Nonexistence of Majorana fermions in Kerr-Newman type spacetimes with nontrivial charge*

He-Qun Zhang (张鹤群)[2†] Xiao Zhang (张晓)[1,2‡]

[1]Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China
[2]School of Mathematical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

Abstract: We show that the Dirac equation is separated into four differential equations for time-periodic Majorana fermions in Kerr-Newman and Kerr-Newman-(A)dS spacetimes. Although they cannot be transformed into radial and angular equations, the four differential equations yield two algebraic identities. When the electric or magnetic charge is nonzero, they conclude that there is no differentiable time-periodic Majorana fermions outside the event horizon in Kerr-Newman and Kerr-Newman-AdS spacetimes, or between the event horizon and the cosmological horizon in Kerr-Newman-dS spacetime.

Keywords: Dirac equation, Majorana fermion, Kerr-Newman-type spacetime

DOI: 10.1088/1674-1137/ad709e

I. INTRODUCTION

The Dirac equation in black hole spacetimes plays a significant role in the study of general relativity and quantum cosmology. A Dirac fermion is a spinor Ψ in spacetimes satisfying the Dirac equation

$$\left(D+\mathrm{i}\lambda\right)\Psi=0, \tag{1}$$

where λ is a certain real number. In 1968, Kinnersley introduced the null basis to study the Petrov type D metric [1]. In [2], Chandrasekhar separated the Dirac equation in Kerr spacetime when Dirac fermions are time-periodic and given by

$$\Psi=S^{-1}\psi,\quad \psi=\mathrm{e}^{-\mathrm{i}(\omega t+(k+\frac{1}{2})\phi)}\begin{pmatrix} R_-(r)\Theta_-(\theta)\\ R_+(r)\Theta_+(\theta)\\ R_+(r)\Theta_-(\theta)\\ R_-(r)\Theta_+(\theta)\end{pmatrix}, \tag{2}$$

where S is a diagonal matrix,

$$S=\Delta_r^{\frac{1}{4}}\mathrm{diag}\left((r+\mathrm{i}a\cos\theta)^{\frac{1}{2}}I_{2\times2},\ (r-\mathrm{i}a\cos\theta)^{\frac{1}{2}}I_{2\times2}\right),$$

and Page extended his method to Kerr-Newman spacetime [3]. Since then, various studies have been conducted to investigate Hawking radiation and the numerical solutions of Dirac fermions in various spacetime backgrounds (for examples, see [4–7]. In [8], Finster, Kamran, Smoller, and Yau applied Chandrasekhar's separation to prove the nonexistence of the L^2 integrable, time-periodic solutions of the Dirac equation in non-extreme Kerr-Newman spacetime. This indicates that the normalizable time-periodic Dirac fermions must either disappear into the black hole or escape to infinity. In [9, 10], Belgiorno and Cacciatori applied the spectral properties to prove the non-existence of the L^2 integrable, time-periodic solutions of the Dirac equation with mass greater than $\frac{1}{2}\sqrt{\frac{|\Lambda|}{3}}$ in non-extreme Kerr-Newman-(A)dS spacetimes, where Λ is the cosmological constant. In [11], Wang and Zhang applied Chandrasekhar's separation to prove the nonexistence of the L^p integrable, time-periodic solutions of the Dirac equation with arbitrary mass and $0<p\le\frac{4}{3}$, or with mass greater than $q\sqrt{-\frac{\Lambda}{3}}$ and $\frac{4}{3}<p\le\frac{4}{3-2q}$, $0<q<\frac{3}{2}$ in non-extreme Kerr-Newman-AdS spacetime. In non-extreme Kerr-Newman-dS spacetime, the nonexistence of L^p integrable, time-periodic Dirac fermions hold true for arbitrary mass and $p\ge2$ [12]. In particular, taking $p=2$, they confirmed Belgiorno and Cacciatori's results in which normalizable time-periodic Dirac fermions with mass greater than

Received 5 July 2024; Accepted 16 August 2024; Published online 17 August 2024
* Supported by the National Natural Science Foundation of China (12326602)
† E-mail: zhanghequn@amss.ac.cn
‡ E-mail: xzhang@amss.ac.cn

$\frac{1}{2}\sqrt{-\frac{\Lambda}{3}}$ must either disappear into the black hole or escape to infinity.

