

Cosmological constraints on neutrino mass within consistent cosmological models*

Ye-Huang Pang (庞业煌)^{1,2,3†} Xue Zhang (张雪)^{4‡} Qing-Guo Huang (黄庆国)^{1,2,3§}

¹School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China

²School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

³CAS Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

⁴Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, Yangzhou 225009, China

Abstract: Recently, the emergence of cosmological tension has raised doubts about the consistency of the Λ CDM model. To constrain the neutrino mass within a consistent cosmological framework, we investigate three massive neutrinos with normal hierarchy (NH) and inverted hierarchy (IH) in both the axion-like early dark energy (Axi-EDE) and AdS-EDE models. We use joint datasets including the cosmic microwave background power spectrum from *Planck* 2018, Pantheon of type Ia supernova, baryon acoustic oscillation, and H_0 data from SH0ES. For the ν Axi-EDE model, we obtain $\sum m_{\nu, \text{NH}} < 0.152$ eV and $\sum m_{\nu, \text{IH}} < 0.178$ eV, whereas for the ν AdS-EDE model, we find $\sum m_{\nu, \text{NH}} < 0.135$ eV and $\sum m_{\nu, \text{IH}} < 0.167$ eV. Our results exhibit a preference for NH in both the ν Axi-EDE and ν AdS-EDE models.

Keywords: neutrino mass, cosmological tension, early dark energy

DOI: 10.1088/1674-1137/ad34c0

1. INTRODUCTION

In the standard model of particle physics, three generations of neutrinos, namely, ν_e, ν_μ , and ν_τ , are associated with three neutrino flavor eigenstates. The phenomenon of neutrino oscillation provides evidence that the neutrinos are massive [1, 2]. Neutrino mass constraints can be derived from various particle physics experiments; however, the precise value of the neutrino mass remains unclear to date. Among these measurements, the kinematics of tritium decay can provide an upper limit on the neutrino mass [3]. Neutrino oscillation provides an approach to exploring the splittings of the neutrino mass-squared Δm_{ij} . The resulting mass-squared splittings are [4]

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \simeq 7.5 \times 10^{-5} \text{eV}^2, \quad (1)$$

$$|\Delta m_{31}^2| \equiv |m_3^2 - m_1^2| \simeq 2.5 \times 10^{-3} \text{eV}^2, \quad (2)$$

where m_1, m_2 , and m_3 are neutrino mass eigenvalues that

can be transformed into flavor eigenstates with a neutrino mixing matrix [4]. The sign of Δm_{31}^2 remains undetermined; thus, the above results present two possibilities of neutrino mass hierarchy: the normal hierarchy (NH), characterized by $m_1 < m_2 < m_3$, and the inverted hierarchy (IH), characterized by $m_3 < m_1 < m_2$. We denote the sum of the neutrino mass as $\sum m_\nu$. In the NH case, there is a lower limit on the total neutrino mass, which is $\sum m_{\nu, \text{NH}} > 0.06$ eV. Similarly, for the IH case, we have $\sum m_{\nu, \text{IH}} > 0.10$ eV.

Cosmological observations can also provide constraints on neutrino mass, particularly the total neutrino mass $\sum m_\nu$ [5–9]. In the early universe, massive neutrinos are ultra-relativistic, which can be regarded as radiation. However, in the late-time universe, massive neutrinos become non-relativistic and behave like cold dark matter. The special characteristic of massive neutrinos would have significant implications on cosmological evolution at both background and perturbation levels, consequently leading to a reshaping of the cosmic microwave background (CMB) power spectrum [10–14]. To remain consistent with CMB observations, there is an

Received 29 January 2024; Accepted 15 March 2024; Published online 16 March 2024

* We acknowledge the use of HPC Cluster of ITP-CAS. XZ is supported by the National Natural Science Foundation of China (12005183). QGH is supported by the National Natural Science Foundation of China (12250010, 11975019, 11991052, 12047503), and the Key Research Program of Frontier Sciences, CAS, (ZDBS-LY-7009)

[†] E-mail: pangyehuang22@mailsucas.ac.cn

[‡] E-mail: zhangxue@yzu.edu.cn

[§] E-mail: huangqg@itp.ac.cn

©2024 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

upper limit on the sum of neutrino masses $\sum m_\nu$. Note that the inference of total neutrino mass constraints from the CMB is model-dependent. In concordance Λ CDM model, it is found that $\sum m_\nu < 0.26$ eV at the 95% confidence level (C.L.) using Planck 2018 TT, TE, EE+lowE and assuming degeneracy hierarchy ($m_1 = m_2 = m_3$), where lowE refers to low- ℓ ($2 \leq \ell \leq 29$) EE polarization power spectra [5]. Adding baryon acoustic oscillation (BAO) data results in a more stringent constraint $\sum m_\nu < 0.12$ eV at the 95% C.L. The neutrino mass hierarchy can be distinguished through strict cosmological constraints derived from cosmological observations. Cosmological constraints on the neutrino mass hierarchy were first proposed in [15]. Subsequently, an updated constraint was derived using Planck 2018 TT, TE, and EE+lowE+BAO, yielding $\sum m_{\nu, \text{NH}} < 0.15$ eV and $\sum m_{\nu, \text{IH}} < 0.17$ eV at the 95% C.L. [16].

