

Mass mixing between QCD axions*

Hai-Jun Li (李海军)^{1†} Yu-Feng Zhou (周宇峰)^{1,2,3,4‡}

¹Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

²School of Physical Sciences, University of Chinese Academy of Sciences, Beijing 100049, China

³School of Fundamental Physics and Mathematical Sciences, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, China

⁴International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China

Abstract: We introduce a novel level crossing phenomenon in the mass mixing between QCD axions: one canonical QCD axion and one Z_N axion. The level crossing can occur at or slightly before the QCD phase transition critical temperature, depending on the ratio of the axion decay constants, ~ 1.69 . The cosmological evolution of the mass eigenvalues in these two scenarios is similar; however, the transition of axion energy density differs significantly. Finally, we estimate the relic density of the QCD axion dark matter in this context. Additionally, this level crossing may have some interesting cosmological implications.

Keywords: axion, dark matter, QCD phase transition

DOI: 10.1088/1674-1137/aded01 **CSTR:** 32044.14.ChinesePhysicsC.49115101

I. INTRODUCTION

QCD axions are attractive candidates for cold dark matter (DM). The canonical QCD axion was predicted by the Peccei-Quinn (PQ) mechanism [1, 2] to solve the strong CP problem in the Standard Model (SM) [3–8]. It obtains a tiny mass from the QCD non-perturbative effects [9, 10]. As a DM candidate, it can be non-thermally produced in the early Universe through the misalignment mechanism [11–13]. At high cosmic temperatures, the QCD axion is massless and obtains a non-zero mass during the QCD phase transition. The oscillation of the axion when its mass is comparable to the Hubble parameter explains the observed DM abundance. See, *e.g.*, Ref. [14] for a recent review. The reduced-mass Z_N axion [15] can also solve the strong CP problem with $N \geq 3$ [16] and account for DM through the trapped plus kinetic misalignment mechanism [17, 18]. Here, N is an odd number, and N mirror worlds that are nonlinearly realized by the axion field under a Z_N symmetry can coexist in nature. Due to the suppressed non-perturbative effects on axion potential from the N degenerate QCD groups, the Z_N axion mass is exponentially suppressed at the QCD phase transition. To avoid confusion, in what follows, "QCD axion"

only represents the canonical QCD axion.

The mass mixing in the multiple axions model [19] has attracted extensive attention in recent years.¹⁾ Considering the non-zero mass mixing between these axions, the cosmological evolution process of the mass eigenvalues, called the level crossing, can occur and induce the adiabatic transition of the axion energy density, which is similar to the MSW effect [23–25] in neutrino oscillations. The level crossing has some interesting cosmological implications, such as the modification of the axion relic density and isocurvature perturbations [26–29], domain wall formation [30], gravitational wave emission and primordial black hole formation [31–33], and dark energy composition [34, 35].

In this study, we investigate the mass mixing between QCD axions: one QCD axion and one Z_N axion. We find that level crossing can occur at the QCD phase transition critical temperature or slightly before it, depending on the relationship between the axion decay constants. The conditions for level crossing to occur in these two cases are discussed in detail. The cosmological evolution of the mass eigenvalues in both cases is similar, whereas the transition of energy density among the axions differs. Finally, we estimate the relic density of the QCD axion and

Received 13 May 2025; Accepted 8 July 2025; Published online 9 July 2025

* Supported by the CAS Project for Young Scientists in Basic Research YSBR-006, the National Key R&D Program of China (2017YFA0402204) and the National Natural Science Foundation of China (11821505, 11825506, 12047503)

[†] E-mail: lihajun@itp.ac.cn

[‡] E-mail: yfzhou@itp.ac.cn

1) For instance, the mass mixing between the QCD axion and axionlike particle (ALP) [20] or sterile axion [21], as well as the mixing between the Z_N axion and ALP [22].



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP³ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

Z_N axion DM through the misalignment mechanism. To the best of our knowledge, this is the first investigation of the effect of level crossing in the mass mixing within a multiple QCD axions model.

II. THE MODEL

Here, we consider a minimal multiple QCD axion model, with one QCD axion ϕ and one Z_N axion φ . These two axions are charged under two copies of the SM, yet both are coupled to QCD. We expect ϕ to receive the dominant contribution from the QCD potential, leading to the cancellation of the potential that occurs in the usual Z_N axion case, while such cancellation would not occur for the single φ .¹⁾ The low-energy effective Lagrangian is given by

$$\mathcal{L} \supset \frac{1}{2} \sum_{\phi, \varphi} \partial_\mu \Phi \partial^\mu \Phi - m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi}{f_a}\right) \right] - \frac{m_N^2(T) F_a^2}{N^2} \left[1 - \cos\left(\frac{\phi}{f_a} + N \frac{\varphi}{F_a}\right) \right], \quad (1)$$

where $m_a(T)$ and $m_N(T)$ are the temperature-dependent QCD axion and Z_N axion masses, respectively, and f_a and F_a are the QCD axion and Z_N axion decay constants, respectively. The QCD axion mass $m_a(T)$ can be described by

$$\begin{cases} \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{z}}{1+z}, & T \leq T_{\text{QCD}} \\ \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{z}}{1+z} \left(\frac{T}{T_{\text{QCD}}} \right)^{-b}, & T > T_{\text{QCD}} \end{cases} \quad (2)$$

where m_π and f_π represent the mass and decay constant of the pion, respectively, $z \equiv m_u/m_d \simeq 0.48$ signifies the ratio of the up to down quark masses, $T_{\text{QCD}} \simeq 150 \text{ MeV}$ is the QCD phase transition critical temperature, and $b \simeq 4.08$ is an index taken from the dilute instanton gas approximation. The first term in Eq. (2) corresponds to the zero-temperature QCD axion mass $m_{a,0}$. The Z_N axion mass $m_N(T)$ can be described by

$$\begin{cases} \frac{m_\pi f_\pi}{\sqrt[4]{\pi} F_a} \sqrt[4]{\frac{1-z}{1+z}} N^{3/4} z^{N/2}, & T \leq T_{\text{QCD}} \\ \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}}, & T_{\text{QCD}} < T \leq \frac{T_{\text{QCD}}}{\gamma} \\ \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}} \left(\frac{\gamma T}{T_{\text{QCD}}} \right)^{-b}, & T > \frac{T_{\text{QCD}}}{\gamma} \end{cases} \quad (3)$$

where $\gamma \in (0, 1)$ is a temperature parameter. The first term in Eq. (3) also corresponds to the zero-temperature Z_N axion mass $m_{N,0}$. Notably, this mass undergoes a sudden exponential suppression at T_{QCD} due to the Z_N symmetry. Subsequently, the equations of motion of ϕ and φ are given by

$$\ddot{\phi} + 3H\dot{\phi} + m_a^2(T) f_a \sin\left(\frac{\phi}{f_a}\right) + \frac{m_N^2(T) F_a^2}{N^2 f_a} \sin\left(\frac{\phi}{f_a} + N \frac{\varphi}{F_a}\right) = 0, \quad (4)$$

and

$$\ddot{\varphi} + 3H\dot{\varphi} + \frac{m_N^2(T) F_a}{N} \sin\left(\frac{\phi}{f_a} + N \frac{\varphi}{F_a}\right) = 0, \quad (5)$$

where $H(T)$ is the Hubble parameter, and the dots represent derivatives with respect to physical time t . By diagonalizing the mass mixing matrix