For Chandrasekhar's separation, the Dirac equation can be transformed into radial and angular equations. In [13], Kraniotis observed that the radial and angular equations could be reduced to generalized Heun's equations in Kerr-Newman spacetime, which provide local time-periodic solutions in terms of holomorphic functions, whose power series coefficients are determined by a four-term recurrence relation. Using the four-term recursion formula, he also proved that there is no time-periodic solution with a fermion energy strictly less than its mass in Kerr-Newman spacetime.

In the recent search for neutrinos, which are one of the most mysterious particles in the universe, we are interested in discovering whether they are Majorana fermions. The most promising method to date is through double beta decay [14]. Various approaches have been studied to distinguish between Majorana and Dirac fermions (for examples, see [15−22]).

A Majorana fermion is a Dirac fermion whose antiparticle is itself. To define a Majorana fermion precisely, let us introduce the 4-component charge conjugate operator

$$C = \begin{pmatrix} \epsilon_{\beta\alpha} & \\ & \epsilon^{\dot\beta\dot\alpha} \end{pmatrix}$$

with the Pauli matrix σ_2 and antisymmetric operator on spin indices, where

$$\epsilon^{\dot\alpha\dot\beta} = -\epsilon_{\alpha\beta} = \mathrm{i}\sigma_2 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}.$$

The charge conjugation of the Dirac fermion Ψ is defined by

$$\Psi^C = C\bar{\Psi}^T.$$

Therefore, Majorana fermions are given by

$$\Psi_{\rm Maj} = \begin{pmatrix} \Psi_{\rm Weyl} \\ \mathrm{i}\sigma_2\Psi^*_{\rm Weyl} \end{pmatrix} \tag{3}$$

and satisfy the Dirac equation [23−25], where $\Psi_{\rm Weyl}$ is the Weyl spinor, and $\Psi^*_{\rm Weyl}$ is its complex conjugate. Time-period Majorana fermions can be given by

$$\Psi = S^{-1}E\psi, \quad \psi = \begin{pmatrix} R_-(r)\Theta_-(\theta) \\ R_+(r)\Theta_+(\theta) \\ \bar{R}_+(r)\bar{\Theta}_+(\theta) \\ -\bar{R}_-(r)\bar{\Theta}_-(\theta) \end{pmatrix}, \tag{4}$$

where S and E are the diagonal matrices

$$S = \Delta_r^{\frac{1}{4}}\mathrm{diag}\left((r+\mathrm{i}a\cos\theta)^{\frac{1}{2}}I_{2\times2},\ (r-\mathrm{i}a\cos\theta)^{\frac{1}{2}}I_{2\times2}\right),$$
$$E = \mathrm{diag}\left(\mathrm{e}^{-\mathrm{i}(\omega t+(k+\frac{1}{2})\phi)}I_{2\times2},\ \mathrm{e}^{\mathrm{i}(\omega t+(k+\frac{1}{2})\phi)}I_{2\times2}\right).$$

In this short paper, we show that the Dirac equation is separated into four differential equations for time-periodic Majorana fermions given by (4) in Kerr-Newman and Kerr-Newman-(A)dS spacetimes. Although they cannot be transformed into radial and angular equations, the four differential equations yield two algebraic identities. When the electric or magnetic charge is nonzero, they conclude that there are no differentiable time-periodic Majorana fermions outside the event horizon in Kerr-Newman and Kerr-Newman-AdS spacetimes, or between the event horizon and the cosmological horizon in Kerr-Newman-dS spacetime.

We remark that Dirac fermions taking form (2) are not consistent with Majorana condition (3). Thus, previous results on the existence or non-existence of time-periodic Dirac fermions cannot be applied to the current situation for time-periodic Majorana fermions.

