

Topological amplitudes of bottom baryon decays in the $SU(3)_F$ limit*

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Abstract: Motivated by the first observation of CP violation in baryon decays, we study the topological amplitudes of bottom baryon decays in the $SU(3)_F$ limit. The topological diagrams of the charmless two-body decays of bottom baryons are presented in detail. The linear relations between topologies and $SU(3)$ irreducible amplitudes are derived through tensor contraction and $SU(3)$ decomposition. Four amplitudes among the 13 independent amplitudes are critical to the CP asymmetries. The small CP asymmetries might indicate small relative strong phases between amplitudes A_2 and $A'_{12,14}$. To avoid them, we suggest measuring CP asymmetries in the $\Xi_b^0 \rightarrow pK^-$ and $\Xi_b^- \rightarrow \Lambda^0 K^-$ decays. Furthermore, the Körner-Pati-Woo theorem can be tested by measuring the branching fractions of the $\Lambda_b^0 \rightarrow \Sigma^0 K_S^0$ and $\Lambda_b^0 \rightarrow \Sigma^- K^+$ modes.

Keywords: bottom baryon, topological diagram, flavor symmetry, CP violation

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I. INTRODUCTION

Bottom baryon decays play an important role in studying perturbative and nonperturbative strong interactions, extracting the quark-mixing Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and searching for new physics beyond the Standard Model. Compared with the bottom meson decays, the non-zero spin of baryons provides opportunities to study decay dynamics via more observables beyond branching fractions [1–10]. As bottom baryons are increasingly generated at the Large Hadron Collider (LHC), several charmless decay channels of bottom baryons have been observed in experiments [11–20]. Very recently, the LHCb Collaboration has reported the first observation of CP violation in bottom baryon decays [21]

$$A_{CP}(\Lambda_b^0 \rightarrow pK^- \pi^+ \pi^-) = (2.45 \pm 0.46 \pm 0.10)\%. \quad (1)$$

It is a milestone in particle physics as CP asymmetries are well established in meson systems [22–25], whereas CP violation in baryon decays has not been observed until now.

From a theoretical perspective, the QCD in baryon non-leptonic decays are complicated owing to the presence of three valence quarks [26–30]. Because of the ab-

sence of nonperturbative input, only a few decay channels can be calculated in the QCD inspired approach. To extract the decay information of bottom baryons, it is significant to analyze the topological diagrams in the flavor $SU(3)$ symmetry. A topological diagram provides an intuitive description of the dynamics of heavy hadron decays and a theoretical framework that allows not only model-dependent data analysis but also model calculations. The topological diagrams of charmed baryon decays have been widely analyzed in recent studies [31–38]. However, the topological amplitudes of bottom baryon decays have not been systematically studied so far.

Topological diagrams in heavy hadron decays can be formalized as invariant tensors constructed from four-quark operators and initial/final states [39], allowing us to study topologies with mathematical tools. Owing to the different spin wave functions of the $q_1 \leftrightarrow q_2$ symmetric and antisymmetric octets $\mathcal{B}_8^S$ and $\mathcal{B}_8^A$, the topological diagrams of baryon decays into octet baryons have two distinct sets. Furthermore, the relations between topological diagrams and $SU(3)$ irreducible amplitudes are not evident, as the topological amplitudes of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays are constructed from third-rank octet tensors, whereas $SU(3)$ irreducible amplitudes are constructed from (1,1)-rank octet tensors. In Ref. [40], we established a framework to analyze the topological amplitudes of charmed baryon decays in the $SU(3)_F$ limit. In this study,

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we extend this framework to the bottom baryon decays. The linear relations between topologies and $SU(3)$ irreducible amplitudes are derived through tensor contraction and $SU(3)$ decomposition. Among the 13 independent amplitudes contributing to $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays, four amplitudes associated with three-dimensional irreducible representations are critical to the CP asymmetries of bottom baryon decays. Recent measurements of the CP asymmetries of the $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ modes [20] have indicated small strong phases between A_2 and $A'_{12,14}$. To avoid the small strong phases, we suggest measuring CP asymmetries in the $\Xi_b^0 \rightarrow pK^-$ and $\Xi_b^- \rightarrow \Lambda^0 K^-$ decays. Furthermore, the Körner-Pati-Woo theorem [41, 42] can be tested by measuring the branching fractions of $\Lambda_b^0 \rightarrow \Sigma^0 K_S^0$ and $\Lambda_b^0 \rightarrow \Sigma^- K^+$ decays under isospin symmetry.

The remainder of this paper is organized as follows. In Sec. II, we present the topological amplitudes of bottom baryon decays in the $SU(3)_F$ limit. The phenomenological analysis of the topological amplitudes is presented in Sec. III. Sec. IV provides a brief summary of this paper. The $SU(3)$ irreducible amplitudes constructed by the third-rank octet tensors are discussed in Appendix A.

II. TOPOLOGICAL AMPLITUDES OF BOTTOM BARYON DECAYS

The effective Hamiltonian of the $b \rightarrow u\bar{u}q$ transition is given by [43]

$$\begin{aligned} \mathcal{H}_{\text{eff}} = & \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left[V_{ub} V_{uq}^* \left(\sum_{i=1}^2 C_i^u(\mu) O_i^u(\mu) \right) \right. \\ & \left. + V_{cb} V_{cq}^* \left(\sum_{i=1}^2 C_i^c(\mu) O_i^c(\mu) \right) \right] \\ & - \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left[V_{tb} V_{tq}^* \left(\sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right) \right. \\ & \left. + C_{7\gamma}(\mu) O_{7\gamma}(\mu) + C_{8g}(\mu) O_{8g}(\mu) \right] + \text{h.c.} \quad (2) \end{aligned}$$

The magnetic-penguin contributions can be included in the Wilson coefficients for the penguin operators [44–46]. In the flavor $SU(3)$ limit, the weak Hamiltonian of bottom decays can be written as [47]

$$\mathcal{H}_{\text{eff}} = \sum_{i,j,k=1}^3 \{ H_{ij}^{(u)k} O_{ij}^{(u)k} + H_i^{(c)} O_i^{(c)} + H_{ij}^{(p)k} O_{ij}^{(p)k} \}, \quad (3)$$

where $O_{ij}^{(u)k}$, $O_i^{(c)}$, and $O_{ij}^{(p)k}$ denote the four-quark operators including the Fermi coupling constant G_F and the Wilson coefficients. The superscripts u , c , and p are used to distinguish the tree operators $O_{1,2}^{(u)}$, $O_{1,2}^{(c)}$ and the pen-

guin operators O_{3-10} . Indices i, j , and k are flavor indices. The color indices and current structures of the four quark operators are summed into $O_{ij}^{(u)k}$, $O_i^{(c)}$, and $O_{ij}^{(p)k}$. The matrices $H^{(u,c,p)}$ are the coefficient matrices. According to the effective Hamiltonian of $b \rightarrow u\bar{u}q$ decays, the non-zero CKM coefficients include

$$\begin{aligned} H_{21}^{(u)1} &= V_{ub} V_{ud}^*, & H_{31}^{(u)1} &= V_{ub} V_{us}^*, \\ H_2^{(c)} &= V_{cb} V_{cd}^*, & H_3^{(c)} &= V_{cb} V_{cs}^*, \\ H_{12}^{(p)1} &= H_{22}^{(p)2} = H_{32}^{(p)3} = -V_{tb} V_{td}^*, \\ H_{13}^{(p)1} &= H_{23}^{(p)2} = H_{33}^{(p)3} = -V_{tb} V_{ts}^*. \end{aligned} \quad (4)$$

Similar to the effective Hamiltonian, the initial and final states, for instance M , can be written as

$$|M^\alpha\rangle = (M^\alpha)_j^i |M_j^i\rangle, \quad (5)$$

where $|M_j^i\rangle$ is the quark composition of the meson state, $|M_j^i\rangle = |q_i \bar{q}_j\rangle$, and (M^α) is the coefficient matrix.

The decay amplitude of the $\mathcal{B}_{b\bar{3}}^\gamma \rightarrow \mathcal{B}_8^\alpha M^\beta$ mode is constructed as follows:

$$\begin{aligned} \mathcal{A}(\mathcal{B}_{b\bar{3}}^\gamma \rightarrow \mathcal{B}_8^\alpha M^\beta) &= \langle \mathcal{B}_8^\alpha M^\beta | \mathcal{H}_{\text{eff}} | \mathcal{B}_{b\bar{3}}^\gamma \rangle \\ &= \sum_{\omega} (\mathcal{B}_8^\alpha)^{ijk} \langle \mathcal{B}_8^{ijk} | (M^\beta)_m^l \langle M_m^l | H_{np}^q O_{np}^q | (\mathcal{B}_{b\bar{3}}^\gamma)_{rs} \rangle | [\mathcal{B}_{b\bar{3}}]_{rs} \rangle \\ &= \sum_{\omega} \langle \mathcal{B}_8^{ijk} M_m^l | O_{np}^q | [\mathcal{B}_{b\bar{3}}]_{rs} \rangle \times (\mathcal{B}_8^\alpha)^{ijk} (M^\beta)_m^l H_{np}^q (\mathcal{B}_{b\bar{3}}^\gamma)_{rs} \\ &= \sum_{\omega} X_{\omega} (C_{\omega})_{\alpha\beta\gamma}. \end{aligned} \quad (6)$$

In the above formula, $\sum_{\omega}$ represents summing over all possible full contractions. $X_{\omega} = \langle \mathcal{B}_8^{ijk} M_m^l | O_{np}^q | [\mathcal{B}_{b\bar{3}}]_{rs} \rangle$ is the reduced matrix element, and $(C_{\omega})_{\alpha\beta\gamma} = (\mathcal{B}_8^\alpha)^{ijk} (M^\beta)_m^l H_{np}^q (\mathcal{B}_{b\bar{3}}^\gamma)_{rs}$ is the Clebsch-Gordan (CG) coefficient. According to the Wigner-Eckart theorem [48, 49], X_{ω} is independent of the indices α , β , and γ . All the information about the initial and final states is absorbed into the Clebsch-Gordan coefficient $(C_{\omega})_{\alpha\beta\gamma}$. There are two different octets with symmetric and antisymmetric flavor wavefunctions under $q_1 \leftrightarrow q_2$. They are labeled by $\mathcal{B}_8^S$ and $\mathcal{B}_8^A$ and their flavor and spin wave functions are $\phi_S \chi_S$ and $\phi_A \chi_A$, respectively. The total amplitude for the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decay is obtained by summing the amplitudes of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^S M$ and $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^A M$ transitions,

$$\mathcal{A}(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M) = \mathcal{A}^S(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^S M) + \mathcal{A}^A(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^A M). \quad (7)$$

The topological amplitudes for the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays are similar to those for the $\mathcal{B}_{c\bar{3}} \rightarrow \mathcal{B}_8 M$ decays, which have been studied in Ref. [40]. Hence, the details of the model-independent analysis of topological amplitudes are presented in Appendix A. The difference between the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ and $\mathcal{B}_{c\bar{3}} \rightarrow \mathcal{B}_8 M$ decays is that the d - and s -quark loop diagrams in the $\mathcal{B}_{c\bar{3}} \rightarrow \mathcal{B}_8 M$ decays are identical under the $SU(3)_F$ symmetry, whereas the u - and c -quark loop diagrams in the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays are distinct, as the c quark is not a basis of the flavor $SU(3)$ group. Hence, the charm-loop amplitudes induced by operator $\mathcal{O}_i^{(c)}$ are presented explicitly in Appendix A.

To derive the linear relations between the topological diagrams, in which the baryon octet is written as a tensor with three covariant indices, and the $SU(3)$ irreducible amplitudes, in which the baryon octet is written as a tensor with a covariant index and a contravariant index, the key step is to determine the relation between the two different tensor representations. As noted in Ref. [40], the third-rank tensors $(\mathcal{B}_8^S)^{ijk}$ and $(\mathcal{B}_8^A)^{ijk}$ are expressed in

terms of the $(1,1)$ -rank tensor $(\mathcal{B}_8)_j^i$ as

$$(\mathcal{B}_8^S)^{ijk} = \frac{1}{\sqrt{6}} [\epsilon^{kil}(\mathcal{B}_8)_l^j + \epsilon^{kjl}(\mathcal{B}_8)_l^i], \quad (\mathcal{B}_8^A)^{ijk} = \frac{1}{\sqrt{2}} \epsilon^{ijl}(\mathcal{B}_8)_l^k. \quad (8)$$

One can verify that the indices i and j are symmetric in $(\mathcal{B}_8^S)^{ijk}$ and antisymmetric in $(\mathcal{B}_8^A)^{ijk}$. The detailed derivation of the linear relations between the topologies and the $SU(3)$ irreducible amplitudes is shown in Appendix A.

The topological amplitudes of $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays are presented in Tables 1–8. We use the superscript P to distinguish the penguin-induced diagrams from the tree-induced diagrams. The decay amplitudes constructed by $(\mathcal{B}_8)_j^i$ are listed in Tables 9 and 10. Note that the tree-induced amplitudes $A_{15} \sim A_{18}$ and penguin-induced amplitudes A_1^P and A_{10}^P do not contribute to the bottom baryon decays. One can verify that Tables 9 and 10 are consistent with Tables 1 to 8 using the decomposition in Eq. (A12). For example, the topological amplitudes contributing to the $\Lambda_b^0 \rightarrow p\pi^-$ decay are

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow p\pi^-) &= \lambda_u A_2 - \lambda_t A'_{12} = \lambda_u (A_2 + A_{12}) + \lambda_c A_1^c - \lambda_t (A_4^P + A_6^P + A_{12}^P + 3A_{15}^P) \\ &= \frac{1}{\sqrt{2}} \lambda_u (-a_1^A + a_3^A - a_4^A - a_5^A - a_7^A - a_8^A + a_{14}^A - a_{18}^A + a_{20}^A + a_{21}^A + 2a_{28}^A + 2a_{30}^A) \\ &\quad - \frac{1}{\sqrt{6}} \lambda_u (a_1^S - 2a_2^S + a_3^S - a_4^S - a_5^S + 2a_6^S - a_7^S - a_8^S + 2a_9^S + 3a_{14}^S + a_{18}^S - 2a_{19}^S + a_{20}^S + 3a_{21}^S) \\ &\quad + \frac{1}{\sqrt{2}} (-a_1^{c,A} + a_3^{c,A} + a_4^{c,A} + 2a_6^{c,A}) - \frac{1}{\sqrt{6}} (a_1^{c,S} - 2a_2^{c,S} + a_3^{c,S} + 3a_4^{c,S}) \\ &\quad - \frac{1}{\sqrt{2}} \lambda_t (-a_2^{A,P} - a_3^{A,P} + a_4^{A,P} - a_6^{A,P} - a_7^{A,P} + a_9^{A,P} - 2a_{10}^{A,P} - a_{11}^{A,P} + a_{12}^{A,P} + a_{13}^{A,P} - a_{18}^{A,P} + a_{20}^{A,P} + a_{21}^{A,P} - 3a_{23}^{A,P} + 3a_{25}^{A,P} \\ &\quad + 3a_{26}^{A,P} + 2a_{29}^{A,P} + 2a_{30}^{A,P} + 6a_{32}^{A,P}) - \frac{1}{\sqrt{6}} \lambda_t (-2a_1^{S,P} + a_2^{S,P} + a_3^{S,P} - a_4^{S,P} + 2a_5^{S,P} - a_6^{S,P} + a_7^{S,P} - 2a_8^{S,P} + a_9^{S,P} \\ &\quad - 3a_{11}^{S,P} + 3a_{12}^{S,P} + 3a_{13}^{S,P} + a_{18}^{S,P} - 2a_{19}^{S,P} + a_{20}^{S,P} + 3a_{21}^{S,P} + 3a_{23}^{S,P} - 6a_{24}^{S,P} + 3a_{25}^{S,P} + 9a_{26}^{S,P}). \end{aligned} \quad (9)$$

Moreover, one can verify the isospin relations derived in Ref. [50] and the U -spin or $SU(3)_F$ relations provided in Refs. [51–54] using Tables 9 and 10.

The linear correlation of decay amplitudes contributing to the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ modes in the Standard Model is beyond the model-independent analysis in Appendix A. The nonzero coefficients induced by tree operators $\mathcal{O}_{1,2}^{(u)}$ in the $SU(3)$ irreducible representations are

$$\begin{aligned} H(\bar{6})^{(u)31} &= -V_{ub} V_{ud}^*, & H(\bar{6})^{(u)12} &= V_{ub} V_{us}^*, \\ H(15)_{12}^{(u)1} &= 3V_{ub} V_{ud}^*, & H(15)_{22}^{(u)2} &= -2V_{ub} V_{ud}^*, \\ H(15)_{32}^{(u)3} &= -V_{ub} V_{ud}^*, & H(15)_{13}^{(u)1} &= 3V_{ub} V_{us}^*, \\ H(15)_{33}^{(u)3} &= -2V_{ub} V_{us}^*, & H(15)_{32}^{(u)2} &= -V_{ub} V_{us}^* \end{aligned}$$

$$H^{(u)}(3)_2 = V_{ub} V_{ud}^*, \quad H^{(u)}(3)_3 = V_{ub} V_{us}^*. \quad (10)$$

The nonzero coefficients induced by tree operators $\mathcal{O}_{1,2}^{(c)}$ in the $SU(3)$ irreducible representations are

$$H(3)_2^{(c)} = V_{cb} V_{cd}^*, \quad H(3)_3^{(c)} = V_{cb} V_{cs}^*. \quad (11)$$

The nonzero coefficients induced by penguin operators in the $SU(3)$ irreducible representations are

$$\begin{aligned} H(3)_2^{(p)} &= -V_{tb} V_{td}^*, & H(3)_2^{(p)} &= -3V_{tb} V_{td}^*, \\ H(3)_3^{(p)} &= -V_{tb} V_{ts}^*, & H(3)_3^{(p)} &= -3V_{tb} V_{ts}^*. \end{aligned} \quad (12)$$

Table 1. Topological amplitudes of $\Xi_b^{-,0}$ decays into $\mathcal{B}_8^S M$ from the $b \rightarrow d$ transition.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 \pi^-$	$\frac{1}{2\sqrt{3}} \lambda_c(a_1^{S,c} + a_2^{S,c} - 2a_3^{S,c} - 3a_4^{S,c}) - \frac{1}{2\sqrt{3}} \lambda_t(a_1^{S,p} + 3a_{11}^{S,p} - 3a_{12}^{S,p} - 3a_{13}^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} - 3a_{21}^{S,p} \\ + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} - 9a_{26}^{S,p} - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) \\ + \frac{1}{2\sqrt{3}} \lambda_u(a_1^S - 3a_{14}^S + a_{18}^S + a_{19}^S + a_2^S - 2a_{20}^S - 3a_{21}^S - 2a_3^S)$
$\Xi_b^- \rightarrow \Sigma^- \pi^0$	$-\frac{1}{2\sqrt{3}} \lambda_c(a_1^{S,c} + a_2^{S,c} - 2a_3^{S,c} - 3a_4^{S,c}) + \frac{1}{2\sqrt{3}} \lambda_t(a_1^{S,p} + 3a_{11}^{S,p} - 3a_{12}^{S,p} - 3a_{13}^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} - 3a_{21}^{S,p} \\ + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} - 9a_{26}^{S,p} - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) - \frac{1}{2\sqrt{3}} \lambda_u(3a_{13}^S + a_{18}^S + a_{19}^S - 2a_{20}^S - 3a_{21}^S)$
$\Xi_b^- \rightarrow \Sigma^- \eta_8$	$-\frac{1}{2} \lambda_c(a_1^{S,c} - a_2^{S,c} + a_4^{S,c}) + \frac{1}{2} \lambda_t(-a_1^{S,p} - a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + a_{18}^{S,p} - a_{19}^{S,p} + a_2^{S,p} \\ + a_{21}^{S,p} + 3a_{23}^{S,p} - 3a_{24}^{S,p} + 3a_{26}^{S,p} - a_4^{S,p} + a_5^{S,p} + a_7^{S,p} - a_8^{S,p}) - \frac{1}{2} \lambda_u(a_{13}^S + a_{18}^S - a_{19}^S + a_{21}^S)$
$\Xi_b^- \rightarrow \Sigma^- \eta_1$	$\frac{1}{\sqrt{2}} \lambda_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c} - 3a_5^{S,c}) + \frac{1}{\sqrt{2}} \lambda_t(-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + 3a_{14}^{S,p} - 3a_{16}^{S,p} + 3a_{17}^{S,p} - a_{18}^{S,p} - a_2^{S,p} \\ + a_{20}^{S,p} + a_{21}^{S,p} + 3a_{22}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p} + 3a_{26}^{S,p} + 9a_{27}^{S,p} + a_3^{S,p} + a_4^{S,p} - a_6^{S,p} - a_7^{S,p} + a_9^{S,p}) - \frac{1}{\sqrt{2}} \lambda_u(a_{13}^S - a_{18}^S + a_{20}^S + a_{21}^S + 3a_{22}^S)$
$\Xi_b^- \rightarrow n K^-$	$\frac{1}{\sqrt{6}} \lambda_c(-2a_1^{S,c} + a_2^{S,c} + a_3^{S,c}) - \frac{1}{\sqrt{6}} \lambda_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_2^{S,p} + a_{20}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} \\ + a_3^{S,p} + 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda_u(-2a_1^S - 2a_{18}^S + a_{19}^S + a_2^S + a_{20}^S + a_3^S)$
$\Xi_b^- \rightarrow \Xi^- K^0$	$\frac{1}{\sqrt{6}} \lambda_c(-a_1^{S,c} + 2a_2^{S,c} - a_3^{S,c} - 3a_4^{S,c}) + \frac{1}{\sqrt{6}} \lambda_t(-2a_1^{S,p} - 3a_{11}^{S,p} + 3a_{12}^{S,p} + 3a_{13}^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{21}^{S,p} + 3a_{23}^{S,p} \\ - 6a_{24}^{S,p} + 3a_{25}^{S,p} + 9a_{26}^{S,p} + a_3^{S,p} - a_4^{S,p} + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) - \frac{1}{\sqrt{6}} \lambda_u(a_{18}^S - 2a_{19}^S + a_{20}^S + 3a_{21}^S)$
$\Xi_b^- \rightarrow \Lambda^0 \pi^-$	$-\frac{1}{2} \lambda_c(a_1^{S,c} - a_2^{S,c} + a_4^{S,c}) - \frac{1}{2} \lambda_t(a_1^{S,p} + a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - a_{18}^{S,p} + a_{19}^{S,p} - a_2^{S,p} - a_{21}^{S,p} - 3a_{23}^{S,p} \\ + 3a_{24}^{S,p} - 3a_{26}^{S,p} + a_4^{S,p} - a_5^{S,p} - a_7^{S,p} + a_8^{S,p}) - \frac{1}{2} \lambda_u(a_1^S + a_{14}^S + a_{18}^S - a_{19}^S - a_2^S + a_{21}^S)$
$\Xi_b^0 \rightarrow \Sigma^+ \pi^-$	$\frac{3}{\sqrt{6}} \lambda_c a_4^{S,c} + \frac{3}{\sqrt{6}} \lambda_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - a_{21}^{S,p} - 3a_{26}^{S,p}) + \frac{1}{\sqrt{6}} \lambda_u(3a_{14}^S + 3a_{21}^S - a_7^S - a_8^S + 2a_9^S)$
$\Xi_b^0 \rightarrow \Sigma^0 \pi^0$	$\frac{1}{2\sqrt{6}} \lambda_c(a_1^{S,c} + a_2^{S,c} - 2a_3^{S,c} + 3a_4^{S,c}) - \frac{1}{2\sqrt{6}} \lambda_t(a_1^{S,p} - 3a_{11}^{S,p} + 3a_{12}^{S,p} + 3a_{13}^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} \\ - 2a_{20}^{S,p} + 3a_{21}^{S,p} + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} + 9a_{26}^{S,p} - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) \\ + \frac{1}{2\sqrt{6}} \lambda_u(a_1^S + a_{10}^S + a_{11}^S - 2a_{12}^S - 3a_{13}^S + a_{18}^S + a_{19}^S + a_2^S - 2a_{20}^S + 3a_{21}^S - 2a_3^S - a_7^S - a_8^S + 2a_9^S)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{6\sqrt{2}} (-3\lambda_c(a_1^{S,c} - a_2^{S,c} + a_4^{S,c})) - \frac{3}{6\sqrt{2}} \lambda_t(a_1^{S,p} + a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - a_{18}^{S,p} + a_{19}^{S,p} \\ - a_2^{S,p} - a_{21}^{S,p} - 3a_{23}^{S,p} + 3a_{24}^{S,p} - 3a_{26}^{S,p} + a_4^{S,p} - a_5^{S,p} - a_7^{S,p} + a_8^{S,p}) + \frac{1}{6\sqrt{2}} \lambda_u(a_1^S + a_{10}^S + a_{11}^S - 2a_{12}^S \\ - 3a_{13}^S - 3a_{18}^S + 3a_{19}^S + a_2^S - 3a_{21}^S - 2a_3^S + 4a_4^S - 2a_5^S - 2a_6^S + a_7^S + a_8^S - 2a_9^S)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_1$	$\frac{3}{6} \lambda_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c} - 3a_5^{S,c}) - \frac{3}{6} \lambda_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - 3a_{14}^{S,p} + 3a_{16}^{S,p} - 3a_{17}^{S,p} + a_{18}^{S,p} + a_2^{S,p} - a_{20}^{S,p} - a_{21}^{S,p} \\ - 3a_{22}^{S,p} + 3a_{23}^{S,p} - 3a_{25}^{S,p} - 3a_{26}^{S,p} - 9a_{27}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) + \frac{1}{6} \lambda_u(a_1^S + a_{10}^S + a_{11}^S - 2a_{12}^S - 3a_{13}^S \\ + 3a_{15}^S + 3a_{16}^S - 6a_{17}^S + 3a_{18}^S + a_2^S - 3a_{20}^S - 3a_{21}^S - 9a_{22}^S - 2a_3^S - 2a_4^S + a_5^S + a_6^S + a_7^S + a_8^S - 2a_9^S)$
$\Xi_b^0 \rightarrow \Sigma^- \pi^+$	$\frac{1}{\sqrt{6}} \lambda_c(a_1^{S,c} + a_2^{S,c} + 2a_3^{S,c}) - \frac{1}{\sqrt{6}} \lambda_t(a_1^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} - 2a_3^{S,p} \\ - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda_u(a_{10}^S + a_{11}^S - 2a_{12}^S + a_{18}^S + a_{19}^S - 2a_{20}^S)$
$\Xi_b^0 \rightarrow p K^-$	$\frac{1}{\sqrt{6}} \lambda_c(-a_1^{S,c} + 2a_2^{S,c} - a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda_t(-2a_1^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{23}^{S,p} - 6a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p} - a_4^{S,p} \\ + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) - \frac{1}{\sqrt{6}} \lambda_u(a_1^S + a_{18}^S - 2a_{19}^S - 2a_2^S + a_{20}^S + a_3^S - a_4^S - a_5^S + 2a_6^S)$
$\Xi_b^0 \rightarrow n \bar{K}^0$	$\frac{1}{\sqrt{6}} \lambda_c(a_1^{S,c} + a_2^{S,c} - 2a_3^{S,c}) - \frac{1}{\sqrt{6}} \lambda_t(a_1^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} \\ - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda_u(a_{18}^S + a_{19}^S - 2a_{20}^S - a_4^S + 2a_5^S - a_6^S)$