The constraints on neutrino mass in alternative cosmological models beyond the standard Λ CDM model have been extensively investigated [17–31]. The new physics in these models may impose distinct constraints on the neutrino mass compared to those derived from the Λ CDM model. In recent years, a multitude of cosmological models have been proposed to alleviate the Hubble tension [32, 33], which refers to the discrepancy between the local distance ladder measurement and the early-time CMB measurement. According to the CMB observations by *Planck* [34], $H_0 = 67.27 \pm 0.60$ km s⁻¹Mpc⁻¹ in the Λ CDM model. However, the latest H_0 measurement by SH0ES yields $H_0 = 73.04 \pm 1.04$ km s⁻¹Mpc⁻¹ [35]. The discrepancy between these two results reaches a 5σ significance level. Therefore, the Λ CDM model may potentially exhibit inconsistencies; thus, we need to constrain the neutrino masses within a consistent cosmological framework. Early dark energy (EDE) is a promising solution to alleviate Hubble tension by increasing the Hubble rate $H(z)$ prior to recombination. It reduces the sound horizon at recombination while increasing the H_0 value without shifting the angular scale of the sound horizon. Because the CMB has tight constraints on the angular scale of the sound horizon, introducing EDE can increase the value of H_0 without violating the fit to the CMB power spectrum. In recent years, several EDE models with different mechanisms have been proposed [36–46], such as the axion-like EDE model (Axi-EDE) [37, 39] and the EDE model with an anti-de Sitter phase around recombination (AdS-EDE) [43]. In this study, we aim to constrain neutrino mass within the Axi-EDE and AdS-EDE models while investigating the preferred hierarchy based on current observational data.

This paper is organized as follows. In Sec. II, we introduce the datasets and methods for constraining neutrino mass. Models and their parameters are summarized in Sec. III. The results of parameter constraints and discussions are presented in Sec. IV. Finally, our conclu-

sions are outlined in Sec. V.

II. DATASETS

We use a combination of the following datasets to constrain the neutrino mass and other cosmological parameters:

- CMB

CMB data include the *Planck* 2018 high- ℓ `Planck` likelihood of the TT power spectrum ($30 \leq \ell \leq 2500$), TE cross correlation power spectrum and EE power spectra ($30 \leq \ell \leq 2000$) [47], and low- ℓ TT and EE power spectrum ($2 \leq \ell \leq 30$) and lensing power spectra [48].

- SN Ia

We use the data on the 1048 type Ia supernova in the redshift range $0.01 < z < 2.3$ from Pantheon [49], which provides information on the distance modulus versus redshift for each supernova.

- BAO

The BAO data include the r_d/D_V value of 6dFGS at $z_{\text{eff}} = 0.15$ [50], the D_V/r_d value of SDSS DR7 at $z_{\text{eff}} = 0.15$ [51], and D_A/r_d , Hr_d at $z_{\text{eff}} = 0.38, 0.51, 0.61$ from BOSS DR12 [52], where r_d is the sound horizon at the baryon drag epoch,

$$r_d = \int_{z_d}^{\infty} \frac{c_s(z)}{H(z)} dz, \quad (3)$$

and $c_s(z)$ is the sound speed of the baryon-photon fluid. The angular diameter distance D_A is given by

$$D_A(z) = \frac{1}{1+z} \int_0^z \frac{dz'}{H(z')}. \quad (4)$$

Then, the definition of D_V is

$$D_V(z) = \left[(1+z)^2 D_A^2(z) \frac{z}{H(z)} \right]^{1/3}. \quad (5)$$

- H_0 measurement

Supernovae and H_0 from the Equation of State of dark energy (SH0ES) are measured through a three-step local distance ladder, and the latest H_0 measurement result is $H_0 = 73.04 \pm 1.04$ km s⁻¹Mpc⁻¹ [35]. We adopt the Gaussian prior of H_0 in our analysis.

We perform Markov-chain Monte Carlo (MCMC) sampling using the Python package `cobaya` [53], and the chains is considered converged if the Gelman-Rubin criterion [54] $R-1 < 0.05$ is satisfied.