II. GEOMETRY OF KERR-NEWMAN-TYPE SPACETIMES

For convenience of discussion, we unify the Kerr-Newman and Kerr-Newman-(A)dS metrics

$$\mathrm{d}s^2_{\rm KNType} = -\frac{\Delta_r}{U}\left(\mathrm{d}t - \frac{a\sin^2\theta}{\Xi}\mathrm{d}\phi\right)^2 + \frac{U}{\Delta_r}\mathrm{d}r^2 + \frac{U}{\Delta_\theta}\mathrm{d}\theta^2 + \frac{\Delta_\theta\sin^2\theta}{U}\left(a\mathrm{d}t - \frac{r^2+a^2}{\Xi}\mathrm{d}\phi\right)^2, \tag{5}$$

by taking κ as zero, pure imaginary, and real, where $\Lambda = -3\kappa^2$ is the cosmological constant, and

$$\Delta_r = (r^2+a^2)(1+\kappa^2r^2) - 2mr + P^2 + Q^2,$$
$$\Delta_\theta = 1-\kappa^2a^2\cos^2\theta, \quad U = r^2+a^2\cos^2\theta, \quad \Xi = 1-\kappa^2a^2 > 0.$$

The metric (5) solves the Einstein-Maxwell field equations with the electromagnetic potential

$$A=-\frac{Qr}{U}\left(\mathrm{d}t-\frac{a\sin^2\theta}{\Xi}\mathrm{d}\varphi\right)-\frac{P\cos\theta}{U}\left(a\mathrm{d}t-\frac{r^2+a^2}{\Xi}\mathrm{d}\varphi\right), \tag{6}$$

where P and Q are real numbers representing the magnetic and electric charges, respectively.

In the following, we let $0\le\mu,\nu\le 3$, and $1\le i,j\le 3$. On a 4-dimensional Lorentzian manifold, we choose the frame $\{e_\mu\}$ such that e_0 is timelike and e_i are spacelike. We denote $\{e^\alpha\}$ as the dual coframe. The Cartan structure equations are

$$de^\mu=-\omega^\mu{}_\nu\wedge e^\nu,\quad \omega_{\mu\nu}=g_{\mu\gamma}\omega^\gamma{}_\nu=-\omega_{\nu\mu}.$$

If it is spin, we use the cotangent bundle to define the Clifford multiplication, spin connection, and Dirac operator [11, 12]. We fix the Clifford multiplications as the following Weyl representation:

$$e^0\mapsto\begin{pmatrix} & I_{2\times2}\\ I_{2\times2} & \end{pmatrix},\quad e^i\mapsto\begin{pmatrix} & \sigma_i\\ -\sigma_i & \end{pmatrix}, \tag{7}$$

where σ_i are Pauli matrices,

$$\sigma_1=\begin{pmatrix} & 1\\ 1 & \end{pmatrix},\ \sigma_2=\begin{pmatrix} & -i\\ i & \end{pmatrix},\ \sigma_3=\begin{pmatrix} 1 & \\ & -1\end{pmatrix}.$$

We fix our discussion in the region $\Delta_r>0$. The coframe is

$$\begin{aligned}
e^0&=\sqrt{\frac{\Delta_r}{U}}\left(\mathrm{d}t-\frac{a\sin^2\theta}{\Xi}\mathrm{d}\phi\right),\\
e^1&=\sqrt{\frac{U}{\Delta_\theta}}\mathrm{d}\theta\\
e^2&=\sqrt{\frac{\Delta_\theta}{U}}\sin\theta\left(a\mathrm{d}t-\frac{r^2+a^2}{\Xi}\mathrm{d}\phi\right),\\
e^3&=\sqrt{\frac{U}{\Delta_r}}\mathrm{d}r.
\end{aligned}$$

with the dual frame

$$\begin{aligned}
e_0&=\frac{r^2+a^2}{\sqrt{U\Delta_r}}\left(\partial_t+\frac{a\Xi}{r^2+a^2}\partial_\phi\right),\ e_1=\sqrt{\frac{\Delta_r}{U}}\partial_r,\\
e_2&=\sqrt{\frac{\Delta_\theta}{U}}\partial_\theta,\ e_3=-\frac{1}{\sqrt{U\Delta_\theta}}\left(a\sin\theta\partial_t+\frac{\Xi}{\sin\theta}\partial_\phi\right).
\end{aligned}$$