Continued on next page

Table 1-continued from previous page

Channel	Amplitude
$\Xi_b^0 \rightarrow \Xi^0 K^0$	$\frac{1}{\sqrt{6}}\lambda_c 3a_4^{S,c} + \frac{3}{\sqrt{6}}(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - a_{21}^{S,p} - 3a_{26}^{S,p}) + \frac{1}{\sqrt{6}}\lambda_u(3a_{21}^S + a_7^S - 2a_8^S + a_9^S)$
$\Xi_b^0 \rightarrow \Xi^- K^+$	$\frac{1}{\sqrt{6}}\lambda_c(-a_1^{S,c} + 2a_2^{S,c} - a_3^{S,c}) + \frac{1}{\sqrt{6}}\lambda_t(-2a_1^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{23}^{S,p} - 6a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p} - a_4^{S,p} + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) - \frac{1}{\sqrt{6}}\lambda_u(a_{10}^S - 2a_{11}^S + a_{12}^S + a_{18}^S - 2a_{19}^S + a_{20}^S)$
$\Xi_b^0 \rightarrow \Lambda^0 \pi^0$	$-\frac{1}{2\sqrt{2}}\lambda_c(a_1^{S,c} - a_2^{S,c} + a_4^{S,c}) - \frac{1}{2\sqrt{2}}\lambda_t(a_1^{S,p} + a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - a_{18}^{S,p} + a_{19}^{S,p} - a_2^{S,p} - a_{21}^{S,p} - 3a_{23}^{S,p} + 3a_{24}^{S,p} - 3a_{26}^{S,p} + a_4^{S,p} - a_5^{S,p} - a_7^{S,p} + a_8^{S,p}) - \frac{1}{2\sqrt{2}}\lambda_u(a_1^S + a_{10}^S - a_{11}^S - a_{13}^S + a_{18}^S - a_{19}^S - a_2^S + a_{21}^S + a_7^S - a_8^S)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_8$	$-\frac{1}{2\sqrt{6}}\lambda_c(a_1^{S,c} - 3a_2^{S,c} + 2a_3^{S,c} - a_4^{S,c}) - \frac{1}{2\sqrt{6}}\lambda_t(3a_1^{S,p} - a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} - a_{18}^{S,p} + 3a_{19}^{S,p} - a_2^{S,p} - 2a_{20}^{S,p} + a_{21}^{S,p} - 3a_{23}^{S,p} + 9a_{24}^{S,p} - 6a_{25}^{S,p} + 3a_{26}^{S,p} - 2a_3^{S,p} + a_4^{S,p} - 3a_5^{S,p} + 2a_6^{S,p} - a_7^{S,p} + 3a_8^{S,p} - 2a_9^{S,p}) - \frac{1}{2\sqrt{6}}\lambda_u(a_1^S + a_{10}^S - a_{11}^S - a_{13}^S + a_{18}^S - 3a_{19}^S - a_2^S + 2a_{20}^S - a_{21}^S - 2a_{23}^S + 2a_{24}^S - a_7^S + a_8^S)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_1$	$-\frac{1}{2\sqrt{3}}\lambda_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c} - 3a_5^{S,c}) - \frac{1}{2\sqrt{3}}\lambda_t(-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + 3a_{14}^{S,p} - 3a_{16}^{S,p} + 3a_{17}^{S,p} - a_{18}^{S,p} - a_2^{S,p} + a_{20}^{S,p} + a_{21}^{S,p} + 3a_{22}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p} + 9a_{26}^{S,p} + a_3^{S,p} + a_4^{S,p} - a_6^{S,p} - a_7^{S,p} + a_9^{S,p}) - \frac{1}{2\sqrt{3}}\lambda_u(a_1^S + a_{10}^S - a_{11}^S - a_{13}^S + 3a_{15}^S - 3a_{16}^S + a_{18}^S - a_2^S - a_{20}^S - a_{21}^S - 3a_{22}^S + a_5^S - a_6^S - a_7^S + a_8^S)$

Table 2. Topological amplitudes of Λ_b^0 decays into $\mathcal{B}_s^S M$ from the $b \rightarrow d$ transition.

Channel	Amplitude
$\Lambda_b^0 \rightarrow \Sigma^0 K^0$	$\frac{1}{2\sqrt{3}}\lambda_c(2a_1^{S,c} - a_2^{S,c} - a_3^{S,c}) + \frac{1}{2\sqrt{3}}\lambda_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} + a_{20}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p} + 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda_u(2a_{18}^S - a_{19}^S - a_{20}^S - 2a_4^S + a_5^S + a_6^S - 2a_7^S + a_8^S + a_9^S)$
$\Lambda_b^0 \rightarrow \Sigma^- K^+$	$\frac{1}{\sqrt{6}}\lambda_c(-2a_1^{S,c} + a_2^{S,c} + a_3^{S,c}) - \frac{1}{\sqrt{6}}\lambda_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p} + 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{\sqrt{6}}\lambda_u(-2a_{10}^S + a_{11}^S + a_{12}^S - 2a_{18}^S + a_{19}^S + a_{20}^S)$
$\Lambda_b^0 \rightarrow p \pi^-$	$\frac{1}{\sqrt{6}}\lambda_c(-a_1^{S,c} + 2a_2^{S,c} - a_3^{S,c} - 3a_4^{S,c}) + \frac{1}{\sqrt{6}}\lambda_t(-2a_1^{S,p} - 3a_{11}^{S,p} + 3a_{12}^{S,p} + 3a_{13}^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{21}^{S,p} + 3a_{23}^{S,p} - 6a_{24}^{S,p} + 3a_{25}^{S,p} + 9a_{26}^{S,p} + a_3^{S,p} - a_4^{S,p} + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) - \frac{1}{\sqrt{6}}\lambda_u((a_1^S + 3a_{14}^S + a_{18}^S - 2a_{19}^S - 2a_2^S + a_{20}^S + 3a_{21}^S + a_3^S - a_4^S - a_5^S + 2a_6^S - a_7^S - a_8^S + 2a_9^S)$
$\Lambda_b^0 \rightarrow n \pi^0$	$\frac{1}{2\sqrt{3}}\lambda_c(a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c} + 3a_4^{S,c}) + \frac{1}{2\sqrt{3}}\lambda_t(2a_1^{S,p} + 3a_{11}^{S,p} - 3a_{12}^{S,p} - 3a_{13}^{S,p} - a_{18}^{S,p} + 2a_{19}^{S,p} - a_2^{S,p} - a_{20}^{S,p} - 3a_{21}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - 9a_{26}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p} - a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda_u(2a_1^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S + a_{18}^S - 2a_{19}^S - a_2^S + a_{20}^S + 3a_{21}^S - a_3^S + a_4^S - 2a_5^S + a_6^S + a_7^S - 2a_8^S + a_9^S)$
$\Lambda_b^0 \rightarrow n \eta_8$	$\frac{1}{2}(a_1^{S,c} - a_3^{S,c} - a_4^{S,c}) - \frac{1}{2}\lambda_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} + a_{18}^{S,p} + a_2^{S,p} - a_{20}^{S,p} - a_{21}^{S,p} + 3a_{23}^{S,p} - 3a_{25}^{S,p} - 3a_{26}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) + \frac{1}{2}\lambda_u(2a_1^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S + 3a_{18}^S - a_2^S - 3a_{20}^S - 3a_{21}^S - a_3^S - a_4^S + 2a_5^S - a_6^S - a_7^S + 2a_8^S - a_9^S)$
$\Lambda_b^0 \rightarrow n \eta_1$	$\frac{1}{2}\lambda_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c}) - \frac{1}{2}\lambda_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} + a_{18}^{S,p} + a_2^{S,p} - a_{20}^{S,p} - a_{21}^{S,p} + 3a_{23}^{S,p} - 3a_{25}^{S,p} - 3a_{26}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) + \frac{1}{6}\lambda_u(2a_1^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S + 3a_{18}^S - a_2^S - 3a_{20}^S - 3a_{21}^S - a_3^S - a_4^S + 2a_5^S - a_6^S - a_7^S + 2a_8^S - a_9^S)$
$\Lambda_b^0 \rightarrow \Lambda^0 K^0$	$-\frac{1}{2}\lambda_c(a_2^{S,c} - a_3^{S,c} - 2a_4^{S,c}) - \frac{1}{2}\lambda_t(-a_1^{S,p} - 2a_{11}^{S,p} + 2a_{12}^{S,p} + 2a_{13}^{S,p} - a_{19}^{S,p} + a_{20}^{S,p} + 2a_{21}^{S,p} - 3a_{24}^{S,p} + 3a_{25}^{S,p} + 6a_{26}^{S,p} + a_3^{S,p} + a_5^{S,p} - a_6^{S,p} - a_8^{S,p} + a_9^{S,p}) - \frac{1}{2}\lambda_u(a_{19}^S - a_{20}^S - 2a_{21}^S + a_5^S - a_6^S + a_8^S - a_9^S)$

Table 3. Topological amplitudes of $\Xi_b^{-,0}$ decays into $\mathcal{B}_8^s M$ from the $b \rightarrow s$ transition.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 K^-$	$-\frac{1}{2\sqrt{3}}\lambda'_c(a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c} + 3a_4^{S,c}) - \frac{1}{2\sqrt{3}}\lambda'_t(2a_1^{S,p} + 3a_{11}^{S,p} - 3a_{12}^{S,p} - 3a_{13}^{S,p} - 3a_{18}^{S,p}$ $+ 2a_{19}^{S,p} - a_2^{S,p} - a_{20}^{S,p} - 3a_{21}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - 9a_{26}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p}$ $- a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda'_u(-a_1^S - 3a_{14}^S - a_{18}^S + 2a_{19}^S + 2a_{20}^S - a_{21}^S - 3a_{25}^S)$
$\Xi_b^- \rightarrow \Sigma^- \bar{K}^0$	$-\frac{1}{\sqrt{6}}\lambda'_c(a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c} + 3a_4^{S,c}) + \frac{1}{\sqrt{6}}\lambda'_t(-2a_1^{S,p} - 3a_{11}^{S,p} + 3a_{12}^{S,p} + 3a_{13}^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{21}^{S,p}$ $+ 3a_{23}^{S,p} - 6a_{24}^{S,p} + 3a_{25}^{S,p} + 9a_{26}^{S,p} + a_3^{S,p} - a_4^{S,p} + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) + \frac{1}{\sqrt{6}}\lambda'_u(-a_{18}^S + 2a_{19}^S - a_{20}^S - 3a_{21}^S)$
$\Xi_b^- \rightarrow \Xi^0 \pi^-$	$\frac{1}{\sqrt{6}}\lambda'_c(-2a_1^{S,c} + a_2^{S,c} + a_3^{S,c}) - \frac{1}{\sqrt{6}}\lambda'_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} + a_{21}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p}$ $+ 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{\sqrt{6}}\lambda'_u(-2a_1^S - 2a_{18}^S + a_{19}^S + a_2^S + a_{20}^S + a_3^S)$
$\Xi_b^- \rightarrow \Xi^- \pi^0$	$\frac{1}{2\sqrt{3}}\lambda'_c(-2a_1^{S,c} + a_2^{S,c} + a_3^{S,c}) - \frac{1}{2\sqrt{3}}\lambda'_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} + a_{21}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p}$ $+ 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda'_u(-3a_{13}^S - 2a_{18}^S + a_{19}^S + a_{20}^S)$
$\Xi_b^- \rightarrow \Xi^- \eta_8$	$\frac{1}{6}\lambda'_c(-3a_2^{S,c} + 3a_3^{S,c} + 6a_4^{S,c}) + \frac{1}{6}\lambda'_t(a_1^{S,p} + 2a_{11}^{S,p} - 2a_{12}^{S,p} - 2a_{13}^{S,p} + a_{19}^{S,p} - a_{20}^{S,p} - 2a_{21}^{S,p} + 3a_{24}^{S,p}$ $- 3a_{25}^{S,p} - 6a_{26}^{S,p} - a_3^{S,p} - a_5^{S,p} + a_6^{S,p} + a_8^{S,p} - a_9^{S,p}) + \frac{1}{6}\lambda'_u(3a_{13}^S - 3a_{19}^S + 3a_{20}^S + 6a_{21}^S)$
$\Xi_b^- \rightarrow \Xi^- \eta_1$	$\frac{1}{\sqrt{2}}\lambda'_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c} - 3a_5^{S,c}) + \frac{1}{3\sqrt{2}}\lambda'_t(-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + 3a_{14}^{S,p} - 3a_{16}^{S,p} + 3a_{17}^{S,p} - a_{18}^{S,p} - a_2^{S,p} + a_{20}^{S,p} + a_{21}^{S,p}$ $+ 3a_{22}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p} + 3a_{26}^{S,p} + 9a_{27}^{S,p} + a_3^{S,p} + a_4^{S,p} - a_6^{S,p} - a_7^{S,p} + a_9^{S,p}) + \frac{1}{3\sqrt{2}}\lambda'_u(-3a_{13}^S + 3a_{18}^S - 3a_{20}^S - 3a_{21}^S - 9a_{22}^S)$
$\Xi_b^- \rightarrow \Lambda^0 K^-$	$-\frac{1}{2}\lambda'_c(-a_1^{S,c} + a_3^{S,c} + a_4^{S,c}) - \frac{1}{2}\lambda'_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} + a_{18}^{S,p} + a_2^{S,p} - a_{20}^{S,p} - a_{21}^{S,p} + 3a_{23}^{S,p} - 3a_{25}^{S,p}$ $- 3a_{26}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) - \frac{1}{2}\lambda'_u(-a_1^S + a_{14}^S - a_{18}^S + a_{20}^S + a_{21}^S + a_3^S)$
$\Xi_b^0 \rightarrow \Sigma^+ K^-$	$\frac{1}{\sqrt{6}}\lambda'_c(a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c} + 3a_4^{S,c}) + \frac{1}{\sqrt{6}}\lambda'_t(2a_1^{S,p} + 3a_{11}^{S,p} - 3a_{12}^{S,p} - 3a_{13}^{S,p} - a_{18}^{S,p} + 2a_{19}^{S,p}$ $- a_2^{S,p} - a_{20}^{S,p} - 3a_{21}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - 9a_{26}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p} - a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p})$ $+ \frac{1}{\sqrt{6}}\lambda'_u(a_1^S + 3a_{14}^S + a_{18}^S - 2a_{19}^S - 2a_{20}^S + 3a_{21}^S + a_3^S - a_4^S - a_5^S + 2a_6^S - a_7^S - a_8^S + 2a_9^S)$
$\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0$	$-\frac{1}{2\sqrt{3}}\lambda'_c(a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c} + 3a_4^{S,c}) + \frac{1}{2\sqrt{3}}\lambda'_t(-2a_1^{S,p} - 3a_{11}^{S,p} + 3a_{12}^{S,p} + 3a_{13}^{S,p} + a_{18}^{S,p} - 2a_{19}^{S,p} + a_2^{S,p} + a_{20}^{S,p} + 3a_{21}^{S,p} + 3a_{23}^{S,p} - 6a_{24}^{S,p} + 3a_{25}^{S,p}$ $+ 9a_{26}^{S,p} + a_3^{S,p} - a_4^{S,p} + 2a_5^{S,p} - a_6^{S,p} + a_7^{S,p} - 2a_8^{S,p} + a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda'_u(-a_{18}^S + 2a_{19}^S - a_{20}^S - 3a_{21}^S + a_4^S + a_5^S - 2a_6^S + a_7^S + a_8^S - 2a_9^S)$
$\Xi_b^0 \rightarrow \Xi^0 \pi^0$	$\frac{1}{2\sqrt{3}}\lambda'_c(-2a_1^{S,c} + a_2^{S,c} + a_3^{S,c}) - \frac{1}{2\sqrt{3}}\lambda'_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} + a_{21}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p} + 2a_4^{S,p}$ $- a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{2\sqrt{3}}\lambda'_u(-2a_1^S - 2a_{10}^S + a_{11}^S + a_{12}^S + 3a_{13}^S - 2a_{18}^S + a_{19}^S + a_2^S + a_{20}^S + a_3^S)$
$\Xi_b^0 \rightarrow \Xi^0 \eta_8$	$\frac{1}{2}\lambda'_c(a_2^{S,c} - a_3^{S,c} - 2a_4^{S,c}) + \frac{1}{2}\lambda'_t(a_1^{S,p} + 2a_{11}^{S,p} - 2a_{12}^{S,p} - 2a_{13}^{S,p} + a_{19}^{S,p} - a_{20}^{S,p} - 2a_{21}^{S,p} + 3a_{24}^{S,p} - 3a_{25}^{S,p} - 6a_{26}^{S,p} - a_3^{S,p} - a_5^{S,p} + a_6^{S,p}$ $+ a_8^{S,p} - a_9^{S,p}) + \frac{1}{6}\lambda'_u(-2a_1^S - 2a_{10}^S + a_{11}^S + a_{12}^S + 3a_{13}^S + 3a_{19}^S + a_2^S - 3a_{20}^S - 6a_{21}^S + a_3^S - 2a_4^S + 4a_5^S - 2a_6^S - 2a_7^S + 4a_8^S - 2a_9^S)$
$\Xi_b^0 \rightarrow \Xi^0 \eta_1$	$+\frac{1}{\sqrt{2}}\lambda'_c(-a_1^{S,c} + a_3^{S,c} + a_4^{S,c} + 3a_5^{S,c}) + \frac{1}{\sqrt{2}}\lambda'_t(a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - 3a_{14}^{S,p} + 3a_{16}^{S,p} - 3a_{17}^{S,p} + a_{18}^{S,p} + a_{20}^{S,p} - a_{21}^{S,p}$ $- a_{22}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p} - 3a_{26}^{S,p} - 9a_{27}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) + \frac{1}{3\sqrt{2}}\lambda'_u(-2a_1^S - 2a_{10}^S + a_{11}^S$ $+ a_{12}^S + 3a_{13}^S - 6a_{15}^S + 3a_{16}^S + 3a_{17}^S - 3a_{18}^S + a_2^S + 3a_{20}^S + 3a_{21}^S + 9a_{22}^S + a_3^S + a_4^S - 2a_5^S + a_6^S + a_7^S - 2a_8^S + a_9^S)$
$\Xi_b^0 \rightarrow \Xi^- \pi^+$	$\frac{1}{\sqrt{6}}\lambda'_c(2a_1^{S,c} - a_2^{S,c} - a_3^{S,c}) + \frac{1}{\sqrt{6}}\lambda'_t(a_1^{S,p} - 2a_{18}^{S,p} + a_{19}^{S,p} - 2a_{20}^{S,p} + a_{21}^{S,p} - 6a_{23}^{S,p} + 3a_{24}^{S,p} + 3a_{25}^{S,p} + a_3^{S,p}$ $+ 2a_4^{S,p} - a_5^{S,p} - a_6^{S,p} - 2a_7^{S,p} + a_8^{S,p} + a_9^{S,p}) + \frac{1}{\sqrt{6}}\lambda'_u(2a_{10}^{S,p} - a_{11}^{S,p} - a_{12}^{S,p} + 2a_{18}^{S,p} - a_{19}^{S,p} - a_{20}^{S,p})$
$\Xi_b^0 \rightarrow \Lambda^0 \bar{K}^0$	$-\frac{1}{2}\lambda'_c(a_1^{S,c} - a_3^{S,c} - a_4^{S,c}) - \frac{1}{2}\lambda'_t(-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} - a_{18}^{S,p} - a_2^{S,p} + a_{20}^{S,p} + a_{21}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p}$ $+ 3a_{26}^{S,p} + a_3^{S,p} + a_4^{S,p} - a_6^{S,p} - a_7^{S,p} + a_9^{S,p}) - \frac{1}{2}\lambda'_u(a_{18}^S - a_{20}^S - a_{21}^S - a_4^S + a_5^S - a_7^S + a_8^S)$

Table 4. Topological amplitudes of Λ_b^0 decays into $\mathcal{B}_8^S M$ from the $b \rightarrow s$ transition.

