

# Two non-perturbative $\alpha'$ or loop corrected string cosmological solutions\*

Li Song (宋丽)<sup>1†</sup> Deyou Chen (陈德友)<sup>2‡</sup> 

<sup>1</sup>College of Physics, Sichuan University, Chengdu 610065, China

<sup>2</sup>School of Science, Xihua University, Chengdu 610039, China

**Abstract:** In this paper, we present two non-perturbative string cosmological solutions without curvature singularities for the bosonic gravi-dilaton system. These solutions are general in that they can straightforwardly match the perturbative solution to arbitrarily high orders in the perturbative region. The first solution includes non-perturbative  $\alpha'$  corrections based on Hohm-Zwiebach action. We then use the simple phenomenological map between the  $\alpha'$  and loop corrected theories in string cosmology to construct a non-perturbative loop corrected non-singular solution. Both solutions are non-singular everywhere. Therefore, the pre- and post-big-bangs are smoothly connected by these solutions.

**Keywords:** Hohm-Zwiebach action, string cosmological solutions, regular solutions

**DOI:** 10.1088/1674-1137/acddd5

## I. INTRODUCTION

Resolving the big-bang singularity is a widely concerned problem. In Einstein's general theory of gravity, spacetime is generated from the big-bang, whereas in string cosmology, the story is different owing to the famous scale-factor duality [1–4]. Scale-factor duality was first obtained from the equations of motion (EOM) of a string's low energy gravi-dilaton effective action. It shows that the EOM with the FLRW-like ansatz is invariant under the transformations between the scale factor and its inversion, namely,  $a(t) \leftrightarrow 1/a(t)$ . Keep in mind that string dilaton plays a central role in this duality. The main differences between  $T$ -duality and scale-factor duality are that scale-factor duality does not require a compactified background and belongs to a continuous group of  $O(d,d)$ . Remarkably, scale-factor duality introduces pre-big-bang cosmology [5–9]. It implies that the story of our universe was not only born from the initial big-bang singularity, but that there is a long duration of evolution in a pre-big-bang region. Pre- and post-big-bang scenarios are disconnected by the big-bang singularity. However, when the universe evolves to the big-bang singularity, the growth of the string coupling  $g_s = \exp(2\phi)$  and Hubble parameter  $H(t)$  invalidates perturbative theory. The theory of quantum gravity around this region requires the low energy effective action to include two types of corrections: (1) The string curvature scale, which includes higher-derivative  $\alpha'$  corrections, and (2) the

strong coupling regime, which requires quantum loop corrections.

In literature, several phenomenological higher loop models [9–11] have been proposed to resolve the big-bang singularity. For the higher-derivative  $\alpha'$  corrections, at the cost of losing scale-factor duality or  $O(d,d)$  symmetry, several simplified models were proposed [5, 12] to smoothly connect the pre- and post-big-bang scenarios. However, can a natural global picture of the non-singular universe be obtained when the action includes  $\alpha'$  corrections to all orders?

To answer this question, let us focus on recent developments in non-perturbative string cosmology to all orders in  $\alpha'$ . In Refs. [13, 14], in addition to manifesting scale-factor duality in the EOM, Meissner and Veneziano found that when the massless closed string fields only depend on time, the low energy effective action of closed strings with the zeroth and first orders in  $\alpha'$  could be simplified and rewritten in an  $O(d,d)$  invariant form. Subsequently, Sen proved that this result could be extended to all orders in the  $\alpha'$  corrections of full string field theory [2, 15]. In other words, closed string spacetime action can be rewritten in the  $O(d,d)$  covariant form to all orders in  $\alpha'$  without imposing any extra constraint or symmetry. Based on these works, Hohm and Zwiebach [16–18] demonstrated that  $O(d,d)$  covariant spacetime action could be dramatically simplified. This result provides possible non-perturbative dS or AdS vacua to bosonic string theory [18–22]. Moreover, Hohm-

Received 30 May 2023; Accepted 13 June 2023; Published online 14 June 2023

\* Li Song is supported by the Sichuan Science and Technology Program (2022YFG0317). Deyou Chen is supported by the Tianfu talent plan and FXHU.

