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# Time-reversal invariance violation effect in *dd* scattering*

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**Abstract:** A formalism is developed for calculating the signal of the violation of time-reversal invariance, provided that space-reflection (parity) invariance is conserved during the scattering of tensor-polarized deuterons on vector-polarized deuterons. The formalism is based on Glauber theory and fully considers the spin dependence of *NN* elastic scattering amplitudes and the spin structure of colliding deuterons. Numerical calculations are performed in the laboratory proton energy range $T_p = 0.1$–$1.2$ GeV using the SAID database for spin amplitudes and in the energy region of the SPD NICA experiment corresponding to the invariant mass of the interacting nucleon pairs $\sqrt{s_{NN}} = 2.5$–$25$ GeV, using two phenomenological models of *pN* elastic scattering. It is found that only one type of the time-reversal non-invariant parity conserving *NN* interaction gives a non-zero contribution to the signal in question, which is important for isolating an unknown constant of this interaction from the corresponding data.

**Keywords:** time-reversal invariance, spin observables, deuteron-deuteron scattering

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## I. INTRODUCTION

Discrete symmetries with respect to time reversal (T), space reflection (P), and charge conjugation (C) play a key role in the theory of fundamental interactions and astrophysics. Under CPT-symmetry, which takes place in local quantum field theory [1, 2], the violation of T-invariance also indicates the violation of CP-symmetry, which is necessary to explain the baryon asymmetry of the Universe [3]. CP violation observed in the decays of *K*, *B*, and *D* mesons is consistent with the standard model (SM) of fundamental interactions; however, it is far from sufficient to explain the observed baryon asymmetry [4]. Therefore, there must be other sources of CP violation in nature beyond the SM.

One of these sources is associated with the electric dipole moments (EDMs) of free elementary particles, neutral atoms, and the lightest nuclei, the search for which has attracted significant attention over the last few decades [5]. The observation of a non-zero EDM value indicates that *T*-invariance and parity are violated simultaneously. Considerably less attention has been paid to experiments on the search for the effects of T-invariance violation with parity conservation (TVPC) and flavor conservation. This type of interaction was introduced in [6] to explain the CP violation observed in kaon decays and is related to physics beyond of the SM [7, 8]. As demonstrated in a model-independent manner within the effective field theory [9], owing to an unknown mechanism of EDM generation, the available experimental limitations on EDMs cannot be used to estimate the appropriate restrictions on TVPC effects. The detection of these at the current level of experimental sensitivity would represent direct evidence of physics beyond the SM.

In the scattering of two polarized nuclei, the signal of the violation of T-invariance while conserving parity is the component of the total cross section, which corresponds to the interaction of a transversely polarized ($P_y$) incident nucleus with a tensor-polarized ($P_{xz}$) target nucleus [10]. This observable cannot be simulated by the interaction in the initial or final states and is not zero only in the presence of the discussed TVPC interaction, just as EDMs are a signal of a T- and P-violating interaction.

Following the description of the experimental COSY project for studying the TVPC effect in *pd* interactions [11], this type of component of the total cross section (known in the literature as the TVPC null-test signal) can also be measured in *dd* scattering by measuring the asymmetry of the event counting rate in this process. This asymmetry appears, when the sign of the vector polariza-

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tion of one of the colliding deuterons ($P_y^{(1)}$) is changed, whereas the tensor polarization ($P_{xz}^{(2)}$) of the second deuteron is unchanged.

When using this method, the transverse vector polarization $P_y^{(2)}$ of the second (tensor-polarized) deuteron must be zero [12]. Another method of measurement that does not require such a restriction on $P_y^{(2)}$ but uses the rotating polarization of the incoming beam in combination with Fourier analysis of the time-dependent counting rate of the number of events was proposed in Ref. [13]. A possible measurement procedure for the TVPC null-test signal in *dd* scattering was recently discussed in Ref. [14].

Here, we focus on the theoretical calculation of the TVPC null-test signal. Its dependence on the collision energy for *pd* [15, 16] and $^3$He*d* [17] scattering was investigated within Glauber theory in the laboratory energy range 0.1–1 GeV considering the full spin dependence of *NN* scattering amplitudes and the *S* and *D* components of the deuteron wave function.

In this paper, we calculate the TVPC null-test signal in *dd* scattering for the first time using fully spin-dependent Glauber theory for this process and generalize the method developed in [14–16]. In the following, Sec. II provides the basic mathematical formalism for this calculation, Sec. III presents and analyzes the results of numerical calculations, and Sec. IV provides the conclusions. A detailed derivation of the final formulas for the TVPC signal is given in the appendix.

## II. CALCULATION OF THE TVPC SIGNAL IN *dd* SCATTERING

In *pd* collisions, the TVPC signal is determined by the component of the total cross section corresponding to a vector-polarized proton interacting with a tensor-polarized deuteron [13]. Unlike *pd* scattering, *dd* scattering has two symmetric components of the total cross section corresponding to the vector polarization of one deuteron and the tensor polarization of the other. Accordingly, the TVPC transition operator $dd \to dd$ at zero angle includes two terms:

$$\hat{M}_{\rm TVPC}(0) = g_1\hat{O}_1 + g_2\hat{O}_2. \tag{1}$$

Here, the operators $\hat{O}_1$ and $\hat{O}_2$ are defined as

$$\hat{O}_1 = \hat{k}_m\hat{Q}_{mn}^{(1)}\varepsilon_{nlr}S_l^{(2)}\hat{k}_r,$$

$$\hat{O}_2 = \hat{k}_m\hat{Q}_{mn}^{(2)}\varepsilon_{nlr}S_l^{(1)}\hat{k}_r, \tag{2}$$

where $\hat{\mathbf{k}}$ is a unit vector directed along the incident beam, $S_l^{(j)}$ are the components of the spin operator of the *j*-th deuteron, $\hat{Q}_{mn}^{(j)} = \frac{1}{2}\left(S_m^{(j)}S_n^{(j)} + S_n^{(j)}S_m^{(j)} - \frac{4}{3}\delta_{mn}I\right)$ is the symmetric tensor operator, and $\varepsilon_{nlr}$ is the fully antisymmetric tensor ($m$, $n$, $l$, $r = x, y, z$). Henceforth, we assume $j = 1$ for the incident deuteron and $j = 2$ for the target deuteron.

We find the TVPC signal using the optical theorem:

$$\sigma_{\rm TVPC} = 4\sqrt{\pi}{\rm Im\,Tr}(\hat{\rho}_i\hat{M}_{\rm TVPC}(0)) = \sigma_{\rm TVPC}^{(1)} + \sigma_{\rm TVPC}^{(2)}, \tag{3}$$

where $\hat{\rho}_i$ is the spin density matrix of the initial state, which includes vector and tensor polarizations of both deuterons, and the cross sections $\sigma_{\rm TVPC}^{(i)}$ $(i = 1, 2)$ are expressed through the amplitudes $g_i$ as follows:

$$\sigma_{\rm TVPC}^{(1)} = 4\sqrt{\pi}{\rm Im}\left(\frac{g_1}{9}\right)(P_{xz}^{(1)}P_y^{(2)} - P_{zy}^{(1)}P_x^{(2)}),$$

$$\sigma_{\rm TVPC}^{(2)} = 4\sqrt{\pi}{\rm Im}\left(\frac{g_2}{9}\right)(P_{xz}^{(2)}P_y^{(1)} - P_{zy}^{(2)}P_x^{(1)}). \tag{4}$$

In turn, the amplitudes $g_1$ and $g_2$ can be expressed in terms of matrix elements from the transition operator over the spin states of the incident and target deuterons in the initial and final states, $< m_1', m_2'|\hat{M}_{\rm TVPC}(0)|m_1, m_2 >$:

$$< -1, 1|\hat{M}_{\rm TVPC}(0)|0, 0 >= i\frac{g_1 + g_2}{2},$$

$$< 1, 0|\hat{M}_{\rm TVPC}(0)|0, 1 >= i\frac{g_1 - g_2}{2}. \tag{5}$$

Let us find the transition operator $\hat{M}_{\rm TVPC}(0)$ in the Glauber model, taking spin effects into account. A single-scattering mechanism, as well as in the case of *pd* collisions, does not contribute to the TVPC signal because the corresponding TVPC *NN* amplitude is vanishing at the zero scattering angle [12]. In this paper, we calculate the TVPC signal in the double-scattering approximation, neglecting the contributions of triple and quadruple *NN* collisions, which give only a small correction to the *dd* elastic differential cross section at forward scattering angles [14, 18].