$$\mathbf{M}^2 = \begin{pmatrix} m_a^2(T) + \frac{m_N^2(T) F_a^2}{N^2 f_a^2} & \frac{m_N^2(T) F_a}{N f_a} \\ \frac{m_N^2(T) F_a}{N f_a} & m_N^2(T) \end{pmatrix}, \quad (6)$$

we can derive the heavy (a_h) and light (a_l) axion mass eigenstates

$$\begin{pmatrix} a_h \\ a_l \end{pmatrix} = \begin{pmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} \phi \\ \varphi \end{pmatrix}, \quad (7)$$

with the corresponding mass eigenvalues $m_{h,l}(T)$

$$\begin{aligned} m_{h,l}^2(T) = & \frac{1}{2} \left[m_a^2(T) + m_N^2(T) + \frac{m_N^2(T) F_a^2}{N^2 f_a^2} \right] \\ & \pm \frac{1}{2N^2 f_a^2} \left[-4N^4 m_a^2(T) m_N^2(T) f_a^4 \right. \\ & \left. + \left((m_a^2(T) + m_N^2(T)) N^2 f_a^2 \right. \right. \\ & \left. \left. + m_N^2(T) F_a^2 \right)^2 \right]^{1/2}, \end{aligned} \quad (8)$$

where α is the mass mixing angle

1) There is a widely held belief that string theory can predict a substantial number of axions [36–38], encompassing various types such as the QCD axion and potentially the Z_N axion, known as the axiverse [39–41]. In this theoretical context, a theory that incorporates the QCD axions within our framework emerges as a particularly intriguing physics scenario beyond the Standard Model (BSM). This pursuit not only enhances our understanding of their characteristics but may also uncover new physics within the BSM framework.

$$\cos^2 \alpha = \frac{1}{2} \left(1 + \frac{m_N^2(T) - m_a^2(T) - \frac{m_N^2(T) F_a^2}{N^2 f_a^2}}{m_h^2(T) - m_l^2(T)} \right). \quad (9)$$

The so-called axion level crossing occurs when, considering the interaction between two axion fields, there exists a nontrivial mass mixing of their temperature-dependent mass eigenvalues $m_{h,l}(T)$ that is a function of the cosmic temperature T . This can occur when the difference of $m_h^2(T) - m_l^2(T)$ is minimized.

Case I: Level crossing at T_{QCD}

We first consider a case where the zero-temperature QCD axion mass $m_{a,0}$ is smaller than the second term in Eq. (3) (we can define this as the mass $m_{N,\pi}$)

$$\frac{m_\pi f_\pi}{f_a} \frac{\sqrt{z}}{1+z} < \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}}, \quad (10)$$

which can be characterized as

$$\zeta \equiv \frac{F_a}{f_a} < \sqrt{\frac{1+z}{1-z}} \simeq 1.69, \quad (11)$$

where we have defined the ratio ζ . Meanwhile, for level crossing to occur in this case, the zero-temperature QCD axion mass should be larger than the zero-temperature Z_N axion mass $m_{N,0}$, *i.e.*,

$$\frac{m_\pi f_\pi}{f_a} \frac{\sqrt{z}}{1+z} > \frac{m_\pi f_\pi}{\sqrt[4]{\pi} F_a} \sqrt[4]{\frac{1-z}{1+z}} N^{3/4} z^{N/2}, \quad (12)$$

which can also be characterized as

$$\zeta > \frac{1}{\sqrt[4]{\pi}} \sqrt[4]{\frac{(1-z)(1+z)^3}{z^2}} N^{3/4} z^{N/2} \simeq 1.24 \times 0.48^{N/2} N^{3/4}, \quad (13)$$

which depends on the value of N . Fig. 1 (left) illustrates level crossing in this case. We have set $f_a = 10^{12}$ GeV, $F_a = 10^{11.5}$ GeV, $N = 17$, and $\gamma = 0.25$. The red and blue lines correspond to the temperature-dependent mass eigenvalues $m_h(T)$ and $m_l(T)$, respectively. Note that the level crossing occurs at the QCD phase transition critical temperature

$$T_\times = T_{\text{QCD}}. \quad (14)$$

Here, we discuss the cosmological evolution of the mass eigenvalues in this scenario. At high temperatures, the heavy mass eigenvalue $m_h(T)$ corresponds to the Z_N axion, whereas the light one $m_l(T)$ corresponds to the QCD axion. They will approach each other at T_{QCD} and then move away from each other. When $T < T_{\text{QCD}}$, $m_h(T)$ represents the QCD axion, and $m_l(T)$ represents the Z_N axion. For a more intuitive description, we can express them as follows:

$$m_h(T) \Rightarrow \begin{cases} m_a(T), & T < T_\times \\ m_N(T), & T > T_\times \end{cases} \quad (15)$$

and

$$m_l(T) \Rightarrow \begin{cases} m_N(T), & T < T_\times \\ m_a(T), & T > T_\times \end{cases} \quad (16)$$

The above discussion is focused on Case I, where level crossing occurs at T_{QCD} .

To compare with another case discussed below, we show the condition for level crossing to occur in Case I

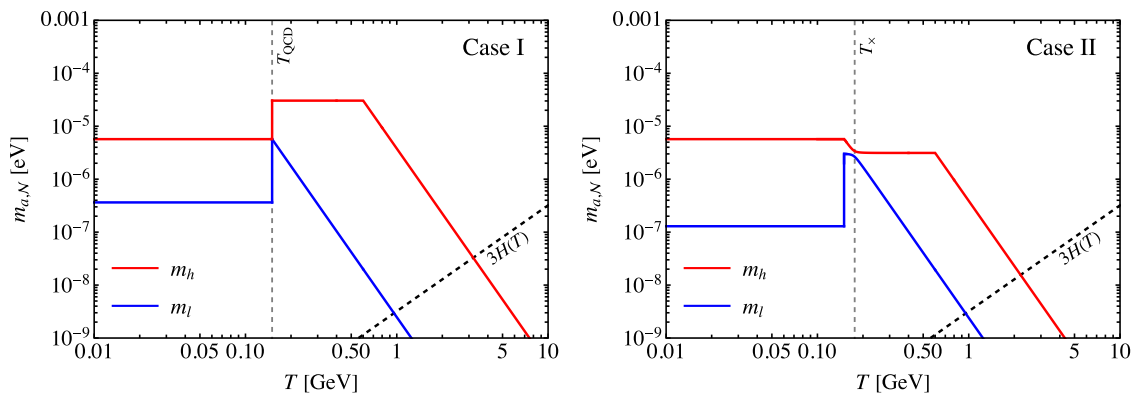


Fig. 1. (color online) Illustrations of level crossing in Cases I (left) and II (right). The red and blue lines represent the mass eigenvalues $m_h(T)$ and $m_l(T)$, respectively. The black dashed line represents the Hubble parameter $H(T)$. Left: We set $f_a = 10^{12}$ GeV, $F_a = 10^{11.5}$ GeV ($\Rightarrow \zeta \simeq 0.32$), $N = 17$, and $\gamma = 0.25$. The gray dashed line represents the temperature T_{QCD} . Right: We set $f_a = 10^{12}$ GeV, $F_a = 10^{12.5}$ GeV ($\Rightarrow \zeta \simeq 3.16$), $N = 13$, and $\gamma = 0.25$. The gray dashed line represents the level crossing temperature $T_\times \simeq 176.3$ MeV.

by using Eqs. (11) and (13). See Fig. 2 with the red shaded region in the $\{N, \zeta\}$ plane. The unshaded regions represent areas where no level crossing occurs.