† E-mail: songli1984@scu.edu.cn

‡ E-mail: deyouchen@hotmail.com

©2023 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

Zwiebach action makes it possible to seriously analyze the stringy effects on the cosmological singularity and black hole singularity. In Ref. [23], a set of non-singular non-perturbative string cosmological solutions was constructed for the first time. This set of solutions was subsequently extended to more general solutions matching the perturbative solution to an arbitrary order in  $\alpha'$  expansion [24]. For more recent developments in smoothing cosmological and black hole singularities, refer to Refs. [25–34].

The purpose of this paper is to give non-singular non-perturbative solutions with  $\alpha'$  corrections or loop corrections. We first construct an  $\alpha'$  corrected solution based on Hohm-Zwiebach action. The solution we present is non-perturbative in that it covers the entire region from the pre-big-bang to the post-big-bang and is non-singular everywhere. In the perturbative region  $t \rightarrow \infty$ , this solution matches the perturbative solution to an arbitrary order in  $\alpha'$ . Therefore, though we currently only know the first two orders in  $\alpha'$  corrections, once higher orders are calculated, our solution can be straightforwardly matched

to them by fixing the parameters. Our solution is parallel to those given in [24]. In Ref. [24], a phenomenological map between  $\alpha'$  corrected theory and loop corrected theory are found. With the help of this map, we construct a non-perturbative loop corrected solution, which is non-singular everywhere and includes higher loop contributions. Hence, both the  $\alpha'$  corrected and loop corrected solutions we construct resolve the big-bang singularity and connect the pre-big-bang and post-big-bang scenarios smoothly.

The remainder of this paper is organized as follows: We give a non-singular non-perturbative solution to any order in  $\alpha'$  expansion in Section II. In Section III, we present a loop corrected non-singular non-perturbative solution. Section IV contains our conclusions.

## II. GENERAL $\alpha'$ CORRECTED SOLUTION

Because we are going to find both  $\alpha'$  and loop corrected solutions, we write the full perturbative structure of the closed string effective action,

$$I = \int d^{d+1}x \sqrt{-g} \left\{ e^{-2\phi} \left[ (R + 4(\partial\phi)^2 - \frac{1}{12}\mathcal{H}^2) + \frac{1}{4}(R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho} + \dots) + \mathcal{O}(\alpha'^2) \right] \right. \\ \left. + \left[ (c_R^1 R + c_\phi^1 (\partial\phi)^2 + c_{\mathcal{H}}^1 \mathcal{H}^2) + \alpha' (c_{\alpha'R}^1 R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} + \dots) + \mathcal{O}(\alpha'^2) \right] \right. \\ \left. + e^{2\phi} \left[ (c_R^2 R + c_\phi^2 (\partial\phi)^2 + c_{\mathcal{H}}^2 \mathcal{H}^2) + \alpha' (c_{\alpha'R}^2 R_{\mu\nu\sigma\rho} R^{\mu\nu\sigma\rho} + \dots) + \mathcal{O}(\alpha'^2) \right] + \dots \right\}. \quad (1)$$

This action contains three massless fields, the metric  $g_{\mu\nu}$ , dilaton  $\phi$ , and antisymmetric field  $b_{\mu\nu}$ , whose field strength is  $\mathcal{H}_{\mu\nu\rho} = 3\partial_{[\mu}b_{\nu\rho]}$ . We set  $b_{\mu\nu} = 0$  in this study. All  $c_{[\dots]}$  are unknown up to now. We first consider the  $\alpha'$  corrected solutions. To this end, we focus on the loop tree level, that is, the first line of the above action. In the FLRW background,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j. \quad (2)$$