With respect to the above coframe, we obtain

$$\begin{aligned}
de^0&=C^0_{10}e^1\wedge e^0+C^0_{30}e^3\wedge e^0+C^0_{12}e^1\wedge e^2,\\
de^1&=C^1_{31}e^3\wedge e^1,\\
de^2&=C^2_{30}e^3\wedge e^0+C^2_{32}e^3\wedge e^2+C^2_{12}e^1\wedge e^2,\\
de^3&=C^3_{31}e^3\wedge e^1,
\end{aligned}$$

where

$$\begin{aligned}
&C^0_{10}=-\frac{a^2}{U}\sqrt{\frac{\Delta_\theta}{U}}\sin\theta\cos\theta,\quad C^0_{30}=\partial_r\sqrt{\frac{\Delta_r}{U}},\\
&C^0_{12}=\frac{2a}{U}\sqrt{\frac{\Delta_r}{U}}\cos\theta,\quad C^1_{31}=\frac{r}{U}\sqrt{\frac{\Delta_r}{U}},\\
&C^2_{12}=\frac{1}{\sin\theta}\partial_\theta\left(\sqrt{\frac{\Delta_\theta}{U}}\sin\theta\right),\quad C^2_{32}=\frac{r}{U}\sqrt{\frac{\Delta_r}{U}},\\
&C^2_{30}=-\frac{2ar}{U}\sqrt{\frac{\Delta_\theta}{U}}\sin\theta,\quad C^3_{31}=\frac{a^2}{U}\sqrt{\frac{\Delta_\theta}{U}}\sin\theta\cos\theta.
\end{aligned}$$

Thus, the connection 1-forms are

$$\begin{aligned}
\omega^0{}_1&=-\omega_{01}=C^0_{10}e^0+\frac{1}{2}C^0_{12}e^2,\\
\omega^0{}_2&=-\omega_{02}=-\frac{1}{2}C^2_{30}e^3-\frac{1}{2}C^0_{12}e^1,\\
\omega^0{}_3&=-\omega_{03}=C^0_{30}e^0-\frac{1}{2}C^2_{30}e^2,\\
\omega^1{}_2&=\omega_{12}=\frac{1}{2}C^0_{12}e^0-C^2_{12}e^2,\\
\omega^1{}_3&=\omega_{13}=C^3_{31}e^3+C^1_{31}e^1,\\
\omega^2{}_3&=\omega_{23}=\frac{1}{2}C^2_{30}e^0+C^2_{32}e^2.
\end{aligned} \tag{8}$$

The spin connection is defined as

$$\tilde\nabla_X\Psi=X(\Psi)-\frac{1}{4}\omega_{\mu\nu}(X)e^\mu\cdot e^\nu\cdot\Psi,$$

where X is a vector, Φ is a spinor, and $e^\mu\cdot$ is the Clifford multiplication. Therefore, using (8), we obtain

$$\begin{aligned}
\tilde\nabla_{e_0}\Psi=&e_0\Psi-\frac{1}{2}\omega_{01}(e_0)e^0\cdot e^1\cdot\Psi-\frac{1}{2}\omega_{03}(e_0)e^0\cdot e^3\cdot\Psi\\
&-\frac{1}{2}\omega_{12}(e_0)e^1\cdot e^2\cdot\Psi-\frac{1}{2}\omega_{23}(e_0)e^2\cdot e^3\cdot\Psi,\\
\tilde\nabla_{e_1}\Psi=&e_1\Psi-\frac{1}{2}\omega_{02}(e_1)e^0\cdot e^2\cdot\Psi-\frac{1}{2}\omega_{12}(e_1)e^1\cdot e^2\cdot\Psi,\\
\tilde\nabla_{e_2}\Psi=&e_2\Psi-\frac{1}{2}\omega_{01}(e_2)e^0\cdot e^1\cdot\Psi-\frac{1}{2}\omega_{03}(e_2)e^0\cdot e^3\cdot\Psi\\
&-\frac{1}{2}\omega_{12}(e_2)e^1\cdot e^2\cdot\Psi-\frac{1}{2}\omega_{23}(e_2)e^2\cdot e^3\cdot\Psi,\\
\tilde\nabla_{e_3}\Psi=&e_3\Psi-\frac{1}{2}\omega_{02}(e_3)e^0\cdot e^2\cdot\Psi-\frac{1}{2}\omega_{13}(e_3)e^1\cdot e^3\cdot\Psi.
\end{aligned}$$