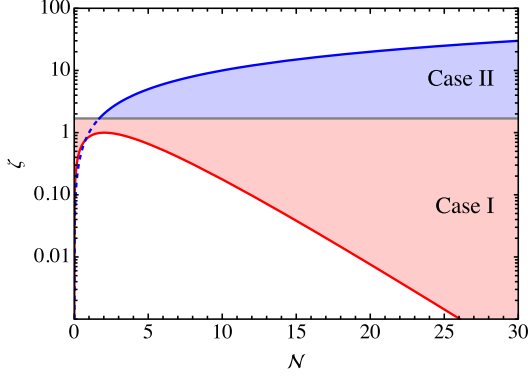


Fig. 2. (color online) Conditions for level crossing to occur in the $\{N, \zeta\}$ plane. The red and blue shaded regions correspond to Cases I and II, respectively. The gray line represents $\zeta = 1.69$. Note that $N = 2k + 1$, $k \in \mathbb{N}^+$.

Case II: Level crossing slightly before T_{QCD}

Next, we consider the case where the zero-temperature QCD axion mass is larger than $m_{N,\pi}$. We have

$$\zeta > \sqrt{\frac{1+z}{1-z}} \simeq 1.69. \quad (17)$$

We present an illustration of this case in Fig. 1 (right). We have set $f_a = 10^{12} \text{ GeV}$, $F_a = 10^{12.5} \text{ GeV}$, $N = 13$, and $\gamma = 0.25$. Here, in contrast to that in Case I, the level crossing takes place slightly before the temperature T_{QCD} . By solving $d(m_h^2(T) - m_l^2(T))/dT = 0$ at $T_{\text{QCD}} < T < T_{\text{QCD}}/\gamma$, we obtain the level crossing temperature

$$T_x = T_{\text{QCD}} \left(\sqrt{\frac{1-z}{1+z}} \zeta \right)^{1/b} \left(1 - \frac{\zeta^2}{N^2} \right)^{-1/(2b)}, \quad (18)$$

which is slightly higher than T_{QCD} . The QCD axion mass at T_x is given by

$$m_a(T_x) \simeq m_{N,\pi} \sqrt{1 - \frac{\zeta^2}{N^2}}. \quad (19)$$

Note that the condition $\zeta < N$ should be satisfied in Eq. (18). This level crossing persists for a parametric duration given by

1) Since in the Case I the QCD axion energy density at T_{QCD} is non-adiabatic, its relic density is not straightforward to determine. As a result, we focus our attention on the Z_N axion.

$$\Delta T_x = \left| \frac{1}{\cos \alpha(T_x)} \frac{d \cos \alpha(T_x)}{dT} \right|^{-1}, \quad (20)$$

where $\alpha(T)$ represents the mass mixing angle given by Eq. (9). This scenario is referred to as Case II. The temperature-dependent behavior of the mass eigenvalues is similar to that in Case I. However, the transition of energy density between them differs significantly, as will be discussed in detail later. It is worth noting that Case I can transition into Case II, depending on the value of the ratio $\zeta \sim 1.69$.

Then, in Fig. 2 (blue shaded region), we illustrate the condition for level crossing to occur in Case II, using Eq. (17) and the inequality $\zeta < N$. Additionally, we note that at T_{QCD}/γ , the QCD axion mass should be smaller than the Z_N axion mass

$$\frac{m_{\pi} f_{\pi}}{f_a} \frac{\sqrt{z}}{1+z} \gamma^b < \frac{m_{\pi} f_{\pi}}{F_a} \sqrt{\frac{z}{1-z^2}}, \quad (21)$$

which can be characterized as

$$\zeta < \sqrt{\frac{1+z}{1-z}} \gamma^{-b} \simeq 1.69 \gamma^{-b}, \quad (22)$$

which depends on the value of γ . For the small γ , the constraint of $\zeta < N$ is more stringent than Eq. (22). Thus, we do not include it in Fig. 2.

III. AXION RELIC DENSITY

Here, we estimate the relic density of the axion DM in Cases I and II through the misalignment mechanism. We consider the pre-inflationary scenario, in which the PQ symmetry is spontaneously broken during inflation.

We first discuss Case I, focusing on the Z_N axion DM.¹⁾ We begin with the QCD axion field, which is frozen at high temperatures with an arbitrary initial misalignment angle $\theta_{1,a}$ and starts to oscillate at $T_{1,a}$. The oscillation temperature T_1 is given by $m(T) = 3H(T)$. In the following, the subscript "x" stands for the physical quantity at T_x or corresponding to x . The initial energy density in the QCD axion field is given by

$$\rho_{a,1} = \frac{1}{2} m_{a,1}^2 f_a^2 \theta_{1,a}^2. \quad (23)$$

At $T_{\text{QCD}} < T < T_{1,a}$, the QCD axion energy density is adiabatic invariant with the comoving number $N_a \equiv \rho_a a^3 / m_a$, where a is the scale factor. Then, we have the QCD axion energy density just before T_{QCD} as

$$\rho_{a,\text{QCD}} = \frac{1}{2} m_{a,0} m_{a,1} f_a^2 \theta_{1,a}^2 \left(\frac{a_{1,a}}{a_{\text{QCD}}} \right)^3. \quad (24)$$

In this case, the axion is trapped around $\theta_{\text{tr}} \equiv \theta_a(T_{\text{QCD}})$ until T_{QCD} , with the initial axion velocity $\dot{\theta}_{\text{tr}} \equiv \dot{\theta}_a(T_{\text{QCD}})$. At $T = T_{\text{QCD}}$, the light mass eigenvalue $m_l(T)$ will comprise the Z_N axion, the axion mass is suddenly suppressed, and the true minimum develops. Then, we have the mean velocity of the Z_N axion

$$\sqrt{\langle \dot{\theta}_{\text{tr}}^2 \rangle} = \frac{1}{\sqrt{2}\zeta} \sqrt{m_{a,0} m_{a,1}} \theta_{1,a} \left(\frac{a_{1,a}}{a_{\text{QCD}}} \right)^{3/2}. \quad (25)$$

Note that the Z_N axion energy density at T_{QCD} is non-adiabatic, which just after T_{QCD} is given by

$$\rho_{N,\text{tr}} = \frac{1}{2} F_a^2 \dot{\theta}_{\text{tr}}^2 + 2 \frac{m_N^2 F_a^2}{N^2}, \quad (26)$$

where we consider a case that the axion has a large mean velocity. Then, the Z_N axion will start to oscillate at T_2 , which is defined as the temperature when the kinetic energy is equal to the barrier height. At $T_2 < T < T_{\text{QCD}}$, the Z_N axion energy density is conserved with the comoving PQ charge $q_{\text{kin}} = \dot{\theta}_N a^3$, and we have the scale factor at T_2 as $a_2 = (N \dot{\theta}_{\text{tr}} / (2m_N))^{1/3} a_{\text{QCD}}$. Then, at $T_0 < T < T_2$, the adiabatic approximation is valid, and we have the Z_N axion energy density at present T_0 as

$$\rho_{N,0} = \frac{m_{N,0} F_a^2 \dot{\theta}_{\text{tr}}}{N} \left(\frac{a_{\text{QCD}}}{a_0} \right)^3. \quad (27)$$