The loop tree level action with all orders in the  $\alpha'$  corrections is given by Hohm and Zwiebach in Refs. [17, 18]:

$$I_{\alpha'} = \int d^Dx \sqrt{-g} e^{-2\phi} \left( R + 4(\partial\phi)^2 \right. \\ \left. + \frac{1}{4}\alpha' (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} + \dots) + \alpha'^2(\dots) + \dots \right), \quad (3)$$

$$= \int dt e^{-\Phi} \left( -\dot{\Phi}^2 + \sum_{k=1}^{\infty} (\alpha')^{k-1} c_k \text{tr}(\dot{S}^{2k}) \right), \quad (4)$$

where the notation  $S$  is defined as

$$S = \begin{pmatrix} 0 & a^2(t) \\ a^{-2}(t) & 0 \end{pmatrix}. \quad (5)$$

In the action (4), we can only determine the coefficients  $c_1 = -\frac{1}{8}$  and  $c_2 = \frac{1}{64}$  for bosonic string theory through the one-loop and two-loop beta functions of the non-linear sigma model, and  $c_{k \geq 3}$  are undetermined constants. We define

$$H(t) = \frac{\dot{a}(t)}{a(t)}, \\ f(H) = d \sum_{k=1}^{\infty} (-\alpha')^{k-1} 2^{2(k+1)} k c_k H^{2k-1} \\ = -2dH - 2d\alpha' H^3 + \mathcal{O}(\alpha'^3), \\ g(H) = d \sum_{k=1}^{\infty} (-\alpha')^{k-1} 2^{2k+1} (2k-1) c_k H^{2k} \\ = -dH^2 - \frac{3}{2}d\alpha' H^4 + \mathcal{O}(\alpha'), \quad (6)$$

where  $H(t)$  is the Hubble parameter. Note that

$$g'(H) = Hf'(H), \quad \text{and} \quad g(H) = Hf(H) - \int_0^H f(x)dx,$$

where  $f'(H) \equiv \frac{d}{dH}f(H)$ . The action (4) is simplified to the Hohm-Zwiebach action

$$I_{HZ} = \int dt e^{-\Phi} (-\dot{\Phi}^2 + g(H) - Hf(H)), \quad (7)$$

and after variation, the EOM of Hohm-Zwiebach action (7) is

$$\ddot{\Phi} + \frac{1}{2}Hf(H) = 0, \quad \frac{d}{dt}(e^{-\Phi}f(H)) = 0, \quad \dot{\Phi}^2 + g(H) = 0. \quad (8)$$

In the perturbative region  $t \rightarrow \infty$ , the perturbative solution can be obtained iteratively using (6),

$$\begin{aligned} H(t) &= \frac{\sqrt{2}}{\sqrt{\alpha'}} \left[ \frac{t_0}{t} - 160c_2 \frac{t_0^3}{t^3} + \frac{256(770c_2^2 + 19c_3)}{3} \frac{t_0^5}{t^5} \right. \\ &\quad \left. - \frac{2048(88232c_2^3 + 4644c_3c_2 + 41c_4)}{5} \frac{t_0^7}{t^7} + O\left(\frac{t_0^9}{t^9}\right) \right], \\ \Phi(t) &= -\frac{1}{2} \log \left( \beta^2 \frac{t^2}{t_0^2} \right) - 32c_2 \frac{t_0^2}{t^2} + \frac{256(44c_2^2 + c_3)}{3} \frac{t_0^4}{t^4} \\ &\quad - \frac{2048(6976c_2^3 + 352c_3c_2 + 3c_4)}{15} \frac{t_0^6}{t^6} + O\left(\frac{t_0^8}{t^8}\right), \end{aligned} \quad (9)$$