Channel	Amplitude
$\Lambda_b^0 \rightarrow \Sigma^+ \pi^-$	$\frac{1}{\sqrt{6}} \lambda'_c (a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda'_t (2a_1^{S,p} - a_{18}^{S,p} + 2a_{19}^{S,p} - a_2^{S,p} - a_{20}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p} - a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (a_1^S + a_{18}^S - 2a_{19}^S - 2a_2^S + a_{20}^S + a_3^S - a_4^S - a_5^S + 2a_6^S)$
$\Lambda_b^0 \rightarrow \Sigma^0 \pi^0$	$\frac{1}{\sqrt{6}} \lambda'_c (a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda'_t (2a_1^{S,p} - a_{18}^{S,p} + 2a_{19}^{S,p} - a_2^{S,p} - a_{20}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p} - a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p}) + \frac{1}{2\sqrt{6}} \lambda'_u (a_1^S + a_{10}^S - 2a_{11}^S + a_{12}^S + 2a_{18}^S - 4a_{19}^S - 2a_2^S + 2a_{20}^S + a_3^S - a_4^S - a_5^S + 2a_6^S)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{6\sqrt{2}} \lambda'_u (a_1^S + a_{10}^S - 2a_{11}^S + a_{12}^S - 2a_2^S + a_3^S + a_4^S + a_5^S - 2a_6^S + 4a_7^S - 2a_8^S - 2a_9^S)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_1$	$\frac{1}{6} \lambda'_u (a_1^S + a_{10}^S - 2a_{11}^S + a_{12}^S + 3a_{15}^S - 6a_{16}^S + 3a_{17}^S - 2a_2^S + a_3^S + a_4^S + a_5^S - 2a_6^S - 2a_7^S + a_8^S + a_9^S)$
$\Lambda_b^0 \rightarrow \Sigma^- \pi^+$	$\frac{1}{\sqrt{6}} \lambda'_c (a_1^{S,c} - 2a_2^{S,c} + a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda'_t (2a_1^{S,p} - a_{18}^{S,p} + 2a_{19}^{S,p} - a_2^{S,p} - a_{20}^{S,p} - 3a_{23}^{S,p} + 6a_{24}^{S,p} - 3a_{25}^{S,p} - a_3^{S,p} + a_4^{S,p} - 2a_5^{S,p} + a_6^{S,p} - a_7^{S,p} + 2a_8^{S,p} - a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (a_{10}^S - 2a_{11}^S + a_{12}^S + a_{18}^S - 2a_{19}^S + a_{20}^S)$
$\Lambda_b^0 \rightarrow p K^-$	$-\frac{3}{\sqrt{6}} \lambda'_c a_4^{S,c} + \frac{3}{\sqrt{6}} \lambda'_t (-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + a_{21}^{S,p} + 3a_{26}^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (-3a_{14}^S - 3a_{21}^S + a_7^S + a_8^S - 2a_9^S)$
$\Lambda_b^0 \rightarrow n \bar{K}^0$	$-\frac{3}{\sqrt{6}} \lambda'_c a_4^{S,c} + \frac{3}{\sqrt{6}} \lambda'_t (-a_{11}^{S,p} + a_{12}^{S,p} + a_{13}^{S,p} + a_{21}^{S,p} + 3a_{26}^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (-3a_{21}^S - a_7^S + 2a_8^S - a_9^S)$
$\Lambda_b^0 \rightarrow \Xi^0 K^0$	$-\frac{1}{\sqrt{6}} \lambda'_c (a_1^{S,c} + a_2^{S,c} - 2a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda'_t (a_1^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (-a_{18}^S - a_{19}^S + 2a_{20}^S + a_4^S - 2a_5^S + a_6^S)$
$\Lambda_b^0 \rightarrow \Xi^- K^+$	$\frac{1}{\sqrt{6}} \lambda'_c (-a_1^{S,c} - a_2^{S,c} + 2a_3^{S,c}) + \frac{1}{\sqrt{6}} \lambda'_t (a_1^{S,p} + a_{18}^{S,p} + a_{19}^{S,p} + a_2^{S,p} - 2a_{20}^{S,p} + 3a_{23}^{S,p} + 3a_{24}^{S,p} - 6a_{25}^{S,p} - 2a_3^{S,p} - a_4^{S,p} - a_5^{S,p} + 2a_6^{S,p} + a_7^{S,p} + a_8^{S,p} - 2a_9^{S,p}) + \frac{1}{\sqrt{6}} \lambda'_u (-a_{10}^S - a_{11}^S + 2a_{12}^S - a_{18}^S - a_{19}^S + 2a_{20}^S)$
$\Lambda_b^0 \rightarrow \Lambda^0 \pi^0$	$-\frac{1}{2\sqrt{2}} \lambda'_u (a_1^S + a_{10}^S - a_{12}^S - 2a_{13}^S - a_3^S + a_4^S - a_5^S)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_8$	$-\frac{1}{\sqrt{6}} \lambda'_c (a_1^{S,c} - a_3^{S,c} + 2a_4^{S,c}) - \frac{1}{\sqrt{6}} \lambda'_t (2a_{11}^{S,p} - 2a_{12}^{S,p} - 2a_{13}^{S,p} - a_{18}^{S,p} - a_2^{S,p} + a_{20}^{S,p} - 2a_{21}^{S,p} - 3a_{23}^{S,p} + 3a_{25}^{S,p} - 6a_{26}^{S,p} + a_3^{S,p} + a_4^{S,p} - a_6^{S,p} - a_7^{S,p} + a_9^{S,p}) - \frac{1}{2\sqrt{6}} \lambda'_u (a_1^S + a_{10}^S - a_{12}^S - 2a_{13}^S + 2a_{18}^S - 2a_{20}^S + 4a_{21}^S - a_3^S - a_4^S + a_5^S - 2a_8^S + 2a_9^S)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_1$	$-\frac{1}{\sqrt{3}} \lambda'_c (a_1^{S,c} - a_3^{S,c} - a_4^{S,c} - 3a_5^{S,c}) + \frac{1}{\sqrt{3}} \lambda'_t (a_{11}^{S,p} - a_{12}^{S,p} - a_{13}^{S,p} - 3a_{14}^{S,p} + 3a_{16}^{S,p} - 3a_{17}^{S,p} + a_{18}^{S,p} + a_2^{S,p} - a_{20}^{S,p} - a_{21}^{S,p} - 3a_{22}^{S,p} + 3a_{23}^{S,p} - 3a_{25}^{S,p} - 3a_{26}^{S,p} - 9a_{27}^{S,p} - a_3^{S,p} - a_4^{S,p} + a_6^{S,p} + a_7^{S,p} - a_9^{S,p}) - \frac{1}{2\sqrt{3}} \lambda'_u (a_1^S + a_{10}^S - a_{12}^S - 2a_{13}^S + 3a_{15}^S - 3a_{17}^S + 2a_{18}^S - 2a_{20}^S - 2a_{21}^S - 6a_{22}^S - a_3^S - a_4^S + a_5^S + a_8^S - a_9^S)$

Table 5. Topological amplitudes of Ξ_b^{-0} decays into $\mathcal{B}_8^A M$ from the $b \rightarrow d$ transition.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 \pi^-$	$-\frac{1}{2} \lambda_c (a_1^{A,c} + a_2^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{2} \lambda_t (a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) - \frac{1}{2} \lambda_u (a_1^A - a_{14}^A + a_{18}^A + a_{19}^A + a_2^A - a_{21}^A - 2a_{28}^A - 2a_{30}^A)$
$\Xi_b^- \rightarrow \Sigma^- \pi^0$	$\frac{1}{2} \lambda_c (a_1^{A,c} + a_2^{A,c} - a_4^{A,c} - 2a_6^{A,c}) - \frac{1}{2} \lambda_t (a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) + \frac{1}{2} \lambda_u (a_{13}^A + a_{18}^A + a_{19}^A - a_{21}^A + 2a_{29}^A - 2a_{30}^A)$
$\Xi_b^- \rightarrow \Sigma^- \eta_8$	$\frac{1}{2\sqrt{3}} \lambda_c (-a_1^{A,c} + a_2^{A,c} + 2a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) - \frac{1}{2\sqrt{3}} \lambda_t (a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p} + a_{19}^{A,p} - a_2^{A,p} + 2a_{20}^{A,p} + a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{24}^{A,p} + 6a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - 2a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p} - a_5^{A,p} - 2a_6^{A,p} - a_7^{A,p} + a_8^{A,p} + 2a_9^{A,p}) + \frac{1}{2\sqrt{3}} \lambda_u (a_{13}^A - a_{18}^A + a_{19}^A + 2a_{20}^A + a_{21}^A + 2a_{29}^A + 2a_{30}^A)$
$\Xi_b^- \rightarrow \Sigma^- \eta_1$	$\frac{1}{\sqrt{6}} \lambda_c (-a_1^{A,c} - 2a_2^{A,c} - a_3^{A,c} + a_4^{A,c} + 3a_5^{A,c} + 2a_6^{A,c} + 6a_7^{A,c}) + \frac{1}{\sqrt{6}} \lambda_t (2a_{10}^{A,p} + 2a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} - 3a_{14}^{A,p} + 6a_{15}^{A,p} + 3a_{16}^{A,p} - a_{17}^{A,p} - 2a_{18}^{A,p} + 2a_{19}^{A,p} - a_{20}^{A,p} - a_{21}^{A,p} - 3a_{22}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} - 3a_{26}^{A,p} - 9a_{27}^{A,p} - a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) - \frac{1}{2\sqrt{3}} \lambda'_u (a_1^S + a_{10}^S - a_{12}^S - 2a_{13}^S + 3a_{15}^S - 3a_{17}^S + 2a_{18}^S - 2a_{20}^S - 2a_{21}^S - 6a_{22}^S - a_3^S - a_4^S + a_5^S + a_8^S - a_9^S)$

Continued on next page

Table 5-continued from previous page

Channel	Amplitude
	$-3a_{17}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} - 3a_{22}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 9a_{27}^{A,p} - 6a_{28}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p}$ $- 6a_{31}^{A,p} - 6a_{32}^{S,p} - 18a_{33}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) + \frac{1}{\sqrt{6}}\lambda_u(a_{13}^A - a_{18}^A - 2a_{19}^A - a_{20}^A + a_{21}^A + 3a_{22}^A + 2a_{29}^A + 2a_{30}^A + 6a_{31}^A)$
$\Xi_b^- \rightarrow nK^-$	$\frac{1}{\sqrt{2}}\lambda_c(a_2^{A,c} + a_3^{A,c}) - \frac{1}{\sqrt{2}}\lambda_t(a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p}) + \frac{1}{\sqrt{2}}\lambda_u(a_{19}^A + a_2^A + a_{20}^A + a_3^A)$
$\Xi_b^- \rightarrow \Xi^- K^0$	$\frac{1}{\sqrt{2}}\lambda_c(-a_1^{A,c} + a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p}$ $- 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} + a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) + \frac{1}{\sqrt{2}}\lambda_u(-a_{18}^A + a_{20}^A + a_{21}^A + 2a_{30}^A)$
$\Xi_b^- \rightarrow \Lambda^0 \pi^-$	$\frac{1}{2\sqrt{3}}\lambda_c(-a_1^{A,c} - a_2^{A,c} + 2a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) - \frac{1}{2\sqrt{3}}\lambda_t(a_1^{A,p} - a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p} + a_{19}^{A,p}$ $- a_2^{A,p} + 2a_{20}^{A,p} + a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{24}^{A,p} + 6a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - 2a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p} - a_5^{A,p}$ $- 2a_6^{A,p} - a_7^{A,p} + a_8^{A,p} + 2a_9^{A,p}) + \frac{1}{2\sqrt{3}}\lambda_u(-a_1^A + a_{14}^A - a_{18}^A + a_{19}^A + a_2^A + 2a_{20}^A + a_{21}^A + 2a_{28}^A + 2a_3^A + 2a_{30}^A)$
$\Xi_b^0 \rightarrow \Sigma^+ \pi^-$	$- \frac{1}{\sqrt{2}}\lambda_c(a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(-2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} + a_{21}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p})$ $- \frac{1}{\sqrt{2}}\lambda_u(a_{14}^A + a_{21}^A + 2a_{28}^A + 2a_{30}^A - a_7^A - a_8^A)$
$\Xi_b^0 \rightarrow \Sigma^0 \pi^0$	$- \frac{1}{2\sqrt{2}}\lambda_c(a_1^{A,c} + a_2^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{2\sqrt{2}}(a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p}$ $+ a_{21}^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) - \frac{1}{2\sqrt{2}}\lambda_u(a_1^A + a_{10}^A$ $+ a_{11}^A - a_{13}^A + a_{18}^A + a_{19}^A + a_2^A + a_{21}^A - 2a_{29}^A + 2a_{30}^A - a_7^A - a_8^A)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{2\sqrt{6}}\lambda_c(-a_1^{A,c} + a_2^{A,c} + 2a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) - \frac{1}{2\sqrt{6}}\lambda_t(a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p}$ $+ a_{19}^{A,p} - a_2^{A,p} + 2a_{20}^{A,p} + a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{24}^{A,p} + 6a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - 2a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p}$ $- a_5^{A,p} - 2a_6^{A,p} - a_7^{A,p} + a_8^{A,p} + 2a_9^{A,p}) - \frac{1}{2\sqrt{6}}\lambda_u(a_1^A + a_{10}^A + a_{11}^A - a_{13}^A + a_{18}^A - a_{19}^A + a_2^A$ $- 2a_{20}^A - a_{21}^A - 2a_{29}^A - 2a_{30}^A + 2a_3^A + 2a_6^A + a_7^A + a_8^A)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_1$	$- \frac{1}{2\sqrt{3}}\lambda_c(a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 3a_5^{A,c} - 2a_6^{A,c} - 6a_7^{A,c}) - \frac{1}{2\sqrt{3}}\lambda_t(-2a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p}$ $+ 3a_{14}^{A,p} - 6a_{15}^{A,p} - 3a_{16}^{A,p} + 3a_{17}^{A,p} - a_{18}^{A,p} - 2a_{19}^{A,p} - a_2^{A,p} - a_{20}^{A,p} + a_{21}^{A,p} + 3a_{22}^{A,p} - 3a_{23}^{A,p} - 6a_{24}^{A,p} - 3a_{25}^{A,p} + 3a_{26}^{A,p} + 9a_{27}^{A,p}$ $+ 6a_{28}^{A,p} + 2a_{29}^{A,p} + a_3^{A,p} + 2a_{30}^{A,p} + 6a_{31}^{A,p} + 6a_{32}^{A,p} + 18a_{33}^{A,p} + a_4^{A,p} + 2a_5^{A,p} + a_6^{A,p} - a_7^{A,p} - 2a_8^{A,p} - a_9^{A,p}) - \frac{1}{2\sqrt{3}}\lambda_u(a_1^A + a_{10}^A$ $+ a_{11}^A - a_{13}^A + 3a_{15}^A + 3a_{16}^A + a_{18}^A + 2a_{19}^A + a_2^A + a_{20}^A - a_{21}^A - 3a_{22}^A - 2a_{29}^A - 2a_{30}^A - 6a_{31}^A - a_3^A - a_6^A + a_7^A + a_8^A)$
$\Xi_b^0 \rightarrow \Sigma^- \pi^+$	$- \frac{1}{\sqrt{2}}\lambda_c(a_1^{A,c} + a_2^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(a_1^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) - \frac{1}{\sqrt{2}}\lambda_u(a_{10}^A + a_{11}^A + a_{18}^A - a_{19}^A)$
$\Xi_b^0 \rightarrow pK^-$	$\frac{1}{\sqrt{2}}\lambda_c(-a_1^{A,c} + a_3^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} + a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p})$ $- \frac{1}{\sqrt{2}}\lambda_u(a_1^A + a_{18}^A - a_{20}^A - a_3^A + a_4^A + a_5^A)$
$\Xi_b^0 \rightarrow n\bar{K}^0$	$\frac{1}{\sqrt{2}}\lambda_c(-a_1^{A,c} - a_2^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(a_1^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) - \frac{1}{\sqrt{2}}\lambda_u(a_{18}^A + a_{19}^A + a_4^A - a_6^A)$
$\Xi_b^0 \rightarrow \Xi^0 K^0$	$\frac{1}{\sqrt{2}}\lambda_c(-a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(-2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} + a_{21}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p}) - \frac{1}{\sqrt{2}}\lambda_u(a_{21}^A + 2a_{30}^A - a_7^A + a_9^A)$
$\Xi_b^0 \rightarrow \Xi^- K^+$	$\frac{1}{\sqrt{2}}\lambda_c(-a_1^{A,c} + a_3^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} + a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) + \frac{1}{\sqrt{2}}\lambda_u(-a_{10}^A + a_{12}^A - a_{18}^A + a_{20}^A)$
$\Xi_b^0 \rightarrow \Lambda^0 \pi^0$	$\frac{1}{2\sqrt{6}}\lambda_c(-a_1^{A,c} + a_2^{A,c} + 2a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) - \frac{1}{2\sqrt{6}}\lambda_t(a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p} + a_{19}^{A,p} - a_2^{A,p} + 2a_{20}^{A,p}$ $+ a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{24}^{A,p} + 6a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - 2a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p} - a_5^{A,p} - 2a_6^{A,p} - a_7^{A,p} + a_8^{A,p} + 2a_9^{A,p})$ $+ \frac{1}{2\sqrt{6}}\lambda_u(-a_1^A - a_{10}^A + a_{11}^A + 2a_{12}^A - a_{13}^A - a_{18}^A + a_{19}^A + a_2^A + 2a_{20}^A + a_{21}^A - 2a_{29}^A + 2a_3^A + 2a_{30}^A - a_7^A + a_8^A + 2a_9^A)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_8$	$\frac{1}{6\sqrt{2}}\lambda_c(-5a_1^{A,c} - a_2^{A,c} + 4a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{6\sqrt{2}}\lambda_t(a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} + 5a_{18}^{A,p}$ $+ a_{19}^{A,p} + 5a_{22}^{A,p} - 4a_{20}^{A,p} + a_{21}^{A,p} + 15a_{23}^{A,p} + 3a_{24}^{A,p} - 12a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} + 4a_3^{A,p} + 2a_{30}^{A,p}$

Table 5-continued from previous page

Channel	Amplitude
$\Xi_b^0 \rightarrow \Lambda^0 \eta_1$	$+6a_{32}^{A,p} - 5a_4^{A,p} - a_5^{A,p} + 4a_6^{A,p} + 5a_7^{A,p} + a_8^{A,p} - 4a_9^{A,p} - \frac{1}{6\sqrt{2}}\lambda_u(a_1^A + a_{10}^A - a_{11}^A - 2a_{12}^A + a_{13}^A + 5a_{18}^A$ $+ a_{19}^A - a_2^A - 4a_{20}^A + a_{21}^A + 2a_{29}^A - 2a_3^A + 2a_{30}^A + 4a_4^A + 2a_5^A - 2a_6^A - a_7^A + a_8^A + 2a_9^A)$ $+ \frac{1}{6}\lambda_c(a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 3a_5^{A,c} - 2a_6^{A,c} - 6a_7^{A,c}) + \frac{1}{6}\lambda_t(-2a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p}$ $+ 3a_{14}^{A,p} - 6a_{15}^{A,p} - 3a_{16}^{A,p} + 3a_{17}^{A,p} - a_{18}^{A,p} - 2a_{19}^{A,p} - a_2^{A,p} - a_{20}^{A,p} + a_{21}^{A,p} + 3a_{22}^{A,p} - 3a_{23}^{A,p} - 6a_{24}^{A,p} - 3a_{25}^{A,p}$ $+ 3a_{26}^{A,p} + 9a_{27}^{A,p} + 6a_{28}^{A,p} + 2a_{29}^{A,p} + a_3^{A,p} + 2a_{30}^{A,p} + 6a_{31}^{A,p} + 6a_{32}^{A,p} + 18a_{33}^{A,p} + a_4^{A,p} + 2a_5^{A,p} + a_6^{A,p} - a_7^{A,p} - 2a_8^{A,p}$ $- a_9^{A,p}) + \frac{1}{6}\lambda_u(-a_1^A - a_{10}^A + a_{11}^A + 2a_{12}^A - a_{13}^A - 3a_{15}^A + 3a_{16}^A + 6a_{17}^A + a_{18}^A + 2a_{19}^A + a_2^A$ $+ a_{20}^A - a_{21}^A - 3a_{22}^A - 2a_{29}^A + 2a_3^A - 2a_{30}^A - 6a_{31}^A + 2a_4^A + a_5^A - a_6^A + a_7^A - a_8^A - 2a_9^A)$

Table 6. Topological amplitudes of Λ_b^0 decays into $\mathcal{B}_8^A M$ from the $b \rightarrow d$ transition.