The amplitude of the double-scattering mechanism in an elastic *dd* collision consists of two terms, the so-called "normal" and "abnormal" terms. The first ("normal") corresponds to the sequential scattering of both nucleons of the incident deuteron on one of the nucleons of the target deuteron, and similarly, of one of the nucleons in the incident beam on both nucleons of the target. The second ("abnormal") is the simultaneous collision of one nucleon from the incident beam with one of the target nucleons and another nucleon of the beam with another nucleon of the target. The corresponding scattering amplitude at zero angle takes the form

$$\hat{M}^{(2)}(0) = \hat{M}^{(2n)}(0) + \hat{M}^{(2a)}(0),$$
$$\hat{M}^{(2n)}(0) = \frac{\mathrm{i}}{2\pi^{3/2}} \int\int\int \mathrm{d}^3\rho \mathrm{d}^3 r \mathrm{d}^2 q \Psi^+_{d(12)}(\mathbf{r})\Psi^+_{d(34)}(\boldsymbol{\rho}) \times \left[\mathrm{e}^{\mathrm{i}\mathbf{q}\boldsymbol{\delta}}\hat{O}^{(2n)}(\mathbf{q}) + \mathrm{e}^{\mathrm{i}\mathbf{q}\mathbf{s}}\hat{O}'^{(2n)}(\mathbf{q})\right]\Psi_{d(34)}(\boldsymbol{\rho})\Psi_{d(12)}(\mathbf{r}),$$
$$\hat{M}^{(2a)}(0) = \frac{\mathrm{i}}{2\pi^{3/2}} \int\int\int \mathrm{d}^3\rho \mathrm{d}^3 r \mathrm{d}^2 q \Psi^+_{d(12)}(\mathbf{r})\Psi^+_{d(34)}(\boldsymbol{\rho})\mathrm{e}^{\mathrm{i}\mathbf{q}(\mathbf{s}-\boldsymbol{\delta})} \times \left[\hat{O}^{(2a)}(\mathbf{q}) + \hat{O}'^{(2a)}(\mathbf{q})\right]\Psi_{d(34)}(\boldsymbol{\rho})\Psi_{d(12)}(\mathbf{r}). \tag{6}$$

The operators $\hat{O}^{(2n)}(\mathbf{q})$, $\hat{O}'^{(2n)}(\mathbf{q})$, $\hat{O}^{(2a)}(\mathbf{q})$ , and $\hat{O}'^{(2a)}(\mathbf{q})$ are expressed in terms of spin-dependent $NN$ amplitudes:

$$\hat{O}^{(2n)}(\mathbf{q}) = \frac{1}{2}\{M_{31}(\mathbf{q}), M_{41}(-\mathbf{q})\} + \frac{1}{2}\{M_{32}(\mathbf{q}), M_{42}(-\mathbf{q})\},$$
$$\hat{O}'^{(2n)}(\mathbf{q}) = \frac{1}{2}\{M_{31}(\mathbf{q}), M_{32}(-\mathbf{q})\} + \frac{1}{2}\{M_{41}(\mathbf{q}), M_{42}(-\mathbf{q})\},$$
$$\hat{O}^{(2a)}(\mathbf{q}) = M_{31}(\mathbf{q})M_{42}(-\mathbf{q}),$$
$$\hat{O}'^{(2a)}(\mathbf{q}) = M_{32}(\mathbf{q})M_{41}(-\mathbf{q}). \tag{7}$$

Here, the subscripts 1 and 2 refer to the nucleons of the target deuteron, and 3 and 4 refer to the nucleons of the incoming deuteron; $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\boldsymbol{\rho} = \mathbf{r}_3 - \mathbf{r}_4$ by $\mathbf{s}$ and $\boldsymbol{\delta}$ are the components of the vectors $\mathbf{r}$ and $\boldsymbol{\rho}$, respectively, perpendicular to the direction of the incident beam. In the Glauber approximation, $\mathbf{qr} = \mathbf{qs}$ and $\mathbf{q}\boldsymbol{\rho} = \mathbf{q}\boldsymbol{\delta}$. In (7), $\{,\}$ denotes the anticommutator of two spin NN amplitudes.

The deuteron wave function is represented in a standard way:

$$\Psi_{d(ij)} = \frac{1}{\sqrt{4\pi}r}\left(u(r) + \frac{1}{2\sqrt{2}}w(r)\hat{S}_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)\right), \tag{8}$$

where $u(r)$ and $w(r)$ are the $S$- and $D$-wave radial functions, $\hat{S}_{12}(\hat{\mathbf{r}};\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j) = 3(\boldsymbol{\sigma}_i\cdot\hat{r})(\boldsymbol{\sigma}_j\cdot\hat{r}) - \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j$ is the tensor operator, and $\frac{1}{2}\boldsymbol{\sigma}_i$ is the spin operator of the $i$th nucleon.

For $T$-even $P$-even $NN$ amplitudes, we use the following representation [19]:

$$M_{ij}(\mathbf{q}) = A_N + C_N(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{n}}) + C'_N(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{n}}) + B_N(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{k}})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{k}}) + (G_N + H_N)(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{q}}) + (G_N - H_N)(\boldsymbol{\sigma}_i\cdot\hat{\mathbf{n}})(\boldsymbol{\sigma}_j\cdot\hat{\mathbf{n}}). \tag{9}$$

Here, the unit vectors $\hat{\mathbf{k}},\hat{\mathbf{q}},\hat{\mathbf{n}}$ correspond to the vectors

$$\mathbf{k} = \frac{1}{2}(\mathbf{p}+\mathbf{p}'), \quad \mathbf{q} = \mathbf{p}-\mathbf{p}', \quad \mathbf{n} = [\mathbf{p}'\times\mathbf{p}], \tag{10}$$

where $\mathbf{p}$ and $\mathbf{p}'$ are the momenta of the incident and scattered nucleons, respectively, and the invariant amplitudes $A_N, C_N, C'_N, B_N, G_N, H_N$ (which correspond to $pN$ amplitudes with $N = p$ for $\{ij\} = \{31\},\{42\}$ and $N = n$ for $\{ij\} = \{32\},\{41\}$) depend on the momentum $q = |\mathbf{q}|$. To calculate $M_{ij}(-\mathbf{q})$, we replace $\mathbf{q}\to-\mathbf{q}$, $\mathbf{n}\to-\mathbf{n}$ in Eq. (9). In the laboratory frame traditionally used to derive scattering amplitudes in the Glauber model, the amplitudes $C_N$ and $C'_N$ are different.

The amplitudes (9) are normalized in such a way that

$$\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}t} = \frac{1}{4}\mathrm{Tr}(M_{ij}M^+_{ij}). \tag{11}$$

In turn, the amplitudes $M$ of $dd$ elastic scattering are related to the differential cross section as follows:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{9}\mathrm{Tr}(\hat{M}\hat{M}^+). \tag{12}$$

This relationship is consistent with the optical theorem (3).

