

Nuclear Dynamics and the Impact of ${}^7\text{Li}$ Breakup and Triton Transfer in ${}^7\text{Li} + {}^{28}\text{Si}$ Systems*

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Abstract: Our previous study [A. H. Al-Ghamdi et al, JTUSCI 16 (2022) 1026] provided a comprehensive analysis of elastic scattering angular distributions (ADs) for the ${}^7\text{Li} + {}^{28}\text{Si}$ system. This analysis aimed to identify the types of threshold anomaly, specifically normal or breakup, by examining the energy dependence of volume integrals across various interaction potentials. The current study extends this work by investigating the effects of ${}^7\text{Li}$ breakup into a valence particle (triton) orbiting a core (alpha) in the context of a ${}^{28}\text{Si}$ target, as well as the influence of the ${}^{28}\text{Si}({}^7\text{Li}, \alpha){}^{31}\text{P}$ triton transfer reaction on the elastic ADs of the ${}^7\text{Li} + {}^{28}\text{Si}$ system. The results demonstrate the significance of coupling to the ${}^7\text{Li}$ breakup channel and its subsequent impact on the elastic scattering channel. This strong coupling generates a dynamic polarization potential (DPP), leading to a reduction in potential strengths. A semi-microscopic DPP approach was utilized to model this effect, employing the continuum discretized coupled channels (CDCC) method. An effective potential (U^{eff}), considered as the sum of cluster folding and dynamic polarization potentials, the later was generated using the trivially equivalent local potential (TELP) approach, was employed successfully to reproduce the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs data. Furthermore, the analysis was broadened to assess the effect of the triton stripping reaction ${}^{28}\text{Si}({}^7\text{Li}, \alpha){}^{31}\text{P}$ on the elastic ${}^7\text{Li} + {}^{28}\text{Si}$ scattering.

Keywords: Dynamical polarization potential (DPP), Threshold anomaly, Elastic scattering

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I. INTRODUCTION

The lithium-7 isotope is a weakly bound (WB) nucleus, notable for having a single-bound excited state at an excitation energy (E_x) of 0.4776 MeV [1]. This state precedes the formation of the ${}^7\text{Li}$ cluster structure, which transitions to $\alpha + t$ at $E_x = 2.467$ MeV [2]. These properties highlight the importance of breakup processes in systems involving ${}^7\text{Li}$. Research has shown that systems with WB nuclei like ${}^7\text{Li}$ exhibit distinct characteristics that differ markedly from those with tightly bound nuclei. One key difference is the absence of the normal threshold anomaly (TA) [3], where the real part of the nuclear potential displays a localized peak near the Coulomb barrier energy (V_B), and the imaginary part increases progressively with energy until reaching a plateau. In contrast, systems involving ${}^7\text{Li}$ projectiles demonstrate a breakup threshold anomaly (BTA) [4], characterized by an increasing depth of the imaginary potential as the energy decreases below the V_B .

Another key finding in systems with ${}^7\text{Li}$ projectiles is

the need to reduce the real potential strength by $\sim 30\text{--}60\%$ (depending on the bombarded target and incident energy) to accurately reproduce the measured angular distributions (ADs) data [5–8]. These phenomena are attributed to the breakup of the ${}^7\text{Li}$ nucleus into $\alpha + t$ in the presence of the studied targets, where strong coupling to the breakup channel creates a dynamic polarization potential (DPP) that considerably reduces the strength of the real potential. Recent studies on systems induced by ${}^7\text{Li}$ projectiles [9–10] have also explored the influence of triton transfer rather than ${}^7\text{Li}$ breakup on the elastic scattering channel for the ${}^7\text{Li} + {}^{93}\text{Nb}$ [9], ${}^7\text{Li} + {}^{209}\text{Bi}$ [10], and ${}^7\text{Li} + {}^{144}\text{Sm}$ [11] systems.

This study revisits the ${}^7\text{Li} + {}^{28}\text{Si}$ system, which has been the subject of extensive experimental and theoretical investigations [12–24] focused on its characteristics at energies below and near the V_B . Pakou et al. [12] measured the elastic scattering ADs for the ${}^7\text{Li} + {}^{28}\text{Si}$ system at various energies close to the barrier. Their analysis, which employed the continuum discretized coupled channels (CDCC) method in conjunction with the double-fold-

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ing optical model (DFOM) using BDM3Y1 interaction, revealed significant insights into the interaction dynamics. Notably, the depth of the imaginary potential was found to decrease as the energy approached the barrier, while the real part of the scattering potential remained largely constant, with only a minor reduction.