Channel	Amplitude
$\Lambda_b^0 \rightarrow \Sigma^0 K^0$	$\frac{1}{2}\lambda_c(-a_2^{A,c} - a_3^{A,c}) + \frac{1}{2}\lambda_t(a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p})$ $+ \frac{1}{2}\lambda_u(-a_{19}^A - a_{20}^A + a_5^A + a_6^A + a_8^A + a_9^A)$
$\Lambda_b^0 \rightarrow \Sigma^- K^+$	$\frac{1}{\sqrt{2}}\lambda_c(a_2^{A,c} + a_3^{A,c}) - \frac{1}{\sqrt{2}}\lambda_t(a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p}$ $+ a_9^{A,p}) + \frac{1}{\sqrt{2}}\lambda_u(a_{11}^A + a_{12}^A + a_{19}^A + a_{20}^A)$
$\Lambda_b^0 \rightarrow p \pi^-$	$\frac{1}{\sqrt{2}}\lambda_c(-a_1^{A,c} + a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}}\lambda_t(2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_2^{A,p}$ $- a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} + a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p})$ $- \frac{1}{\sqrt{2}}\lambda_u(a_1^A - a_{14}^A + a_{18}^A - a_{20}^A - a_{21}^A - 2a_{28}^A - a_3^A - 2a_{30}^A + a_4^A + a_5^A + a_7^A + a_8^A)$
$\Lambda_b^0 \rightarrow n \pi^0$	$\frac{1}{2}\lambda_c(a_1^{A,c} - a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{2}\lambda_t(-2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p} - a_2^{A,p} + a_{20}^{A,p}$ $+ a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p} - a_6^{A,p} - a_7^{A,p} + a_9^{A,p})$ $- \frac{1}{2}\lambda_u(a_{11}^A + a_{12}^A - a_{13}^A - a_{18}^A + a_2^A + a_{20}^A + a_{21}^A - 2a_{29}^A + a_3^A + 2a_{30}^A - a_4^A + a_6^A - a_7^A + a_9^A)$
$\Lambda_b^0 \rightarrow n \eta_8$	$- \frac{1}{2\sqrt{3}}\lambda_c(a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{2\sqrt{3}}\lambda_t(2a_{10}^{A,p} + 2a_{11}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p}$ $+ a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p}$ $- a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) - \frac{1}{2\sqrt{3}}\lambda_u(a_{11}^A + a_{12}^A - a_{13}^A + a_{18}^A + 2a_{19}^A + a_2^A$ $+ a_{20}^A - a_{21}^A - 2a_{29}^A + a_3^A - 2a_{30}^A + a_4^A - a_6^A + a_7^A - a_9^A)$
$\Lambda_b^0 \rightarrow n \eta_1$	$- \frac{1}{2\sqrt{3}}\lambda_c(a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{2\sqrt{3}}\lambda_t(2a_{10}^{A,p} + 2a_{11}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p}$ $+ a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p}$ $- 6a_{32}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) - \frac{1}{2\sqrt{3}}\lambda_u(a_{11}^A + a_{12}^A - a_{13}^A + a_{18}^A$ $+ 2a_{19}^A + a_2^A + a_{20}^A - a_{21}^A - 2a_{29}^A + a_3^A - 2a_{30}^A + a_4^A - a_6^A + a_7^A - a_9^A)$
$\Lambda_b^0 \rightarrow \Lambda^0 K^0$	$\frac{1}{2\sqrt{3}}\lambda_c(2a_1^{A,c} + a_2^{A,c} - a_3^{A,c} - 2a_4^{A,c} - 4a_6^{A,c}) - \frac{1}{2\sqrt{3}}\lambda_t(a_1^{A,p} + 4a_{10}^{A,p} + 2a_{11}^{A,p} - 2a_{12}^{A,p} - 2a_{13}^{A,p} + 2a_{18}^{A,p} + a_{19}^{A,p}$ $+ 2a_{20}^{A,p} - 2a_{21}^{A,p} + 6a_{23}^{A,p} + 3a_{24}^{A,p} - 3a_{25}^{A,p} - 6a_{26}^{A,p} - 4a_{29}^{A,p} + a_3^{A,p} - 4a_{30}^{A,p} - 12a_{32}^{A,p} - 2a_4^{A,p} - a_5^{A,p}$ $+ a_6^{A,p} + 2a_7^{A,p} + a_8^{A,p} - a_9^{A,p}) + \frac{1}{2\sqrt{3}}\lambda_u(2a_{18}^A + a_{19}^A - a_{20}^A - 2a_{21}^A - 4a_{30}^A + 2a_4^A + a_5^A - a_6^A + 2a_7^A + a_8^A - a_9^A)$

According to Eqs. (10)–(12), there are no penguin-induced amplitudes in the 15 and $\bar{6}$ irreducible representations. Furthermore, only the second component of 3-dimensional presentation in the $\Delta S = 0$ transition is non-

zero. Therefore, the amplitudes induced by the 3-dimensional representations always appear simultaneously and can be absorbed into four amplitudes,

Table 7. Topological amplitudes of Ξ_b^{-0} decays into $\mathcal{B}_8^A M$ from the $b \rightarrow s$ transition.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 K^-$	$\frac{1}{2} \lambda'_c (-a_1^{A,c} + a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{2} \lambda'_t (2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} + a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) + \frac{1}{2} \lambda'_u (-a_1^A + a_{14}^A - a_{18}^A + a_{20}^A + a_{21}^A + 2a_{28}^A + a_3^A + 2a_{30}^A)$
$\Xi_b^- \rightarrow \Sigma^- \bar{K}^0$	$\frac{1}{\sqrt{2}} \lambda'_c (-a_1^{A,c} + a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}} \lambda'_t (2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} + a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (-a_{18}^A + a_{20}^A + a_{21}^A + 2a_{30}^A)$
$\Xi_b^- \rightarrow \Xi^0 \pi^-$	$\frac{1}{\sqrt{2}} \lambda'_c (a_2^{A,c} + a_3^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{19}^A + a_2^A + a_{20}^A + a_3^A)$
$\Xi_b^- \rightarrow \Xi^- \pi^0$	$\frac{1}{2} \lambda'_c (a_2^{A,c} + a_3^{A,c}) - \frac{1}{2} \lambda'_t (a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p}) + \frac{1}{2} \lambda'_u (a_{13}^A + a_{19}^A + a_{20}^A + 2a_{29}^A)$
$\Xi_b^- \rightarrow \Xi^- \eta_8$	$\frac{1}{2\sqrt{3}} \lambda'_c (2a_1^{A,c} + a_2^{A,c} - a_3^{A,c} - 2a_4^{A,c} - 4a_6^{A,c}) - \frac{1}{2\sqrt{3}} \lambda'_t (a_1^{A,p} + 4a_{10}^{A,p} + 2a_{11}^{A,p} - 2a_{12}^{A,p} - 2a_{13}^{A,p} + 2a_{18}^{A,p} + a_{19}^{A,p} + 2a_2^{A,p} - a_{20}^{A,p} - 2a_{21}^{A,p} + 6a_{23}^{A,p} + 3a_{24}^{A,p} - 3a_{25}^{A,p} - 6a_{26}^{A,p} - 4a_{29}^{A,p} + a_3^{A,p} - 4a_{30}^{A,p} - 12a_{32}^{A,p} - 2a_4^{A,p} - a_5^{A,p} + a_6^{A,p} + 2a_7^{A,p} + a_8^{A,p} - a_9^{A,p}) + \frac{1}{2\sqrt{3}} \lambda'_u (a_{13}^A + 2a_{18}^A + a_{19}^A - a_{20}^A - 2a_{21}^A + 2a_{29}^A - 4a_{30}^A)$
$\Xi_b^- \rightarrow \Xi^- \eta_1$	$\frac{1}{\sqrt{6}} \lambda'_c (-a_1^{A,c} - 2a_2^{A,c} - a_3^{A,c} + a_4^{A,c} + 3a_5^{A,c} + 2a_6^{A,c} + 6a_7^{A,c}) + \frac{1}{\sqrt{6}} \lambda'_t (2a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} - 3a_{14}^{A,p} + 6a_{15}^{A,p} + 3a_{16}^{A,p} - 3a_{17}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} - 3a_{22}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 9a_{27}^{A,p} - 6a_{28}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p} - 6a_{31}^{A,p} - 6a_{32}^{A,p} - 18a_{33}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) + \frac{1}{\sqrt{6}} \lambda'_u (a_{13}^A - a_{18}^A - 2a_{19}^A - a_{20}^A + a_{21}^A + 3a_{22}^A + 2a_{29}^A + 2a_{30}^A + 6a_{31}^A)$
$\Xi_b^0 \rightarrow \Lambda^0 K^-$	$\frac{1}{2\sqrt{3}} \lambda'_c (-a_1^{A,c} - 2a_2^{A,c} - a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{2\sqrt{3}} \lambda'_t (2a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) - \frac{1}{2\sqrt{3}} \lambda'_u (a_1^A - a_{14}^A + a_{18}^A + 2a_{19}^A + 2a_2^A + a_{20}^A - a_{21}^A - 2a_{28}^A + a_3^A - 2a_{30}^A)$
$\Xi_b^0 \rightarrow \Sigma^+ K^-$	$\frac{1}{\sqrt{2}} \lambda'_c (a_1^{A,c} - a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) + \frac{1}{\sqrt{2}} \lambda'_t (-2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} - a_{18}^{A,p} - a_2^{A,p} + a_{20}^{A,p} + a_{21}^{A,p} - 3a_{23}^{A,p} + 3a_{25}^{A,p} + 3a_{26}^{A,p} + 2a_{29}^{A,p} - a_3^{A,p} + 2a_{30}^{A,p} + 6a_{32}^{A,p} + a_4^{A,p} - a_6^{A,p} - a_7^{A,p} + a_9^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_1^A - a_{14}^A + a_{18}^A - a_{20}^A - a_{21}^A - 2a_{28}^A - a_3^A - 2a_{30}^A + a_4^A + a_5^A + a_7^A + a_8^A)$
$\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0$	$\frac{1}{2} \lambda'_c (-a_1^{A,c} + a_3^{A,c} + a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{2} \lambda'_t (2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} + a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) - \frac{1}{2} \lambda'_u (a_{18}^A - a_{20}^A - a_{21}^A - 2a_{30}^A + a_4^A + a_5^A + a_7^A + a_8^A)$
$\Xi_b^0 \rightarrow \Xi^0 \pi^0$	$\frac{1}{2} \lambda'_c (a_2^{A,c} + a_3^{A,c}) - \frac{1}{2} \lambda'_t (a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p}) + \frac{1}{2} \lambda'_u (a_{11}^A + a_{12}^A - a_{13}^A + a_{19}^A + a_2^A + a_{20}^A - 2a_{29}^A + a_3^A)$
$\Xi_b^0 \rightarrow \Xi^0 \eta_8$	$\frac{1}{2\sqrt{3}} \lambda'_c (-2a_1^{A,c} - a_2^{A,c} + a_3^{A,c} + 2a_4^{A,c} + 4a_6^{A,c}) + \frac{1}{2\sqrt{3}} \lambda'_t (a_1^{A,p} + 4a_{10}^{A,p} + 2a_{11}^{A,p} - 2a_{12}^{A,p} - 2a_{13}^{A,p} + 2a_{18}^{A,p} + a_{19}^{A,p} + 2a_2^{A,p} - a_{20}^{A,p} - 2a_{21}^{A,p} + 6a_{23}^{A,p} + 3a_{24}^{A,p} - 3a_{25}^{A,p} - 6a_{26}^{A,p} - 4a_{29}^{A,p} + a_3^{A,p} - 4a_{30}^{A,p} - 12a_{32}^{A,p} - 2a_4^{A,p} - a_5^{A,p} + a_6^{A,p} + 2a_7^{A,p} + a_8^{A,p} - a_9^{A,p}) + \frac{1}{2\sqrt{3}} \lambda'_u (a_{11}^A + a_{12}^A - a_{13}^A - 2a_{18}^A - a_{19}^A + a_2^A + a_{20}^A + 2a_{21}^A - 2a_{29}^A + a_3^A + 4a_{30}^A - 2a_4^A + 2a_6^A - 2a_7^A + 2a_9^A)$
$\Xi_b^0 \rightarrow \Xi^0 \eta_1$	$\frac{1}{\sqrt{6}} \lambda'_c (a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 3a_5^{A,c} - 2a_6^{A,c} - 6a_7^{A,c}) + \frac{1}{\sqrt{6}} \lambda'_t (-2a_1^{A,p} - 2a_{10}^{A,p} - a_{11}^{A,p} + a_{12}^{A,p} + a_{13}^{A,p} + 3a_{14}^{A,p} - 6a_{15}^{A,p} - 3a_{16}^{A,p} + 3a_{17}^{A,p} - a_{18}^{A,p} - 2a_{19}^{A,p} - a_2^{A,p} - a_{20}^{A,p} + a_{21}^{A,p} + 3a_{22}^{A,p} - 3a_{23}^{A,p} - 6a_{24}^{A,p} - 3a_{25}^{A,p} + 3a_{26}^{A,p} + 9a_{27}^{A,p} + 6a_{28}^{A,p} + 2a_{29}^{A,p} + a_3^{A,p} + 2a_{30}^{A,p} + 6a_{31}^{A,p} + 6a_{32}^{A,p} + 18a_{33}^{A,p} + a_4^{A,p} + 2a_5^{A,p} + a_6^{A,p} - a_7^{A,p} - 2a_8^{A,p} - a_9^{A,p}) + \frac{1}{\sqrt{6}} \lambda'_u (a_{11}^A + a_{12}^A - a_{13}^A + 3a_{16}^A + 3a_{17}^A + a_{18}^A + 2a_{19}^A + a_2^A + a_{20}^A - a_{21}^A - 3a_{22}^A - 2a_{29}^A + a_3^A - 2a_{30}^A - 6a_{31}^A + a_4^A - a_6^A + a_7^A - a_9^A)$
$\Xi_b^0 \rightarrow \Xi^- \pi^+$	$\frac{1}{\sqrt{2}} \lambda'_c (-a_2^{A,c} - a_3^{A,c}) + \frac{1}{\sqrt{2}} \lambda'_t (a_1^{A,p} + a_{19}^{A,p} + a_{20}^{A,p} + 3a_{24}^{A,p} + 3a_{25}^{A,p} - a_3^{A,p} - a_5^{A,p} - a_6^{A,p} + a_8^{A,p} + a_9^{A,p}) - \frac{1}{\sqrt{2}} \lambda'_u (a_{11}^A + a_{12}^A + a_{19}^A + a_{20}^A)$
$\Xi_b^0 \rightarrow \Lambda^0 \bar{K}^0$	$\frac{1}{2\sqrt{3}} \lambda'_c (a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 2a_6^{A,c}) - \frac{1}{2\sqrt{3}} \lambda'_t (2a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) + \frac{1}{2\sqrt{3}} \lambda'_u (a_{18}^A + 2a_{19}^A + a_{20}^A - a_{21}^A - 2a_{30}^A + a_4^A - a_5^A - 2a_6^A + a_7^A - a_8^A - 2a_9^A)$

Table 8. Topological amplitudes of Λ_b^0 decays into $\mathcal{B}_8^A M$ from the $b \rightarrow s$ transition.

Channel	Amplitude
$\Lambda_b^0 \rightarrow \Sigma^+ \pi^-$	$\frac{1}{\sqrt{2}} \lambda'_c (a_1^{A,c} - a_3^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} + a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p})$ $+ \frac{1}{\sqrt{2}} \lambda'_u (a_1^A + a_{18}^A - a_{20}^A - a_3^A + a_4^A + a_5^A)$
$\Lambda_b^0 \rightarrow \Sigma^0 \pi^0$	$\frac{1}{2\sqrt{2}} \lambda'_c (2a_1^{A,c} - 2a_3^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} + a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p})$ $+ \frac{1}{2\sqrt{2}} \lambda'_u (a_1^A + a_{10}^A - a_{12}^A + 2a_{18}^A - 2a_{20}^A - a_3^A + a_4^A + a_5^A)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{2\sqrt{6}} \lambda'_u (a_1^A + a_{10}^A - a_{12}^A - a_3^A - a_4^A - a_5^A - 2a_8^A - 2a_9^A)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_1$	$\frac{1}{2\sqrt{3}} \lambda'_u (a_1^A + a_{10}^A - a_{12}^A + 3a_{15}^A - 3a_{17}^A - a_3^A - a_4^A - a_5^A + a_8^A + a_9^A)$
$\Lambda_b^0 \rightarrow \Sigma^- \pi^+$	$\frac{1}{\sqrt{2}} \lambda'_c (a_1^{A,c} - a_3^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_{18}^{A,p} + a_2^{A,p} - a_{20}^{A,p} + 3a_{23}^{A,p} - 3a_{25}^{A,p} + a_3^{A,p} - a_4^{A,p} + a_6^{A,p} + a_7^{A,p} - a_9^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{10}^A - a_{12}^A + a_{18}^A - a_{20}^A)$
$\Lambda_b^0 \rightarrow p K^-$	$\frac{1}{\sqrt{2}} \lambda'_c (a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}} \lambda'_t (2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} - a_{21}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{14}^A + a_{21}^A + 2a_{28}^A + 2a_{30}^A - a_7^A - a_8^A)$
$\Lambda_b^0 \rightarrow n \bar{K}^0$	$\frac{1}{\sqrt{2}} \lambda'_c (a_4^{A,c} + 2a_6^{A,c}) + \frac{1}{\sqrt{2}} \lambda'_t (2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} - a_{21}^{A,p} - 3a_{26}^{A,p} - 2a_{29}^{A,p} - 2a_{30}^{A,p} - 6a_{32}^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{21}^A + 2a_{30}^A - a_7^A + a_9^A)$
$\Lambda_b^0 \rightarrow \Xi^0 K^0$	$\frac{1}{\sqrt{2}} \lambda'_c (a_1^{A,c} + a_2^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_1^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{18}^A + a_{19}^A + a_4^A - a_6^A)$
$\Lambda_b^0 \rightarrow \Xi^- K^+$	$\frac{1}{\sqrt{2}} \lambda'_c (a_1^{A,c} + a_2^{A,c}) - \frac{1}{\sqrt{2}} \lambda'_t (a_1^{A,p} + a_{18}^{A,p} + a_{19}^{A,p} + a_2^{A,p} + 3a_{23}^{A,p} + 3a_{24}^{A,p} - a_4^{A,p} - a_5^{A,p} + a_7^{A,p} + a_8^{A,p}) + \frac{1}{\sqrt{2}} \lambda'_u (a_{10}^A + a_{11}^A + a_{18}^A + a_{19}^A)$
$\Lambda_b^0 \rightarrow \Lambda^0 \pi^0$	$\frac{1}{2\sqrt{6}} \lambda'_u (a_1^A + a_{10}^A + 2a_{11}^A + a_{12}^A - 2a_{13}^A + 2a_{15}^A - 4a_{29}^A + a_3^A - a_4^A + a_5^A + 2a_6^A)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_8$	$\frac{1}{3\sqrt{2}} \lambda'_c (a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} + 2a_4^{A,c} + 4a_6^{A,c}) - \frac{1}{3\sqrt{2}} \lambda'_t (2a_1^{A,p} - 4a_{10}^{A,p} - 2a_{11}^{A,p} + 2a_{12}^{A,p} + 2a_{13}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p})$ $+ a_2^{A,p} + a_{20}^{A,p} + 2a_{21}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} + 6a_{26}^{A,p} + 4a_{29}^{A,p} - a_3^{A,p} + 4a_{30}^{A,p} + 12a_{32}^{A,p} - a_4^{A,p} - 2a_5^{A,p}$ $- a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p}) + \frac{1}{6\sqrt{2}} \lambda'_u (a_1^A + a_{10}^A + 2a_{11}^A + a_{12}^A - 2a_{13}^A + 2a_{18}^A + 4a_{19}^A + 2a_{21}^A$ $+ 2a_{20}^A + 4a_{21}^A - 4a_{29}^A + a_3^A + 8a_{30}^A + a_4^A - a_5^A - 2a_6^A - 4a_7^A - 2a_8^A + 2a_9^A)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_1$	$\frac{1}{3} \lambda'_c (a_1^{A,c} + 2a_2^{A,c} + a_3^{A,c} - a_4^{A,c} - 3a_5^{A,c} - 2a_6^{A,c} - 6a_7^{A,c}) - \frac{1}{3} \lambda'_t (2a_1^{A,p} + 2a_{10}^{A,p} + a_{11}^{A,p} - a_{12}^{A,p} - a_{13}^{A,p} - 3a_{14}^{A,p} + 6a_{15}^{A,p})$ $+ 3a_{16}^{A,p} - 3a_{17}^{A,p} + a_{18}^{A,p} + 2a_{19}^{A,p} + a_2^{A,p} + a_{20}^{A,p} - a_{21}^{A,p} - 3a_{22}^{A,p} + 3a_{23}^{A,p} + 6a_{24}^{A,p} + 3a_{25}^{A,p} - 3a_{26}^{A,p} - 9a_{27}^{A,p}$ $- 6a_{28}^{A,p} - 2a_{29}^{A,p} - a_3^{A,p} - 2a_{30}^{A,p} - 6a_{31}^{A,p} - 6a_{32}^{A,p} - 18a_{33}^{A,p} - a_4^{A,p} - 2a_5^{A,p} - a_6^{A,p} + a_7^{A,p} + 2a_8^{A,p} + a_9^{A,p})$ $+ \frac{1}{6} \lambda'_u (a_1^A + a_{10}^A + 2a_{11}^A + a_{12}^A - 2a_{13}^A + 3a_{15}^A + 6a_{16}^A + 3a_{17}^A + 2a_{18}^A + 4a_{19}^A + 2a_{21}^A + 2a_{20}^A$ $- 2a_{21}^A - 6a_{22}^A - 4a_{29}^A + a_3^A - 4a_{30}^A - 12a_{31}^A + a_4^A - a_5^A - 2a_6^A + 2a_7^A + a_8^A - a_9^A)$

Table 9. Decay amplitudes of $\mathcal{B}_{\bar{b}3}^- \rightarrow \mathcal{B}_8 M$ decays from the $b \rightarrow d$ transition, in which the baryon octet is expressed as a (1,1)-rank tensor.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 \pi^-$	$\frac{1}{\sqrt{2}} \lambda_t (A'_{11} - A'_{12}) + \frac{1}{\sqrt{2}} \lambda_u (A_2 + A_3 - A_7 + A_8)$
$\Xi_b^- \rightarrow \Sigma^- \pi^0$	$\frac{1}{\sqrt{2}} \lambda_t (-A'_{11} + A'_{12}) + \frac{1}{\sqrt{2}} \lambda_u (A_1 + A_4 + A_7 - A_8)$
$\Xi_b^- \rightarrow \Sigma^- \eta_8$	$-\frac{1}{\sqrt{6}} \lambda_t (A'_{11} + A'_{12}) + \frac{1}{\sqrt{6}} \lambda_u (A_1 + A_4 + A_7 + A_8)$
$\Xi_b^- \rightarrow \Sigma^- \eta_1$	$-\frac{1}{\sqrt{3}} \lambda_t (A'_{11} + A'_{12} + 3A'_{13}) + \frac{1}{\sqrt{3}} \lambda_u (A_1 + 3A_{10} + A_4 + A_7 + A_8)$
$\Xi_b^- \rightarrow n K^-$	$-\lambda_t A'_{11} + \lambda_u A_7$
$\Xi_b^- \rightarrow \Xi^- K^0$	$-\lambda_t A'_{12} + \lambda_u A_8$
$\Xi_b^- \rightarrow \Lambda^0 \pi^-$	$-\frac{1}{\sqrt{6}} \lambda_t (A'_{11} + A'_{12}) + \frac{1}{\sqrt{6}} \lambda_u (A_2 + A_3 + A_7 + A_8)$