Furthermore, we take the TVPC $NN\to NN$ transition operator in the form [12]

$$t_{ij} = h_N[(\boldsymbol{\sigma}_i\cdot\mathbf{k})(\boldsymbol{\sigma}_j\cdot\mathbf{q}) + (\boldsymbol{\sigma}_i\cdot\mathbf{q})(\boldsymbol{\sigma}_j\cdot\mathbf{k}) - \frac{2}{3}(\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)(\mathbf{q}\cdot\mathbf{k})]/m^2 + g_N[\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_j]\cdot[\mathbf{q}\times\mathbf{k}](\boldsymbol{\tau}_i-\boldsymbol{\tau}_j)_z/m^2 + g'_N(\boldsymbol{\sigma}_i-\boldsymbol{\sigma}_j)\cdot i[\mathbf{q}\times\mathbf{k}][\boldsymbol{\tau}_i\times\boldsymbol{\tau}_j]_z/m^2, \tag{13}$$

where $m$ is the nucleon mass. In the calculations, we use the TVPC $NN$ amplitudes $T_{ij}$ normalized in the same manner as the $T$-even $P$-even amplitudes (9) and related to the amplitudes (13) as [12]

$$T_{ij} = \frac{m}{4\sqrt{\pi}k_{NN}}t_{ij}, \tag{14}$$

where $k_{NN}$ is the nucleon momentum in the $NN$ center-of-mass frame. Considering TVPC interactions, the products of $NN$ amplitudes included in the operators of normal and abnormal double scattering (7) take the form

$$
\begin{aligned}
&[M_{ij}(\mathbf{q})+T_{ij}(\mathbf{q})][M_{kl}(-\mathbf{q})+T_{kl}(-\mathbf{q})]\\
&=M_{ij}(\mathbf{q})M_{kl}(-\mathbf{q})+T_{ij}(\mathbf{q})T_{kl}(-\mathbf{q})\\
&\quad+T_{ij}(\mathbf{q})M_{kl}(-\mathbf{q})+M_{ij}(\mathbf{q})T_{kl}(-\mathbf{q}),
\end{aligned}
\tag{15}
$$

where the first two terms correspond to the spin-dependent $T$-even $P$-even amplitude of $dd$ scattering (the second term can be neglected), and the last two correspond to the $T$-odd $P$-even (TVPC) amplitude.

Let us separately consider the contributions of three types of TVPC $NN$ interactions.

i) The $NN$ amplitude of the $g'$ type contributes only to the charge-exchange process $pn\to np$. In $dd$ collisions, a double scattering process is possible with two sequential (or simultaneous in the case of abnormal scattering) charge-exchange collisions: $pn\to np$ and $np\to pn$. The product of the corresponding amplitudes has a form similar to Eq. (15), where $NN$ amplitudes are charge-exchange ones. In this case, $T$-even amplitudes are the same for the processes $pn\to np$ and $np\to pn$, whereas $T$-odd amplitudes have an equal magnitude but an opposite sign for these two processes. Therefore, the net contribution of the $g'$-type amplitude to the TVPC signal becomes zero, as in the case of $pd$ scattering [15].

ii) The contribution of the $g$-type $NN$ amplitude becomes zero for identical nucleons, owing to the isospin factor $(\boldsymbol{\tau}_i-\boldsymbol{\tau}_j)_z$ (see Eq. (13)). In this case, the operators of normal (or abnormal) double scattering, considering the decomposition (15), contain the sum of $g$-type $pn$ and $np$ amplitudes multiplied by the same $T$-even $pp$ (or $pn$) amplitude. Because the $g$-type amplitudes for $pn$ and $np$ elastic scattering have different signs owing to the same isospin factor, the net $g$-type contribution to the TVPC signal also tends to zero. This can be easily shown by explicitly writing the operators $\hat{O}^{(2n)}$, $\hat{O}'^{(2n)}$, $\hat{O}^{(2a)}$ , and $\hat{O}'^{(2a)}$ and employing the symmetry of the deuteron wave functions with respect to index permutations $1\leftrightarrow 2$ and $3\leftrightarrow 4$.

iii) Thus, among the three types of TVPC $NN$ interactions, only the $h$-type amplitude contributes to the TVPC signal in $dd$ scattering. To calculate the respective contribution, we substitute the expansion (15) with the $h$-type TVPC $NN$ amplitude into the operators (7) and then perform integration by the nucleon coordinates in the expressions for double-scattering amplitudes (6). It is a straightforward but rather cumbersome procedure in the case of spin $NN$ amplitudes and the $D$-wave included in the deuteron wave functions. Finally, by calculating the spin matrix elements (5), we find the TVPC amplitudes of $dd$ scattering $g_i$ $(i=1,2)$. The detailed derivation of the $h$-type TVPC signal is given in the appendix.

As a result, we obtain the following expressions for the amplitudes $g_1$ and $g_2$:

$$
g_1=\frac{\mathrm{i}}{2\pi m}\int_0^\infty \mathrm{d}q q^2\left[Z_0+Z(q)\right]\zeta(q)h_N(q)\left(C_n(q)+C_p(q)\right),
$$

$$
g_2=\frac{\mathrm{i}}{2\pi m}\int_0^\infty \mathrm{d}q q^2\left[Z_0+Z(q)\right]\zeta(q)h_N(q)\left(C'_n(q)+C'_p(q)\right), \tag{16}
$$

where the first term in square brackets refers to normal, and the second term refers to abnormal double scattering. In Eq. (16), we assume $h_p=h_n=h_N$, which is justified in the beginning of the next section. The quantities $Z_0$, $Z(q)$ , and $\zeta(q)$ in Eq. (16) are the linear combinations of the deuteron form factors:

$$
\begin{aligned}
Z_0&=S_0^{(0)}(0)-\frac{1}{2}S_0^{(2)}(0)=1-\frac{3}{2}P_D,\\
Z(q)&=S_0^{(0)}(q)-\frac{1}{2}S_0^{(2)}(q)\\
&\quad-\frac{1}{\sqrt{2}}S_2^{(1)}(q)+\sqrt{2}S_2^{(2)}(q),\\
\zeta(q)&=S_0^{(0)}(q)+\frac{1}{10}S_0^{(2)}(q)\\
&\quad-\frac{1}{\sqrt{2}}S_2^{(1)}(q)+\frac{\sqrt{2}}{7}S_2^{(2)}(q)+\frac{18}{35}S_4^{(2)}(q),
\end{aligned}
\tag{17}
$$

where $P_D$ is the $D$-state probability in the deuteron. Moreover, note that $Z_0=Z(0)$. If the $D$-wave contribution is neglected, both $Z(q)$ and $\zeta(q)$ are reduced to a purely $S$-wave form factor $S_0^{(0)}(q)$, and $Z_0$ turns to unity. The deuteron form factors arising in (17) are defined as follows:

$$
\begin{aligned}
S_0^{(0)}(q)&=\int_0^\infty \mathrm{d}r u^2(r)j_0(qr),\\
S_0^{(2)}(q)&=\int_0^\infty \mathrm{d}r w^2(r)j_0(qr),\\
S_2^{(1)}(q)&=2\int_0^\infty \mathrm{d}r u(r)w(r)j_2(qr),\\
S_2^{(2)}(q)&=-\frac{1}{\sqrt{2}}\int_0^\infty \mathrm{d}r w^2(r)j_2(qr),\\
S_4^{(2)}(q)&=\frac{1}{2}\int_0^\infty \mathrm{d}r w^2(r)j_4(qr).
\end{aligned}
\tag{18}
$$

Note that the form factor $S_4^{(2)}(q)$ is absent in the electromagnetic structure of the deuteron.

The TVPC signal is eventually found from the amplitudes $g_1$ and $g_2$ using the formulas (3)-(4) for a given combination of polarizations of colliding deuterons.