Further experimental investigations have examined different energy ranges. For instance, Sinha et al. [13] measured ADs in the laboratory energy (E_{lab}) range of 11.5–26 MeV, analyzing their data with an optical potential (OP) model that included both imaginary surface and volume potentials. Additionally, CDCC calculations were conducted, focusing solely on the projectile breakup. In a study [14], scattering data for the ${}^7\text{Li} + {}^{28}\text{Si}$ system at energies $E_{\text{lab}} = 20$ and 36 MeV were compared to optical model calculations, demonstrating good agreement.

At higher energies, specifically $E_{\text{lab}} = 45$ and 48 MeV [15], the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs were analyzed using real folded potentials, achieving a satisfactory renormalization that matched the experimental cross sections. Notably, at even higher energy, $E_{\text{lab}} = 350$ MeV [16], the ADs exhibited diffractive oscillations at forward angles, followed by an exponential falloff at larger angles. Zerva et al. [17] expanded this analysis by investigating the excitation functions for quasielastic scattering of ${}^7\text{Li} + {}^{28}\text{Si}$ at angles of 150° and 170° , further exploring barrier distributions through both coupled reaction channel (CRC) and CDCC methods, which take into account transfer reactions and breakup.

Numerous theoretical studies [18–24] have also explored the ${}^7\text{Li} + {}^{28}\text{Si}$ system. Farid and Hassanain [18] employed a real double-folded (DF) potential alongside an imaginary Woods-Saxon (WS) potential to analyze the elastic scattering ADs of ${}^7\text{Li}$ on various targets, including ${}^{28}\text{Si}$. The same authors applied the S1Y effective nucleon-nucleon (NN) interaction using the coupled channels (CC) method to fit the experimental data [19]. Pakou et al. [20] introduced a phenomenological DPP to explain the observed reduction in potential strength in their analysis of the ${}^7\text{Li} + {}^{28}\text{Si}$ system. In another investigation [21], the elastic scattering AD at $E_{\text{lab}} = 350$ MeV was analyzed using the CDCC method, achieving a reasonable fit to the data. Kuterbekov et al. [22] studied the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs within the energy range of 7.5–32 MeV using the WS optical model with the **SPI-GENOA** code. W. Chen et al. [23] applied a microscopic approach to model the OP for ${}^7\text{Li}$ scattering from various nuclei, predicting the ADs for ${}^7\text{Li}$ scattered from targets ranging from ${}^{27}\text{Al}$ to ${}^{208}\text{Pb}$ without any free parameters.

The ${}^7\text{Li} + {}^{27}\text{Al}$ system, closely related to the ${}^7\text{Li} + {}^{28}\text{Si}$ system, has also been extensively studied through both experimental [25–29] and theoretical [23, 30–33] studies in the energy range of 6–89 MeV. Pakou et al. [31] explored the interaction mechanisms of WB ${}^6,{}^7\text{Li}$ and ${}^7,{}^9\text{Be}$ nuclei as they scatter off ${}^{27}\text{Al}$ and ${}^{28}\text{Si}$ targets, utilizing

the microscopic BDM3Y1 framework. Their CDCC calculations effectively described the experimental elastic scattering data, revealing that only the ${}^6\text{Li}$ projectile exhibits significant coupling to the continuum. In contrast, the ${}^7\text{Li}$ and ${}^7\text{Be}$ projectiles show weak couplings, while the ${}^9\text{Be}$ projectile has an extremely weak coupling. The reaction cross-section (σ_R) values did not show significant enhancement when appropriately reduced, indicating that continuum effects vary across different projectiles and systems. In a study [33], the ADs for ${}^6\text{Li}$ and ${}^7\text{Li}$ projectiles on heavy targets ($A \geq 40$) were studied using a single-folding model based on the Jeukenne–Lejeune–Mahaux (*JLM*) potential, accurately reproducing the ADs and σ_R for both stable and unstable nuclei.