Continued on next page

Table 9-continued from previous page

Channel	Amplitude
$\Xi_b^0 \rightarrow \Sigma^+ \pi^-$	$\lambda_t(A'_{12} + A'_{14}) - \lambda_u(A_2 + A_5)$
$\Xi_b^0 \rightarrow \Sigma^0 \pi^0$	$\frac{1}{2} \lambda_t(A'_{11} + A'_{12} + 2A'_{14}) + \frac{1}{2} \lambda_u(A_3 + A_4 - A_5 - A_6)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_8$	$-\frac{1}{2\sqrt{3}} \lambda_t(A'_{11} + A'_{12}) - \frac{1}{2\sqrt{3}} \lambda_u(A_3 - A_4 + A_5 + A_6)$
$\Xi_b^0 \rightarrow \Sigma^0 \eta_1$	$-\frac{1}{\sqrt{6}} \lambda_t(A'_{11} + A'_{12} + 3A'_{13}) - \frac{1}{\sqrt{6}} \lambda_u(A_3 - A_4 + A_5 + A_6 + 3A_9)$
$\Xi_b^0 \rightarrow \Sigma^- \pi^+$	$\lambda_t(A'_{11} + A'_{14}) - \lambda_u(A_1 + A_6)$
$\Xi_b^0 \rightarrow p K^-$	$\lambda_t A'_{14} - \lambda_u A_5$
$\Xi_b^0 \rightarrow n \bar{K}^0$	$\lambda_t(A'_{11} + A'_{14})$
$\Xi_b^0 \rightarrow \Xi^0 K^0$	$\lambda_t(A'_{12} + A'_{14})$
$\Xi_b^0 \rightarrow \Xi^- K^+$	$\lambda_t A'_{14} - \lambda_u A_6$
$\Xi_b^0 \rightarrow \Lambda^0 \pi^0$	$-\frac{1}{2\sqrt{3}} \lambda_t(A'_{11} + A'_{12}) + \frac{1}{2\sqrt{3}} \lambda_u(A_3 - A_4 - A_5 - A_6)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_8$	$\frac{1}{6} \lambda_t(A'_{11} + A'_{12} + 6A'_{14}) - \frac{1}{6} \lambda_u(A_3 + A_4 + A_5 + A_6)$
$\Xi_b^0 \rightarrow \Lambda^0 \eta_1$	$\frac{1}{3\sqrt{2}} \lambda_t(A'_{11} + A'_{12} + 3A'_{13}) - \frac{1}{3\sqrt{2}} \lambda_u(A_3 + A_4 + A_5 + A_6 + 3A_9)$
$\Lambda_b^0 \rightarrow \Sigma^0 K^0$	$\frac{1}{\sqrt{2}} \lambda_t A'_{11} + \frac{1}{\sqrt{2}} \lambda_u A_3$
$\Lambda_b^0 \rightarrow \Sigma^- K^+$	$-\lambda_t A'_{11} + \lambda_u A_1$
$\Lambda_b^0 \rightarrow p \pi^-$	$-\lambda_t A'_{12} + \lambda_u A_2$
$\Lambda_b^0 \rightarrow n \pi^0$	$\frac{1}{\sqrt{2}} \lambda_t A'_{12} + \frac{1}{\sqrt{2}} \lambda_u A_4$
$\Lambda_b^0 \rightarrow n \eta_8$	$\frac{1}{\sqrt{6}} \lambda_t(2A'_{11} - A'_{12}) + \frac{1}{\sqrt{6}} \lambda_u A_4$
$\Lambda_b^0 \rightarrow n \eta_1$	$\frac{1}{\sqrt{6}} \lambda_t(2A'_{11} - A'_{12}) + \frac{1}{\sqrt{6}} \lambda_u A_4$
$\Lambda_b^0 \rightarrow \Lambda^0 K^0$	$-\frac{1}{\sqrt{6}} \lambda_t(A'_{11} - 2A'_{12}) + \frac{1}{\sqrt{6}} \lambda_u A_3$

Table 10. Decay amplitudes of $\mathcal{B}_{b3}^- \rightarrow \mathcal{B}_8 M$ decays from the $b \rightarrow s$ transition, in which the baryon octet is expressed as a (1,1)-rank tensor.

Channel	Amplitude
$\Xi_b^- \rightarrow \Sigma^0 K^-$	$-\frac{1}{\sqrt{2}} \lambda'_t A'_{12} + \frac{1}{\sqrt{2}} \lambda'_u(A_2 + A_3 + A_8)$
$\Xi_b^- \rightarrow \Sigma^- \bar{K}^0$	$-\lambda'_t A'_{12} + \lambda'_u A_8$
$\Xi_b^- \rightarrow \Xi^0 \pi^-$	$-\lambda'_t A_{11} + \lambda'_u A_7$
$\Xi_b^- \rightarrow \Xi^- \pi^0$	$-\frac{1}{\sqrt{2}} \lambda'_t A'_{11} + \frac{1}{\sqrt{2}} \lambda'_u(A_1 + A_4 + A_7)$
$\Xi_b^- \rightarrow \Xi^- \eta_8$	$-\frac{1}{\sqrt{6}} \lambda'_t(A'_{11} - 2A'_{12}) + \frac{1}{\sqrt{6}} \lambda'_u(A_1 + A_4 + A_7 - 2A_8)$
$\Xi_b^- \rightarrow \Xi^- \eta_1$	$-\frac{1}{\sqrt{3}} \lambda'_t(A'_{11} + A'_{12} + 3A'_{13}) + \frac{1}{\sqrt{3}} \lambda'_u(A_1 + 3A_{10} + A_4 + A_7 + A_8)$
$\Xi_b^- \rightarrow \Lambda^0 K^-$	$\frac{1}{\sqrt{6}} \lambda'_t(2A'_{11} - A'_{12}) + \frac{1}{\sqrt{6}} \lambda'_u(A_2 + A_3 - 2A_7 + A_8)$
$\Xi_b^0 \rightarrow \Sigma^+ K^-$	$\lambda'_t A'_{12} - \lambda'_u A_2$
$\Xi_b^0 \rightarrow \Sigma^0 \bar{K}^0$	$-\frac{1}{\sqrt{2}} \lambda'_t A'_{12} - \frac{1}{\sqrt{2}} \lambda'_u A_3$
$\Xi_b^0 \rightarrow \Xi^0 \pi^0$	$-\frac{1}{\sqrt{2}} \lambda'_t A'_{11} - \frac{1}{\sqrt{2}} \lambda'_u A_4$
$\Xi_b^0 \rightarrow \Xi^0 \eta_8$	$\frac{1}{\sqrt{6}} \lambda'_t(A'_{11} - 2A'_{12}) - \frac{1}{\sqrt{6}} \lambda'_u A_4$

Continued on next page

Table 10-continued from previous page

Channel	Amplitude
$\Xi_b^0 \rightarrow \Xi^0 \eta_1$	$\frac{1}{\sqrt{3}} \lambda'_t (A'_{11} + A'_{12} + 3A'_{13}) - \frac{1}{\sqrt{3}} \lambda'_u A_4$
$\Xi_b^0 \rightarrow \Xi^- \pi^+$	$\lambda'_t A'_{11} - \lambda'_u A_1$
$\Xi_b^0 \rightarrow \Lambda^0 \bar{K}^0$	$\frac{1}{\sqrt{6}} \lambda'_t (-2A'_{11} + A'_{12}) - \frac{1}{\sqrt{6}} \lambda'_u A_3$
$\Lambda_b^0 \rightarrow \Sigma^+ \pi^-$	$-\lambda'_t A'_{14} + \lambda'_u A_5$
$\Lambda_b^0 \rightarrow \Sigma^0 \pi^0$	$-\lambda'_t A'_{14} + \frac{1}{2} \lambda'_u (A_5 + A_6)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_8$	$\frac{1}{2\sqrt{3}} \lambda'_t (-2A_3 + A_5 + A_6)$
$\Lambda_b^0 \rightarrow \Sigma^0 \eta_1$	$\frac{1}{\sqrt{6}} \lambda'_t (A_3 + A_5 + A_6 + 3A_9)$
$\Lambda_b^0 \rightarrow \Sigma^- \pi^+$	$-\lambda'_t A'_{14} + \lambda'_u A_6$
$\Lambda_b^0 \rightarrow p K^-$	$-\lambda'_t (A'_{12} + A'_{14}) + \lambda'_u (A_2 + A_5)$
$\Lambda_b^0 \rightarrow n \bar{K}^0$	$-\lambda'_t (A'_{12} + A'_{14})$
$\Lambda_b^0 \rightarrow \Xi^0 K^0$	$-\lambda'_t (A'_{11} + A'_{14})$
$\Lambda_b^0 \rightarrow \Xi^- K^+$	$-\lambda'_t (A'_{11} + A'_{14}) + \lambda'_u (A_1 + A_6)$
$\Lambda_b^0 \rightarrow \Lambda^0 \pi^0$	$\frac{1}{2\sqrt{3}} \lambda'_t (-2A_4 + A_5 + A_6)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_8$	$-\frac{1}{3} \lambda'_t (2(A'_{11} + A'_{12}) + 3A'_{14}) + \frac{1}{6} \lambda'_u (-2A_3 - 2A_4 + A_5 + A_6)$
$\Lambda_b^0 \rightarrow \Lambda^0 \eta_1$	$\frac{\sqrt{2}}{3} \lambda'_t (A'_{11} + A'_{12} + 3A'_{13}) + \frac{1}{3\sqrt{2}} \lambda'_u (A_3 - 2A_4 + A_5 + A_6 + 3A_9)$

$$\begin{aligned}
b'_{11} &= -\frac{\lambda_u}{\lambda_t} b_{15} - \frac{\lambda_c}{\lambda_t} b_4^c + 3b_{11}^p + b_{15}^p, \\
b'_{12} &= -\frac{\lambda_u}{\lambda_t} b_{16} - \frac{\lambda_c}{\lambda_t} b_2^c + 3b_{12}^p + b_{12}^p, \\
b'_{13} &= -\frac{\lambda_u}{\lambda_t} b_{17} - \frac{\lambda_c}{\lambda_t} b_3^c + 3b_{13}^p + b_{17}^p, \\
b'_{14} &= -\frac{\lambda_u}{\lambda_t} b_{18} - \frac{\lambda_c}{\lambda_t} b_1^c + 3b_{14}^p + b_{18}^p,
\end{aligned} \quad (13)$$

where $\lambda_u = V_{ub} V_{ud}^*$, $\lambda_c = V_{cb} V_{cd}^*$, $\lambda_t = V_{tb} V_{td}^*$. According to Eqs. (13) and (A15), the tree-induced amplitudes $A_{11} \sim A_{14}$, $A_1^c \sim A_4^c$, and all the penguin-induced amplitudes can be absorbed into four amplitudes in the $SU(3)_F$ limit,

$$\begin{aligned}
A'_{11} &= -\frac{\lambda_u}{\lambda_t} A_{11} - \frac{\lambda_c}{\lambda_t} A_3^c + A_3^p + A_5^p + A_{11}^p + 3A_{17}^p, \\
A'_{12} &= -\frac{\lambda_u}{\lambda_t} A_{12} - \frac{\lambda_c}{\lambda_t} A_1^c + A_4^p + A_6^p + A_{12}^p + 3A_{15}^p, \\
A'_{13} &= -\frac{\lambda_u}{\lambda_t} A_{13} - \frac{\lambda_c}{\lambda_t} A_2^c + A_2^p + A_9^p + A_{13}^p + 3A_{16}^p, \\
A'_{14} &= -\frac{\lambda_u}{\lambda_t} A_{14} - \frac{\lambda_c}{\lambda_t} A_4^c + A_7^p + A_8^p + A_{14}^p + 3A_{18}^p.
\end{aligned} \quad (14)$$

For the $\Delta S = -1$ transition, $\lambda_{u,c,t}$ in Eqs. (13) and (14) are replaced by $\lambda'_{u,c,t}$ and $\lambda'_u = V_{ub} V_{us}^*$, $\lambda'_c = V_{cb} V_{cs}^*$, $\lambda'_t = V_{tb} V_{ts}^*$. According to Eq. (14), all the penguin induced amplitudes are determined once the tree induced amplitudes with quark loops are known. There is no degree of free-

dom of penguin induced diagrams in the SM. Combining Eqs. (A19) and (13), there are 13 independent amplitudes contributing to the $\mathcal{B}_{\bar{b}s} \rightarrow \mathcal{B}_8 M$ decays in the $SU(3)_F$ limit.

III. PHENOMENOLOGICAL ANALYSIS

First, we compare the topological amplitudes in bottom and charmed baryon decays. For charmed baryon decays, the contributions from the d - and s -quark loop diagrams cancel each other out because $V_{cd}^* V_{ud} \sim -V_{cs}^* V_{us}$. The penguin-induced amplitudes are much smaller than the tree-induced amplitudes under the $SU(3)_F$ limit, as $|V_{cb}^* V_{ub} / V_{cd}^* V_{ud}| \times (\alpha_s / \pi) \sim 10^{-4}$. Note that all the penguin-induced diagrams and tree-induced diagrams with quark-loops are included in $A'_{11} \sim A'_{14}$. Consequently, $A'_{11} \sim A'_{14}$ are negligible in the branching fractions of charmed baryon decays. For bottom baryon decays, the $b \rightarrow d$ transition is dominated by tree-induced amplitudes, as $|V_{tb} V_{td}^* / V_{ub} V_{ud}^*| \times (\alpha_s / \pi) \sim 0.1$. The $b \rightarrow s$ transition is dominated by the penguin-induced and quark-loop diagrams, as $|V_{tb} V_{ts}^* / V_{ub} V_{us}^*| \times (\alpha_s / \pi) \sim 2$. The amplitudes $A'_{11} \sim A'_{14}$ are essential to the branching fractions of bottom baryon decays. The CP asymmetries in the bottom baryon decays arise from the interference between tree amplitudes $A_1 \sim A_{10}$ and $A'_{11} \sim A'_{14}$. The CP asymmetries of charmless bottom baryon decays could reach $\mathcal{O}(0.1)$, whereas the CP asymmetries of charmed bottom baryon

decays are estimated to be $O(10^{-4})$. Because of the different CKM matrix elements, the U -spin relations between two U -spin conjugate modes differ for charmed and bottom baryon decays [55].

Owing to the limited experimental data, we cannot determine decay amplitudes by global fitting. However, some dynamical information can still be extracted from the available data. The CP asymmetries in the $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ decays have been measured by the LHCb Collaboration as [20]

$$\begin{aligned} A_{CP}(\Lambda_b^0 \rightarrow p\pi^-) &= (0.2 \pm 0.8 \pm 0.4)\%, \\ A_{CP}(\Lambda_b^0 \rightarrow pK^-) &= (-1.1 \pm 0.7 \pm 0.4)\%. \end{aligned} \quad (15)$$

The branching fractions for these decays are given by [56]

$$\begin{aligned} \mathcal{B}r(\Lambda_b^0 \rightarrow p\pi^-) &= (4.6 \pm 0.8) \times 10^{-6}, \\ \mathcal{B}r(\Lambda_b^0 \rightarrow pK^-) &= (5.5 \pm 1.0) \times 10^{-6}. \end{aligned} \quad (16)$$

The branching fractions of the $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ decays are of the same order, indicating that the penguin amplitudes are significant. The small CP asymmetries in these two decay modes, particularly in $\Lambda_b^0 \rightarrow pK^-$, might result from small relative phases between the tree and penguin amplitudes. According to the PQCD calculations [27], the color-favored emitted diagram T , denoted as a_{28}^A in this paper, is one order larger than the other diagrams. Eq. (A12) shows that a_{28}^A contributes only to the amplitude A_2 . Consequently, A_2 is expected to be larger than the other amplitudes. The small CP violation in the $\Lambda_b^0 \rightarrow pK^-$ mode might be explained by small relative strong phases $\delta_{A'_{12}} - \delta_{A_2}$ and $\delta_{A'_{14}} - \delta_{A_2}$. Hence, we suggest measuring CP asymmetries in the $\Xi_b^0 \rightarrow pK^-$ and $\Xi_b^- \rightarrow \Lambda^0 K^-$ decays on LHCb. In these processes, the tree and penguin amplitudes may be of comparable magnitudes after accounting for the CKM matrix elements, and the small strong phases $\delta_{A'_{12}} - \delta_{A_2}$ and $\delta_{A'_{14}} - \delta_{A_2}$ are avoided.

In the literature, the Körner-Pati-Woo theorem [41, 42] plays an important role in analyzing heavy baryon weak decays. It states that the two quarks produced by weak operators must be antisymmetric in flavor if they enter the same low-lying baryon. In diagrams $a_1^{S,A} \sim a_{12}^{S,A}$, $a_{15}^{S,A} \sim a_{17}^{S,A}$, the two quarks emitted from the weak vertex enter the final-state or resonance-state baryon. They should be antisymmetric in flavor if the Körner-Pati-Woo theorem holds. If two of the diagrams $a_1^{S,A} \sim a_{12}^{S,A}$, $a_{15}^{S,A} \sim a_{17}^{S,A}$ are connected with each other by interchanging two emitted quarks, these two diagrams are opposite. Then, we have

$$a_1^{S,A} = -a_2^{S,A}, \quad a_5^{S,A} = -a_6^{S,A}, \quad a_7^{S,A} = -a_{10}^{S,A}, \quad (17)$$

$$a_8^{S,A} = -a_{11}^{S,A}, \quad a_9^{S,A} = -a_{12}^{S,A}, \quad a_{15}^{S,A} = -a_{16}^{S,A}. \quad (18)$$

Furthermore, the antisymmetry of the two emitted quarks conflicts with the requirement that these two quarks are symmetric in diagrams a_3^S, a_4^S, a_{17}^S , which results in

$$a_3^S = a_4^S = a_{17}^S = 0. \quad (19)$$

Applying the above equations to Eq. (A12), we obtain the following relations:

$$A_1 = -A_3, \quad A_5 = -A_7, \quad A_6 = -A_8, \quad A_9 = -A_{10}. \quad (20)$$

The decay amplitudes of the $\Lambda_b^0 \rightarrow \Sigma^0 K^0$ and $\Lambda_b^0 \rightarrow \Sigma^- K^+$ modes are

$$\begin{aligned} \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^0 K^0) &= \frac{1}{\sqrt{2}} \lambda_t A'_{11} + \frac{1}{\sqrt{2}} \lambda_u A_3, \\ \mathcal{A}(\Lambda_b^0 \rightarrow \Sigma^- K^+) &= -\lambda_t A'_{11} + \lambda_u A_1. \end{aligned} \quad (21)$$

If the Körner-Pati-Woo theorem is valid, the relation $A_1 = -A_3$ leads to

$$\mathcal{B}r(\Lambda_b^0 \rightarrow \Sigma^- K^+) = 4 \mathcal{B}r(\Lambda_b^0 \rightarrow \Sigma^0 K_S^0) \quad (22)$$

under the isospin symmetry. Isospin breaking is naively predicted to be $O(1\%)$. If Eq. (22) is violated, it indicates that the Körner-Pati-Woo theorem is not valid in topological diagrams.

IV. SUMMARY

In summary, we studied the topological amplitudes of $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays in the $SU(3)_F$ limit. The linear relations between topologies and $SU(3)$ irreducible amplitudes are derived through tensor contraction and $SU(3)$ decomposition. Thirteen independent amplitudes contribute to $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decays in the Standard Model, of which four are crucial to the CP asymmetries. The small CP asymmetries in the $\Lambda_b^0 \rightarrow p\pi^-$ and $\Lambda_b^0 \rightarrow pK^-$ modes might indicate small strong phases between amplitudes A_2 and $A'_{12,14}$. To avoid the small strong phases, we suggest measuring CP asymmetries in the $\Xi_b^0 \rightarrow pK^-$ and $\Xi_b^- \rightarrow \Lambda^0 K^-$ decays. Moreover, the Körner-Pati-Woo theorem can be tested by measuring the branching fractions of the $\Lambda_b^0 \rightarrow \Sigma^0 K_S^0$ and $\Lambda_b^0 \rightarrow \Sigma^- K^+$ decays under isospin symmetry.