III. MODELS

It is argued that the activation of an axion-like scalar field ϕ around recombination can serve as EDE, potentially resolving the H_0 tension [37]. This Axi-EDE can be described by the potential function

$$V(\phi) = m^2 f^2 [1 - \cos(\phi/f)]^n, \quad (6)$$

where n is a phenomenological parameter. When $n = 1$, m represents the axion mass and f denotes the axion decay constant. In the context of alleviating H_0 tension, n is usually chosen to be 3. The field is frozen at an initial value owing to Hubble friction and rolls down the potential when the Hubble parameter decreases to a critical level. The field acts as an EDE component before recombination, leading to a reduction the sound horizon at recombination $r_s(z_*)$. When the field oscillates around the minimum, its equation of state changes to $w_n = (n-1)/(n+1)$. Its energy density dilutes rapidly and hardly affects subsequent cosmological evolution. The fractional energy density of Axi-EDE is defined as $f_{\text{EDE}}(z) \equiv \rho_{\text{EDE}}(z)/\rho_{\text{tot}}(z)$, and the redshift at which $f_{\text{EDE}}(z)$ reaches its maximum is denoted as z_c . For simplicity, we use f_{EDE} to represent $f_{\text{EDE}}(z_c)$. If m and f are given, f_{EDE} and z_c can be solved numerically. Thus, the Axi-EDE model has 3 additional parameters, namely, f_{EDE} , z_c and the initial field value ϕ_i . The robustness of the Axi-EDE model has been evaluated using various datasets [36, 55–57].

In [43], the author proposes an EDE model with an AdS phase around recombination to effectively drive the values of f_{EDE} and H_0 higher. In this model, the potential of the scalar field ϕ is

$$V(\phi) = V_0(\phi/M_{\text{Pl}})^{2n} - V_{\text{AdS}}, \quad (7)$$

where V_0 is a constant, V_{AdS} is the depth of the AdS well, and $M_{\text{Pl}} = 1/\sqrt{8\pi G_N}$ is the reduced Planck mass. The potential function behaves similarly to the Rock 'n' Roll EDE model (RnR-EDE) [40] when the field is allowed to drop into an AdS phase and climb out of the potential well in the subsequent evolution. The AdS-EDE field is also initially frozen, and z_c denotes the redshift at which the field begins rolling down. The AdS-EDE model also introduces three extra parameters, namely, V_0 , ϕ_i , and V_{AdS} . These parameters are typically transformed to f_{EDE} , $\ln(1+z_c)$, and α_{AdS} . The relationship between α_{AdS} and V_{AdS} is $V_{\text{AdS}} \equiv \alpha_{\text{AdS}}[\rho_m(z_c) + \rho_r(z_c)]$, where ρ_m and ρ_r are the densities of matter and radiation, respectively. The AdS-EDE model requires a suitable selection of f_{EDE} and $\ln(1+z_c)$ parameters, i.e., V_{AdS} and V_0 , to ensure the field does not escape the AdS potential well and prevent a col-

lapsing universe, which is undesirable. This feature will result in a non-zero f_{EDE} when constraining AdS-EDE with the CMB alone [58]. It has been shown that the H_0 value inferred from AdS-EDE with fixed $\alpha_{\text{AdS}} = 3.79 \times 10^{-4}$ significantly alleviates the tension [43, 58–60]. Note that the constraints on α_{AdS} may be different depending on the datasets used. Therefore, we consider α_{AdS} as a free parameter in our analysis.

It is commonly assumed that one neutrino has a mass of $m_\nu = 0.06$ eV and the other two neutrinos are massless in cosmological models [5]. We follow this assumption for both the ν Axi-EDE and ν AdS-EDE base models. However, to constrain the neutrino mass in the NH or IH case, it is necessary to consider three massive neutrino cases in the ν Axi-EDE and ν AdS-EDE models. To investigate the inclination toward the NH or IH, we also use the neutrino mass hierarchy parameter defined as

$$\Delta \equiv \frac{m_3 - m_1}{m_1 + m_3}. \quad (8)$$

With this parameter, we can integrate the NH and IH into a unified model, in which $\Delta > 0$ corresponds to the NH and $\Delta < 0$ represents the IH. Consequently, the ratio of probabilities between $\Delta > 0$ and $\Delta < 0$ can reflect the preference inclination. The neutrino masses can be expressed in terms of Δ [61]:

$$m_1 = \frac{1 - \Delta}{2\sqrt{|\Delta|}} \sqrt{|\Delta m_{31}^2|}, \quad (9)$$

$$m_2 = \sqrt{m_1^2 + \Delta m_{21}^2}, \quad (10)$$

$$m_3 = \frac{1 + \Delta}{2\sqrt{|\Delta|}} \sqrt{|\Delta m_{31}^2|}. \quad (11)$$