## III. NUMERICAL RESULTS

For numerical calculations, spin amplitudes of $pp$ and $pn$ elastic scattering are required, that is, both $T$-even $P$-even amplitudes from (9) and $T$-odd $P$-even amplitudes from (13). As shown in the previous section, the $g$ and $g'$ type interactions do not contribute to the TVPC signal in $dd$ scattering. Therefore, we consider only the $h$-type interaction. The numerical value of the constant in the respective amplitude $h_N$ (13) is unknown; therefore, it is impossible to calculate the absolute value of the TVPC signal, but it is possible to calculate its dependence on the collision energy.

The amplitude $h_N$ dependence on the momentum $q$ can be given under the assumption that the $h$-type $NN$ interaction is determined by the exchange of the $h_1(1170)$ meson with quantum numbers $I^G(J^{PC})=0^-(1^{+-})$ between nucleons (see [15] and references therein). Under this assumption, according to the studies [15, 20], we take the following expression for the amplitude $h_N$:

$$h_N = -\mathrm{i}\phi_h \frac{2G_h^2}{m_h^2+\mathbf{q}^2} F_{hNN}(\mathbf{q}^2), \tag{19}$$

where $\phi_h=\tilde{G}_h/G_h$ is the ratio of the coupling constant of the $h_1$-meson with a nucleon for the T-non-invariant interaction ($\tilde{G}_h$) to the corresponding constant of the T-invariant interaction ($G_h$), and $F_{hNN}(\mathbf{q}^2)=(\Lambda^2-m_h^2)/(\Lambda^2-\mathbf{q}^2)$ is the phenomenological monopole form factor at the $hNN$ vertex. The numerical parameters are taken from Ref. [20]: $m_h=1.17$ GeV, $G_h=4\pi\times1.56$, and $\Lambda=2$ GeV, from the Bonn $NN$-interaction potential. At the same time, owing to the isoscalar nature of this meson, we have the equality of the amplitudes $h_p=h_n=h_N$, which is taken into account in the formulas (16) for the $dd$ TVPC amplitudes $g_i$ $(i=1,2)$.

In the range of laboratory proton beam energies 0.1–1.2 GeV in $pN$ scattering (corresponding to the interval of the invariant mass of colliding nucleons $\sqrt{s_{pN}}=1.9$–$2.4$ GeV), the $T$-even $P$-even amplitudes $A_N,\cdots,H_N$ are available in the SAID database [21], which we use in the numerical calculations of the TVPC signal at these energies. In the calculations at higher energies $\sqrt{s_{NN}}\gtrsim2.5$ GeV, corresponding to the conditions of the NICA SPD experiment, we employ the phenomenological models for the spin amplitudes of $pN$ elastic scattering available in the literature.

In the formulation of $pp$ scattering models in the high-energy region, the helicity amplitudes $\phi_1\div\phi_5$ are used, with the conventional notation (see [22]). The spin amplitudes $A_N$, $B_N$, $C_N$, $C'_N$, $G_N$ , and $H_N$, defined in (9), are related to the helicity amplitudes via the following relations, which are valid at small momentum transfers and high energies specific for the Glauber model (see [19] and references therein):

$$\begin{aligned} &A_N=(\phi_1+\phi_3)/2, \quad B_N=(\phi_3-\phi_1)/2,\\ &C_N=\mathrm{i}\phi_5,\ \ G_N=\phi_2/2,\ \ H_N=\phi_4/2;\\ &C'_N=C_N+\frac{\mathrm{i}q}{2m}A_N. \end{aligned} \tag{20}$$

Here, we use two different models for the helicity amplitudes of $pN$ elastic scattering. The first one [22] involves the Regge parameterization of data on the $pp$ differential cross section and spin correlations $A_N$, $A_{NN}$ in the range of laboratory momenta $3\div50$ GeV/$c$. This model includes the contributions from four Regge trajectories, $\omega$, $\rho$, $f_2$, $a_2$, and the $P$ pomeron exchange. As noted in [22], in the Regge model, because of isospin symmetry and relations due to $G$ parity, the $pp$ and $pn$ scattering amplitudes can be represented as the following linear combinations of these five contributions:

$$\begin{aligned} &\phi(pp)=-\phi_\omega-\phi_\rho+\phi_{f_2}+\phi_{a_2}+\phi_P,\\ &\phi(pn)=-\phi_\omega+\phi_\rho+\phi_{f_2}-\phi_{a_2}+\phi_P; \end{aligned} \tag{21}$$

$\phi_\omega$ is the contribution of the $\omega$ Regge trajectory, etc. The energy domain, in which Regge parameterization was performed in [22], corresponds to the range of the $pp$ invariant mass $\sqrt{s_{pp}}=2.8$–$10$ GeV.

The second model is based on the Regge-eikonal model developed by Selyugin (see [23] and references therein) and was coined the High Energy Generalized Structure (HEGS) model by its author. This model considers $pp$, $p\bar{p}$ , and $pn$ elastic scattering at small angles and the nucleon structure based on data on the generalized parton distributions of nucleons. The helicity amplitudes of $NN$ elastic scattering obtained in this model allow us to describe the available experimental data on the differential cross section and single-spin asymmetry $A_N(s,t)$ in $pp$ scattering in the energy range $\sqrt{s}$ from 3.6 to 10 TeV with a minimal number of variable parameters [24]. In both models, at the energies $\sqrt{s_{pp}}\geq3$ GeV considered in this study, the following approximate relationships hold for the helicity amplitudes of $pp$ elastic scattering: $\phi_1=\phi_3$, $\phi_2=0, \phi_4=0$.

When calculating the TVPC signal according to the optical theorem, the Coulomb contributions are excluded from $pp$ amplitudes. The explanation for this is given in [15, 25]. The reason is that the Coulomb interaction does not violate $T$ invariance and therefore cannot directly contribute to the TVPC signal. Indeed, the spin structure of the transition operator for scattering on the deuteron at zero angle is such that the spin-independent amplitude $A_N$ and the amplitudes $B_N$, $G_N$, and $H_N$, additively containing the Coulomb contribution, do not enter the expressions for $dd$ TVPC amplitudes (16). At the same time, the Coulomb term enters the spin-flip amplitude $C'_N$ through the amplitude $A_N$; however, $A_N$ is multiplied by the transferred momentum $q$ (see Eq. (20)), which compensates for the Coulomb singularity at $q \to 0$ when integrating over $q$ in Eq. (16). Numerically, the contribution of the Coulomb interaction to the TVPC signal is negligible [15].

The figures below show the results of our calculations of the TVPC signal in $dd$ scattering using the SAID database (Fig. 1) and two phenomenological models for spin $NN$ amplitudes: Regge parameterization (Fig. 2) and the HEGS model (Fig. 3) in the energy intervals corresponding to these parameterizations.