with

$$\begin{aligned} f(H(t)) &= -2dH - 128c_2d\alpha'H^3 + 768c_3d\alpha'^2H^5 - 4096c_4d\alpha'^3H^7 + O(\alpha'^4H^9), \\ &= \frac{\sqrt{d}}{t_0} \left[ -\frac{2t_0}{t} + 64c_2 \frac{t_0^3}{t^3} - \frac{512(50c_2^2 + c_3)}{3} \frac{t_0^5}{t^5} + \frac{4096(2632c_2^3 + 124c_3c_2 + c_4)}{5} \frac{t_0^7}{t^7} + O\left(\frac{t_0^9}{t^9}\right) \right], \\ g(H(t)) &= -dH^2 - 96c_2d\alpha'H^4 + 640c_3d\alpha'^2H^6 - 3584c_4d\alpha'^3H^8 + O(\alpha'^4H^{10}), \\ &= \frac{1}{t_0^2} \left[ -\frac{t_0^2}{t^2} + 128c_2 \frac{t_0^4}{t^4} - \frac{2048(50c_2^2 + c_3)}{3} \frac{t_0^6}{t^6} + \frac{8192(24448c_2^3 + 1136c_3c_2 + 9c_4)}{15} \frac{t_0^8}{t^8} + O\left(\frac{t_0^{10}}{t^{10}}\right) \right], \end{aligned}$$

where  $\beta$  is an integration constant,  $t_0 \equiv \frac{\sqrt{\alpha'}}{\sqrt{2d}}$ , and the universal  $c_1 = -\frac{1}{8}$  is used. This solution is singular at the non-perturbative region  $t = 0$ . From scale factor duality,  $H(t) \rightarrow -H(t)$ ,  $\Phi(t) \rightarrow \Phi(t)$ ,  $f(t) \rightarrow -f(t)$ , and  $g(t) \rightarrow g(t)$  is also a solution.

A non-perturbative non-singular solution of the EOM (8) should meet two conditions: (1) It must match the perturbative solution (9) in the perturbative region  $\frac{\sqrt{\alpha'}}{t} \rightarrow 0$ , and (2) it must be non-singular for any  $\frac{\sqrt{\alpha'}}{t}$ . Because  $c_{k \geq 3}$  are still unknown up to now, a good solution only needs to match the first two orders in  $\alpha'$ . Such a solution

was constructed in Ref. [23]. However, an important question is can we construct a general solution that can easily match all given  $c_{k \leq n}$  for some  $n > 2$ ? In other words, a general solution expressed in terms of  $c_k$ . In ref. [24], two such solutions were given. In this study, we obtain another simpler solution. Every term in our solution is non-singular. Let us first define a dimensionless parameter:

$$\tau \equiv \frac{t}{t_0} = \sqrt{\frac{2d}{\alpha'}} t. \quad (10)$$

After extensive calculation, we find a non-perturbative solution of the EOM (8),

$$H(t) = \frac{\sqrt{\frac{2}{\alpha'\beta^2\lambda_0}} \left( e^{-\sum_{k=1}^{\infty} \frac{\lambda_k}{\tau^{2k+1}}} \left( (\tau^2 + 1)^2 \sum_{k=1}^{\infty} \left( \frac{8k^2\lambda_k\tau^{4k-2}}{(\tau^{2k+1})^3} - \frac{2k(2k-1)\lambda_k\tau^{2k-2}}{(\tau^{2k+1})^2} \right) + \tau^2 - 1 \right) \right)}{(\tau^2 + 1)^{3/2}}, \quad (11)$$

$$\Phi(t) = \frac{1}{2} \log \frac{\lambda_0}{1 + \tau^2} + \sum_{n=1}^{\infty} \frac{\lambda_n}{1 + \tau^{2n}}, \quad (12)$$

$$f(H(t)) = -2d \sqrt{\frac{2\beta^2\lambda_0}{(\tau^2 + 1)\alpha'}} e^{\sum_{k=1}^{\infty} \frac{\lambda_k}{\tau^{2k+1}}} = -2dH(t) - 2d\alpha'H(t)^3 + O(\alpha'^2), \quad (13)$$

$$\begin{aligned}
g(H(t)) &= -\frac{2d}{\alpha'} \left( \sum_{k=1}^{\infty} \frac{2k\lambda_k \tau^{2k-1}}{(\tau^{2k} + 1)^2} + \frac{\tau}{\tau^2 + 1} \right)^2 \\
&= -dH(t)^2 - \frac{3}{2}d\alpha'H(t)^4 + O(\alpha'^2). \quad (14)
\end{aligned}$$