Substituting the mean velocity, it reads

$$\rho_{N,0} = C \frac{m_{N,0} \sqrt{m_{a,0} m_{a,1}} \theta_{1,a} F_a^2}{\sqrt{2} N \zeta} \left(\frac{\sqrt{a_{1,a} a_{\text{QCD}}}}{a_0} \right)^3, \quad (28)$$

where $C \simeq 2$ is a constant. Compared with the no mixing case, the relic density can be modified by

$$R_\rho \sim \sqrt[4]{\frac{1-z}{1+z}} \sqrt[4]{\frac{m_{N,1}}{m_{a,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \sqrt{\zeta}}. \quad (29)$$

Then, in the left panel of Fig. 3, we show the distribution of R_ρ in the $\{\zeta, \log(\eta)\}$ plane, where $\eta \equiv m_{N,1}/m_{a,1}$ and the initial misalignment angles are assumed to be of order one. Considering the significant impact of the temperature parameter γ on the oscillation temperature of the Z_N axion, we incorporate the variations in γ into the observed changes in η to provide a more intuitive illustration of how the relic density varies across different parametric conditions. We find that the ratio R_ρ is enhanced in most regions, whereas it is suppressed in a small subset of regions. Note that, in Case I, the parameter N also exhibits a significant property related to ζ , as illustrated in Fig. 2. However, this property cannot be directly observed from Fig. 3, and here, we roughly set $0 < \zeta < 1.69$.

Next, we discuss the Z_N axion DM in Case II. Because of the insights gained from the discussion in the last paragraph, we can simplify our discussion somewhat. We also begin with the QCD axion field, and its initial energy density at $T_{1,a}$ is given by Eq. (23). Then, at $T_x < T < T_{1,a}$, the QCD axion energy density is adiabatic invariant, and at T_x , it can be described by

$$\rho_{a,x} = \frac{1}{2} m_{N,\pi} m_{a,1} f_a^2 \theta_{1,a}^2 \left(\frac{a_{1,a}}{a_x} \right)^3. \quad (30)$$

At $T = T_x$, level crossing occurs, the light mass eigenvalue $m_l(T)$ will comprise the Z_N axion, and the energy density $\rho_{a,x}$ is transferred to the Z_N axion $\rho_{N,x}$. To ensure this axion energy transition is adiabatic at T_x , we require the following condition to be satisfied:

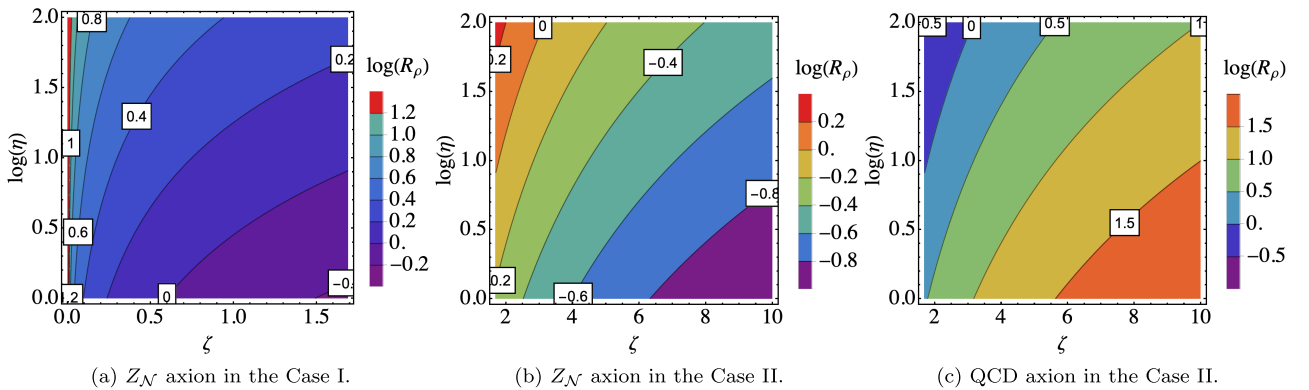


Fig. 3. (color online) Distributions of the axion relic density ratio $\log(R_\rho)$ in the $\{\zeta, \log(\eta)\}$ plane. The left, middle, and right panels represent the Z_N axion in Case I, the Z_N axion in Case II, and the QCD axion in Case II, respectively. Note that the initial misalignment angles are assumed to be of order one.

$$\Delta t_x \gg \max \left[\frac{2\pi}{m_l(T_x)}, \frac{2\pi}{m_h(T_x) - m_l(T_x)} \right], \quad (31)$$

where Δt_x is the time interval corresponding to the temperature change described in Eq. (20). Additionally, we have verified this condition using the parameter set illustrated in the right panel of Fig. 1, and the results are presented in Fig. 4. Then, at $T_{\text{QCD}} < T < T_x$, the adiabatic approximation is valid again, and the Z_N axion energy density just before T_{QCD} is given by

$$\rho_{N,\text{QCD}} = \frac{1}{2} m_{N,\pi} m_{a,1} f_a^2 \theta_{1,a}^2 \left(\frac{a_{1,a}}{a_{\text{QCD}}} \right)^3, \quad (32)$$

with mean velocity

$$\sqrt{\langle \dot{\theta}_{1,a}^2 \rangle} = \frac{1}{\sqrt{2}\zeta} \sqrt{m_{N,\pi} m_{a,1}} \theta_{1,a} \left(\frac{a_{1,a}}{a_{\text{QCD}}} \right)^{3/2}. \quad (33)$$

The subsequent steps ($T < T_{\text{QCD}}$) are similar to those discussed in the previous paragraph. Substituting the mean velocity into Eq. (27), we have the present Z_N axion energy density

$$\rho_{N,0} = C \frac{m_{N,0} \sqrt{m_{N,\pi} m_{a,1}} \theta_{1,a} F_a^2}{\sqrt{2} N \zeta} \left(\frac{\sqrt{a_{1,a} a_{\text{QCD}}}}{a_0} \right)^3. \quad (34)$$

Now, compared with the no mixing case, we find that the relic density can be modified by

$$R_\rho \sim \sqrt[4]{\frac{m_{N,1}}{m_{a,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \zeta}. \quad (35)$$

As previously demonstrated, in the middle panel of

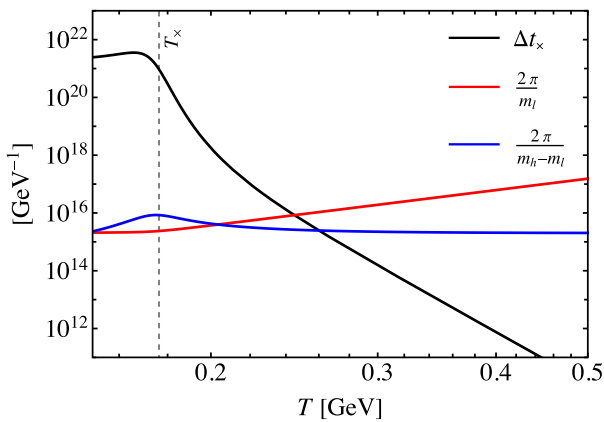


Fig. 4. (color online) The three terms in Eq. (31) as functions of temperature T . The black, red, and blue lines represent Δt_x , $2\pi/m_l(T)$, and $2\pi/(m_h(T) - m_l(T))$, respectively. The gray dashed line represents $T_x \approx 176.3 \text{ MeV}$.