Using Clifford multiplication (7), these can be written as the matrix forms

$$\tilde{\nabla}_{e_\mu}\Psi = e_\mu\Psi + E_\mu\cdot\Psi,\quad E_\mu = -\frac{1}{2}\begin{pmatrix}\epsilon_\mu & 0\\ 0 & -\epsilon_\mu^h\end{pmatrix},\qquad (9)$$

where ϵ_μ^h is the Hermitian conjugate of ϵ_μ and

$$\begin{aligned}\epsilon_0 &= \left(C_{10}^0 - \frac{\mathrm{i}}{2}C_{30}^2\right)\sigma_1 + \left(C_{30}^0 - \frac{\mathrm{i}}{2}C_{12}^0\right)\sigma_3,\\ \epsilon_1 &= \left(-\frac{1}{2}C_{12}^0 + \mathrm{i}C_{31}^1\right)\sigma_2,\\ \epsilon_2 &= \left(\frac{1}{2}C_{12}^0 - \mathrm{i}C_{32}^2\right)\sigma_1 - \left(\frac{1}{2}C_{30}^2 - \mathrm{i}C_{12}^2\right)\sigma_3,\\ \epsilon_3 &= \left(-\frac{1}{2}C_{30}^2 + \mathrm{i}C_{31}^3\right)\sigma_2.\end{aligned}$$

III. TIME-PERIODIC MAJORANA FERMIONS

In this section, we prove the nonexistence of time-periodic Majorana fermions in Kerr-Newman type spacetime when the electric or magnetic charge is nonzero.

First, we simplify the Dirac equation (1) on metric (5) when Ψ is given by (4). The Dirac operator with electromagnetic potential A is

$$D = e^\mu\cdot\left(\tilde{\nabla}_{e_\mu} + \mathrm{i}A(e_\mu)\right).\qquad (10)$$

We denote $\mathcal{J} = \mathrm{diag}(I_{2\times2}, -I_{2\times2})$. In terms of (6) and (9), we obtain

$$\begin{aligned}& e^\mu\cdot e_\mu(\Psi) = \sqrt{\frac{\Delta_r}{U}}e^3\cdot S^{-1}E\partial_r\psi + \sqrt{\frac{\Delta_\theta}{U}}e^1\cdot S^{-1}E\partial_\theta\psi\\ & -\mathrm{i}\frac{r^2+a^2}{\sqrt{U\Delta_r}}\left(\omega + \frac{a\Xi}{r^2+a^2}\left(k+\frac{1}{2}\right)\right)e^0\cdot\mathcal{J}S^{-1}E\psi\\ & + \frac{1}{2U^{\frac{3}{2}}}e^3\cdot\left(S - \partial_r(U\sqrt{\Delta_r})S^{-1}\right)E\psi\\ & + \frac{a\sin\theta}{2U^{\frac{3}{2}}}\sqrt{\frac{\Delta_\theta}{\Delta_r}}e^1\cdot\left(\mathrm{i}\mathcal{J}S + 2a\cos\theta\sqrt{\Delta_r}S^{-1}\right)E\psi\\ & + \frac{\mathrm{i}}{\sqrt{U\Delta_\theta}}\left(a\omega\sin\theta + \frac{\Xi}{\sin\theta}\left(k+\frac{1}{2}\right)\right)e^2\cdot S^{-1}\mathcal{J}E\psi,\\ & e^\mu\cdot E_\mu\Psi = \frac{2\Delta_\theta - \Xi}{2\sqrt{U\Delta_\theta}}\cot\theta e^1\cdot S^{-1}E\psi - \frac{\mathrm{i}a}{2}\sqrt{\frac{\Delta_r\Delta_\theta}{U}}\sin\theta e^1\\ & \cdot\mathcal{J}S^{-3}E\psi + \frac{\partial_r\sqrt{\Delta_r}}{2\sqrt{U}}e^3\cdot S^{-1}E\psi + \frac{\Delta_r}{2\sqrt{U}}e^3\cdot S^{-3}E\psi,\\ & e^\mu\cdot\left(\mathrm{i}A(e_\mu)\right)\Psi = -\frac{\mathrm{i}Qr}{\sqrt{U\Delta_r}}e^0\cdot S^{-1}E\psi - \frac{\mathrm{i}P\cot\theta}{\sqrt{U\Delta_\theta}}e^2\cdot S^{-1}E\psi.\end{aligned}$$

Note that

$$\begin{aligned}& e^\mu\mathcal{J} = -\mathcal{J}e^\mu,\quad e^\mu E = E^{-1}e^\mu,\quad e^\mu E^{-1} = Ee^\mu,\\ & e^\mu S^{-1} = \frac{1}{\sqrt{U\Delta_r}}Se^\mu,\quad e^\mu S = \sqrt{U\Delta_r}S^{-1}e^\mu.\end{aligned}$$