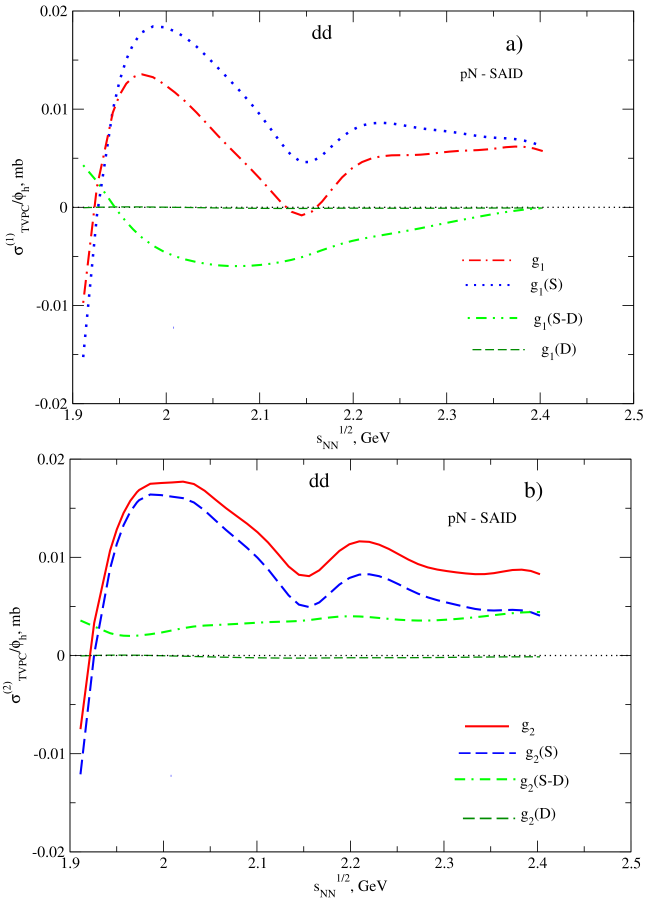


**Fig. 1.** (color online) Energy dependence of TVPC signals (cross sections) corresponding to the amplitudes $g_1$ (a) and $g_2$ (b) in $dd$ scattering for the spin $pN$ amplitudes taken from the SAID database [21]. (a) $g_1$: *S*-wave (dotted line), *D*-wave (thin dashed line), *S-D* interference (dash-dot-dotted line), and total $S+D$ (dash-dotted line). (b) $g_2$: *S*-wave (dashed line), *D*-wave (thin dashed line), *S-D* interference (dash-dash-dotted line), and total $S+D$ (solid line). The invariant mass of the interacting $NN$ pair (one nucleon from the beam and another from the target) is shown along the *X*-axis. On both panels, the straight thin dotted line shows the zero level for easy visualization.

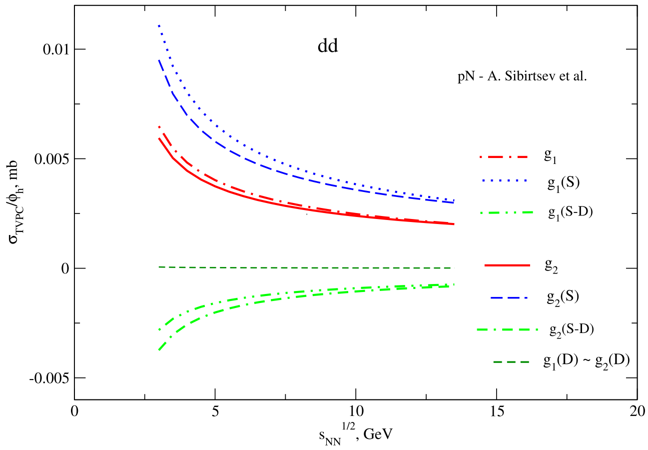


**Fig. 2.** (color online) Energy dependence of TVPC signals corresponding to $g_1$ and $g_2$ amplitudes in $dd$ scattering for the spin $pN$ amplitudes taken from Ref. [22]. The notations are the same as in Fig. 1, panels (a) and (b).

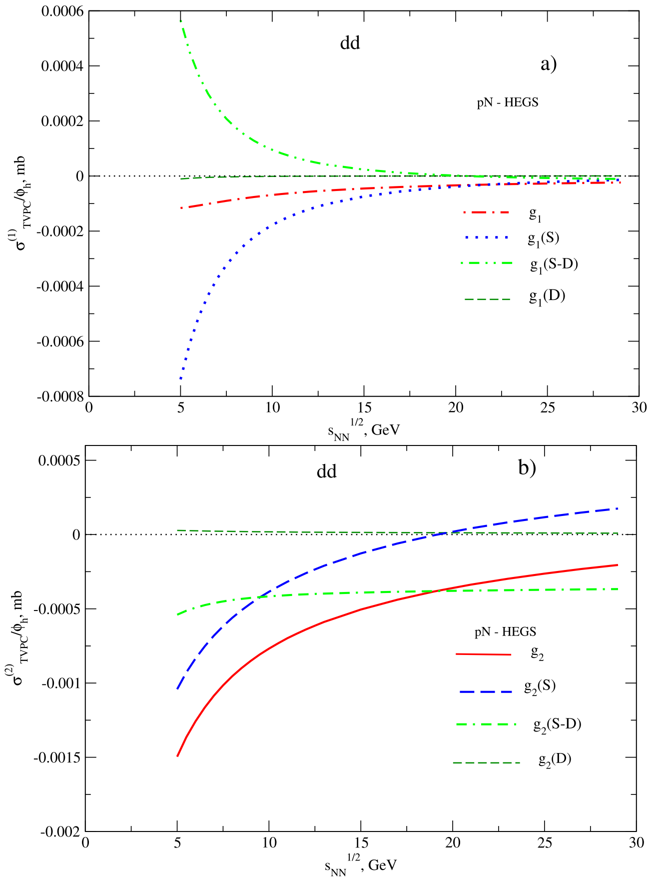


**Fig. 3.** (color online) Same as in Fig. 1 but for spin $pN$ amplitudes from the HEGS model [24].

As shown in Fig. 1 , the maximum of the signal is located in the energy range 1.95–2.05 GeV, and its absolute value unevenly decreases with further increase in collision energy and demonstrates a second local maximum at ~ 2.2 GeV in $\sigma^{(2)}_{\rm TVPC}$ and a plateau in $\sigma^{(1)}_{\rm TVPC}$.

Note that the *S*-wave of the deuteron dominates in both amplitudes $g_1$ and $g_2$ in the entire range of the invariant mass $\sqrt{s_{NN}} = 1.9$–$2.4$ GeV covered by the SAID database, whereas the contribution of the pure *D*-wave is negligible. The $S-D$ interference is essential and destructive for the $g_1$ amplitude but constructive for the $g_2$ amplitude. The numerical difference between the amplitudes $g_1$ and $g_2$ occurs because one of them ($g_2$) is calculated in the rest frame of a tensor-polarized ($P^{(2)}_{xz}$) deuteron target $d_2$, on which a vector-polarized ($P^{(1)}_y$) deuteron beam $d_1$ scatters, and the other ($g_1$) is calculated in a collision when a tensor-polarized ($P^{(1)}_{xz}$) deuteron beam $d_1$ falls on a vector-polarized ($P^{(2)}_y$) target $d_2$.

The results obtained using the Regge parameterization of *pN* amplitudes from [22] are shown in Fig. 2. With this *pN* input, the amplitudes $g_1$ and $g_2$ are numerically similar to each other for the *S*- and *D*-wave contributions and for the total $S+D$ calculation. The *D* wave is negligible and the $S-D$ interference is destructive for both the $g_1$ and $g_2$ amplitudes. Note that the maximum of the TVPC signal is obtained at the minimal energy $\sqrt{s_{NN}} = 2.666$ GeV from the range considered, and the signal decreases monotonically with an increase in the collision energy $\sqrt{s_{NN}}$.

With the HEGS parameterization [23, 24], at energies $\sqrt{s_{NN}} \sim 5$ GeV, the TVPC signal is obtained to be approximately an order of magnitude lower than that with the parameterization [22] and decreases with increasing energy (see Fig. 3). As for the parameterization from Refs. [21] and [22], when using the HEGS model, the contribution of the deuteron *D* wave to the TVPC signal is negligible in magnitude compared to the *S*-wave contribution, and $S-D$ interference is significant. Furthermore, as for the parameterization from Ref. [21], the $S-D$ interference is destructive for the $g_1$ amplitude and constructive for the $g_2$ amplitude.

## IV. CONCLUSION

In this study, the TVPC signal is calculated (up to an unknown constant) for *dd* scattering. The calculation is based on the Glauber diffraction theory with full consideration of the spin dependence of the *NN* scattering amplitudes. We consider the contributions of the single and double scattering mechanisms dominating in the amplitude of the elastic process $dd \to dd$ in the region of the first diffraction maximum, which gives the main contribution to the TVPC signal [14]. For the first time, the *D*-component of the deuteron wave function is considered in the calculation of this effect together with the *S*-component previously accounted for in [14]. The *S*-*D* interference is found to be significant in the TVPC signal.