Fig. 3, we present the distribution of R_ρ in the $\{\zeta, \log(\eta)\}$ plane. In most regions, the ratio R_ρ can be suppressed, whereas in a small subset of regions, it can be enhanced. Note also that ζ is constrained by the set of N , and here, we only present a small range of zeta values, specifically $1.69 < \zeta < 10$. Meanwhile, note that Eq. (35) resembles the single level crossing scenario, where the Z_N axion mixes with ALP, leading to the suppression of the Z_N axion relic density, as indicated by Eq. (25) of Ref. [22]. However, due to the differing ranges of parameters, here, ζ exceeds 1.69, and consequently, the Z_N axion relic density ratio R_ρ can be either enhanced or suppressed.

Finally, we discuss the QCD axion DM in Case II. We should begin with the Z_N axion field. It starts to oscillate at $T_{1,N}$ with the initial misalignment angle $\theta_{1,N}$, and the initial energy density is given by

$$\rho_{N,1} = \frac{1}{2} m_{N,1}^2 F_a^2 \theta_{1,N}^2. \quad (36)$$

At $T_x < T < T_{1,N}$, the Z_N axion energy density is adiabatic invariant, and at T_x , we have

$$\rho_{N,x} = \frac{1}{2} m_{N,\pi} m_{N,1} F_a^2 \theta_{1,N}^2 \left(\frac{a_{1,N}}{a_x} \right)^3. \quad (37)$$

At $T = T_x$, level crossing occurs, the heavy mass eigenvalue $m_h(T)$ will comprise the QCD axion, and the energy density $\rho_{N,x}$ is transferred to the QCD axion $\rho_{a,x}$. Then, at $T_0 < T < T_x$, the adiabatic approximation is valid again, and we have the present QCD axion energy density

$$\rho_{a,0} = \frac{1}{2} m_{a,0} m_{N,1} F_a^2 \theta_{1,N}^2 \left(\frac{a_{1,N}}{a_0} \right)^3. \quad (38)$$

Compared with the no mixing case, the relic density can be modified by

$$R_\rho \sim \sqrt{\frac{m_{a,1}}{m_{N,1}}} \frac{\theta_{1,N}^2 \zeta^2}{\theta_{1,a}^2}. \quad (39)$$

In the right panel of Fig. 3, we also present the distribution of R_ρ in the $\{\zeta, \log(\eta)\}$ plane. We find that, within the parameter space we consider, the ratio R_ρ is enhanced in most regions, whereas it is suppressed in a minority of those regions. We also note that Eq. (39) resembles the level crossing that occurs in the mixing between the QCD axion and ALP, as illustrated by Eq. (3.26) of Ref. [29]. In such a case, the QCD axion relic density can be significantly suppressed. However, as mentioned earlier, due to the varying ranges of parameters, the QCD axion relic density ratio R_ρ in this context can be either enhanced or suppressed.

IV. CONCLUSION

In summary, we have investigated the mass mixing between the canonical QCD axion and Z_N axion and introduced a novel level crossing phenomenon. During the mixing, we find that level crossing can occur at the QCD phase transition critical temperature T_{QCD} (Case I) or slightly before T_{QCD} (Case II), depending on the ratio of the axion decay constants $\zeta \sim 1.69$. The conditions for level crossing to occur in Cases I and II are discussed in detail. In Case I, the zero-temperature QCD axion mass $m_{a,0}$ should be smaller than the mass $m_{N,\pi}$ and larger than the zero-temperature Z_N axion mass $m_{N,0}$, which can be characterized by the inequality $1.24 \times 0.48^{N/2} N^{3/4} < \zeta < 1.69$. In Case II, the zero-temperature QCD axion mass $m_{a,0}$ should be larger than $m_{N,\pi}$ and the condition $\zeta < N$ should be satisfied, which can also be characterized by the inequality $1.69 < \zeta < N$. Note that, in this case, we also have a relatively weak constraint given by $\zeta < 1.69\gamma^{-b}$, which becomes more stringent for large values of γ .

Despite the similarity in the cosmological evolution of mass eigenvalues in both cases, the transition of energy density between the axions differs. Specifically, in Case I, the axion energy transition at the level crossing is non-adiabatic (at T_{QCD}), while in Case II, it is considered to be adiabatic (at $T_\times$). Finally, we estimate the relic density of the axion DM through the misalignment mechanism, focusing on the Z_N axion DM in Cases I and II, as well as the QCD axion DM in Case II. We also compare this with other axion mixing scenarios and find that, within the parameter space we consider, the axion relic density can be either enhanced or suppressed. Furthermore, the level crossing phenomenon in this scenario may have other intriguing cosmological implications, such as gravitational waves and primordial black holes.

The appendix is organized as follows. In App. A, we provide an illustrative example of the UV completion for the Z_N axion. In App. B, we present the temperature-dependent mass of the Z_N axion. In App. C, we describe how our axion mixing potential is derived. Finally, in App. D, we present some details about the axion relic density, considering both the no-mixing and mixing scenarios.

APPENDIX A: UV COMPLETION FOR THE Z_N AXION

Here, we provide an illustrative example of the UV completion for the Z_N axion. We consider the KSVZ Z_N axion scenario [16], with N copies of vector-like Dirac fermions Q_k and a gauge singlet complex scalar S . The Z_N symmetry reads $Q_k \rightarrow Q_{k+1}$, $S \rightarrow e^{2\pi i/N} S$. The general Lagrangian is given by

$$\mathcal{L}_{\text{UV}} = \partial_\mu S \partial^\mu S + \sum_{k=0}^{N-1} \left[\bar{Q}_k \not{D} Q_k + y e^{2\pi i k/N} S \bar{Q}_k \frac{1+\gamma_5}{2} Q_k + \text{h.c.} \right] - V(S, H_k), \quad (\text{A1})$$

which exhibits an accidental $U(1)_{\text{PQ}}$ symmetry that is spontaneously broken by the vacuum expectation value v_S through the potential $V(S, H_k)$. The complex scalar S can be expressed as

$$S = \frac{1}{\sqrt{2}} (v_S + \rho) e^{i\frac{\varphi}{v_S}}, \quad (\text{A2})$$

where ρ and φ represent the radial and angular (axion) modes, respectively. Through an axial transformation, the axion mode can be rotated away from the Yukawa term. After integrating out the heavy quarks, the resulting effective Lagrangian for the axion is given by

$$\delta \mathcal{L}_{\text{UV}} = \sum_{k=0}^{N-1} \frac{\alpha_s}{8\pi} \left(\frac{\varphi}{F_a} + \frac{2\pi k}{N} \right) G_k \tilde{G}_k, \quad (\text{A3})$$

where we set $v_S = F_a$, F_a is the axion decay constant, α_s is the strong fine structure constant, and G_k and $\tilde{G}_k$ are the gluon field strength tensor and dual tensor, respectively.

In the context of the KSVZ Z_N axion model, the discrete symmetry is inherently present due to its construction. By elevating the inherent Z_N symmetry to a gauge symmetry, an accidental $U(1)_{\text{PQ}}$ invariance arises, which, for large values of N , is efficiently shielded from additional sources of explicit breaking. Furthermore, the lowest-dimensional PQ-violating operator in the scalar potential that is compatible with the Z_N symmetry is S^N , which contributes an explicitly PQ-breaking term to the potential.