In addition to the 6 parameters in the Λ CDM model, the extra sampling parameters are $\{f_{\text{EDE}}, \log_{10} a_c, \phi_i, m_{\text{min}}\}$ for the NH or IH case in the ν Axi-EDE model, whereas $\sum m_\nu$ is a derived parameter. For the ν Axi-EDE model with the Δ parameter, the extra sampling parameters are $\{f_{\text{EDE}}, \log_{10} a_c, \phi_i, \Delta\}$, whereas m_{min} and $\sum m_\nu$ are derived parameters. For the NH or IH case in the ν AdS-EDE model, the extra sampling parameters are $\{f_{\text{EDE}}, \ln(1+z_c), \alpha_{\text{AdS}}, m_{\text{min}}\}$, whereas $\sum m_\nu$ is a derived parameter. For the ν AdS-EDE model with the parameter Δ , the extra sampling parameters are $\{f_{\text{EDE}}, \ln(1+z_c), \alpha_{\text{AdS}}, \Delta\}$, whereas m_{min} and $\sum m_\nu$ are derived parameters. To calculate the theoretical prediction of observable quantities, we use modified versions of the `class` package for the ν Axi-EDE [56]¹⁾ and ν AdS-EDE models²⁾.

1) <https://github.com/PoulinV/AxiCLASS>

2) https://github.com/genye00/class_multiscf

IV. RESULTS AND DISCUSSION

We use the `getdist` package [62] to plot the posterior distribution of cosmological parameters and analyze the best-fit parameters. The parameter constraints obtained from MCMC chains are summarized in Table 1. The combination dataset used for analysis is Planck+BAO+Pantheon+SH0ES. The corresponding best-fit values of χ^2 are listed in the final line of the table. The one-dimensional posterior distributions of m_{\min} and $\sum m_\nu$ for the ν Axi-EDE and ν AdS-EDE models are shown in Fig. 1. In addition, Fig. 2 shows the posterior of the Δ parameter.

Axion-like early dark energy (Axi-EDE): For the NH case of the ν Axi-EDE model, $m_{\min} < 0.0429$ eV and $\sum m_\nu < 0.152$ eV at the 95% C.L., whereas for the IH case, $m_{\min} < 0.0437$ eV and $\sum m_\nu < 0.178$ eV at the 95% C.L. Moreover, the constraints on other parameters for the NH or IH case of the ν Axi-EDE model exhibit negligible deviations from those in the base case. The NH and IH cases of the ν Axi-EDE model exhibit a better fit to the observational datasets, as evidenced by their smaller χ^2 values when compared to the base case. Furthermore, the fitting performance of the NH case is better than that of the IH case because $\Delta\chi^2 = \chi^2(\nu\text{Axi-EDE, NH}) - \chi^2(\nu\text{Axi-EDE, IH}) = -2.1$.

The marginalized probability densities $p(\Delta)$ for the NH and IH cases of the ν Axi-EDE model with the parameter Δ are plotted in Fig. 2. We can then calculate these two probabilities:

$$P(\text{NH}) \equiv P(\Delta > 0) = \int_0^1 p(\Delta) d\Delta, \quad (12)$$

and

$$P(\text{IH}) \equiv P(\Delta > 0) = \int_{-1}^0 p(\Delta) d\Delta. \quad (13)$$

The ratio of these two probabilities can be calculated as

$$P(\text{NH}) : P(\text{IH}) = 2.11 : 1, \quad (14)$$

indicating that the NH is more favorable than the IH, which aligns with the aforementioned result of the $\Delta\chi^2$ value.

Early dark energy with AdS phase (AdS-EDE): The constraints on the minimum neutrino mass in the ν AdS-EDE model are $m_{\min} < 0.0363$ eV for the NH case and $m_{\min} < 0.0391$ eV for the IH case at the 95% C.L. The constraints on the total neutrino mass are $\sum m_\nu < 0.135$ eV for the NH case and $\sum m_\nu < 0.167$ eV for the IH case at the 95% C.L. The constraints on parameters other than neutrino parameters in the NH and IH cases of the ν AdS-EDE model are not significantly different from those in the base case. Moreover, the NH and IH cases of the ν AdS-EDE model fit better than the base case. The discrepancy between the χ^2 values in the NH and IH cases is $\Delta\chi^2 = \chi^2(\nu\text{AdS-EDE, NH}) - \chi^2(\nu\text{AdS-EDE, IH}) = -2.7$. Therefore, the NH case still provides a better fit than the IH case within the framework of ν AdS-EDE.

Table 1. Mean values and 1σ constraints on the cosmological parameters in the base case, considering one massive neutrino, as well as the NH and IH cases of the ν Axi-EDE and ν AdS-EDE models. The upper limits of α_{AdS} , m_{\min} , and $\sum m_\nu$ are at the 95% C.L.