The TVPC scattering amplitude is considerably smaller in magnitude than the corresponding *T*-even hadron amplitude. However, owing to the different symmetry properties of these amplitudes, the *T*-odd amplitude of elastic scattering does not interfere with the corresponding *T*-even amplitude. Therefore, the typical accuracy of a Glauber theory calculation of the total cross section is similar to that of the TVPC signal calculation. To a large extent, this accuracy is determined by our knowledge of *NN* elastic scattering amplitudes, which are included in the TVPC signal as multipliers.

Here, for the *pN* amplitudes, we use the database [21] at lower energies as well as an available parameterization [22] and a phenomenological model [24] at higher energies. The energy ranges of *pN* collisions correspond to the intervals of the invariant mass of the *NN* pair $\sqrt{s_{NN}} = 1.9$–$2.4$ GeV (the laboratory kinetic energy of the proton $T_l = 0.1$–$1.2$ GeV) and $\sqrt{s_{NN}} = 2.5$–$25$ GeV (the laboratory momentum of the proton beam $P_l = 2.2$–$332$ GeV/*c*.)

The maximum value of the TVPC signal corresponds to the invariant mass $\sqrt{s_{NN}} \sim 1.95$–$2.05$ GeV. At the collision energies corresponding to the conditions of the SPD NICA experiment, $\sqrt{s_{NN}} \gtrsim 2.5$ GeV, the magnitude of the signal essentially depends on the model used for the *T*-even *P*-even spin amplitudes of *pN* scattering and decreases with increasing energy, under the assumption that the TVPC interaction constant does not depend on energy. This is consistent with the general trend of spin phenomena, that is, the decrease in the *T*-even *P*-even spin effect in magnitude with increasing energy. However, at the energies of the NICA complex corresponding to the conditions of the early baryon Universe, the possible growth of an unknown TVPC constant is not excluded.

We find that only one of the three types of the TVPC *NN* interaction that do not disappear on the mass shell, *i.e.*, $h_N$, gives a non-zero contribution to the TVPC signal, whereas the contributions of other two ($g_N$ and $g'_N$) vanish owing to their specific symmetry properties. Therefore, the search for a TVPC signal in *dd* scattering differs from the previously considered processes of *pd* and $^3$He*d* scattering, where two types of the TVPC *NN* interactions, $h_N$ and $g_N$, give non-zero contributions [15–17]. This is one of the main results of this study, which is important for extracting the unknown constant of the TVPC interaction from the data.

## ACKNOWLEDGMENTS

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## APPENDIX: DERIVATION FOR *h*-type TVPC AMPLITUDES

To find the operators of normal and abnormal double scattering (7) in the case of the TVPC *NN* interaction of the *h* type, we use expression (15) and omit the linear terms in $\hat{\mathbf{q}}$ (or $\hat{\mathbf{n}}$), which become zero when integrated over the direction of the vector $\mathbf{q}$ in (6). Then, considering the symmetry of the deuteron wave functions with respect to permutation of the nucleon indices, the spin dependence of the operators (7) can be represented as (henceforth, by $\hat{O}^{(2n)}$, $\hat{O}'^{(2n)}$, etc., we refer to operators for the *h*-type TVPC interaction)

$$\begin{aligned}\hat{O}^{(2n)}(\mathbf{q}) &= \boldsymbol{\sigma}_1\cdot\mathbf{V}_n(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4),\\ \hat{O}'^{(2n)}(\mathbf{q}) &= \boldsymbol{\sigma}_3\cdot\mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2),\\ \hat{O}^{(2a)}(\mathbf{q}) &= \boldsymbol{\sigma}_1\cdot\mathbf{V}_a(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4),\\ \hat{O}'^{(2a)}(\mathbf{q}) &= \boldsymbol{\sigma}_3\cdot\mathbf{V}'_a(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2),\end{aligned}\tag{A1}$$

where

$$\begin{aligned}\mathbf{V}_n(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4) =& -2\Pi(h_pC_n+h_nC_p)\\ &\times[\hat{\mathbf{k}}(\boldsymbol{\sigma}_3\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_4\cdot\hat{\mathbf{n}})+\hat{\mathbf{q}}(\boldsymbol{\sigma}_3\cdot\hat{\mathbf{k}})(\boldsymbol{\sigma}_4\cdot\hat{\mathbf{n}})],\\ \mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) =& -2\Pi(h_pC'_n+h_nC'_p)\\ &\times[\hat{\mathbf{k}}(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{q}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}})+\hat{\mathbf{q}}(\boldsymbol{\sigma}_1\cdot\hat{\mathbf{k}})(\boldsymbol{\sigma}_2\cdot\hat{\mathbf{n}})]\end{aligned}\tag{A2}$$

and $\Pi=\dfrac{q}{4\sqrt{\pi}m}$. The vector operators $\mathbf{V}_a(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4)$ and $\mathbf{V}'_a(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)$ are similar to $\mathbf{V}_n(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4)$ and $\mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)$, respectively, with the replacement $h_n\leftrightarrow h_p$.

Such a representation allows us to easily integrate over the coordinates of nucleons inside one of the colliding deuterons. Thus, after integrating the normal double-scattering operator $\hat{O}^{(2n)}$ over the coordinates of nucleons in the target, we obtain the operator

$$\begin{aligned}\hat{\Omega}^{(2n)}(\mathbf{q}) &= \int d^3r\Psi^+_{d(12)}(\mathbf{r})\hat{O}^{(2n)}(\mathbf{q})\Psi_{d(12)}(\mathbf{r})\\ &= Z_0\mathbf{S}^{(2)}\cdot\mathbf{V}_n(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4),\end{aligned}\tag{A3}$$

where $\mathbf{S}^{(2)}$ is the spin operator of the target deuteron, and the factor $Z_0$ is defined in (17). For $\hat{O}'^{(2n)}$, we obtain a similar expression after integration by the coordinates of nucleons in the beam:

$$\begin{aligned}\hat{\Omega}'^{(2n)}(\mathbf{q}) &= \int d^3\rho\Psi^+_{d(34)}(\boldsymbol{\rho})\hat{O}'^{(2n)}(\mathbf{q})\Psi_{d(34)}(\boldsymbol{\rho})\\ &= Z_0\mathbf{S}^{(1)}\cdot\mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2),\end{aligned}\tag{A4}$$

where $\mathbf{S}^{(1)}$ is the spin operator of the incident deuteron.

In the same manner, the abnormal double-scattering operator $\hat{O}^{(2a)}$ is integrated by $d^3r$ (with the factor $e^{i\mathbf{q}\mathbf{r}}$), and $\hat{O}'^{(2a)}$ by $d^3\rho$ (with the factor $e^{-i\mathbf{q}\boldsymbol{\rho}}$), and we obtain the following expressions:

$$\begin{aligned}\hat{\Omega}^{(2a)}(\mathbf{q}) &= \int d^3r\Psi^+_{d(12)}(\mathbf{r})e^{i\mathbf{q}\mathbf{r}}\hat{O}^{(2a)}(\mathbf{q})\Psi_{d(12)}(\mathbf{r})\\ &= [S_0^{(0)}(q)-\frac{1}{2}S_0^{(2)}(q)]\mathbf{S}^{(2)}\cdot\mathbf{V}_a(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4)\\ &\quad+\frac{1}{\sqrt{2}}[S_2^{(2)}(q)-\frac{1}{\sqrt{2}}S_2^{(1)}(q)]\\ &\quad\times\hat{S}_{12}(\hat{\mathbf{q}};\mathbf{S}^{(2)},\mathbf{V}_a(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4)),\end{aligned}\tag{A5}$$

$$\begin{aligned}\hat{\Omega}'^{(2a)}(\mathbf{q}) &= \int d^3\rho\Psi^+_{d(34)}(\boldsymbol{\rho})e^{-i\mathbf{q}\boldsymbol{\rho}}\hat{O}'^{(2a)}(\mathbf{q})\Psi_{d(34)}(\boldsymbol{\rho})\\ &= [S_0^{(0)}(q)-\frac{1}{2}S_0^{(2)}(q)]\mathbf{S}^{(1)}\cdot\mathbf{V}'_a(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)\\ &\quad+\frac{1}{\sqrt{2}}[S_2^{(2)}(q)-\frac{1}{\sqrt{2}}S_2^{(1)}(q)]\\ &\quad\times\hat{S}_{12}(\hat{\mathbf{q}};\mathbf{S}^{(1)},\mathbf{V}'_a(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)),\end{aligned}\tag{A6}$$

where the deuteron form factors $S_i^{(j)}(q)$ are defined in Eq. (18).