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APPENDIX A: TOPOLOGICAL AMPLITUDES OF BOTTOM BARYON DECAYS

The bottom baryon anti-triplet is

$$|\mathcal{B}_{b\bar{3}}\rangle = \begin{pmatrix} 0 & \Lambda_b^0 & \Xi_b^0 \\ -\Lambda_b^0 & 0 & \Xi_b^- \\ -\Xi_b^0 & -\Xi_b^- & 0 \end{pmatrix}. \quad (A1)$$

The light pseudoscalar meson nonet is

$$|M\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{2/3}\eta_8 \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \eta_1 & 0 \\ 0 & 0 & \eta_1 \end{pmatrix}. \quad (A2)$$

The light baryon octets $\mathcal{B}_8^S$ and $\mathcal{B}_8^A$ can be written as tensors with three covariant indices. $\mathcal{B}_8^S$ is given by

$$\begin{aligned} \Sigma^+ &= \frac{1}{\sqrt{6}}(2\mathcal{B}_8^{113} - \mathcal{B}_8^{311} - \mathcal{B}_8^{131}), \\ p &= \frac{1}{\sqrt{6}}(-2\mathcal{B}_8^{112} + \mathcal{B}_8^{211} + \mathcal{B}_8^{121}), \\ \Sigma^- &= \frac{1}{\sqrt{6}}(-2\mathcal{B}_8^{223} + \mathcal{B}_8^{322} + \mathcal{B}_8^{232}), \\ n &= \frac{1}{\sqrt{6}}(2\mathcal{B}_8^{221} - \mathcal{B}_8^{122} - \mathcal{B}_8^{212}), \end{aligned}$$

$$\begin{aligned} \Xi^- &= \frac{1}{\sqrt{6}}(2\mathcal{B}_8^{332} - \mathcal{B}_8^{233} - \mathcal{B}_8^{323}), \\ \Xi^0 &= \frac{1}{\sqrt{6}}(-2\mathcal{B}_8^{331} + \mathcal{B}_8^{133} + \mathcal{B}_8^{313}), \\ \Sigma^0 &= \frac{1}{\sqrt{12}}(\mathcal{B}_8^{321} + \mathcal{B}_8^{231} + \mathcal{B}_8^{312} + \mathcal{B}_8^{132} - 2\mathcal{B}_8^{123} - 2\mathcal{B}_8^{213}), \\ \Lambda^0 &= \frac{1}{2}(\mathcal{B}_8^{132} - \mathcal{B}_8^{231} + \mathcal{B}_8^{312} - \mathcal{B}_8^{321}), \end{aligned} \quad (A3)$$

and $\mathcal{B}_8^A$ is given by

$$\begin{aligned} \Sigma^+ &= \frac{1}{\sqrt{2}}(\mathcal{B}_8^{311} - \mathcal{B}_8^{131}), \quad p = \frac{1}{\sqrt{2}}(\mathcal{B}_8^{121} - \mathcal{B}_8^{211}), \\ \Sigma^- &= \frac{1}{\sqrt{2}}(\mathcal{B}_8^{232} - \mathcal{B}_8^{322}), \quad n = \frac{1}{\sqrt{2}}(\mathcal{B}_8^{122} - \mathcal{B}_8^{212}), \\ \Xi^- &= \frac{1}{\sqrt{2}}(\mathcal{B}_8^{233} - \mathcal{B}_8^{323}), \quad \Xi^0 = \frac{1}{\sqrt{2}}(\mathcal{B}_8^{313} - \mathcal{B}_8^{133}), \\ \Sigma^0 &= \frac{1}{2}(\mathcal{B}_8^{132} - \mathcal{B}_8^{312} + \mathcal{B}_8^{231} - \mathcal{B}_8^{321}), \\ \Lambda^0 &= \frac{1}{\sqrt{12}}(2\mathcal{B}_8^{213} - 2\mathcal{B}_8^{123} + \mathcal{B}_8^{231} - \mathcal{B}_8^{321} + \mathcal{B}_8^{312} - \mathcal{B}_8^{132}). \end{aligned} \quad (A4)$$

The decay amplitude of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ decay can be constructed via Eq. (6). The number of topologies can be counted by permutation. For the contributions from operators $\mathcal{O}^{(u,p)}$, there are five covariant/contravariant indices in $\langle \mathcal{B}_8^{ijk} M_m^l | \mathcal{O}_{np}^{(u,p)q} | [\mathcal{B}_{c\bar{3}}]_{rs} \rangle$. The number of full contractions is $N = A_5^3 = 120$. Considering that the two flavor indices in the antitriplet are antisymmetric, the number of terms is $N_S + N_A = N/2 = 60$. As the symmetric light quarks in $\mathcal{B}_8^S$ conflict with the antisymmetric light quarks in $\mathcal{B}_{b\bar{3}}$, the number of topological diagrams for bottom antitriplet baryon decays into $\mathcal{B}_8^S$ is less than that for decays into $\mathcal{B}_8^A$. The difference between them, $N_A - N_S$, is computed to be $A_3^3 = 6$. Then, N_S and N_A are solved to be $N_S = 27$, $N_A = 33$. For the contributions from $\mathcal{O}^{(c)}$, there are four covariant/contravariant indices in $\langle \mathcal{B}_8^{ijk} M_m^l | \mathcal{O}_n^{(c)} | [\mathcal{B}_{b\bar{3}}]_{op} \rangle$. As $N_S^c + N_A^c = A_4^2/2 = 12$ and $N_A^c - N_S^c = A_2^2 = 2$, we have $N_S^c = 5$ and $N_A^c = 7$.

The topological amplitude of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^S M$ decay is constructed as

$$\begin{aligned} \mathcal{A}^S(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^S M) &= a_1^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i (\mathcal{B}_8^S)^{jkl} + a_2^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i (\mathcal{B}_8^S)^{jlk} + a_3^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i (\mathcal{B}_8^S)^{klj} + a_4^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j (\mathcal{B}_8^S)^{klm} \\ &+ a_5^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j (\mathcal{B}_8^S)^{kml} + a_6^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j (\mathcal{B}_8^S)^{lmk} + a_7^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k (\mathcal{B}_8^S)^{jlm} + a_8^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k (\mathcal{B}_8^S)^{jml} \\ &+ a_9^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k (\mathcal{B}_8^S)^{lmj} + a_{10}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l (\mathcal{B}_8^S)^{jkm} + a_{11}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l (\mathcal{B}_8^S)^{jmk} + a_{12}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l (\mathcal{B}_8^S)^{kmj} \\ &+ a_{13}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^l (\mathcal{B}_8^S)^{ikj} + a_{14}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^k (\mathcal{B}_8^S)^{ilj} + a_{15}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^m (\mathcal{B}_8^S)^{jkl} + a_{16}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^m (\mathcal{B}_8^S)^{jlk} \\ &+ a_{17}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^m (\mathcal{B}_8^S)^{klj} + a_{18}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i (\mathcal{B}_8^S)^{jkm} + a_{19}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i (\mathcal{B}_8^S)^{jmk} + a_{20}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i (\mathcal{B}_8^S)^{kmj} \end{aligned}$$

$$\begin{aligned}
& + a_{21}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^k(\mathcal{B}_8^S)^{imj} + a_{22}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^m(\mathcal{B}_8^S)^{ikj} + a_{23}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^S)^{jkm} + a_{24}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^S)^{jmk} \\
& + a_{25}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^S)^{kmj} + a_{26}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^k(\mathcal{B}_8^S)^{imj} + a_{27}^S(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^m(\mathcal{B}_8^S)^{ikj} + a_1^{c,S}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^S)^{jkl} \\
& + a_2^{c,S}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^S)^{jlk} + a_3^{c,S}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^S)^{klj} + a_4^{c,S}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^k(\mathcal{B}_8^S)^{ilj} + a_5^{c,S}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^S)^{ikj}. \quad (A5)
\end{aligned}$$

The topological amplitude of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^A M$ decay is constructed as

$$\begin{aligned}
\mathcal{A}(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^A M) = & a_1^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i(\mathcal{B}_8^A)^{jkl} + a_2^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i(\mathcal{B}_8^A)^{jlk} + a_3^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^i(\mathcal{B}_8^A)^{klj} + a_4^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j(\mathcal{B}_8^A)^{klm} \\
& + a_5^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j(\mathcal{B}_8^A)^{kml} + a_6^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^j(\mathcal{B}_8^A)^{lmk} + a_7^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k(\mathcal{B}_8^A)^{jlm} + a_8^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k(\mathcal{B}_8^A)^{jml} \\
& + a_9^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^k(\mathcal{B}_8^A)^{lmj} + a_{10}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l(\mathcal{B}_8^A)^{jkm} + a_{11}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l(\mathcal{B}_8^A)^{jmk} + a_{12}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^l(\mathcal{B}_8^A)^{kmj} \\
& + a_{13}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^l(\mathcal{B}_8^A)^{ikj} + a_{14}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^k(\mathcal{B}_8^A)^{ilj} + a_{15}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^m(\mathcal{B}_8^A)^{jkl} + a_{16}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^i M_m^m(\mathcal{B}_8^A)^{jlk} \\
& + a_{17}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^m(\mathcal{B}_8^A)^{klj} + a_{18}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i(\mathcal{B}_8^A)^{jkm} + a_{19}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i(\mathcal{B}_8^A)^{jmk} + a_{20}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^i(\mathcal{B}_8^A)^{kmj} \\
& + a_{21}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^k(\mathcal{B}_8^A)^{imj} + a_{22}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^l M_m^m(\mathcal{B}_8^A)^{ikj} + a_{23}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^A)^{jkm} + a_{24}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^A)^{jmk} \\
& + a_{25}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^i(\mathcal{B}_8^A)^{kmj} + a_{26}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^k(\mathcal{B}_8^A)^{imj} + a_{27}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^m(\mathcal{B}_8^A)^{ikj} + a_{28}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^m M_m^k(\mathcal{B}_8^A)^{ijl} \\
& + a_{29}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^l(\mathcal{B}_8^A)^{ijk} + a_{30}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^k(\mathcal{B}_8^A)^{ijm} + a_{31}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{kl}^m M_m^m(\mathcal{B}_8^A)^{ijk} + a_{32}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^k(\mathcal{B}_8^A)^{ijm} \\
& + a_{33}^A(\mathcal{B}_{b\bar{3}})_{ij} H_{lk}^l M_m^m(\mathcal{B}_8^A)^{ijk} + a_1^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^A)^{jkl} + a_2^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^A)^{jlk} + a_3^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^A)^{klj} \\
& + a_4^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^k(\mathcal{B}_8^A)^{ilj} + a_5^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^A)^{ikj} + a_6^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^i(\mathcal{B}_8^A)^{ijl} + a_7^{c,A}(\mathcal{B}_{b\bar{3}})_{ij} H_k^{(c)} M_l^l(\mathcal{B}_8^A)^{ijk}. \quad (A6)
\end{aligned}$$

Each term in Eqs. (A5) and (A6) can be interpreted as a topological diagram if the index contraction is understood as quark flowing. The topological diagrams contributing to the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^A M$ decay are shown in Fig. A1. The topological diagrams contributing to the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8^S M$ decay can be obtained by replacing the antisymmetric quarks of a_1 to a_{27} in Fig. A1 with symmetric ones. In Eqs. (A5) and (A6), we do not distinguish the tree and penguin contributions. Note that the same contractions with different operators in the weak vertex are different topological amplitudes. The light quark-loops in diagrams $a_{18-22}^{S,A}$, $a_{30,31}^A$ can be replaced by charm-loops; then, the $\mathcal{O}^{(c)}$ contributions are included.

The second-rank octet is given by

$$|\mathcal{B}_8\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{2/3}\Lambda^0 \end{pmatrix}. \quad (A7)$$

The bottom baryon antitriplet can be expressed using the Levi-Civita tensor as

$$|\mathcal{B}_{b\bar{3}}\rangle_{ij} = \epsilon_{ijk} |\mathcal{B}_{b\bar{3}}\rangle^k \quad \text{with} \quad |\mathcal{B}_{b\bar{3}}\rangle^k = \begin{pmatrix} \Xi_b^- \\ -\Xi_b^0 \\ \Lambda_b^0 \end{pmatrix}. \quad (A8)$$

The decay amplitude of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ mode could be constructed by a (1,1)-rank baryon octet and first-rank charmed baryon antitriplet. There are four and three covariant/contravariant indices in tensors $\langle(\mathcal{B}_8)_j^i M_l^k | \mathcal{O}_{mn}^{(u,p)q} | \mathcal{B}_{b\bar{3}}^p\rangle$ and $\langle(\mathcal{B}_8)_j^i M_l^k | \mathcal{O}_m^{(c)} | \mathcal{B}_{b\bar{3}}^n\rangle$, respectively. The number of full contractions is $N = A_4^4 = 24$ and $N^c = A_3^3 = 6$. Considering that $i \neq j$ in $(\mathcal{B}_8)_j^i$, the number of terms contributing to the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ mode is $N + N^c = 18 + 4 = 22$. The decay amplitude of the $\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M$ mode constructed by the second-rank octet tensors is

$$\begin{aligned}
\mathcal{A}(\mathcal{B}_{b\bar{3}} \rightarrow \mathcal{B}_8 M) = & A_1(\mathcal{B}_{b\bar{3}})^i H_{kl}^j M_l^i(\mathcal{B}_8)^k_j + A_2(\mathcal{B}_{b\bar{3}})^i H_{kl}^j M_j^l(\mathcal{B}_8)^k_i + A_3(\mathcal{B}_{b\bar{3}})^i H_{lk}^j M_l^i(\mathcal{B}_8)^k_j + A_4(\mathcal{B}_{b\bar{3}})^i H_{kl}^j M_j^l(\mathcal{B}_8)^k_i \\
& + A_5(\mathcal{B}_{b\bar{3}})^i H_{ik}^j M_l^i(\mathcal{B}_8)^k_j + A_6(\mathcal{B}_{b\bar{3}})^i H_{il}^j M_k^l(\mathcal{B}_8)^k_j + A_7(\mathcal{B}_{b\bar{3}})^i H_{ki}^j M_l^i(\mathcal{B}_8)^k_j + A_8(\mathcal{B}_{b\bar{3}})^i H_{li}^j M_k^l(\mathcal{B}_8)^k_j \\
& + A_9(\mathcal{B}_{b\bar{3}})^i H_{ik}^j M_l^i(\mathcal{B}_8)^k_j + A_{10}(\mathcal{B}_{b\bar{3}})^i H_{ki}^j M_l^i(\mathcal{B}_8)^k_j + A_{11}(\mathcal{B}_{b\bar{3}})^i H_{kj}^l M_l^i(\mathcal{B}_8)^k_i + A_{12}(\mathcal{B}_{b\bar{3}})^i H_{ij}^l M_k^l(\mathcal{B}_8)^k_i
\end{aligned}$$

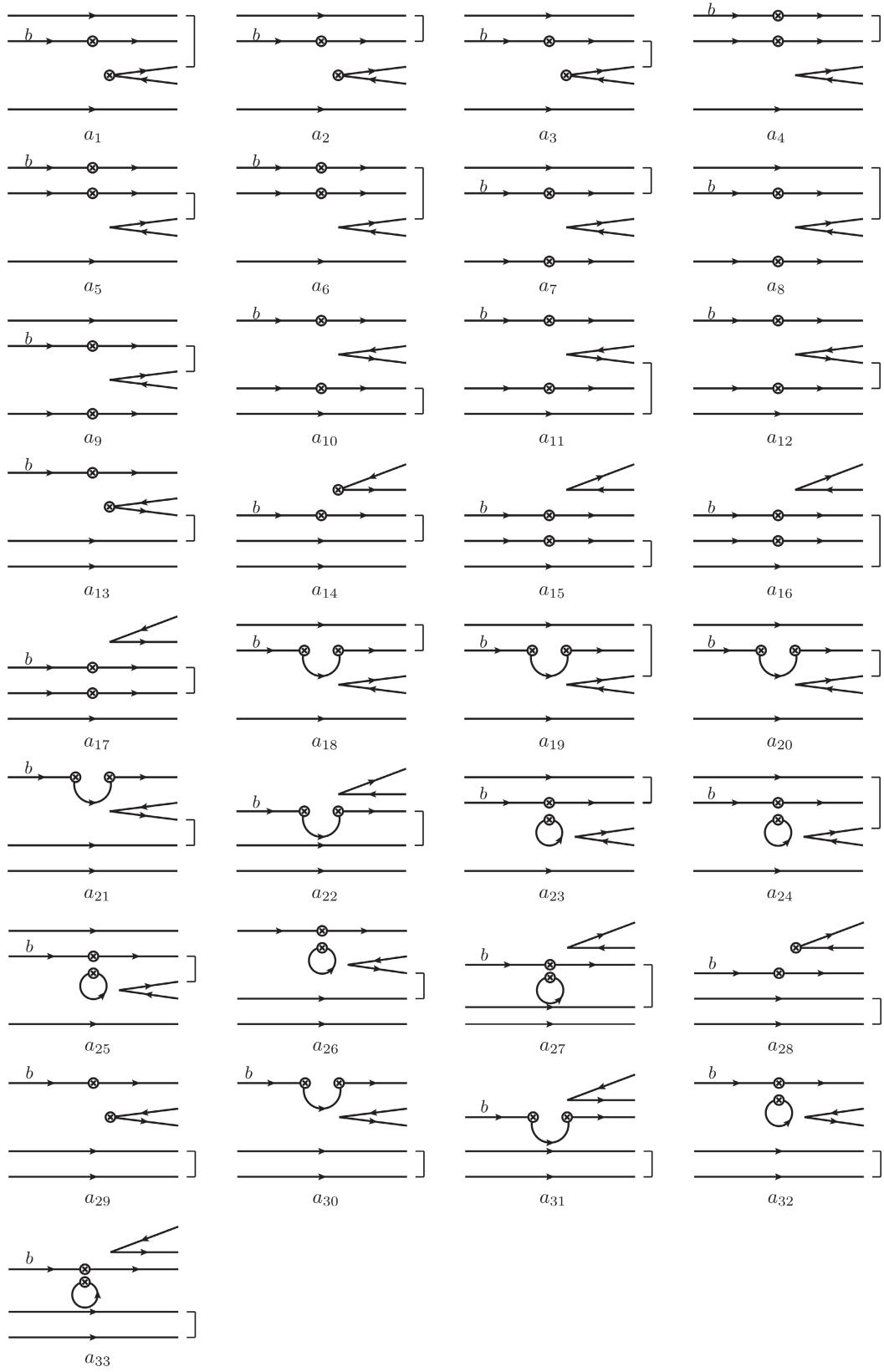


Fig. A1. Topological diagrams of bottom baryon $\mathcal{B}_{b\bar{3}}$ decays into a light baryon octet $\mathcal{B}_8^A$ and a light meson M , in which "j" indicates two antisymmetric light quarks in baryons.

$$\begin{aligned}
& + A_{13}(\mathcal{B}_{\bar{c}\bar{3}})^i H_{kj}^j M_l^l (\mathcal{B}_8)^k_i + A_{14}(\mathcal{B}_{\bar{b}\bar{3}})^i H_{ij}^j M_k^l (\mathcal{B}_8)^k_l + A_{15}(\mathcal{B}_{\bar{b}\bar{3}})^i H_{ji}^j M_k^l (\mathcal{B}_8)^k_i + A_{16}(\mathcal{B}_{\bar{b}\bar{3}})^i H_{jk}^j M_l^l (\mathcal{B}_8)^k_i + A_{17}(\mathcal{B}_{\bar{b}\bar{3}})^i H_{jk}^j M_l^l (\mathcal{B}_8)^k_l \\
& + A_{18}(\mathcal{B}_{\bar{b}\bar{3}})^i H_{ji}^j M_k^l (\mathcal{B}_8)^k_l + A_1^c(\mathcal{B}_{\bar{b}\bar{3}})^i H_j^{(c)} M_k^l (\mathcal{B}_8)^k_i + A_2^c(\mathcal{B}_{\bar{b}\bar{3}})^i H_k^{(c)} M_j^l (\mathcal{B}_8)^k_i + A_3^c(\mathcal{B}_{\bar{b}\bar{3}})^i H_k^{(c)} M_l^j (\mathcal{B}_8)^k_j + A_4^c(\mathcal{B}_{\bar{b}\bar{3}})^i H_i^{(c)} M_k^l (\mathcal{B}_8)^k_j. \quad (A9)
\end{aligned}$$

By substituting Eqs. (8) and (A8) into each term of Eqs. (A5) and (A6), we can derive the relations between decay amplitudes constructed by (1,1)- and third-rank octets. For example, the first term of Eq. (A5) is simplified as

$$\begin{aligned}
a_1^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H_{kl}^m M_m^i (\mathcal{B}_8)^{jkl} &= \frac{1}{\sqrt{6}} a_1^S \epsilon_{ijp} (\mathcal{B}_{\bar{b}\bar{3}})^p H_{kl}^m M_m^i \epsilon^{ljq} (\mathcal{B}_8)^k_q + \frac{1}{\sqrt{6}} a_1^S \epsilon_{ijp} (\mathcal{B}_{\bar{b}\bar{3}})^p H_{kl}^m M_m^i \epsilon^{lkq} (\mathcal{B}_8)^j_q = -\frac{1}{\sqrt{6}} a_1^S (\mathcal{B}_{\bar{b}\bar{3}})^i H_{lk}^j M_j^l (\mathcal{B}_8)^k_i \\
&+ \frac{2}{\sqrt{6}} a_1^S (\mathcal{B}_{\bar{b}\bar{3}})^i H_{kl}^j M_j^l (\mathcal{B}_8)^k_i + \frac{1}{\sqrt{6}} a_1^S (\mathcal{B}_{\bar{b}\bar{3}})^i H_{ik}^j M_j^l (\mathcal{B}_8)^k_l - \frac{2}{\sqrt{6}} a_1^S (\mathcal{B}_{\bar{b}\bar{3}})^i H_{ki}^j M_j^l (\mathcal{B}_8)^k_l. \quad (A10)
\end{aligned}$$

Thus, the diagram a_1^S contributes to Eq. (A9) as

$$A_2 = -\frac{1}{\sqrt{6}} a_1^S + \dots, \quad A_4 = \frac{2}{\sqrt{6}} a_1^S + \dots, \quad A_5 = \frac{1}{\sqrt{6}} a_1^S + \dots, \quad A_7 = -\frac{2}{\sqrt{6}} a_1^S + \dots. \quad (A11)$$

The relations between the third- and (1,1)-rank topological amplitudes for the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8 M$ modes are derived as follows:

$$\begin{aligned}
A_1 &= (a_6^A + a_{11}^A + a_{12}^A) / \sqrt{2} - (a_4^S - a_5^S + 2a_{10}^S - a_{11}^S - a_{12}^S) / \sqrt{6}, \\
A_2 &= (-a_1^A + a_3^A - a_5^A - a_8^A - a_9^A + a_{14}^A + 2a_{28}^A) / \sqrt{2} - (a_1^S - 2a_2^S + a_3^S - a_4^S + a_6^S - 2a_7^S + a_8^S + a_9^S + 3a_{14}^S) / \sqrt{6}, \\
A_3 &= (a_5^A + a_8^A + a_9^A) / \sqrt{2} - (a_4^S - a_6^S + 2a_7^S - a_8^S - a_9^S) / \sqrt{6}, \\
A_4 &= (-a_2^A - a_3^A - a_6^A - a_{11}^A - a_{12}^A + a_{13}^A + 2a_{29}^A) / \sqrt{2} - (-2a_1^S + a_2^S + a_3^S - a_4^S + a_5^S - 2a_{10}^S + a_{11}^S + a_{12}^S + 3a_{13}^S) / \sqrt{6}, \\
A_5 &= (a_1^A - a_3^A + a_5^A) / \sqrt{2} - (-a_1^S + 2a_2^S - a_3^S + a_4^S - a_6^S) / \sqrt{6}, \\
A_6 &= (-a_4^A + a_{10}^A - a_{12}^A) / \sqrt{2} - (-a_5^S + a_6^S - a_{10}^S + 2a_{11}^S - a_{12}^S) / \sqrt{6}, \\
A_7 &= (a_2^A + a_3^A + a_6^A) / \sqrt{2} - (2a_1^S - a_2^S - a_3^S + a_4^S - a_5^S) / \sqrt{6}, \\
A_8 &= (a_4^A + a_7^A - a_9^A) / \sqrt{2} - (a_5^S - a_6^S - a_7^S + 2a_8^S - a_9^S) / \sqrt{6}, \\
A_9 &= (-a_5^A + a_{15}^A - a_{17}^A) / \sqrt{2} - (-a_4^S + a_6^S - a_{15}^S + 2a_{16}^S - a_{17}^S) / \sqrt{6}, \\
A_{10} &= (-a_6^A + a_{16}^A + a_{17}^A) / \sqrt{2} - (-a_4^S + a_5^S + 2a_{15}^S - a_{16}^S - a_{17}^S) / \sqrt{6}, \\
A_{11} &= (-a_6^A + a_{19}^A + a_{20}^A) / \sqrt{2} - (-a_4^S + a_5^S + 2a_{18}^S - a_{19}^S - a_{20}^S) / \sqrt{6}, \\
A_{12} &= (-a_4^A - a_7^A + a_9^A - a_{18}^A + a_{20}^A + a_{21}^A + 2a_{30}^A) / \sqrt{2} - (-a_5^S + a_6^S + a_7^S - 2a_8^S + a_9^S + a_{18}^S - 2a_{19}^S + a_{20}^S + 3a_{21}^S) / \sqrt{6}, \\
A_{13} &= (a_6^A - a_{16}^A - a_{17}^A - a_{19}^A - a_{20}^A + a_{22}^A + 2a_{31}^A) / \sqrt{2} - (a_4^S - a_5^S - 2a_{15}^S + a_{16}^S + a_{17}^S - 2a_{18}^S + a_{19}^S + a_{20}^S + 3a_{22}^S) / \sqrt{6}, \\
A_{14} &= (a_4^A + a_{18}^A - a_{20}^A) / \sqrt{2} - (a_5^S - a_6^S - a_{18}^S + 2a_{19}^S - a_{20}^S) / \sqrt{6}, \\
A_{15} &= (a_4^A - a_{10}^A + a_{12}^A - a_{23}^A + a_{25}^A + a_{26}^A + 2a_{32}^A) / \sqrt{2} - (a_5^S - a_6^S + a_{10}^S - 2a_{11}^S + a_{12}^S + a_{23}^S - 2a_{24}^S + a_{25}^S + 3a_{26}^S) / \sqrt{6}, \\
A_{16} &= (a_5^A - a_{15}^A + a_{17}^A - a_{24}^A - a_{25}^A + a_{27}^A + 2a_{33}^A) / \sqrt{2} - (a_4^S - a_6^S + a_{15}^S - 2a_{16}^S + a_{17}^S - 2a_{23}^S + a_{24}^S + a_{25}^S + 3a_{27}^S) / \sqrt{6}, \\
A_{17} &= (-a_5^A + a_{24}^A + a_{25}^A) / \sqrt{2} - (-a_4^S + a_6^S + 2a_{23}^S - a_{24}^S - a_{25}^S) / \sqrt{6}, \\
A_{18} &= (-a_4^A + a_{23}^A - a_{25}^A) / \sqrt{2} - (-a_5^S + a_6^S - a_{23}^S + 2a_{24}^S - a_{25}^S) / \sqrt{6} \\
A_1^c &= (-a_1^{c,A} + a_3^{c,A} + a_4^A + 2a_6^{c,A}) / \sqrt{2} - (a_1^{c,S} - 2a_2^{c,S} + a_3^{c,S} + 3a_4^{c,S}) / \sqrt{6}, \\
A_2^c &= (-a_2^{c,A} - a_3^{c,A} + a_5^{c,A} + 2a_7^{c,A}) / \sqrt{2} - (-2a_1^{c,S} + a_2^{c,S} + a_3^{c,S} + 3a_5^{c,S}) / \sqrt{6}, \\
A_3^c &= (a_2^{c,A} + a_3^{c,A}) / \sqrt{2} - (2a_1^{c,S} - a_2^{c,S} - a_3^{c,S}) / \sqrt{6}, \\
A_4^c &= (a_1^{c,A} - a_3^{c,A}) / \sqrt{2} - (-a_1^{c,S} + 2a_2^{c,S} - a_3^{c,S}) / \sqrt{6}. \quad (A12)
\end{aligned}$$

According to Eq. (A12), the topological diagrams of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^S M$ and $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^A M$ transitions are not independent. If the topological diagrams of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^S M$ mode are known, the topological diagrams of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^A M$ mode are determined, and vice versa.