Substituting the above formulas into (10), we obtain

$$D\Psi = \frac{1}{U\sqrt{\Delta_r}}SE^{-1}\left(\sqrt{\Delta_r}\mathcal{D}_r - \sqrt{\Delta_\theta}\mathcal{L}_\theta\right)\psi\qquad (11)$$

with

$$\begin{aligned}\mathcal{D}_r &= e^3\partial_r + \frac{\mathrm{i}}{\Delta_r}\left(\omega(r^2+a^2) + a\Xi\left(k+\frac{1}{2}\right)\right)\mathcal{J}e^0 - \frac{\mathrm{i}Qr}{\Delta_r}e^0,\\ \mathcal{L}_\theta &= -e^1\partial_\theta + \frac{\mathrm{i}}{\Delta_\theta}\left(\omega a\sin\theta + \frac{\Xi}{\sin\theta}\left(k+\frac{1}{2}\right)\right)\mathcal{J}e^2\\ & -\left(1 - \frac{\Xi}{2\Delta_\theta}\right)\cot\theta e^1 + \frac{\mathrm{i}P}{\Delta_\theta}\cot\theta e^2.\end{aligned}$$

We denote $\lambda_{\omega k} = \lambda \mathrm{e}^{-2\mathrm{i}(\omega t + (k+\frac{1}{2})\phi)}$. Using (11), we can reduce Dirac equation (1) to

$$D_r\psi = L_\theta\psi,\quad \psi = \begin{pmatrix} R_-(r)\Theta_-(\theta)\\ R_+(r)\Theta_+(\theta)\\ \bar{R}_+(r)\bar{\Theta}_+(\theta)\\ -\bar{R}_-(r)\bar{\Theta}_-(\theta)\end{pmatrix}\qquad (12)$$

where

$$D_r = \begin{pmatrix} -\mathrm{i}\lambda_{\omega k}r & & \sqrt{\Delta_r}D_{r,00} & \\ & -\mathrm{i}\lambda_{\omega k}r & & \sqrt{\Delta_r}D_{r,01}\\ \sqrt{\Delta_r}D_{r,11} & & -\mathrm{i}\overline{\lambda_{\omega k}}r & \\ & \sqrt{\Delta_r}D_{r,10} & & -\mathrm{i}\overline{\lambda_{\omega k}}r\end{pmatrix},$$

$$L_\theta = \begin{pmatrix} a\lambda_{\omega k}\cos\theta & & & \sqrt{\Delta_\theta}L_{\theta,00}\\ & a\lambda_{\omega k}\cos\theta & \sqrt{\Delta_\theta}L_{\theta,01} & \\ & \sqrt{\Delta_\theta}L_{\theta,11} & -a\overline{\lambda_{\omega k}}\cos\theta & \\ \sqrt{\Delta_\theta}L_{\theta,10} & & & -a\overline{\lambda_{\omega k}}\cos\theta\end{pmatrix}$$

and for l, m=0, 1,

$$\begin{aligned} D_{r,lm} &= (-1)^m\partial_r + (-1)^l\frac{\mathrm{i}}{\Delta_r}\left(\omega(r^2+a^2) + \left(k+\frac{1}{2}\right)\Xi a\right) - \frac{\mathrm{i}Qr}{\Delta_r},\\ L_{\theta,lm} &= -(-1)^l\partial_\theta + \frac{(-1)^{l+m}}{\Delta_\theta}\left(\omega a\sin\theta + \frac{\Xi}{\sin\theta}\left(k+\frac{1}{2}\right)\right.\\ & \left. + (-1)^l P\cot\theta - (-1)^m\left(\Delta_\theta - \frac{\Xi}{2}\right)\cot\theta\right).\end{aligned}$$