See also Ref. [16] for another UV completion involving the composite Z_N axion model.

APPENDIX B: TEMPERATURE-DEPENDENT Z_N AXION MASS

Here, we provide a brief overview of the temperature-dependent Z_N axion mass. As derived in Ref. [16], the zero-temperature ($T \leq T_{\text{QCD}}$) Z_N axion potential is given by

$$V_N(\theta) \simeq -\frac{m_{N,0}^2 F_a^2}{N^2} \cos(N\theta), \quad (\text{B1})$$

with the zero-temperature axion mass

$$m_{N,0} \simeq \frac{m_\pi f_\pi}{\sqrt[4]{\pi} F_a} \sqrt[4]{\frac{1-z}{1+z}} N^{3/4} z^{N/2}, \quad (\text{B2})$$

where $z \equiv m_u/m_d \simeq 0.48$. At medium temperatures $T_{\text{QCD}} < T \leq T_{\text{QCD}}/\gamma$, the Z_N axion potential is given by

$$V_N(\theta) \simeq V(\theta) \left(\frac{T}{T_{\text{QCD}}} \right)^{-2b} + \sum_{k=1}^{N-1} V\left(\theta + \frac{2\pi k}{N}\right), \quad (\text{B3})$$

where $V(\theta)$ is the QCD axion zero-temperature chiral potential

$$V(\theta) = -m_\pi^2 f_\pi^2 \sqrt{1 - \frac{4z}{(1+z)^2} \sin^2\left(\frac{\theta}{2}\right)}. \quad (\text{B4})$$

Using

$$m_N(T) = \frac{1}{F_a} \sqrt{\frac{d^2 V_N(\theta)}{d\theta^2} \Big|_{\min}}, \quad (\text{B5})$$

we obtain the axion mass

$$m_N \simeq \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}}, \quad (\text{B6})$$

which is also defined as the mass $m_{N,\pi}$. At high temperatures $T > T_{\text{QCD}}/\gamma$, the Z_N axion potential is given by

$$V_N(\theta) \simeq V(\theta) \left(\frac{T}{T_{\text{QCD}}} \right)^{-2b} + \sum_{k=1}^{N-1} V\left(\theta + \frac{2\pi k}{N}\right) \left(\frac{\gamma T}{T_{\text{QCD}}} \right)^{-2b}. \quad (\text{B7})$$

Then, we have the axion mass

$$m_N \simeq \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}} \left(\frac{\gamma T}{T_{\text{QCD}}} \right)^{-b}. \quad (\text{B8})$$

Note that the temperature parameter γ delineates the high-temperature and intermediate-temperature regions. To be more precise, this parameter can be represented by the inequality $\gamma_{\min} < \gamma < \gamma_{\max}$, where the values of γ at the lower and upper bounds correspond to the mirror copies of the SM that are the coldest and hottest, respectively. For simplicity, we denote it as γ . Finally, the temperature-dependent Z_N axion mass can be described by

$$m_N(T) \simeq \begin{cases} \frac{m_\pi f_\pi}{\sqrt[4]{\pi} F_a} \sqrt[4]{\frac{1-z}{1+z}} N^{3/4} z^{N/2}, & T \leq T_{\text{QCD}} \\ \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}}, & T_{\text{QCD}} < T \leq \frac{T_{\text{QCD}}}{\gamma} \\ \frac{m_\pi f_\pi}{F_a} \sqrt{\frac{z}{1-z^2}} \left(\frac{\gamma T}{T_{\text{QCD}}} \right)^{-b}, & T > \frac{T_{\text{QCD}}}{\gamma} \end{cases} \quad (\text{B9})$$

See also Ref. [17] for more details.

APPENDIX C: AXION MIXING POTENTIAL

In this section, we describe how our axion mixing potential, which is presented in Eq. (1), is derived. Let us first consider the mixing of two axions ϕ_1 and ϕ_2 with the mixing potential

$$V_{\text{mix}} = \Lambda_1^4 \left[1 - \cos\left(n_{11} \frac{\phi_1}{f_1} + n_{12} \frac{\phi_2}{f_2} + \delta_1\right) \right] + \Lambda_2^4 \left[1 - \cos\left(n_{21} \frac{\phi_1}{f_1} + n_{22} \frac{\phi_2}{f_2} + \delta_2\right) \right], \quad (\text{C1})$$

where n_{ij} are the domain wall numbers, and δ_i are the constant phases. To discuss the emergence of the level crossing phenomenon, the domain wall numbers must be set in the following manner:

$$n_{11} \neq 0 (n_{11} = 0), \quad n_{12} = 0 (n_{12} \neq 0), \quad n_{21} \neq 0, \quad n_{22} \neq 0, \quad (\text{C2})$$

or

$$n_{11} \neq 0, \quad n_{12} \neq 0, \quad n_{21} = 0 (n_{21} \neq 0), \quad n_{22} \neq 0 (n_{22} = 0). \quad (\text{C3})$$

In other words, among these domain wall numbers, one must be set to zero, while the rest cannot be zero. This consideration and operation are reasonable, and numerous pieces of literature considering the mixing between two axions have adopted similar approaches, *e.g.*, the mixing between the QCD axion and ALP [20] or sterile axion [21], as well as the mixing between the Z_N axion and ALP [22]. Note that Refs. [20] and [21] can be viewed as corresponding to the considerations in the contexts of Eqs. (C2) and (C3), respectively, if we consider ϕ_1 as the QCD axion and ϕ_2 as the ALP. In these contexts, the domain wall numbers are equal to or less than 1 (and there is exactly one that is less than 1, *i.e.*, 0). In contrast, in Ref. [22] the domain wall numbers can be greater than 1. However, the constant phases must be set to zero:

$$\delta_1 = 0, \quad \delta_2 = 0. \quad (\text{C4})$$

This can be regarded as tantamount to demanding an independent solution for addressing the strong CP problem. Because this assumption has no impact on the evolution of the two axion fields during the mixing process, it will not alter the energy density of axions in the context of mass mixing [29].

Now, consider the scenario of multiple QCD axions mixing in this work, specifically one QCD axion ϕ and one Z_N axion φ . For our purposes, we must consider the following parameter settings:

$$n_{11} = 1, \quad n_{12} = 0, \quad n_{21} = 1, \quad n_{22} = N, \quad \delta_1 = 0, \quad \delta_2 = 0, \quad (C5)$$

Therefore, we obtain the mixing potential in Eq. (1), which is given by

$$V_{\text{mix}} = m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi}{f_a}\right) \right] + \frac{m_N^2(T) F_a^2}{N^2} \left[1 - \cos\left(\frac{\phi}{f_a} + N \frac{\varphi}{F_a}\right) \right]. \quad (C6)$$

As mentioned above, obtaining the axion mixing potential here is both straightforward and reasonable. When we set $n_{11} = 1$ and $n_{12} = 0$, the first term of Eq. (C6) aligns perfectly with the canonical QCD axion potential; however, this alignment is not an absolute requirement in this context. Additionally, it is worth noting that the acquisition of the mixing potential here differs from the acquisition of the single Z_N axion potential. Furthermore, there should exist another mixing scenario with the potential given by

$$V_{\text{mix}} = m_a^2(T) f_a^2 \left[1 - \cos\left(\frac{\phi}{f_a} + N \frac{\varphi}{F_a}\right) \right] + \frac{m_N^2(T) F_a^2}{N^2} \left[1 - \cos\left(N \frac{\varphi}{F_a}\right) \right]. \quad (C7)$$

The corresponding cosmological implications would be entirely distinct and warrant a separate detailed discussion; however, such discussions are beyond the scope of this particular context.