Operator $\mathcal{O}_{ij}^k$ is a reducible representation of the $SU(3)$ group. It can be decomposed as $3 \otimes \bar{3} \otimes 3 = 3_p \oplus 3_t \oplus \bar{6} \oplus 15$. Then, the coefficient matrix H can be written as

$$H_{ij}^k = \frac{1}{8} H(15)_{ij}^k + \frac{1}{4} \epsilon_{ijl} H(\bar{6})^{lk} + \delta_i^k \left(\frac{3}{8} H(3_t)_i - \frac{1}{8} H(3_p)_i \right) + \delta_i^k \left(\frac{3}{8} H(3_p)_j - \frac{1}{8} H(3_t)_j \right), \quad (\text{A13})$$

in which $H(3_p)_i = H_{il}^l$ and $H(3_t)_i = H_{il}^l$. The $SU(3)$ irreducible amplitude of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8 M$ decay is constructed as

$$\begin{aligned} \mathcal{A}^{\text{IR}}(\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8 M) = & b_1 (\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_l^j (\mathcal{B}_8)_k^l + b_2 (\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_k^l (\mathcal{B}_8)_l^j + b_3 (\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_l^j (\mathcal{B}_8)_k^l \\ & + b_4 (\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_i^j (\mathcal{B}_8)_k^l + b_5 (\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_k^l (\mathcal{B}_8)_i^j + b_6 (\mathcal{B}_{\bar{b}\bar{3}})^i H(15)_{ij}^k M_l^j (\mathcal{B}_8)_k^l \\ & + b_7 (\mathcal{B}_{\bar{b}\bar{3}})^i H(15)_{ij}^k M_k^l (\mathcal{B}_8)_l^j + b_8 (\mathcal{B}_{\bar{b}\bar{3}})^i H(15)_{ij}^k M_l^j (\mathcal{B}_8)_k^l + b_9 (\mathcal{B}_{\bar{b}\bar{3}})^i H(15)_{ij}^k M_i^j (\mathcal{B}_8)_k^l \\ & + b_{10} (\mathcal{B}_{\bar{b}\bar{3}})^i H(15)_{ij}^k M_k^l (\mathcal{B}_8)_i^j + b_{11} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_p)_i M_k^j (\mathcal{B}_8)_j^k + b_{12} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_p)_k M_j^i (\mathcal{B}_8)_i^k \\ & + b_{13} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_p)_k M_i^j (\mathcal{B}_8)_j^k + b_{14} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_p)_k M_j^i (\mathcal{B}_8)_i^k + b_{15} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_t)_i M_k^j (\mathcal{B}_8)_j^k \\ & + b_{16} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_t)_k M_j^i (\mathcal{B}_8)_i^k + b_{17} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_t)_k M_i^j (\mathcal{B}_8)_j^k + b_{18} (\mathcal{B}_{\bar{b}\bar{3}})^i H(3_t)_k M_j^i (\mathcal{B}_8)_i^k \\ & + b_1^c (\mathcal{B}_{\bar{b}\bar{3}})^i H(3)_j^{(c)} M_k^j (\mathcal{B}_8)_i^k + b_2^c (\mathcal{B}_{\bar{b}\bar{3}})^i H(3)_k^{(c)} M_j^i (\mathcal{B}_8)_i^k + b_3^c (\mathcal{B}_{\bar{b}\bar{3}})^i H(3)_k^{(c)} M_i^j (\mathcal{B}_8)_j^k \\ & + b_4^c (\mathcal{B}_{\bar{b}\bar{3}})^i H(3)_i^{(c)} M_k^j (\mathcal{B}_8)_j^k. \end{aligned} \quad (\text{A14})$$

By substituting Eq. (A13) into each term of Eq. (A9), the relations between Eqs. (A9) and (A14) are derived as

$$\begin{aligned} b_1 &= \frac{A_6 - A_8}{4}, & b_2 &= \frac{A_5 - A_7}{4}, & b_3 &= \frac{A_9 - A_{10}}{4}, & b_4 &= \frac{-A_1 + A_3}{4}, & b_5 &= \frac{-A_2 + A_4}{4}, \\ b_6 &= \frac{A_6 + A_8}{8}, & b_7 &= \frac{A_5 + A_7}{8}, & b_8 &= \frac{A_9 + A_{10}}{8}, & b_9 &= \frac{A_1 + A_3}{8}, & b_{10} &= \frac{A_2 + A_4}{8}, \\ b_{11} &= -\frac{1}{8} A_5 + \frac{3}{8} A_7 - \frac{1}{8} A_6 + \frac{3}{8} A_8 + A_{18}, & b_{15} &= \frac{3}{8} A_5 - \frac{1}{8} A_7 + \frac{3}{8} A_6 - \frac{1}{8} A_8 + A_{14}, \\ b_{12} &= \frac{3}{8} A_2 - \frac{1}{8} A_4 + \frac{3}{8} A_9 - \frac{1}{8} A_{10} + A_{16}, & b_{16} &= -\frac{1}{8} A_2 + \frac{3}{8} A_4 - \frac{1}{8} A_9 + \frac{3}{8} A_{10} + A_{13}, \\ b_{13} &= -\frac{1}{8} A_1 + \frac{3}{8} A_3 + \frac{3}{8} A_5 - \frac{1}{8} A_7 + A_{17}, & b_{17} &= \frac{3}{8} A_1 - \frac{1}{8} A_3 - \frac{1}{8} A_5 + \frac{3}{8} A_7 + A_{11}, \\ b_{14} &= -\frac{1}{8} A_2 + \frac{3}{8} A_4 + \frac{3}{8} A_6 - \frac{1}{8} A_8 + A_{15}, & b_{18} &= \frac{3}{8} A_2 - \frac{1}{8} A_4 - \frac{1}{8} A_6 + \frac{3}{8} A_8 + A_{12}, \\ b_1^c &= A_1^c, & b_2^c &= A_2^c, & b_3^c &= A_3^c, & b_4^c &= A_4^c. \end{aligned} \quad (\text{A15})$$

The inverse solution of Eq. (A15) is

$$\begin{aligned} A_1 &= -2b_4 + 4b_9, & A_2 &= -2b_5 + 4b_{10}, & A_3 &= 2b_4 + 4b_9, & A_4 &= 2b_5 + 4b_{10}, & A_5 &= 2b_2 + 4b_7, \\ A_6 &= 2b_1 + 4b_6, & A_7 &= -2b_2 + 4b_7, & A_8 &= -2b_1 + 4b_6, & A_9 &= 2b_3 + 4b_8, & A_{10} &= -2b_3 + 4b_8, \\ A_{11} &= b_{17} + b_2 + b_4 - b_7 - b_9, & A_{12} &= b_{18} + b_1 + b_5 - b_6 - b_{10}, & A_{13} &= b_{16} + b_3 - b_5 - b_8 - b_{10}, \\ A_{14} &= b_{15} - b_1 - b_2 - b_6 - b_7, & A_{15} &= b_{14} - b_1 - b_5 - b_6 - b_{10}, & A_{16} &= b_{12} - b_3 + b_5 - b_8 - b_{10}, \\ A_{17} &= b_{13} - b_2 - b_4 - b_7 - b_9, & A_{18} &= b_{11} + b_1 + b_2 - b_6 - b_7 \\ A_1^c &= b_1^c, & A_2^c &= b_2^c, & A_3^c &= b_3^c, & A_4^c &= b_4^c. \end{aligned} \quad (\text{A16})$$

By substituting Eq. (A12) into Eq. (A15), we obtain the equations for $SU(3)$ irreducible amplitudes decomposed in terms of topological amplitudes,

$$\begin{aligned}
b_1 &= -\sqrt{2}(2a_4^A + a_7^A - a_9^A - a_{10}^A + a_{12}^A)/8 + \sqrt{6}(2a_5^S - 2a_6^S - a_7^S + 2a_8^S - a_9^S + a_{10}^S - 2a_{11}^S + a_{12}^S)/24, \\
b_2 &= \sqrt{2}(a_1^A - a_2^A - 2a_3^A + a_5^A - a_6^A)/8 + \sqrt{6}(3a_1^S - 3a_2^S - a_5^S + a_6^S)/24, \\
b_3 &= -\sqrt{2}(a_5^A - a_6^A - a_{15}^A + a_{16}^A + 2a_{17}^A)/8 + \sqrt{6}(a_5^S - a_6^S + 3a_{15}^S - 3a_{16}^S)/24, \\
b_4 &= \sqrt{2}(a_5^A - a_6^A + a_8^A + a_9^A - a_{11}^A - a_{12}^A)/8 - \sqrt{6}(a_5^S - a_6^S + 2a_7^S - a_8^S - a_9^S - 2a_{10}^S + a_{11}^S + a_{12}^S)/24, \\
b_5 &= \sqrt{2}(a_1^A - a_2^A - 2a_3^A + a_5^A - a_6^A + a_8^A + a_9^A - a_{11}^A - a_{12}^A + a_{13}^A - a_{14}^A - 2a_{28}^A + 2a_{29}^A)/8 \\
&\quad + \sqrt{6}(3a_1^S - 3a_2^S - a_5^S + a_6^S - 2a_7^S + a_8^S + a_9^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S + 3a_{14}^S)/24, \\
b_6 &= \sqrt{2}(a_7^A - a_9^A + a_{10}^A - a_{12}^A)/16 + \sqrt{6}(a_7^S - 2a_8^S + a_9^S + a_{10}^S - 2a_{11}^S + a_{12}^S)/48, \\
b_7 &= (a_1^A + a_2^A + a_5^A + a_6^A)/16 - \sqrt{6}(a_1^S + a_2^S - 2a_3^S + 2a_4^S - a_5^S - a_6^S)/48, \\
b_8 &= -\sqrt{2}(a_5^A + a_6^A - a_{15}^A - a_{16}^A)/16 + \sqrt{6}(2a_4^S - a_5^S - a_6^S - a_{15}^S - a_{16}^S + 2a_{17}^S)/48, \\
b_9 &= \sqrt{2}(a_5^A + a_6^A + a_8^A + a_9^A + a_{11}^A + a_{12}^A)/16 + \sqrt{6}(-2a_4^S + a_5^S + a_6^S - 2a_7^S + a_8^S + a_9^S - 2a_{10}^S + a_{11}^S + a_{12}^S)/48, \\
b_{10} &= -\sqrt{2}(a_1^A + a_2^A + a_5^A + a_6^A + a_8^A + a_9^A + a_{11}^A + a_{12}^A - a_{13}^A - a_{14}^A - 2a_{28}^A - 2a_{29}^A)/16 \\
&\quad + \sqrt{6}(a_1^S + a_2^S - 2a_3^S + 2a_4^S - a_5^S - a_6^S + 2a_7^S - a_8^S - a_9^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S - 3a_{14}^S)/48, \\
b_{11} &= -\sqrt{2}(a_1^A - 3a_2^A - 4a_3^A + 4a_4^A + a_5^A - 3a_6^A - 3a_7^A + 3a_9^A + a_{10}^A - a_{12}^A - 8a_{23}^A + 8a_{25}^A)/16 \\
&\quad - \sqrt{6}(7a_1^S - 5a_2^S - 2a_3^S + 2a_4^S - 7a_5^S + 5a_6^S - 3a_7^S + 6a_8^S - 3a_9^S + a_{10}^S - 2a_{11}^S + a_{12}^S - 8a_{23}^S + 16a_{24}^S - 8a_{25}^S)/48, \\
b_{12} &= -\sqrt{2}(3a_1^A - a_2^A - 4a_3^A - 2a_5^A - 2a_6^A + 3a_8^A + 3a_9^A - a_{11}^A - a_{12}^A + a_{13}^A - 3a_{14}^A + 5a_{15}^A + a_{16}^A - 4a_{17}^A \\
&\quad + 8a_{24}^A + 8a_{25}^A - 8a_{27}^A - 6a_{28}^A + 2a_{29}^A - 16a_{33}^A)/16 - \sqrt{6}(5a_1^S - 7a_2^S + 2a_3^S + 4a_4^S - 2a_5^S - 2a_6^S - 6a_7^S + 3a_8^S \\
&\quad + 3a_9^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 3a_{13}^S + 9a_{14}^S + 3a_{15}^S - 9a_{16}^S + 6a_{17}^S - 16a_{23}^S + 8a_{24}^S + 8a_{25}^S + 24a_{27}^S)/48, \\
b_{13} &= \sqrt{2}(3a_1^A - a_2^A - 4a_3^A - 2a_5^A - 2a_6^A + 3a_8^A + 3a_9^A - a_{11}^A - a_{12}^A + 8a_{24}^A + 8a_{25}^A)/16 \\
&\quad + \sqrt{6}(5a_1^S - 7a_2^S + 2a_3^S + 4a_4^S - 2a_5^S - 2a_6^S - 6a_7^S + 3a_8^S + 3a_9^S + 2a_{10}^S - a_{11}^S - a_{12}^S - 16a_{23}^S + 8a_{24}^S + 8a_{25}^S)/48, \\
b_{14} &= \sqrt{2}(a_1^A - 3a_2^A - 4a_3^A + 4a_4^A + a_5^A - 3a_6^A - a_7^A + a_8^A + 2a_9^A - 5a_{10}^A - 3a_{11}^A + 2a_{12}^A + 3a_{13}^A - a_{14}^A - 8a_{23}^A + 8a_{25}^A \\
&\quad + 8a_{26}^A - 2a_{28}^A + 6a_{29}^A + 16a_{32}^A)/16 + \sqrt{6}(7a_1^S - 5a_2^S - 2a_3^S + 2a_4^S - 7a_5^S + 5a_6^S - 3a_7^S + 3a_8^S + a_{10}^S + 7a_{11}^S \\
&\quad - 8a_{12}^S - 9a_{13}^S + 3a_{14}^S - 8a_{23}^S + 16a_{24}^S - 8a_{25}^S - 24a_{26}^S)/48, \\
b_{15} &= \sqrt{2}(3a_1^A - a_2^A - 4a_3^A + 4a_4^A + 3a_5^A - a_6^A - a_7^A + a_9^A + 3a_{10}^A - 3a_{12}^A + 8a_{18}^A - 8a_{20}^A)/16 \\
&\quad + \sqrt{6}(5a_1^S - 7a_2^S + 2a_3^S - 2a_4^S - 5a_5^S + 7a_6^S - a_7^S + 2a_8^S - a_9^S + 3a_{10}^S - 6a_{11}^S + 3a_{12}^S + 8a_{18}^S - 16a_{19}^S + 8a_{20}^S)/48, \\
b_{16} &= \sqrt{2}(a_1^A - 3a_2^A - 4a_3^A + 2a_5^A + 2a_6^A + a_8^A + a_9^A - 3a_{11}^A - 3a_{12}^A + 3a_{13}^A - a_{14}^A - a_{15}^A - 5a_{16}^A - 4a_{17}^A - 8a_{19}^A - 8a_{20}^A \\
&\quad + 8a_{22}^A - 2a_{28}^A + 6a_{29}^A + 16a_{31}^A)/16 + \sqrt{6}(7a_1^S - 5a_2^S - 2a_3^S - 4a_4^S + 2a_5^S + 2a_6^S - 2a_7^S + a_8^S + a_9^S + 6a_{10}^S \\
&\quad - 3a_{11}^S - 3a_{12}^S - 9a_{13}^S + 3a_{14}^S + 9a_{15}^S - 3a_{16}^S - 6a_{17}^S + 16a_{18}^S - 8a_{19}^S - 8a_{20}^S - 24a_{22}^S)/48, \\
b_{17} &= -\sqrt{2}(a_1^A - 3a_2^A - 4a_3^A + 2a_5^A + 2a_6^A + a_8^A + a_9^A - 3a_{11}^A - 3a_{12}^A - 8a_{19}^A - 8a_{20}^A)/16 \\
&\quad - \sqrt{6}(7a_1^S - 5a_2^S - 2a_3^S - 4a_4^S + 2a_5^S + 2a_6^S - 2a_7^S + a_8^S + a_9^S + 6a_{10}^S - 3a_{11}^S - 3a_{12}^S + 16a_{18}^S - 8a_{19}^S - 8a_{20}^S)/48, \\
b_{18} &= -\sqrt{2}(3a_1^A - a_2^A - 4a_3^A + 4a_4^A + 3a_5^A - a_6^A + 5a_7^A + 3a_8^A - 2a_9^A + a_{10}^A - a_{11}^A - 2a_{12}^A + a_{13}^A - 3a_{14}^A + 8a_{18}^A - 8a_{20}^A \\
&\quad - 8a_{21}^A - 6a_{28}^A + 2a_{29}^A - 16a_{30}^A)/16 - \sqrt{6}(5a_1^S - 7a_2^S + 2a_3^S - 2a_4^S - 5a_5^S + 7a_6^S - a_7^S - 7a_8^S + 8a_9^S + 3a_{10}^S \\
&\quad - 3a_{11}^S - 3a_{13}^S + 9a_{14}^S + 8a_{18}^S - 16a_{19}^S + 8a_{20}^S + 24a_{21}^S)/48, \\
b_1^c &= (-a_1^{c,A} + a_3^{c,A} + a_4^A + 2a_6^{c,A})/\sqrt{2} - (a_1^{c,S} - 2a_2^{c,S} + a_3^{c,S} + 3a_4^{c,S})/\sqrt{6},
\end{aligned}$$

$$\begin{aligned}
b_2^c &= (-a_2^{c,A} - a_3^{c,A} + a_5^{c,A} + 2a_7^{c,A})/\sqrt{2} - (-2a_1^{c,S} + a_2^{c,S} + a_3^{c,S} + 3a_5^{c,S})/\sqrt{6}, \\
b_3^c &= (a_2^{c,A} + a_3^{c,A})/\sqrt{2} - (2a_1^{c,S} - a_2^{c,S} - a_3^{c,S})/\sqrt{6}, \\
b_4^c &= (a_1^{c,A} - a_3^{c,A})/\sqrt{2} - (-a_1^{c,S} + 2a_2^{c,S} - a_3^{c,S})/\sqrt{6}.
\end{aligned} \tag{A17}$$

Note that the inverse solution does not exist because the number of terms of topological amplitudes is larger than the $SU(3)$ irreducible amplitudes.