Next, note that in a calculation of the TVPC signal, only non-diagonal spin matrix elements (5) are needed. Therefore, the components of the vectors $\mathbf{V}_a$ and $\mathbf{V}'_a$ parallel to $\hat{\mathbf{k}}$ (see the definition (A.2) and the text below it) do not contribute to the TVPC signal (with the standard choice of $\hat{\mathbf{k}}\|Oz$). For the component of $\mathbf{V}'_a$ parallel to $\hat{\mathbf{q}}$ (we denote it as $\mathbf{V}'^q_a$), we have

$$\hat{S}_{12}(\hat{\mathbf{q}};\mathbf{S}^{(1)},\mathbf{V}'^q_a)=2\mathbf{S}^{(1)}\cdot\mathbf{V}'^q_a,\tag{A7}$$

and a similar relation is fulfilled for the component $\mathbf{V}^q_a$ (with the replacement $\mathbf{S}^{(1)}\to\mathbf{S}^{(2)}$). We denote the parts of the operators (A.5) and (A.6), including only the components $\mathbf{V}^q_a$ and $\mathbf{V}'^q_a$, via $\hat{\Omega}^{(2a)}_q(\mathbf{q})$ and $\hat{\Omega}'^{(2a)}_q(\mathbf{q})$, respectively. Considering the relations (A.7), we obtain expressions for them similar to (A.3) and (A.4), respectively:

$$\hat{\Omega}^{(2a)}_q(\mathbf{q})=Z(q)\mathbf{S}^{(2)}\cdot\mathbf{V}^q_a(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4),\tag{A8}$$

$$\hat{\Omega}'^{(2a)}_q(\mathbf{q})=Z(q)\mathbf{S}^{(1)}\cdot\mathbf{V}'^q_a(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2),\tag{A9}$$

where the factor $Z(q)$ is defined in (17).

We now integrate by the coordinates of the nucleons inside the second deuteron. To do this, it is convenient to represent the vector $\mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2)$ (see Eq. (A.2)) as

$$\mathbf{V}'_n(\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) = \mathbf{W}'^n_{ij}\sigma_{1i}\sigma_{2j}, \tag{A10}$$

where

$$\mathbf{W}'^n_{ij} = -2\Pi(h_pC'_n + h_nC'_p)[\hat{\mathbf{k}}\hat{q}_i\hat{n}_j + \hat{\mathbf{q}}\hat{k}_i\hat{n}_j]. \tag{A11}$$

Similarly, $\mathbf{V}_n(\boldsymbol{\sigma}_3,\boldsymbol{\sigma}_4) = \mathbf{W}^n_{ij}\sigma_{3i}\sigma_{4j}$, where $\mathbf{W}^n_{ij}$ has the same form (A.11), but with the replacement of $C'_N \to C_N$.

In turn, for the vectors $\mathbf{V}_a$ and $\mathbf{V}'_a$, we introduce a similar representation with $\mathbf{W}^a_{ij}$ and $\mathbf{W}'^a_{ij}$, respectively, which differ from $\mathbf{W}^n_{ij}$ and $\mathbf{W}'^n_{ij}$ by replacing $h_n \leftrightarrow h_p$ only. Such a representation allows integration by the nucleon coordinates inside the second deuteron in the same manner as done for *pd* scattering (for example, using formula (12) from Ref. [26]).

Thus, by integrating the operator $\hat{\Omega}'^{2n}(\mathbf{q})$ (A.4) with the factor $\mathrm{e}^{\mathrm{i}\mathbf{qr}}$ over the nucleon coordinates inside the target deuteron and employing the definition of the deuteron form factors (18), we obtain

$$\int \mathrm{d}^3r\Psi^+_{d(12)}(\mathbf{r})\mathrm{e}^{\mathrm{i}\mathbf{qr}}\hat{\Omega}'^{2n}(\mathbf{q})\Psi_{d(12)}(\mathbf{r}) = Z_0\Bigg(\left[S^{(0)}_0(q) - \frac{1}{2}S^{(2)}_0(q) + \frac{1}{2\sqrt{2}}S^{(1)}_2(q) + \frac{7}{2\sqrt{2}}S^{(2)}_2(q)\right]\mathbf{S}^{(1)}\cdot\mathbf{W}'^n_{ij}\{S^{(2)}_i,S^{(2)}_j\}$$
$$+\frac{3}{2\sqrt{2}}\left[S^{(1)}_2(q) + S^{(2)}_2(q)\right]\mathbf{S}^{(1)}\cdot\mathbf{W}'^n_{ij}\{[\mathbf{S}^{(2)}\times\hat{q}]_i,[\mathbf{S}^{(2)}\times\hat{q}]_j\} + 3\mathbf{S}^{(1)}\cdot\mathbf{W}'^n_{ij}\int\mathrm{d}^3r\frac{\mathrm{e}^{\mathrm{i}\mathbf{qr}}}{4\pi r^2}w^2(r)\hat{S}_{12}(\hat{\mathbf{r}};\mathbf{S}^{(2)},\mathbf{S}^{(2)})\hat{r}_i\hat{r}_j\Bigg), \tag{A12}$$

where $\mathbf{W}'^n_{ij}\delta_{ij} = \mathbf{W}'^n_{ij}\hat{q}_i\hat{q}_j = 0$ is taken into account (see Eq. (A.11)). The integral in Eq. (A.12) is easy to calculate if represented as

$$-\frac{\partial}{\partial q_i}\frac{\partial}{\partial q_j}\int\mathrm{d}^3r\frac{\mathrm{e}^{\mathrm{i}\mathbf{qr}}}{4\pi r^2}\frac{w^2(r)}{r^2}\hat{S}_{12}(\hat{\mathbf{r}};\mathbf{S}^{(2)},\mathbf{S}^{(2)})$$
$$= \frac{\partial}{\partial q_i}\frac{\partial}{\partial q_j}\int\mathrm{d}rw^2(r)\frac{j_2(qr)}{(qr)^2}\hat{S}_{12}(\mathbf{q};\mathbf{S}^{(2)},\mathbf{S}^{(2)}), \tag{A13}$$