Writing each row of (12), we obtain

$$-\mathrm{i}\lambda_{\omega k} r R_-\Theta_- + \sqrt{\Delta_r} D_{r,00}\bar{R}_+\bar{\Theta}_+ = a\lambda_{\omega k}\cos\theta R_-\Theta_- - \sqrt{\Delta_\theta} L_{\theta,00}\bar{R}_-\bar{\Theta}_-, \quad (13)$$

$$-\mathrm{i}\lambda_{\omega k} r R_+\Theta_+ - \sqrt{\Delta_r} D_{r,01}\bar{R}_-\bar{\Theta}_- = a\lambda_{\omega k}\cos\theta R_+\Theta_+ + \sqrt{\Delta_\theta} L_{\theta,01}\bar{R}_+\bar{\Theta}_+, \quad (14)$$

$$-\mathrm{i}\overline{\lambda_{\omega k}} r \bar{R}_+\bar{\Theta}_+ + \sqrt{\Delta_r} D_{r,11} R_-\Theta_- = -a\overline{\lambda_{\omega k}}\cos\theta \bar{R}_+\bar{\Theta}_+ + \sqrt{\Delta_\theta} L_{\theta,11} R_+\Theta_+, \quad (15)$$

$$\mathrm{i}\overline{\lambda_{\omega k}} r \bar{R}_-\bar{\Theta}_- + \sqrt{\Delta_r} D_{r,10} R_+\Theta_+ = a\overline{\lambda_{\omega k}}\cos\theta \bar{R}_-\bar{\Theta}_- + \sqrt{\Delta_\theta} L_{\theta,10} R_-\Theta_-. \quad (16)$$

These equations cannot be separated into radial and angular equations. However,

$$\overline{D}_{r,lm} = -D_{r,l\bar{m}}, \quad \overline{L}_{\theta,lm} = L_{\theta,lm}$$

and

$$D_{r,lm} + D_{r,\bar{l}\bar{m}} = -\frac{2\mathrm{i}Qr}{\Delta_r}, \quad L_{\theta,\bar{l}m} + L_{\theta,lm} = 2(-1)^m \frac{P\cot\theta}{\Delta_\theta},$$

where $\bar{l} = (l+1) \mod 2$ and $\bar{m} = (m+1) \mod 2$. Thus, by subtracting the complex conjugation of (13) from (16) and adding the complex conjugation of (14) to (15), we obtain the two algebraic identities

$$\mathrm{i}\alpha(r)R_-\Theta_- - \beta(\theta)R_+\Theta_+ = 0, \quad \beta(\theta)R_-\Theta_- + \mathrm{i}\alpha(r)R_+\Theta_+ = 0,$$

where

$$\alpha(r) = \frac{Qr}{\sqrt{\Delta_r}}, \quad \beta(\theta) = \frac{P\cot\theta}{\sqrt{\Delta_\theta}}.$$

Therefore,

$$\left(\alpha(r)^2 - \beta(\theta)^2\right)R_+\Theta_+ = \left(\alpha(r)^2 - \beta(\theta)^2\right)R_-\Theta_- = 0.$$

If $R_+\Theta_+$ or $R_-\Theta_-$ is nontrival, it must hold that $\alpha(r) = \pm\beta(\theta)$. As $\alpha(r)$ depends only on $r > 0$ (outside the event horizon), and $\beta(\theta)$ depends only on θ, both are constant. Therefore, three cases occur: (i) $P = Q = 0$, (ii) $P \neq 0$, $Q = 0$, and $\theta = \pi/2$, and (iii) $P \neq 0$, $Q \neq 0$, and $r = r_0$ is a positive constant, $\theta = \theta_0$ is a constant. However, $R_+\Theta_+ = R_-\Theta_- = 0$ outside the hypersurface $\theta = \pi/2$ is equipped with the metric

$$\mathrm{d}s_3^2 = -\frac{\Delta_r}{r^2}\left(\mathrm{d}t - \frac{a}{\Xi}\mathrm{d}\phi\right)^2 + \frac{r^2}{\Delta_r}\mathrm{d}r^2 + \frac{1}{r^2}\left(a\mathrm{d}t - \frac{r^2+a^2}{\Xi}\mathrm{d}\phi\right)^2$$

in case (ii), and outside the 2-surface is equipped with the metric

$$\mathrm{d}s_2^2 = -\frac{\Delta_r(r_0)}{U(r_0,\theta_0)}\left(\mathrm{d}t - \frac{a\sin^2\theta_0}{\Xi}\mathrm{d}\phi\right)^2 + \frac{\Delta_\theta(\theta_0)\sin^2\theta_0}{U(r_0,\theta_0)}\left(a\mathrm{d}t - \frac{r_0^2+a^2}{\Xi}\mathrm{d}\phi\right)^2$$

in case (iii). This indicates that Majorana fermions are not differentiable in cases (ii) and (iii). Therefore, if $P \neq 0$ or $Q \neq 0$, we conclude that there is no differentiable time-periodic Majorana fermions in Kerr-Newman-type spacetimes.