	ν Axi-EDE			ν AdS-EDE		
	base	NH	IH	base	NH	IH
$\log(10^{10} A_s)$	3.068 ± 0.015	$3.070^{+0.015}_{-0.016}$	3.075 ± 0.016	3.068 ± 0.015	3.070 ± 0.015	3.075 ± 0.015
n_s	0.9885 ± 0.0063	0.9898 ± 0.0064	0.9908 ± 0.0061	0.9849 ± 0.0056	0.9854 ± 0.0054	0.9871 ± 0.0056
$\Omega_b h^2$	0.02284 ± 0.00021	0.02285 ± 0.00022	0.02285 ± 0.00021	0.02303 ± 0.00020	0.02305 ± 0.00020	0.02310 ± 0.00020
$\Omega_c h^2$	0.1299 ± 0.0033	$0.1309^{+0.0032}_{-0.0036}$	0.1315 ± 0.0035	0.1284 ± 0.0030	0.1286 ± 0.0030	$0.1293^{+0.0029}_{-0.0032}$
τ_{reio}	$0.0583^{+0.0069}_{-0.0078}$	$0.0589^{+0.0072}_{-0.0082}$	$0.0606^{+0.0071}_{-0.0085}$	0.0565 ± 0.0075	0.0573 ± 0.0076	$0.0589^{+0.0068}_{-0.0078}$
H_0 [km s ⁻¹ Mpc ⁻¹]	$71.42^{+0.90}_{-0.79}$	71.51 ± 0.85	71.52 ± 0.85	71.04 ± 0.77	70.98 ± 0.74	70.95 ± 0.76
S_8	0.838 ± 0.012	0.837 ± 0.013	0.835 ± 0.013	0.841 ± 0.013	0.840 ± 0.013	0.838 ± 0.013
f_{EDE}	$0.109^{+0.028}_{-0.022}$	0.119 ± 0.025	$0.124^{+0.026}_{-0.023}$	0.073 ± 0.018	0.087 ± 0.020	0.093 ± 0.020
$\log_{10} a_c$	$-3.64^{+0.13}_{-0.20}$	$-3.63^{+0.12}_{-0.012}$	$-3.613^{+0.084}_{-0.014}$	–	–	–
$\ln(1+z_c)$	–	–	–	$8.181^{+0.095}_{-0.11}$	8.34 ± 0.10	8.335 ± 0.094
α_{AdS}	–	–	–	< 0.000242	< 0.000138	< 0.000149
m_{\min}/eV	–	< 0.0429	< 0.0437	–	< 0.0363	< 0.0391
$\sum m_\nu/\text{eV}$	–	< 0.152	< 0.178	–	< 0.135	< 0.167
χ^2	3868.8	3863.7	3865.8	3864.9	3858.5	3861.2

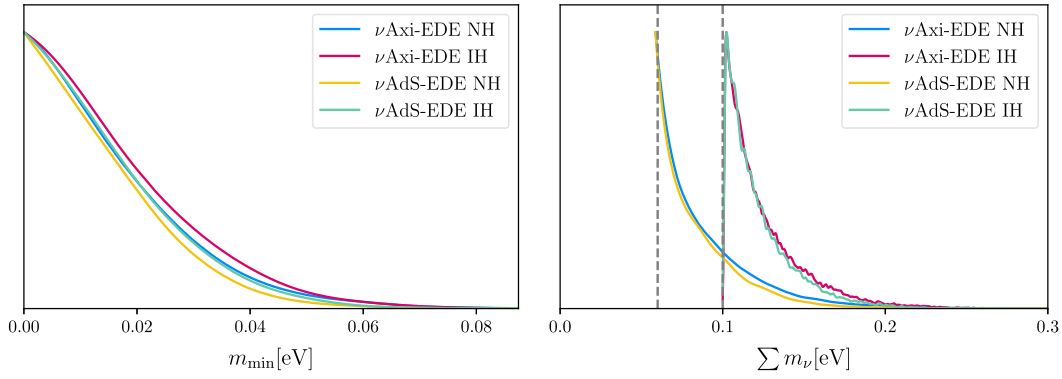


Fig. 1. (color online) 1D posterior distribution of m_{\min} and $\sum m_\nu$ for the NH and IH cases in the ν Axi-EDE and ν AdS-EDE models. The gray dashed lines in the right panel represent the lower limits of $\sum m_\nu$ in the NH case (0.06 eV) and IH case (0.1 eV).

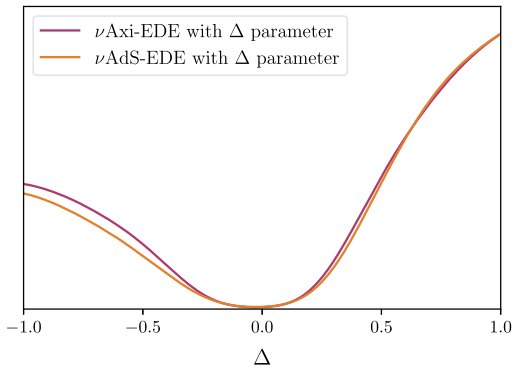


Fig. 2. (color online) 1D posterior distribution of the Δ parameter in the ν Axi-EDE and ν AdS-EDE models.