where $\hat{S}_{12}(\mathbf{q};\mathbf{S}^{(2)},\mathbf{S}^{(2)}) = 3(\mathbf{S}^{(2)}\cdot\mathbf{q})^2 - 2q^2$. After calculating the integral, we obtain four terms proportional to symmetric tensors, $\delta_{ij}$, $\hat{q}_i\hat{q}_j$, $\mathbf{S}^{(2)}_i\hat{q}_j + \mathbf{S}^{(2)}_j\hat{q}_i$ , and $\{S^{(2)}_i,S^{(2)}_j\}$. The first two terms are vanishing when multiplied by the vector $\mathbf{W}'^n_{ij}$, and the third becomes zero when multiplied by its $\hat{q}$-component, which is involved in calculating the TVPC signal (when taking non-diagonal spin matrix elements from the product $\mathbf{S}^{(1)}\cdot\mathbf{W}'^n_{ij}$). Thus, the contribution to the TVPC signal is given only by a term proportional to $\{S^{(2)}_i,S^{(2)}_j\}$, which is obtained by differentiating the operator $\hat{S}_{12}(\mathbf{q};\mathbf{S}^{(2)},\mathbf{S}^{(2)})$ in (A.13). By rewriting $\dfrac{j_2(qr)}{(qr)^2}$ via a linear combination of spherical Bessel functions $j_n(qr), n = 0,2,4$, we obtain the following contribution to the TVPC signal from the last term in Eq. (A.12):

$$\mathbf{S}^{(1)}\cdot\mathbf{W}'^n_{ij}\{S^{(2)}_i,S^{(2)}_j\}\left[\frac{3}{5}S^{(2)}_0(q) - \frac{6\sqrt{2}}{7}S^{(2)}_2(q) + \frac{18}{35}S^{(2)}_4(q)\right], \tag{A14}$$

where the form factor $S^{(2)}_4(q)$ is defined in (18).

When integrating the operator $\hat{\Omega}^{2n}(\mathbf{q})$ with the factor $\mathrm{e}^{\mathrm{i}\mathbf{q}\boldsymbol{\rho}}$ by the coordinates of the nucleons in the incident deuteron, we obtain an expression similar to (A.12), with the replacements $\mathbf{W}'^n_{ij} \to \mathbf{W}^n_{ij}$ and $\mathbf{S}^{(1)} \leftrightarrow \mathbf{S}^{(2)}$. For abnormal scattering, we also obtain similar expressions (with the replacement $Z_0 \to Z(q)$) when integrating $\hat{\Omega}^{2a}_q(\mathbf{q})$ with the factor $\mathrm{e}^{-\mathrm{i}\mathbf{q}\boldsymbol{\rho}}$ over the nucleon coordinates in the beam, and $\hat{\Omega}'^{2a}_q(\mathbf{q})$ with the factor $\mathrm{e}^{\mathrm{i}\mathbf{qr}}$ over the nucleon coordinates in the target.

Now, to obtain the amplitude $M_{\mathrm{TVPC}}(0)$ in the double-scattering approximation, we take the sum of all expressions of the form (A.12) with substitution of (A.14) for normal and abnormal scattering, integrate by the momentum $\mathbf{q}$, and multiply by a factor $\dfrac{\mathrm{i}}{2\pi^{3/2}}$ (see Eq. (6)). From the resulting operator, we calculate the spin matrix elements (5) necessary to find the TVPC amplitudes $g_i$, $i = 1,2$. To do this, we use the following relations:

$$<-1,1|\mathbf{S}^{(1)}\cdot\hat{\mathbf{q}}\{\mathbf{S}^{(2)}\cdot\hat{\mathbf{n}},\mathbf{S}^{(2)}\cdot\hat{\mathbf{k}}\}|0,0>$$
$$= -<-1,1|\mathbf{S}^{(1)}\cdot\hat{\mathbf{q}}\{[\mathbf{S}^{(2)}\times\hat{\mathbf{q}}]\cdot\hat{\mathbf{n}},[\mathbf{S}^{(2)}\times\hat{\mathbf{q}}]\cdot\hat{\mathbf{k}}\}|0,0> = -\frac{\mathrm{i}}{2},$$
$$<1,0|\mathbf{S}^{(1)}\cdot\hat{\mathbf{q}}\{\mathbf{S}^{(2)}\cdot\hat{\mathbf{n}},\mathbf{S}^{(2)}\cdot\hat{\mathbf{k}}\}|0,1>$$
$$= -<1,0|\mathbf{S}^{(1)}\cdot\hat{\mathbf{q}}\{[\mathbf{S}^{(2)}\times\hat{\mathbf{q}}]\cdot\hat{\mathbf{n}},[\mathbf{S}^{(2)}\times\hat{\mathbf{q}}]\cdot\hat{\mathbf{k}}\}|0,1> = \frac{\mathrm{i}}{2}. \tag{A15}$$

When replacing $\mathbf{S}^{(1)} \leftrightarrow \mathbf{S}^{(2)}$, both matrix elements in (A.15) are found to be the same and equal to $-\dfrac{\mathrm{i}}{2}$.

As a result, we obtain formulas (16).

## References


[1] G. Luders, Kgl. Danske Vidensk. Selsk. Mat.-Fys. Medd. **28**, 1 (1954)

[2] W. Pauli, in *Niels Bohr and the Development of Physics* (Pergamon Press, London), 1955

[3] A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. **5**, 32 (1967)

[4] A. Riotto and M. Trodden, Ann. Rev. Nucl. Part. Sci. **49**, 35 (1999), arXiv: hep-ph/9901362

[5] T. Chupp, P. Fierlinger, M. Ramsey-Musolf *et al*., Rev. Mod. Phys. **91**, 015001 (2019), arXiv: 1710.02504

[6] L. B. Okun, Yad. Fiz. **1**, 938 (1965)

[7] V. P. Gudkov, Phys. Rept. **212**, 77 (1992)

[8] S. N. Vergeles, N. N. Nikolaev, Y. N. Obukhov *et al*., Phys. Usp. **66**, 109 (2023), arXiv: 2204.00427

[9] A. Kurylov, G. C. McLaughlin, and M. J. Ramsey-Musolf, Phys. Rev. D **63**, 076007 (2001), arXiv: hep-ph/0011185

[10] A. L. Barabanov, Yad. Fiz. **44**, 1163 (1986)

[11] P. Lenisa *et al*., EPJ Tech. Instrum. **6**, 2 (2019)

[12] A. Temerbayev and Y. Uzikov, Phys. Atom. Nucl. **78**, 35 (2015)

[13] N. Nikolaev, F. Rathmann, A. Silenko *et al*., Phys. Lett. B **811**, 135983 (2020)

[14] Y. Uzikov, M. Platonova, A. Kornev *et al*., Int. Jour. Mod. Phys. E https://doi.org/10.1142/S0218301324410039 (2024)

[15] Y. N. Uzikov and A. Temerbayev, Phys. Rev. C **92**, 014002 (2015), arXiv: 1506.08303

[16] Y. N. Uzikov and J. Haidenbauer, Phys. Rev. C **94**, 035501 (2016), arXiv: 1607.04409

[17] Y. N. Uzikov and M. N. Platonova, JETP Lett. **118**, 785 (2023), arXiv: 2311.10841

[18] G. Alberi and G. Goggi, Phys. Rept. **74**, 1 (1981)

[19] M. N. Platonova and V. I. Kukulin, Phys. Rev. C **81**, 014004 (2010), [Erratum: Phys. Rev. C 94, 069902 (2016)]

[20] M. Beyer, Nucl. Phys. A **560**, 895 (1993), arXiv: nucl-th/9302002

[21] R. A. Arndt, W. J. Briscoe, I. I. Strakovsky *et al*., Phys. Rev. C **76**, 025209 (2007), arXiv: 0706.2195

[22] A. Sibirtsev, J. Haidenbauer, H. W. Hammer *et al*., Eur. Phys. J. A **45**, 357 (2010), arXiv: 0911.4637

[23] O. Selyugin, Symmetry **13**, 164 (2021)

[24] O. V. Selyug, arXiv: 2407.01311

[25] Y.-H. Song, R. Lazauskas, and V. Gudkov, Phys. Rev. C **93**, 065501 (2016), arXiv: 1602.06837

[26] M. N. Platonova and V. I. Kukulin, Phys. Atom. Nucl. **73**, 86 (2010)