IV. CONCLUSION

We note that Chandrasekhar's separation for time-periodic Dirac fermions is not consistent with the condition for Majorana fermions and introduce a new separation for time-periodic Majorana fermions. With this separation, the Dirac equation cannot be transformed into radial and angular equations, as is done in Chandrasekhar's separation. Instead, it is separated into four differential equations, which yield two algebraic identities. When the electric or magnetic charge is nonzero, they conclude that there is no differentiable time-periodic Majorana fermions outside the event horizon in Kerr-Newman and Kerr-Newman-AdS spacetimes, or between the event horizon and the cosmological horizon in Kerr-Newman-dS spacetime. This conclusion plays a role in searching for free Majorana fermions when considering the gravitational effect.

ACKNOWLEDGEMENT

The authors are grateful to the referees for many valuable suggestions.

References

[1] W. Kinnersley, Journal of Mathematical Physics **10**, 1969 (1195)

[2] S. Chandrasekhar, Proceedings of the Royal Society, Series A **349**, 571 (1976)

[3] D. N. Page, Phys. Rev. D **14**, 1509 (1976)
[4] Z. Zhao, Y. X. Guei, and L. Liu, Chinese Astronomy and Astrophysics **5**, 365 (1981)
[5] F. Belgiorno and M. Martellini, Phys. Lett. B **453**, 17 (1999)
[6] S. K. Chakrabarti and B. Mukhopadhyay, Monthly Notices of the Royal Astronomical Society **317**, 979 (2000)
[7] M. Angheben, M. Nadalini, L. Vanzo *et al*., Journal of High Energy Physics **05**, 014 (2005)
[8] F. Finster, N. Kamran, J. Smoller *et al*., Communications on Pure and Applied Mathematics **53**, 90 (2000)
[9] F. Belgiorno and S. L. Cacciatori, Journal of Physics A: Mathematical and Theoretical **42**(13), 135207 (2009)
[10] F. Belgiorno and S. L. Cacciatori, Journal of Mathematical Physics **51**, 033517 (2010)
[11] Y. Wang and X. Zhang, *Nonexistence of time-periodic solutions of the Dirac equation in non-extreme Kerr-Newman-AdS spacetime*, Science China Mathematics, **61**, 73 (2018)
[12] M. Fan, Y. Wang, and X. Zhang, *Nonexistence of time-periodic solutions of the Dirac equation in Kerr-Newman-(A)dS spacetime*, (2024), arXiv: 2404.13255v1[gr-qc]
[13] G.V. Kraniotis, Journal of Physics: Communications **3**, 035026 (2019)
[14] W. Rodejohann, International Journal of Modern Physics E **20**, 1833 (2011)
[15] S. A. Alavi and A. Abbasnezhad, Gravitation and Cosmology **22**, 288 (2016)
[16] T. P. Cheng and L. F. Li, Phys. Rev. Lett. **45**, 1980 (1908)
[17] T. Garavaglia, Phys. Rev. D **29**, 387 (1984)
[18] J. Nieves and P. Pal, Phys. Rev. D **36**, 315 (1987)
[19] S. P. Rosen, Phys. Rev. Lett. **48**, 842 (1982)
[20] R. E. Shrock, Nucl. Phys. B **206**, 359 (1982)
[21] D. Singh, N. Mobed, and G. Papini, Phys. Rev. Lett. **97**, 041101 (2006)
[22] C. S. Kim, M. V. N. Murthy *et al*., Phys. Rev. D **105**, 113006 (2022)
[23] E. Majorana, Nuovo Cimento C-Colloquia and Communications in Physics **14**, 171 (1937)
[24] T. Prokopec and V. H. Unnithan, *Majorana propagator on de Sitter space*, European Physical Journal C, **82** ,1015 (2022)
[25] C-L. Hsieh, V. Memari and M. Halilsoy, D*irac and Majorana fermions in the anti-de Sitter spacetime with tachyonic approaches*, (2024), arXiv: 2403.13810v1[hep-th]