APPENDIX D: AXION RELIC DENSITY

In this section, we present some details about the axion relic density through the misalignment mechanism. We consider the pre-inflationary scenario, in which the PQ symmetry is spontaneously broken during inflation.

QCD axion without mixing

We first show the QCD axion relic density without mixing. At high temperatures $T \gg T_{\text{QCD}}$, the QCD axion

field is frozen at an arbitrary initial misalignment angle $\theta_{1,a}$ and starts to oscillate at $T_{1,a}$. The oscillation temperature $T_{1,a}$ is given by

$$m(T) = 3H(T), \quad (D1)$$

with the Hubble parameter

$$H(T) = \sqrt{\frac{\pi^2 g_*(T)}{90} \frac{T^2}{M_{\text{Pl}}}}, \quad (D2)$$

with the number of effective degrees of freedom of energy density g_* and reduced Planck mass $M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV. The initial energy density in the QCD axion field is

$$\rho_{a,1} = \frac{1}{2} m_{a,1}^2 f_a^2 \theta_{1,a}^2, \quad (D3)$$

where $m_{a,1}$ is the axion mass at $T_{1,a}$. At $T_0 < T < T_{1,a}$, the QCD axion energy density is adiabatic invariant with the comoving number $N_a \equiv \rho_a a^3 / m_a$, where a is the scale factor. Using

$$\frac{\rho_{a,1} a_{1,a}^3}{m_{a,1}} = \frac{\rho_{a,0} a_0^3}{m_{a,0}}, \quad (D4)$$

the present QCD axion energy density (T_0) can be described by

$$\rho_{a,0} = \frac{1}{2} m_{a,0} m_{a,1} f_a^2 \theta_{1,a}^2 \left(\frac{a_{1,a}}{a_0} \right)^3, \quad (D5)$$

where $a_{1,a}$ and a_0 are the scale factors at $T_{1,a}$ and T_0 , respectively.

Z_N axion without mixing

Next, we show the Z_N axion relic density without mixing. At high temperatures $T > T_{\text{QCD}}$, the Z_N axion field is frozen at an initial misalignment angle $\theta_{1,N}$ and starts to oscillate at the oscillation temperature $T_{1,N}$. Here, we consider a case that the Z_N axion mass is temperature-dependent at the first oscillation, *i.e.*, $T_{1,N} > 1/\gamma T_{\text{QCD}}$. The initial energy density in the Z_N axion field is

$$\rho_{N,1} = \frac{1}{2} m_{N,1}^2 F_a^2 \theta_{1,N}^2, \quad (D6)$$

where $m_{N,1}$ is the axion mass at $T_{1,N}$. At $T_{\text{QCD}} < T < T_{1,N}$, the Z_N axion energy density is adiabatic invariant with the comoving number $N_N \equiv \rho_N a^3 / m_N$. Then, we have the Z_N axion energy density at T_{QCD} as

$$\rho_{N,\text{QCD}} = \frac{1}{2} m_{N,1} m_{N,\pi} F_a^2 (\theta_{1,N} - \pi)^2 \left(\frac{a_{1,N}}{a_{\text{QCD}}} \right)^3, \quad (\text{D7})$$

where $a_{N,a}$ is the scale factor at $T_{1,N}$. In this period, the Z_N axion is trapped around $\theta_{\text{tr}} \equiv \theta_N(T_{\text{QCD}}) \sim \pi$ until the QCD phase transition with the initial axion velocity $\dot{\theta}_{\text{tr}} \equiv \dot{\theta}_N(T_{\text{QCD}})$ occurs. Then, the mean velocity is given by

$$\sqrt{\langle \dot{\theta}_{\text{tr}}^2 \rangle} = \frac{1}{\sqrt{2}} \sqrt{m_{N,1} m_{N,\pi}} |\theta_{1,N} - \pi| \left(\frac{a_{1,N}}{a_{\text{QCD}}} \right)^{3/2}. \quad (\text{D8})$$

At $T = T_{\text{QCD}}$, the Z_N axion mass is exponentially suppressed due to the Z_N symmetry, and the true minimum develops. Note that the axion energy density at T_{QCD} is non-adiabatic, which can then be described by

$$\rho_{N,\text{tr}} = \frac{1}{2} F_a^2 \dot{\theta}_{\text{tr}}^2 + 2 \frac{m_{N,1}^2 F_a^2}{N^2}. \quad (\text{D9})$$

The follow-up is determined by the velocity $\dot{\theta}_{\text{tr}} \sim 2m_N/N$, and we consider a case that the axion has a large initial velocity, *i.e.*, $\dot{\theta}_{\text{tr}} \gg 2m_N/N$. Then, at $T_2 < T < T_{\text{QCD}}$, the Z_N axion energy density is conserved with the PQ charge $q_{\text{kin}} = \dot{\theta}_N a^3$, and here, T_2 is defined as the temperature at which the kinetic energy is comparable to the barrier height, $\dot{\theta}_N(T_2) = 2m_N/N$. Using the conserved q_{kin} , the scale factor at T_2 is given by

$$a_2 = \left(\frac{N \dot{\theta}_{\text{tr}}}{2m_N} \right)^{1/3} a_{\text{QCD}}. \quad (\text{D10})$$

At $T < T_2$, the Z_N axion will start to oscillate until the present, and the axion energy density is adiabatic invariant. Then, using

$$\frac{\rho_{N,2} a_2^3}{m_{N,2}} = \frac{\rho_{N,0} a_0^3}{m_{N,0}}, \quad (\text{D11})$$

the Z_N axion energy density at present can be described by

$$\rho_{N,0} = \frac{m_{N,0} F_a^2 \dot{\theta}_{\text{tr}}}{N} \left(\frac{a_{\text{QCD}}}{a_0} \right)^3. \quad (\text{D12})$$

Substituting the mean velocity, we obtain the axion en-

ergy density

$$\rho_{N,0} = C \frac{m_{N,0} \sqrt{m_{N,1} m_{N,\pi}} |\theta_{1,N} - \pi| F_a^2}{\sqrt{2} N} \left(\frac{\sqrt{a_{1,N} a_{\text{QCD}}}}{a_0} \right)^3, \quad (\text{D13})$$

where $C \simeq 2$ is a constant.