For the ν AdS-EDE model with the parameter Δ , the ratio of $P(\text{NH})$ and $P(\text{IH})$ can be calculated from $p(\Delta)$ plotted in Fig. 2,

$$P(\text{NH}) : P(\text{IH}) = 2.38 : 1. \quad (15)$$

It can be concluded that the NH is more favorable than the IH, which is consistent with the aforementioned result of the $\Delta\chi^2$ value. Comparing the constraints of the neutrino mass and the χ^2 values of the above models, we find that the results are congruent. The stringency of the neutrino mass constraints depends on each specific model's characteristics; however, these constraints remain reliable for these two different EDE scenarios.

V. SUMMARY AND CONCLUSIONS

In this study, we analyze the constraints on neutrino mass within consistent cosmological models, namely, the ν Axi-EDE and ν AdS-EDE models. In both models, we assume three massive neutrino mass eigenstates with NH and IH. Additionally, we use the hierarchy parameter Δ in our analysis to evaluate the preference between the NH and IH. We employ a combination of *Planck* + BAO +

Pantheon + SH0ES measurements to constrain the parameters of these models.

The resulting constraint on the minimum neutrino mass in the ν Axi-EDE model with the NH is $m_{\min} < 0.0429$ eV at the 95% C.L., and the constraint on the sum of neutrino masses is $\sum m_\nu < 0.152$ eV. For the ν Axi-EDE model with the IH, the neutrino mass constraints are $m_{\min} < 0.0437$ eV and $\sum m_\nu < 0.178$ eV both at the 95% C.L. The constraint on the minimum neutrino mass in the ν AdS-EDE model with the NH is $m_{\min} < 0.0363$ eV at the 95% C.L., and the constraint on the sum of neutrino masses is $\sum m_\nu < 0.135$ eV at the 95% C.L. In the case of ν AdS-EDE with the IH, the constraints are $m_{\min} < 0.0391$ eV and $\sum m_\nu < 0.167$ eV at the 95% C.L.

Compared to the ν Axi-EDE base model with only one massive neutrino and a total neutrino mass of $\sum m_\nu = 0.06$ eV, there is no apparent shift in the constraints on cosmological parameters in the ν Axi-EDE models with NH or IH. The χ^2 values of the ν Axi-EDE model with the NH and IH are both smaller than that of the base case. Additionally, we find that the ν Axi-EDE model with the NH fits better with the combination datasets compared to the other two cases. By integrating the probability distribution of the Δ parameter, we find $P(\text{NH}) : P(\text{IH}) = 2.11 : 1$ for ν Axi-EDE, indicating a preference for the ν Axi-EDE model with the NH over the IH. Similarly, there seems to be no significant deviation of the cosmological parameters in the ν AdS-EDE model with the NH or IH compared to those in the base case. The χ^2 values of both the NH and IH cases in the ν AdS-EDE model are smaller than those in the base case. Furthermore, $P(\text{NH}) : P(\text{IH}) = 2.38 : 1$ indicates that the NH case of the ν AdS-EDE model provides a better fit to the joint datasets compared to the IH case.

By comparing the overall results of the ν Axi-EDE and ν AdS-EDE models, we find that different hierarchies of three massive neutrino eigenstates give similar results. Thus, it is proved that the parameter constraints of these two EDE models are consistent and reliable.