Applying  $c_1 = -1/8$  and  $c_2 = 1/64$  and expanding this solution in the perturbative region  $t/\sqrt{\alpha'} \rightarrow \infty$  to match the perturbative solution (9), we identify

$$\begin{aligned}
\lambda_0 &= 1/\beta^2, \quad \lambda_1 = 0, \quad \lambda_2 = \frac{11 + 1024c_3}{12}, \\
\lambda_3 &= -\frac{4}{15}(13 + 2816c_3 + 1536c_4)\dots \quad (15)
\end{aligned}$$

It is clear that every single term in the sums of the solution, such as  $\sum_{n=1}^{\infty} \frac{\lambda_n}{1 + \tau^{2n}}$ , is non-singular everywhere. However, note that there could be a small possibility that the summation is not non-singular for fine-tuned parameters,  $\lambda_n, n > 0$ <sup>1)</sup>. If choosing  $\lambda_{n \geq 1} = 0$ , the solution reduces to

$$\begin{aligned}
H(t) &= -\frac{\sqrt{2}}{\sqrt{\alpha'}} \frac{(1 - \tau^2)}{(1 + \tau^2)^{3/2}}, \\
\Phi(t) &= -\frac{1}{2} \log \beta^2 - \frac{1}{2} \log(1 + \tau^2), \\
f(t) &= -\frac{2\sqrt{2}d}{\sqrt{\alpha'}} \frac{1}{\sqrt{1 + \tau^2}}, \\
g(t) &= -\frac{2d}{\alpha'} \frac{\tau^2}{(1 + \tau^2)^2}. \quad (16)
\end{aligned}$$

This is exactly the solution given in Ref. [23].

### III. GENERAL LOOP CORRECTED SOLUTION

After constructing the non-perturbative non-singular  $\alpha'$  corrected solution, we want to find the corresponding loop corrected non-perturbative non-singular solution. We do not know the form and coefficients of the higher loop terms in the full perturbative action (1). For the FLRW background (2), an effective loop corrected action is expressed as

$$\begin{aligned}
I_{\text{Loop}} &= \int d^{d+1}x \sqrt{-g} e^{-2\phi} \left[ R + 4(\partial_\mu \phi)^2 - V(e^{-\Phi(x)}) \right], \\
&= \int dt e^{-\Phi} \left[ -\dot{\Phi} + dH^2 - V(e^{-\Phi}) \right], \quad (17)
\end{aligned}$$

where the  $O(d, d)$  non-local dilaton is [9, 10]

$$\begin{aligned}
e^{-\Phi(t)} &= V_d \int dt' \left| \frac{d(2\phi)}{dt'} \right| \sqrt{-g(t')} e^{-2\phi(t')} \delta(2\phi(t) - 2\phi(t')) \\
&= V_d \sqrt{-g(t)} e^{-2\phi(t)}. \quad (18)
\end{aligned}$$

The EOM is

$$\begin{aligned}
2\ddot{\Phi}_L - 2dH_L^2 - \frac{\partial V}{\partial \Phi_L} &= 0, \\
\dot{\Phi}_L^2 - dH_L^2 - V &= 0, \\
\dot{H}_L - H_L \dot{\Phi}_L &= 0, \quad (19)
\end{aligned}$$

where the subscript  $L$  indicates that the quantities belong to the loop corrected theory. For a positive integer  $n$  and arbitrary parameters  $m_n$  and  $\sigma_n$ , a class of solutions of the above EOM was constructed in Refs. [10, 11],

$$\begin{aligned}
\Phi_L^{(n)}(t) &= \frac{1}{2n} \log \left( \frac{\sigma_n^{2n}}{1 + (m_n t)^{2n}} \right), \\
H_L^{(n)}(t) &= \frac{1}{\sqrt{d}} \frac{m_n}{\sigma_n} e^{\Phi_L^{(n)}(t)} \\
&= \frac{m_n}{\sqrt{d}} \left[ \frac{1}{1 + (m_n t)^{2n}} \right]^{1/2n}. \quad (20)
\end{aligned}$$

This had the potential

$$V_L^{(n)} = \left( \frac{m_n}{\sigma_n} \right)^2 e^{2\Phi_L^{(n)}(t)} \left[ \left( 1 - \sigma_n^{-2n} e^{2n\Phi_L^{(n)}(t)} \right)^{\frac{2n-1}{n}} - 1 \right]. \quad (21)$$

As argued in Refs. [10, 11], because the factor  $e^\Phi$  roughly represents the coupling constant, the integer  $n$  effectively represents the loop number. The potential indicates the non-perturbative contributions by the  $n$ th loop.

