2\Big(m_c^2(xy-1)^2+s(x-1)x(y-1)y^2(xy(35xy+39)-8)\Big)+2(x-1)xy(xy(5xy+8)-3)F(s,x,y)^3+6s^3(x-1)x^2(y-1)^3y^5(xy+1)\Big)\bigg\},$$

$$
\begin{aligned}
\rho^{\langle\bar{s}s\bar{s}s\rangle}_{5,11;A(S)}(s) =&0,\\
\rho^{\langle\bar{s}s\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{5,11;A(S)}(s) =&0,\\
\rho^{\langle g_sG^2\rangle\langle\bar{s}s\rangle}_{5,11;A(S)}(s) =& \int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x^3}{4608\pi^3(x-1)^3(y-1)^4}m_c(y(y(x((x-3)x+3)y-3)+3)-1)\Big(F(s,x,y)\Big(4m_c^2(xy(xy\\
&+2)-1)(xy-1)+3s(x-1)(y-1)y(xy(11xy+14)-3))+6(x-1)(xy(xy+2)-1)F(s,x,y)^2+s(y-1)y\\
&\Big(m_c^2(xy(11xy+14)-3)(xy-1)+6s(x-1)x(y-1)y^2(xy+1)\Big)\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\frac{c_1s^2x^4y^3}{2304\pi^3(x-1)^3(y-1)^2}\\
&m_c^3\left(x^2y^2-1\right)(y(y(x((x-3)x+3)y-3)+3)-1),\\
\rho^{m_s\langle g_sG^2\rangle\langle\bar{s}s\rangle}_{5,11;A(S)}(s) =& \int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x^2}{2304\pi^3(x-1)^3(y-1)^3}m_c^2(xy-1)\Big((x-1)\Big(y\Big(x\Big(y\Big(x\Big(y\Big(x\Big(10((x-3)x+3)y^2+26(x\\
&-3)y+23\Big)+48y-36\Big)+7\Big)-12y+18\Big)-3\Big)-6y+6\Big)-3\Big)F(s,x,y)+m_c^2(xy-1)^2(y(((x-3)x+3)y+x\\
&-3)+1)+s(x-1)x(y-1)y^2\Big(xy\Big(y\Big(35((x-3)x+3)xy^2+(74(x-3)x+117)y+72x-105\Big)+31\Big)-12(y\\
&-1)y-2)\Big)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1sx^3y^2}{4608\pi^3(x-1)^3(y-1)^2}m_c^2(xy-1)(y(((x-3)x+3)y+x-3)+1)\Big(4m_c^2(xy\\
&-1)+s(x-1)(y-1)y(xy(25xy+26)-3)\Big)\Bigg\},\\
\rho^{\langle g_sG^2\rangle^2}_{5,11;A(S)}(s) =& \int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x^5y^4}{46448640\pi^5(x-1)^2(y-1)^2}m_c^4\Big(c_p(2xy(15xy+14)-7)+4xy(25xy+56)-126\Big)\Bigg\}\\
&+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1sx^5y^5}{6635520\pi^5(x-1)^2(y-1)}m_c^4\Big(c_p(xy(15xy+11)-2)+2xy(25xy+39)-24\Big)\Bigg\},\\
\rho^{\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{5,11;A(S)}(s) =& -\int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x^2}{18432\pi^3(x-1)^3(y-1)^3}m_c(y(y(x((x-3)x+3)y-3)+3)-1)\Big(12(x-1)(xy(5xy\\
&+8)-3)F(s,x,y)+4m_c^2(xy(5xy+8)-3)(xy-1)+3s(x-1)(y-1)y(xy(55xy+56)-9)\Big)\\
&+\frac{c_2x^3y^2}{3072\pi^3(x-1)^2(y-1)^2}m_c^3(xy-1)(xy(xy+2)-1)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1sx^2y}{18432\pi^3(x-1)^3(y-1)^2}m_c(y(y(x\\
&((x-3)x+3)y-3)+3)-1)\Big(m_c^2(xy(55xy+56)-9)(xy-1)+6s(x-1)x(y-1)y^2(5xy+4)\Big)\\
&+\frac{c_2sx^3y^3}{12288\pi^3(x-1)^2(y-1)}m_c^3(xy-1)(xy(11xy+14)-3)\Bigg\},\\
\rho^{m_s\langle g_sG^2\rangle\langle\bar{s}\sigma\cdot Gs\rangle}_{5,11;A(S)}(s) =& \int_0^{x_{\max}}\mathrm{d}x\int_{y_{\min}}^{y_{\max}}\mathrm{d}y\Bigg\{\frac{c_1x}{13824\pi^3(x-1)^2(y-1)^2}m_c^2(xy-1)\Big(y\Big(-xc_p(4xy-1)(y(((x-3)x+3)y+x-3)+1)\\
&+y\Big(x\Big(x\Big(50((x-3)x+3)xy^3+6(19(x-3)x+32)y^2+6(19x-29)y+37\Big)-18(y-2)\Big)-18\Big)+18\Big)-9\Big)\\
&+\frac{c_2x^2y}{12288\pi^3(x-1)^2(y-1)^2}m_c^2(xy(xy(xy(2y(5((x-2)x+2)y+8x-26)+7)+4(8y-5)y+7)-12(y-1)y\\
&+3)-3)\Bigg\}+\int_0^1\mathrm{d}x\int_0^1\mathrm{d}y\Bigg\{\frac{c_1x}{13824\pi^3(x-1)^3(y-1)^2}m_c^2\Big(3m_c^2(xy-1)^2(2xy-1)(y(((x-3)x+3)y+x-3)\\
&+1)+s(x-1)x(y-1)y^2\Big(-c_p(xy-1)(11xy-2)(y(((x-3)x+3)y+x-3)+1)+xy\Big(y\Big(x\Big(y\Big(x\Big(175((x\\
&-3)x+3)y^2+156(x-3)y+12\Big)-57y+525\Big)-202\Big)-468y+450\Big)-120\Big)+18(y-1)y-3\Big)\Big)\\
&+\frac{c_2x^2y}{12288\pi^3(x-1)^3(y-1)^2}m_c^2\Big(m_c^2(xy-1)^2(y(((x-2)x+2)y-2)+1)+s(x-1)x(y-1)y^2(xy(y(x(y(35((x\\
&-2)x+2)y+39x-148)+33)+78y-62)+27)-16(y-1)y-2)\Big)\Bigg\},
\end{aligned}
\tag{28}
$$

where the coefficient $c_p = 1$ for current $J^{A(S)}_{5,\mu\nu}$ and $c_p = -1$ for current $J^{A(S)}_{11,\mu\nu}$.

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