The terms constructed by $H(\bar{6})_{ij}^k$ in Eq. (A14) can be written as

$$\begin{aligned}
b_1(\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_l^j(\mathcal{B}_8)_k^l &= b_1(\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_l^j(\mathcal{B}_8)_k^l, \\
b_2(\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_k^l(\mathcal{B}_8)_l^j &= b_2(\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_k^l(\mathcal{B}_8)_l^j, \\
b_3(\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{ij}^k M_l^l(\mathcal{B}_8)_k^j &= b_3(\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_l^l(\mathcal{B}_8)_k^j, \\
b_4(\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{jl}^k M_i^j(\mathcal{B}_8)_k^l &= b_4[(\mathcal{B}_{\bar{b}\bar{3}})_{jl} H(\bar{6})^{ki} M_i^j(\mathcal{B}_8)_k^l - (\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_i^j(\mathcal{B}_8)_k^l + (\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_l^l(\mathcal{B}_8)_k^j], \\
b_5(\mathcal{B}_{\bar{b}\bar{3}})^i H(\bar{6})_{jl}^k M_k^l(\mathcal{B}_8)_i^j &= -b_5[(\mathcal{B}_{\bar{b}\bar{3}})_{ji} H(\bar{6})^{ki} M_k^l(\mathcal{B}_8)_i^j + (\mathcal{B}_{\bar{b}\bar{3}})_{jl} H(\bar{6})^{ki} M_i^j(\mathcal{B}_8)_k^l],
\end{aligned} \tag{A18}$$

where $(\mathcal{B}_{\bar{b}\bar{3}})^i = \epsilon^{ijk}(\mathcal{B}_{\bar{b}\bar{3}})_{jk}/2$ is used. There are four invariant tensors in the RHS of Eq. (A18). It indicates that the degree of freedom of the $SU(3)$ irreducible amplitudes associated with the $\bar{6}$ representation is four. We can define four new parameters to replace $b_1 \sim b_5$,

$$b'_1 = b_1 - b_4, \quad b'_2 = b_2 - b_5, \quad b'_3 = b_3 + b_4, \quad b'_4 = b_4 - b_5. \tag{A19}$$

The amplitude of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8 M$ decay can also be constructed by third-rank octet tensors and $SU(3)$ irreducible operators. The $SU(3)$ irreducible amplitude of the $\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^S M$ decay constructed by third-rank octet tensors is

$$\begin{aligned}
\mathcal{A}^S(\mathcal{B}_{\bar{b}\bar{3}} \rightarrow \mathcal{B}_8^S M) &= c_1^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^m M_m^i(\mathcal{B}_8^S)^{jkl} + c_2^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^m M_m^i(\mathcal{B}_8^S)^{jkl} + c_3^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^m M_m^i(\mathcal{B}_8^S)^{klj} \\
&+ c_4^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^j(\mathcal{B}_8^S)^{klm} + c_5^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^j(\mathcal{B}_8^S)^{kml} + c_6^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^i M_m^j(\mathcal{B}_8^S)^{kml} \\
&+ c_7^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^k(\mathcal{B}_8^S)^{jlm} + c_8^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^k(\mathcal{B}_8^S)^{jml} + c_9^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^k(\mathcal{B}_8^S)^{lmj} \\
&+ c_{10}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^S)^{jlm} + c_{11}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^S)^{jml} + c_{12}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^S)^{lmj} \\
&+ c_{13}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^m M_m^l(\mathcal{B}_8^S)^{ikj} + c_{14}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^m M_m^l(\mathcal{B}_8^S)^{ikj} + c_{15}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^m(\mathcal{B}_8^S)^{jkl} \\
&+ c_{16}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(\bar{6})_{kl}^i M_m^m(\mathcal{B}_8^S)^{jkl} + c_{17}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(15)_{kl}^i M_m^m(\mathcal{B}_8^S)^{klj} + c_{18}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_t)_k M_m^i(\mathcal{B}_8^S)^{jkm} \\
&+ c_{19}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_t)_k M_m^i(\mathcal{B}_8^S)^{jmk} + c_{20}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_t)_k M_m^i(\mathcal{B}_8^S)^{kmj} + c_{21}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_t)_k M_m^k(\mathcal{B}_8^S)^{imj} \\
&+ c_{22}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_t)_k M_m^m(\mathcal{B}_8^S)^{ikj} + c_{23}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_p)_k M_m^i(\mathcal{B}_8^S)^{jkm} + c_{24}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_p)_k M_m^i(\mathcal{B}_8^S)^{jmk} \\
&+ c_{25}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_p)_k M_m^i(\mathcal{B}_8^S)^{kmj} + c_{26}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_p)_k M_m^k(\mathcal{B}_8^S)^{imj} + c_{27}^S(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3_p)_k M_m^m(\mathcal{B}_8^S)^{ikj} \\
&+ c_1^{c,S}(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3)_k^c M_l^i(\mathcal{B}_8^S)^{jkl} + c_2^{c,S}(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3)_k^c M_l^i(\mathcal{B}_8^S)^{jlk} + c_3^{c,S}(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3)_k^c M_l^i(\mathcal{B}_8^S)^{klj} \\
&+ c_4^{c,S}(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3)_k^c M_l^k(\mathcal{B}_8^S)^{ilj} + c_5^{c,S}(\mathcal{B}_{\bar{b}\bar{3}})_{ij} H(3)_k^c M_l^l(\mathcal{B}_8^S)^{ikj}.
\end{aligned} \tag{A20}$$

The relations of the $SU(3)$ irreducible amplitudes and the topological amplitudes are derived as

$$\begin{aligned}
c_1^S &= \frac{a_1^S + a_2^S}{8}, \quad c_2^S = \frac{a_1^S - a_2^S}{4}, \quad c_3^S = \frac{a_3^S}{8}, \quad c_4^S = \frac{a_4^S}{8}, \quad c_5^S = \frac{a_5^S + a_6^S}{8}, \quad c_6^S = \frac{a_5^S - a_6^S}{4}, \\
c_7^S &= \frac{a_7^S + a_{10}^S}{8}, \quad c_{10}^S = \frac{a_7^S - a_{10}^S}{4}, \quad c_8^S = \frac{a_8^S + a_{11}^S}{8}, \quad c_{11}^S = \frac{a_8^S - a_{11}^S}{4}, \quad c_9^S = \frac{a_9^S + a_{12}^S}{8}, \quad c_{12}^S = \frac{a_9^S - a_{12}^S}{4},
\end{aligned}$$

$$\begin{aligned}
c_{13}^S &= \frac{a_{13}^S + a_{14}^S}{8}, \quad c_{14}^S = \frac{a_{13}^S - a_{14}^S}{4}, \quad c_{15}^S = \frac{a_{15}^S + a_{16}^S}{8}, \quad c_{16}^S = \frac{a_{15}^S - a_{16}^S}{4}, \quad c_{17}^S = \frac{a_{17}^S}{8}, \\
c_{18}^S &= \frac{3}{8}a_1^S - \frac{1}{8}a_2^S - \frac{1}{4}a_4^S - \frac{1}{8}a_7^S + \frac{3}{8}a_{10}^S + a_{18}^S, \quad c_{23}^S = -\frac{1}{8}a_1^S + \frac{3}{8}a_2^S - \frac{1}{4}a_4^S + \frac{3}{8}a_7^S - \frac{1}{8}a_{10}^S + a_{23}^S, \\
c_{19}^S &= -\frac{1}{8}a_1^S + \frac{3}{8}a_2^S + \frac{1}{8}a_5^S - \frac{3}{8}a_6^S - \frac{1}{8}a_8^S + \frac{3}{8}a_{11}^S + a_{19}^S, \quad c_{24}^S = \frac{3}{8}a_1^S - \frac{1}{8}a_2^S - \frac{3}{8}a_5^S + \frac{1}{8}a_6^S + \frac{3}{8}a_8^S - \frac{1}{8}a_{11}^S + a_{24}^S, \\
c_{20}^S &= \frac{1}{4}a_3^S - \frac{3}{8}a_5^S + \frac{1}{8}a_6^S - \frac{1}{8}a_9^S + \frac{3}{8}a_{12}^S + a_{20}^S, \quad c_{25}^S = \frac{1}{4}a_3^S + \frac{1}{8}a_5^S - \frac{3}{8}a_6^S + \frac{3}{8}a_9^S - \frac{1}{8}a_{12}^S + a_{25}^S, \\
c_{21}^S &= -\frac{3}{8}a_8^S + \frac{1}{8}a_{11}^S + \frac{3}{8}a_9^S - \frac{1}{8}a_{12}^S - \frac{1}{8}a_{13}^S + \frac{3}{8}a_{14}^S + a_{21}^S, \quad c_{26}^S = \frac{1}{8}a_8^S - \frac{3}{8}a_{11}^S - \frac{1}{8}a_9^S + \frac{3}{8}a_{12}^S + \frac{3}{8}a_{13}^S - \frac{1}{8}a_{14}^S + a_{26}^S, \\
c_{22}^S &= \frac{3}{8}a_{13}^S - \frac{1}{8}a_{14}^S - \frac{3}{8}a_{15}^S + \frac{1}{8}a_{16}^S + \frac{1}{4}a_{17}^S + a_{22}^S, \quad c_{27}^S = -\frac{1}{8}a_{13}^S + \frac{3}{8}a_{14}^S + \frac{1}{8}a_{15}^S - \frac{3}{8}a_{16}^S + \frac{1}{4}a_{17}^S + a_{27}^S, \\
c_1^{c,S} &= a_1^{c,S}, \quad c_2^{c,S} = a_2^{c,S}, \quad c_3^{c,S} = a_3^{c,S}, \quad c_4^{c,S} = a_4^{c,S}, \quad c_5^{c,S} = a_5^{c,S}.
\end{aligned} \tag{A21}$$

The $SU(3)$ irreducible amplitude of the $\mathcal{B}_{c\bar{3}} \rightarrow \mathcal{B}_8^A M$ decay constructed by third-rank octet tensors is

$$\begin{aligned}
\mathcal{A}^A(\mathcal{B}_{c\bar{3}} \rightarrow \mathcal{B}_8^A M) &= c_1^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^m M_m^i(\mathcal{B}_8^A)^{jkl} + c_2^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^m M_m^i(\mathcal{B}_8^A)^{jkl} + c_3^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^m M_m^i(\mathcal{B}_8^A)^{klj} \\
&+ c_4^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^j(\mathcal{B}_8^A)^{klm} + c_5^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^i M_m^j(\mathcal{B}_8^A)^{kml} + c_6^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^j(\mathcal{B}_8^A)^{kml} \\
&+ c_7^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^i M_m^k(\mathcal{B}_8^A)^{ilm} + c_8^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^i M_m^k(\mathcal{B}_8^A)^{iml} + c_9^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^i M_m^k(\mathcal{B}_8^A)^{lmj} \\
&+ c_{10}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^A)^{ilm} + c_{11}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^A)^{iml} + c_{12}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^k(\mathcal{B}_8^A)^{lmj} \\
&+ c_{13}^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^m M_m^l(\mathcal{B}_8^A)^{ikj} + c_{14}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^m M_m^l(\mathcal{B}_8^A)^{ikj} + c_{15}^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^i M_m^m(\mathcal{B}_8^A)^{jkl} \\
&+ c_{16}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^m(\mathcal{B}_8^A)^{jkl} + c_{17}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^i M_m^m(\mathcal{B}_8^A)^{klj} + c_{18}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^i(\mathcal{B}_8^A)^{jkm} \\
&+ c_{19}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^i(\mathcal{B}_8^A)^{jmk} + c_{20}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^i(\mathcal{B}_8^A)^{kmj} + c_{21}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^i(\mathcal{B}_8^A)^{imj} \\
&+ c_{22}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^m(\mathcal{B}_8^A)^{ikj} + c_{23}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^i(\mathcal{B}_8^A)^{jkm} + c_{24}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^i(\mathcal{B}_8^A)^{jmk} \\
&+ c_{25}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^i(\mathcal{B}_8^A)^{kmj} + c_{26}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^i(\mathcal{B}_8^A)^{imj} + c_{27}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^m(\mathcal{B}_8^A)^{ijk} \\
&+ c_{28}^A(\mathcal{B}_{c\bar{3}})_{ij}H(15)_{kl}^m M_m^k(\mathcal{B}_8^A)^{ijl} + c_{29}^A(\mathcal{B}_{c\bar{3}})_{ij}H(\bar{6})_{kl}^m M_m^k(\mathcal{B}_8^A)^{ijl} + c_{30}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^k(\mathcal{B}_8^A)^{ijm} \\
&+ c_{31}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_l)_k M_m^k(\mathcal{B}_8^A)^{ijk} + c_{32}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^k(\mathcal{B}_8^A)^{ijm} + c_{33}^A(\mathcal{B}_{c\bar{3}})_{ij}H(3_p)_k M_m^m(\mathcal{B}_8^A)^{ijk} \\
&+ c_1^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^i(\mathcal{B}_8^A)^{jkl} + c_2^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^i(\mathcal{B}_8^A)^{jlk} + c_3^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^i(\mathcal{B}_8^A)^{klj} \\
&+ c_4^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^k(\mathcal{B}_8^A)^{ilj} + c_5^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^l(\mathcal{B}_8^A)^{ikj} + c_6^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^k(\mathcal{B}_8^A)^{ijl} \\
&+ c_7^{c,A}(\mathcal{B}_{b\bar{3}})_{ij}H(3)_k^c M_l^l(\mathcal{B}_8^A)^{ijk}.
\end{aligned} \tag{A22}$$

The relations of the $SU(3)$ irreducible amplitudes and the topological amplitudes are derived as

$$\begin{aligned}
c_1^A &= \frac{a_1^A + a_2^A}{8}, \quad c_2^A = \frac{a_1^A - a_2^A}{4}, \quad c_3^A = \frac{a_3^A}{4}, \quad c_4^A = \frac{a_4^A}{4}, \quad c_5^A = \frac{a_5^A + a_6^A}{8}, \quad c_6^A = \frac{a_5^A - a_6^A}{4}, \quad c_7^A = \frac{a_7^A + a_{10}^A}{8}, \\
c_{10}^A &= \frac{a_7^A - a_{10}^A}{4}, \quad c_8^A = \frac{a_8^A + a_{11}^A}{8}, \quad c_{11}^A = \frac{a_8^A - a_{11}^A}{4}, \quad c_9^A = \frac{a_9^A + a_{12}^A}{8}, \quad c_{12}^A = \frac{a_9^A - a_{12}^A}{4}, \quad c_{13}^A = \frac{a_{13}^A + a_{14}^A}{8}, \\
c_{14}^A &= \frac{a_{13}^A - a_{14}^A}{4}, \quad c_{15}^A = \frac{a_{15}^A + a_{16}^A}{8}, \quad c_{16}^A = \frac{a_{15}^A - a_{16}^A}{4}, \quad c_{17}^A = \frac{a_{17}^A}{4}, \quad c_{28}^A = \frac{a_{28}^A + a_{29}^A}{8}, \quad c_{29}^A = \frac{a_{28}^A - a_{29}^A}{4}, \\
c_{18}^A &= \frac{3}{8}a_1^A - \frac{1}{8}a_2^A + \frac{1}{2}a_4^A - \frac{1}{8}a_7^A + \frac{3}{8}a_{10}^A + J_{18}^S, \quad c_{23}^A = -\frac{1}{8}a_1^A + \frac{3}{8}a_2^A - \frac{1}{2}a_4^A + \frac{3}{8}a_7^A - \frac{1}{8}a_{10}^A + a_{23}^A, \\
c_{19}^A &= -\frac{1}{8}a_1^A + \frac{3}{8}a_2^A + \frac{1}{8}a_5^A - \frac{3}{8}a_6^A - \frac{1}{8}a_8^A + \frac{3}{8}a_{11}^A + a_{19}^A, \quad c_{24}^A = \frac{3}{8}a_1^A - \frac{1}{8}a_2^A - \frac{3}{8}a_5^A + \frac{1}{8}a_6^A + \frac{3}{8}a_8^A - \frac{1}{8}a_{11}^A + a_{24}^A,
\end{aligned}$$

$$\begin{aligned}
c_{20}^A &= \frac{1}{2}a_3^A - \frac{3}{8}a_5^A + \frac{1}{8}a_6^A - \frac{1}{8}a_9^A + \frac{3}{8}a_{12}^A + a_{20}^A, & c_{25}^A &= -\frac{1}{2}a_3^A + \frac{1}{8}a_5^A - \frac{3}{8}a_6^A + \frac{3}{8}a_9^A - \frac{1}{8}a_{12}^A + a_{25}^S, \\
c_{21}^A &= -\frac{3}{8}a_8^A + \frac{1}{8}a_{11}^A + \frac{3}{8}a_9^A - \frac{1}{8}a_{12}^A - \frac{1}{8}a_{13}^A + \frac{3}{8}a_{14}^A + a_{21}^A, & c_{26}^A &= \frac{1}{8}a_8^A - \frac{3}{8}a_{11}^A - \frac{1}{8}a_9^A + \frac{3}{8}a_{12}^A + \frac{3}{8}a_{13}^A - \frac{1}{8}a_{14}^A + a_{26}^A, \\
c_{22}^A &= \frac{3}{8}a_{13}^A - \frac{1}{8}a_{14}^A - \frac{3}{8}a_{15}^A + \frac{1}{8}a_{16}^A - \frac{1}{2}a_{17}^A + a_{22}^A, & c_{27}^A &= -\frac{1}{8}a_{13}^A + \frac{3}{8}a_{14}^A + \frac{1}{8}a_{15}^A - \frac{3}{8}a_{16}^A + \frac{1}{2}a_{17}^A + a_{27}^A, \\
c_{30}^A &= -\frac{3}{8}a_7^A + \frac{1}{8}a_{10}^A + \frac{3}{8}a_{28}^A - \frac{1}{8}a_{29}^A + a_{30}^A, & c_{32}^A &= \frac{1}{8}a_7^A - \frac{3}{8}a_{10}^A - \frac{1}{8}a_{28}^A + \frac{3}{8}a_{29}^A + a_{32}^A, \\
c_{31}^A &= \frac{1}{8}a_{15}^A - \frac{3}{8}a_{16}^A - \frac{1}{8}a_{28}^A + \frac{3}{8}a_{29}^A + a_{31}^A, & c_{33}^A &= -\frac{3}{8}a_{15}^A + \frac{1}{8}a_{16}^A + \frac{3}{8}a_{28}^A - \frac{1}{8}a_{29}^A + a_{33}^A, \\
c_1^{c,A} &= a_1^{c,A}, & c_2^{c,A} &= a_2^{c,A}, & c_3^{c,A} &= a_3^{c,A}, & c_4^{c,A} &= a_4^{c,A}, & c_5^{c,A} &= a_5^{c,A}, \\
c_6^{c,A} &= a_6^{c,A}, & c_7^{c,A} &= a_7^{c,A}.
\end{aligned} \tag{A23}$$

By substituting Eqs. (8) and (A8) into each term of Eqs. (A20) and (A22), the relations between the $SU(3)$ irreducible amplitudes constructed by 3-rank and (1,1)-rank octets are derived as

$$\begin{aligned}
b_1 &= -(2c_4^A + c_{10}^A - c_{12}^A)/\sqrt{2} + (2c_6^S - c_{10}^S + 2c_{11}^S - c_{12}^S)/\sqrt{6}, \\
b_2 &= (c_2^A - 2c_3^A + c_6^A)/\sqrt{2} + (3c_2^S - c_6^S)/\sqrt{6}, \\
b_3 &= -(c_6^A - c_{16}^A + 2c_{17}^A)/\sqrt{2} + (c_6^S + 3c_{16}^S)/\sqrt{6}, \\
b_4 &= (c_6^A + c_{11}^A + c_{12}^A)/\sqrt{2} - (c_6^S + 2c_{10}^S - c_{11}^S - c_{12}^S)/\sqrt{6}, \\
b_5 &= (c_2^A - 2c_3^A + c_6^A + c_{11}^A + c_{12}^A + c_{14}^A - 2c_{29}^A)/\sqrt{2} + (3c_2^S - c_6^S - 2c_{10}^S + c_{11}^S + c_{12}^S - 3c_{14}^S)/\sqrt{6}, \\
b_6 &= (c_7^A - c_9^A)/\sqrt{2} + (c_7^S - 2c_8^S + c_9^S)/\sqrt{6}, \\
b_7 &= (c_1^A + c_5^A)/\sqrt{2} - (c_1^S - 2c_3^S + 2c_4^S - c_5^S)/\sqrt{6}, \\
b_8 &= -(c_5^A - c_{15}^A)/\sqrt{2} + (2c_4^S - c_5^S - c_{15}^S + 2c_{17}^S)/\sqrt{6}, \\
b_9 &= (c_5^A + c_8^A + c_9^A)/\sqrt{2} - (2c_4^S - c_5^S + 2c_7^S - c_8^S - c_9^S)/\sqrt{6}, \\
b_{10} &= -(c_1^A + c_5^A + c_8^A + c_9^A - c_{13}^A - 2c_{28}^A)/\sqrt{2} + (c_1^S - 2c_3^S + 2c_4^S - c_5^S + 2c_7^S - c_8^S - c_9^S - 3c_{13}^S)/\sqrt{6}, \\
b_{11} &= (c_{23}^A - c_{25}^A)/\sqrt{2} + (c_{23}^S - 2c_{24}^S + c_{25}^S)/\sqrt{6}, \\
b_{12} &= -(c_{24}^A + c_{25}^A - c_{27}^A - 2c_{33}^A)/\sqrt{2} + (2c_{23}^S - c_{24}^S - c_{25}^S - 3c_{27}^S)/\sqrt{6}, \\
b_{13} &= (c_{24}^A + c_{25}^A)/\sqrt{2} - (2c_{23}^S - c_{24}^S - c_{25}^S)/\sqrt{6}, \\
b_{14} &= -(c_{23}^A - c_{25}^A - c_{26}^A - 2c_{32}^A)/\sqrt{2} - (c_{23}^S - 2c_{24}^S + c_{25}^S + 3c_{26}^S)/\sqrt{6}, \\
b_{15} &= (c_{18}^A - c_{20}^A)/\sqrt{2} + (c_{18}^S - 2c_{19}^S + c_{20}^S)/\sqrt{6}, \\
b_{16} &= -(c_{19}^A + c_{20}^A - c_{22}^A - 2c_{31}^A)/\sqrt{2} + (2c_{18}^S - c_{19}^S - c_{20}^S - 3c_{22}^S)/\sqrt{6}, \\
b_{17} &= (c_{19}^A + c_{20}^A)/\sqrt{2} - (2c_{18}^S - c_{19}^S - c_{20}^S)/\sqrt{6}, \\
b_{18} &= -(c_{18}^A - c_{20}^A - c_{21}^A - 2c_{30}^A)/\sqrt{2} - (c_{18}^S - 2c_{19}^S + c_{20}^S + 3c_{21}^S)/\sqrt{6}, \\
b_1^c &= (-c_1^{c,A} + c_3^{c,A} + c_4^{c,A} + 2c_6^{c,A})/\sqrt{2} - (c_1^{c,S} - 2c_2^{c,S} + c_3^{c,S} + 3c_4^{c,S})/\sqrt{6}, \\
b_2^c &= (-c_2^{c,A} - c_3^{c,A} + c_5^{c,A} + 2c_7^{c,A})/\sqrt{2} - (-2c_1^{c,S} + c_2^{c,S} + c_3^{c,S} + 3c_5^{c,S})/\sqrt{6}, \\
b_3^c &= (c_2^{c,A} + c_3^{c,A})/\sqrt{2} - (2c_1^{c,S} - c_2^{c,S} - c_3^{c,S})/\sqrt{6}, \\
b_4^c &= (c_1^{c,A} - c_3^{c,A})/\sqrt{2} - (-c_1^{c,S} + 2c_2^{c,S} - c_3^{c,S})/\sqrt{6}.
\end{aligned} \tag{A24}$$

By substituting Eqs. (A21) and (A23) into Eq. (A24), Eq. (A17) is re-derived.

As noted in Ref. [40], the amplitudes constructed by 3-rank or (1,1)-rank octets can be decomposed by the other amplitudes constructed by 3-rank or (1,1)-rank octets, and the inverse solutions exist. The amplitudes constructed by (1,1)-rank octets can be decomposed by the

amplitudes constructed by 3-rank octets. However, the amplitudes constructed by 3-rank octets cannot be decomposed by the amplitudes constructed by (1,1)-rank octets.

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