Z_N axion in Case I

The axion relic density ratio R_ρ in this case is given by

$$\begin{aligned} R_\rho &\simeq \frac{\sqrt{m_{a,0} m_{a,1}} \theta_{1,a}}{\sqrt{m_{N,1} m_{N,\pi}} |\theta_{1,N} - \pi| \zeta} \left(\frac{\sqrt{a_{1,a}}}{\sqrt{a_{1,N}}} \right)^3 \\ &= \sqrt[4]{\frac{1-z}{1+z}} \sqrt{\frac{m_{a,1}}{m_{N,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \sqrt{\zeta}} \left(\frac{\sqrt{a_{1,a}}}{\sqrt{a_{1,N}}} \right)^3 \\ &= \sqrt[4]{\frac{1-z}{1+z}} \sqrt{\frac{m_{a,1}}{m_{N,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \sqrt{\zeta}} \left(\frac{m_{N,1}}{m_{a,1}} \right)^{3/4} \\ &= \sqrt[4]{\frac{1-z}{1+z}} \sqrt[4]{\frac{m_{N,1}}{m_{a,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \sqrt{\zeta}}. \end{aligned} \quad (\text{D14})$$

Z_N axion in Case II

The axion relic density ratio R_ρ in this case is given by

$$\begin{aligned} R_\rho &\simeq \frac{\sqrt{m_{N,\pi} m_{a,1}} \theta_{1,a}}{\sqrt{m_{N,1} m_{N,\pi}} |\theta_{1,N} - \pi| \zeta} \left(\frac{\sqrt{a_{1,a}}}{\sqrt{a_{1,N}}} \right)^3 \\ &= \sqrt{\frac{m_{a,1}}{m_{N,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \zeta} \left(\frac{\sqrt{a_{1,a}}}{\sqrt{a_{1,N}}} \right)^3 \\ &= \sqrt[4]{\frac{m_{N,1}}{m_{a,1}}} \frac{\theta_{1,a}}{|\theta_{1,N} - \pi| \zeta}. \end{aligned} \quad (\text{D15})$$

QCD axion in Case II

The axion relic density ratio R_ρ in this case is given by

$$R_\rho \simeq \frac{m_{N,1} F_a^2 \theta_{1,N}^2}{m_{a,1} f_a^2 \theta_{1,a}^2} \left(\frac{a_{1,N}}{a_{1,a}} \right)^3 = \sqrt{\frac{m_{a,1}}{m_{N,1}}} \frac{\theta_{1,N}^2 \zeta^2}{\theta_{1,a}^2}. \quad (\text{D16})$$

References

- [1] R.D. Peccei and Helen R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977)
- [2] R.D. Peccei and Helen R. Quinn, *Phys. Rev. D* **16**, 1791 (1977)
- [3] S. Weinberg, *Phys. Rev. Lett.* **40**, 223 (1978)
- [4] F. Wilczek, *Phys. Rev. Lett.* **40**, 279 (1978)
- [5] J. E. Kim, *Phys. Rev. Lett.* **43**, 103 (1979)
- [6] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, *Nucl. Phys. B* **166**, 493 (1980)
- [7] M. Dine, W. Fischler, and M. Srednicki, *Phys. Lett. B* **104**,

- 199 (1981)
- [8] A. R. Zhitnitsky, *Sov. J. Nucl. Phys.* **31**, 260 (1980)
- [9] G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976)
- [10] G. 't Hooft, *Phys. Rev. D* **14**, 3432 (1976)
- [11] J. Preskill, M. B. Wise, and F. Wilczek, *Phys. Lett. B* **120**, 127 (1983)
- [12] L.F. Abbott and P. Sikivie, *Phys. Lett. B* **120**, 133 (1983)
- [13] M. Dine and W. Fischler, *Phys. Lett. B* **120**, 137 (1983)
- [14] C. A. J. O'Hare, *PoS COSMICWISPerS*, 040 (2024), arXiv: 2403.17697[hep-ph]
- [15] A. Hook, *Phys. Rev. Lett.* **120**, 261802 (2018), arXiv: 1802.10093[hep-ph]
- [16] L. D. Luzio, B. Gavela, P. Quilez *et al.*, *JHEP* **05**, 184 (2021), arXiv: 2102.00012[hep-ph]
- [17] L. D. Luzio, B. Gavela, P. Quilez *et al.*, *JCAP* **10**, 001 (2021), arXiv: 2102.01082[hep-ph]
- [18] R. T. Co, L. J. Hall, and K. Harigaya, *Phys. Rev. Lett.* **124**, 251802 (2020), arXiv: 1910.14152[hep-ph]
- [19] C. T. Hill and G. G. Ross, *Nucl. Phys. B* **311**, 253 (1988)
- [20] R. Daido, N. Kitajima, and F. Takahashi, *Phys. Rev. D* **93**, 075027 (2016), arXiv: 1510.06675[hep-ph]
- [21] D. Cyncynates and J. O. Thompson, *Phys. Rev. D* **108**, L091703 (2023), arXiv: 2306.04678[hep-ph]
- [22] H. J. Li, Y. Q. Peng, W. Chao *et al.*, *Phys. Lett. B* **849**, 138444 (2024), arXiv: 2310.02126[hep-ph]
- [23] L. Wolfenstein, *Phys. Rev. D* **17**, 2369 (1978)
- [24] S. P. Mikheyev and A. Yu. Smirnov, *Sov. J. Nucl. Phys.* **42**, 913 (1985)
- [25] S. P. Mikheev and A. Yu. Smirnov, *Nuovo Cim. C* **9**, 17 (1986)
- [26] N. Kitajima and F. Takahashi, *JCAP* **01**, 032 (2015), arXiv: 1411.2011[hep-ph]
- [27] S. Y. Ho, K. Saikawa, and F. Takahashi, *JCAP* **10**, 042 (2018), arXiv: 1806.09551[hep-ph]
- [28] D. Cyncynates, T. Giurgica-Tiron, O. Simon *et al.*, *Phys. Rev. D* **105**, 055005 (2022), arXiv: 2109.09755[hep-ph]
- [29] H. J. Li, *JCAP* **09**, 025 (2024), arXiv: 2307.09245[hep-ph]
- [30] R. Daido, N. Kitajima, and F. Takahashi, *Phys. Rev. D* **92**, 063512 (2015), arXiv: 1505.07670[hep-ph]
- [31] D. Cyncynates, O. Simon, J. O. Thompson *et al.*, *Phys. Rev. D* **106**, 083503 (2022), arXiv: 2208.05501[hep-ph]
- [32] Z. Chen, A. Kobakhidze, C. A. J. O'Hare *et al.*, *Cosmology of the companion-axion model: dark matter, gravitational waves, and primordial black holes*, (2021), arXiv: 2110.11014[hep-ph]
- [33] H. J. Li and Y. F. Zhou, *Chin. Phys. C* **49**, 035101 (2025), arXiv: 2401.09138[hep-ph]
- [34] K. Mürsepp, E. Nardi, and C. Smarra, *Can the QCD axion feed a dark energy component?* (2024), arXiv: 2405.00090[hep-ph]
- [35] K. Mürsepp, *On the viability of the QCD axion - dark energy transition*, (2024), arXiv: 2405.20478[hep-ph]
- [36] E. Witten, *Phys. Lett. B* **149**, 351 (1984)
- [37] M. B. Green and J. H. Schwarz, *Phys. Lett. B* **149**, 117 (1984)
- [38] P. Svrcek and E. Witten, *JHEP* **06**, 051 (2006), arXiv: hep-th/0605206
- [39] A. Arvanitaki, S. Dimopoulos, S. Dubovsky *et al.*, *Phys. Rev. D* **81**, 123530 (2010), arXiv: 0905.4720[hep-th]
- [40] M. Cicoli, M. Goodsell, and A. Ringwald, *JHEP* **10**, 146 (2012), arXiv: 1206.0819[hep-th]
- [41] M. Demirtas, N. Gendler, C. Long *et al.*, *JHEP* **06**, 092 (2023), arXiv: 2112.04503[hep-th]