In Ref. [24], a map was constructed between the  $\alpha'$  and loop corrected theories,

$$\begin{aligned}
H_L &\leftrightarrow f(H_{\alpha'}), \\
-V_L &\leftrightarrow g(H_{\alpha'}) + df(H_{\alpha'})^2 \\
\Phi_L &\leftrightarrow \Phi_{\alpha'} + \Phi_0. \quad (22)
\end{aligned}$$

Here,  $\Phi_0$  is a constant. Through this map, we can find a loop corrected non-perturbative non-singular solution from our  $\alpha'$  corrected solution (14). Because  $e^{n\Phi_L^{(n)}}$  indicates the contribution from the  $n$ -th loop, we use Eq. (20)

1) We are indebted to the anonymous referee to help us clarify this confusion.

to express the solution in terms of  $e^{\Phi_L^{(n)}}$ . Then, after some tedious calculation, we obtain the loop corrected solution,

$$\begin{aligned}\Phi_L(t) &= \Phi_L^{(1)} + \sum_{n=1}^{\infty} e^{2n\Phi_L^{(n)}} \\ H_L(t) &= \frac{m_1}{\sigma_1} \exp \left[ \Phi_L^{(1)} + \sum_{n=1}^{\infty} e^{2n\Phi_L^{(n)}} \right] \\ V_L(\Phi_L^{(n)}(t)) &= \left( \Phi_L^{(1)} + \sum_{n=1}^{\infty} 2n\Phi_L^{(n)} e^{2n\Phi_L^{(n)}} \right)^2 \\ &\quad - \frac{dm_1^2}{\sigma_1^2} \exp \left[ 2\Phi_L^{(1)} + 2 \sum_{n=1}^{\infty} e^{2n\Phi_L^{(n)}} \right], \quad (23)\end{aligned}$$

where we set  $m_n = \sqrt{2d/\alpha'}$ , and  $\sigma_i$  are free parameters to be determined using coefficients calculated from the effective low energy action. Obviously, this solution is non-

singular around the non-perturbative region  $t \sim 0$ .

#### IV. CONCLUSION

In this paper, based on Hohm-Zwiebach action, we first construct a class of general  $\alpha'$  corrected non-perturbative non-singular string cosmology solutions. Currently, only the first two orders in  $\alpha'$  correction are available. Our solution matches these results in the perturbative region as required. Once higher orders in  $\alpha'$  correction are available, we will straightforwardly fix the corresponding parameters in our solution to match the perturbative solutions. There is a phenomenological map between  $\alpha'$  and loop corrected theories, as given in Ref. [24]. We use this map to construct the corresponding loop corrected solution, which is also non-singular and non-perturbative. Because both solutions are non-singular everywhere, the pre- and post-big-bangs are smoothly connected by them.