References

- [1] Y. Fukuda *et al.* (Super-Kamiokande), *Phys. Rev. Lett.* **81**, 1562 (1998), arXiv:hep-ex/9807003
- [2] Q. R. Ahmad *et al.* (SNO), *Phys. Rev. Lett.* **89**, 011301 (2002), arXiv:nucl-ex/0204008
- [3] M. Aker *et al.* (KATRIN), *Phys. Rev. Lett.* **123**, 221802 (2019), arXiv:1909.06048[hep-ex]
- [4] K. A. Olive *et al.* (Particle Data Group), *Chin. Phys. C* **38**, 090001 (2014)
- [5] N. Aghanim *et al.* (Planck), *Planck 2018 results. VI. Cosmological parameters*, *Astron. Astrophys.* **641**, A6 (2020) [Erratum: *Astron. Astrophys.* **652**, C4 (2021)], arXiv:1807.06209[astro-ph.CO]
- [6] Nathalie Palanque-Deslaurier, Christophe Yèche, Nils Schöneberg *et al.*, *JCAP* **04**, 038 (2020), arXiv:1911.09073[astro-ph.CO]
- [7] Shadab Alam *et al.* (eBOSS), *Phys. Rev. D* **103**, 083533 (2021), arXiv:2007.08991[astro-ph.CO]
- [8] T. M. C. Abbott *et al.* (DES), *Phys. Rev. D* **105**, 023520 (2022), arXiv:2105.13549[astro-ph.CO]
- [9] Ziad Sakr, *Universe* **8**, 284 (2022)
- [10] Yvonne Y. Y. Wong, *Ann. Rev. Nucl. Part. Sci.* **61**, 69 (2011), arXiv:1111.1436[astro-ph.CO]
- [11] Julien Lesgourgues and Sergio Pastor, *Adv. High Energy Phys.* **2012**, 608515 (2012), arXiv:1212.6154[hep-ph]
- [12] Julien Lesgourgues, Gianpiero Mangano, Gennaro Miele *et al.*, *Neutrino Cosmology*, (Cambridge University Press, 2013)
- [13] Julien Lesgourgues and Sergio Pastor, *New J. Phys.* **16**, 065002 (2014), arXiv:1404.1740[hep-ph]
- [14] K. N. Abazajian *et al.* (Topical Conveners: K.N. Abazajian, J.E. Carlstrom, A.T. Lee), *Astropart. Phys.* **63**, 66 (2015), arXiv:1309.5383[astro-ph.CO]
- [15] Qing-Guo Huang, Ke Wang, and Sai Wang, *Eur. Phys. J. C* **76**, 489 (2016), arXiv:1512.05899[astro-ph.CO]
- [16] Shouvik Roy Choudhury and Steen Hannestad, *JCAP* **07**, 037 (2020), arXiv:1907.12598[astro-ph.CO]
- [17] Beth A. Reid, Licia Verde, Raul Jimenez *et al.*, *JCAP* **01**, 003 (2010), arXiv:0910.0008[astro-ph.CO]
- [18] Suresh Kumar and Rafael C. Nunes, *Phys. Rev. D* **94**, 123511 (2016), arXiv:1608.02454[astro-ph.CO]
- [19] Weiqiang Yang, Rafael C. Nunes, Supriya Pan *et al.*, *Phys. Rev. D* **95**, 103522 (2017), arXiv:1703.02556[astro-ph.CO]
- [20] Sunny Vagnozzi, Suhail Dhawan, Martina Gerbino *et al.*, *Phys. Rev. D* **98**, 083501 (2018), arXiv:1801.08553[astro-ph.CO]
- [21] Rui-Yun Guo, Jing-Fei Zhang, and Xin Zhang, *Chin. Phys. C* **42**, 095103 (2018), arXiv:1803.06910[astro-ph.CO]
- [22] Shouvik Roy Choudhury and Sandhya Choubey, *JCAP* **09**, 017 (2018), arXiv:1806.10832[astro-ph.CO]
- [23] Lu Feng, Hai-Li Li, Jing-Fei Zhang *et al.*, *Sci. China Phys. Mech. Astron.* **63**, 220401 (2020), arXiv:1903.08848[astro-ph.CO]
- [24] Lu Feng, Dong-Ze He, Hai-Li Li *et al.*, *Sci. China Phys. Mech. Astron.* **63**, 290404 (2020), arXiv:1910.03872[astro-ph.CO]
- [25] Hai-Li Li, Jing-Fei Zhang, and Xin Zhang, *Commun. Theor. Phys.* **72**, 125401 (2020), arXiv:2005.12041[astro-ph.CO]
- [26] Weiqiang Yang, Eleonora Di Valentino, Supriya Pan *et al.*, *Phys. Dark Univ.* **31**, 100762 (2021), arXiv:2007.02927[astro-ph.CO]
- [27] Wilmar Cardona, Rubén Arjona, Alejandro Estrada *et al.*, *JCAP* **05**, 064 (2021), arXiv:2012.05282[astro-ph.CO]
- [28] Adrià G'omez-Valent, Ziyang Zheng, Luca Amendola *et al.*, *Phys. Rev. D* **106**, 103522 (2022), arXiv:2207.14487[astro-ph.CO]
- [29] Vikrant Yadav, Santosh Kumar Yadav, and Anil Kumar Yadav, *Phys. Dark Univ.* **42**, 101363 (2023), arXiv:2307.05155[astro-ph.CO]
- [30] Jianbo Lu, Molin Liu, Yabo Wu *et al.*, *Eur. Phys. J. C* **76**, 679 (2016), arXiv:1606.02987[astro-ph.CO]
- [31] Toyokazu Sekiguchi and Tomo Takahashi, *Phys. Rev. D* **103**, 083516 (2021), arXiv:2011.14481[astro-ph.CO]
- [32] Eleonora Di Valentino, Olga Mena, Supriya Pan *et al.*, *Class. Quant. Grav.* **38**, 153001 (2021), arXiv:2103.01183[astro-ph.CO]
- [33] Elcio Abdalla *et al.*, *JHEP* **34**, 49 (2022), arXiv:2203.06142[astro-ph.CO]
- [34] N. Aghanim *et al.* (Planck), *Astron. Astrophys.* **641**, A1 (2020), arXiv:1807.06205[astro-ph.CO]
- [35] Adam G. Riess *et al.*, *Astrophys. J. Lett.* **934**, L7 (2022), arXiv:2112.04510[astro-ph.CO]
- [36] Vivian Poulin, Tristan L. Smith, and Tanvi Karwal, *The Ups and Downs of Early Dark Energy solutions to the Hubble tension: a review of models, hints and constraints circa 2023*, (2023), arXiv:2302.09032[astro-ph.CO]
- [37] Vivian Poulin, Tristan L. Smith, Tanvi Karwal *et al.*, *Phys. Rev. Lett.* **122**, 221301 (2019), arXiv:1811.04083[astro-ph.CO]
- [38] Meng-Xiang Lin, Giampaolo Benevento, Wayne Hu *et al.*, *Phys. Rev. D* **100**, 063542 (2019), arXiv:1905.12618[astro-ph.CO]
- [39] Tristan L. Smith, Vivian Poulin, and Mustafa A. Amin, *Phys. Rev. D* **101**, 063523 (2020), arXiv:1908.06995[astro-ph.CO]
- [40] Prateek Agrawal, Francis-Yan Cyr-Racine, David Pinner *et al.*, *Phys. Dark Univ.* **42**, 101347 (2023), arXiv:1904.01016[astro-ph.CO]
- [41] Matteo Braglia, William T. Emond, Fabio Finelli *et al.*, *Phys. Rev. D* **102**, 083513 (2020), arXiv:2005.14053[astro-ph.CO]
- [42] Florian Niedermann and Martin S. Sloth, *Phys. Rev. D* **103**, L041303 (2021), arXiv:1910.10739[astro-ph.CO]
- [43] Gen Ye and Yun-Song Piao, *Phys. Rev. D* **101**, 083507 (2020), arXiv:2001.02451[astro-ph.CO]
- [44] Florian Niedermann and Martin S. Sloth, *Phys. Rev. D* **102**, 063527 (2020), arXiv:2006.06686[astro-ph.CO]
- [45] Tanvi Karwal, Marco Raveri, Bhuvnesh Jain *et al.*, *Phys. Rev. D* **105**, 063535 (2022), arXiv:2106.13290[astro-ph.CO]
- [46] Evan McDonough, Meng-Xiang Lin, J. Colin Hill *et al.*, *Phys. Rev. D* **106**, 043525 (2022), arXiv:2112.09128[astro-ph.CO]
- [47] N. Aghanim *et al.* (Planck), *Astron. Astrophys.* **641**, A5 (2020), arXiv:1907.12875[astro-ph.CO]
- [48] N. Aghanim *et al.* (Planck), *Astron. Astrophys.* **641**, A8 (2020), arXiv:1807.06210[astro-ph.CO]
- [49] D. M. Scolnic *et al.* (Pan-STARRS1), *Astrophys. J.* **859**, 101 (2018), arXiv:1710.00845[astro-ph.CO]
- [50] Florian Beutler, Chris Blake, Matthew Colless *et al.*, *Mon. Not. Roy. Astron. Soc.* **416**, 3017 (2011), arXiv:1106.3366[astro-ph.CO]
- [51] Ashley J. Ross, Lado Samushia, Cullan Howlett *et al.*, *Mon. Not. Roy. Astron. Soc.* **449**, 835 (2015),

- arXiv:1409.3242[astro-ph.CO]
- [52] Shadab Alam *et al.* (BOSS), *Mon. Not. Roy. Astron. Soc.* **470**, 2617 (2017), arXiv:1607.03155[astro-ph.CO]
- [53] Jesus Torrado and Antony Lewis, *JCAP* **05**, 057 (2021), arXiv:2005.05290[astro-ph.IM]
- [54] Andrew Gelman and Donald B. Rubin, *Statist. Sci.* **7**, 457 (1992)
- [55] J. Colin Hill, Evan McDonough, Michael W. Toomey *et al.*, *Phys. Rev. D* **102**, 043507 (2020), arXiv:2003.07355[astro-ph.CO]
- [56] Riccardo Murgia, Guillermo F. Abellán, and Vivian Poulin, *Phys. Rev. D* **103**, 063502 (2021), arXiv:2009.10733[astro-ph.CO]
- [57] Mikhail M. Ivanov, Evan McDonough, J. Colin Hill *et al.*, *Phys. Rev. D* **102**, 103502 (2020), arXiv:2006.11235[astro-ph.CO]
- [58] Jun-Qian Jiang and Yun-Song Piao, *Phys. Rev. D* **104**, 103524 (2021), arXiv:2107.07128[astro-ph.CO]
- [59] Gen Ye, Bin Hu, and Yun-Song Piao, *Phys. Rev. D* **104**, 063510 (2021), arXiv:2103.09729[astro-ph.CO]
- [60] Gen Ye and Yun-Song Piao, *Phys. Rev. D* **106**, 043536 (2022), arXiv:2202.10055[astro-ph.CO]
- [61] Lixin Xu and Qing-Guo Huang, *Sci. China Phys. Mech. Astron.* **61**, 039521 (2018), arXiv:1611.05178[astro-ph.CO]
- [62] Antony Lewis, *GetDist: a Python package for analysing Monte Carlo samples*, (2019), arXiv: 1910.13970[astro-ph.IM]