#### References

- [1] G. Veneziano, *Phys. Lett. B* **265**, 287 (1991)
- [2] A. Sen, *Phys. Lett. B* **271**, 295 (1991)
- [3] A. A. Tseytlin, *Mod. Phys. Lett. A* **6**, 1721 (1991)
- [4] A. A. Tseytlin and C. Vafa, *Nucl. Phys. B* **372**, 443 (1992), arXiv:[hep-th/9109048]
- [5] M. Gasperini, M. Maggiore, and G. Veneziano, *Nucl. Phys. B* **494**, 315 (1997), arXiv:[hep-th/9611039]
- [6] G. Veneziano, *String cosmology: The Pre - big bang scenario*, arXiv: hep-th/0002094
- [7] M. Gasperini and G. Veneziano, *Phys. Rept.* **373**, 1 (2003), arXiv:[hep-th/0207130]
- [8] M. Gasperini and G. Veneziano, *Nuovo Cim. C* **38(5)**, 160 (2016), arXiv:[hep-th/0703055]
- [9] M. Gasperini and G. Veneziano, *Astropart. Phys.* **1**, 317 (1993), arXiv:[hep-th/9211021]
- [10] M. Gasperini, M. Giovannini, and G. Veneziano, *Phys. Lett. B* **569**, 113 (2003), arXiv:[hep-th/0306113]
- [11] M. Gasperini, M. Giovannini, and G. Veneziano, *Nucl. Phys. B* **694**, 206 (2004), arXiv:[hep-th/0401112]
- [12] D. A. Easson, *Phys. Rev. D* **68**, 043514 (2003), arXiv:[hep-th/0304168]
- [13] K. A. Meissner and G. Veneziano, *Phys. Lett. B* **267**, 33 (1991)
- [14] K. A. Meissner, *Phys. Lett. B* **392**, 298 (1997), arXiv:[hep-th/9610131]
- [15] A. Sen, *Phys. Lett. B* **274**, 34 (1992), arXiv:[hep-th/9108011]
- [16] O. Hohm and B. Zwiebach, *JHEP* **1604**, 101 (2016), arXiv:1510.00005
- [17] O. Hohm and B. Zwiebach, *Phys. Rev. D* **100**, 126011 (2019), arXiv:1905.06963[hep-th]
- [18] O. Hohm and B. Zwiebach, *Int. J. Mod. Phys. D* **28**, 1943002 (2019), arXiv:1905.06583[hep-th]
- [19] C. Krishnan, *JCAP* **10**, 009 (2019), arXiv:1906.09257[hep-th]
- [20] P. Wang, H. Wu and H. Yang, *Phys. Rev. D* **100**, 046016 (2019), arXiv:1906.09650[hep-th]
- [21] C. A. Núñez and F. E. Rost, *JHEP* **03**, 007 (2021), arXiv:2011.10091[hep-th]
- [22] P. Bieniek, J. Chojnacki, J. H. Kwapisz *et al.*, *Stability of the de-Sitter spacetime. The anisotropic case*, arXiv:2301.06616[hep-th]
- [23] P. Wang, H. Wu, H. Yang *et al.*, *JHEP* **10**, 263 (2019), arXiv:1909.00830[hep-th]
- [24] P. Wang, H. Wu, H. Yang *et al.*, *JHEP* **01**, 164 (2020), arXiv:1910.05808[hep-th]
- [25] S. Ying, *Eur. Phys. J. C* **82(6)**, 523 (2022), arXiv:2112.03087[hep-th]
- [26] S. Ying, *Eur. Phys. J. C* **83**, 577 (2023), arXiv:2212.03808[hep-th]
- [27] F. R. Klinkhamer, *Phys. Rev. D* **101(6)**, 064029 (2020), arXiv:1907.06547[gr-qc]
- [28] H. Bernardo, R. Brandenberger, and G. Franzmann, *JHEP* **02**, 178 (2020), arXiv:1911.00088[hep-th]
- [29] H. Bernardo and G. Franzmann, *JHEP* **05**, 073 (2020), arXiv:2002.09856[hep-th]
- [30] J. Quintin, H. Bernardo, and G. Franzmann, *JHEP* **07**, 149 (2021), arXiv:2105.01083[hep-th]
- [31] T. Codina, O. Hohm, and D. Marques, *Phys. Rev. D* **104(10)**, 106007 (2021), arXiv:2107.00053[hep-th]
- [32] S. Ying, *JHEP* **03**, 044 (2023), arXiv:2212.14785[hep-th]
- [33] T. Codina, O. Hohm, and B. Zwiebach, *2D Black Holes, Bianchi I Cosmologies, and  $\alpha'$* , arXiv: 2304.06763[hep-th]
- [34] M. Gasperini and G. Veneziano, *Non-singular pre-big bang scenarios from all-order  $\alpha'$  corrections*, arXiv: 2305.00222[hep-th]