While some systems induced by ${}^7\text{Li}$ projectiles, such as ${}^7\text{Li} + {}^{12}\text{C}$ [34], ${}^7\text{Li} + {}^{27}\text{Al}$ [25], ${}^7\text{Li} + {}^{28}\text{Si}$ [35], and ${}^7\text{Li} + {}^{138}\text{Ba}$ [36], display a breakup threshold anomaly BTA, other, including ${}^7\text{Li} + {}^{59}\text{Co}$ [37], ${}^7\text{Li} + {}^{80}\text{Se}$ [38], ${}^7\text{Li} + {}^{134}\text{Ba}$ [39], and ${}^7\text{Li} + {}^{208}\text{Pb}$ [40], exhibit the usual TA. These contrasting results regarding the occurrence of the BTA in ${}^7\text{Li}$ -induced systems remain an active area of research [31, 41].

In addition, advancements in theoretical nuclear reaction models continue to evolve. Notably, the CDCC method, traditionally used for three-body scattering problems involving WB projectiles, is now being extended to tackle more complex four-body systems. This includes scenarios where the projectiles consist of three-body nuclei, such as ${}^6\text{He}$ [42, 43] and ${}^9\text{Be}$ [44–46], as well as two-body systems like ${}^{11}\text{Be} + d$ [47]. Overall, the ${}^7\text{Li} + {}^{28}\text{Si}$ system, alongside other related systems, has become a focal point in the investigation of nuclear reactions at sub-barrier energies. Both experimental and theoretical developments are enhancing our understanding of these reactions, shedding light on the role of the Coulomb barrier, the interactions between breakup and transfer reactions, and the ongoing examination of phenomena such as BTA. These advancements significantly contribute to the broader understanding of WB projectiles and their scattering dynamics.

Building upon our previous work [24], where we conducted a comprehensive analysis of the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs using various interaction potentials to identify the nature of threshold anomalies—whether normal or indicative of breakup—we now focus on the effects of both ${}^7\text{Li}$ breakup within ${}^{28}\text{Si}$ target and the ${}^{28}\text{Si}({}^7\text{Li}, \alpha){}^{31}\text{P}$ triton stripping reaction on the elastic ADs for the ${}^7\text{Li} + {}^{28}\text{Si}$ system at energies ranging from 8.5 to 36 MeV [12–14]. The manuscript is organized as: Section II outlines the methodology used in the analyses, Section III presents the results and their implications, and Section IV concludes with a summary and conclusions.

II. METHODOLOGY

The calculations conducted in this study utilize the

CDCC method, which has proven effective for analyzing nuclear reactions involving nuclei with low breakup thresholds, such as ${}^6\text{Li}$ [31]. This method allows researchers to investigate how projectile breakup influences the scattering channels by incorporating explicit couplings to continuum states. The CDCC method can successfully address projectile breakup through various mechanisms, including elastic scattering, breakup, and transfer reactions. Over the years, it has been a powerful tool for replicating experimental results related to the scattering of both unstable and stable nuclei. Originally, the CDCC was designed to handle three-body systems consisting of a target and a two-body projectile. It has since been adapted for four-body reactions, where the projectile dissociates into three fragments within the field of the target, leading to the development of the four-body CDCC. However, solving the CDCC equation becomes quite complex for systems involving more than three bodies. The primary challenge lies in obtaining a set of discretized continuum states for multi-body projectiles. Three main discretization methods have been proposed to address this challenge: the pseudostate (PS) method [48], the average (Av) method [49], and the midpoint (Mid) method [50]. The PS approach replaces states with pseudostates generated by diagonalization of the projectile's internal Hamiltonian, providing an alternative to the Av and Mid techniques.

In our previous study [24], the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs were investigated using cluster folding potential (CFP), which was normalized using two factors: N_{RCF} for the real component and N_{ICF} for the imaginary component. The analysis indicated that the real CFP strength needed to be reduced by approximately 60% to adequately describe the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs. A similar pattern emerged in the analysis using the double folding potential (DFP) and São Paulo potential (SPP), where it was found that the real DFP and SPP strength required reductions of approximately 63% and 48%, respectively, for proper representation of the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs. These reductions in real potential strengths were attributed primarily to the ${}^7\text{Li}$ breakup. To further investigate the impact of ${}^7\text{Li}$ breakup on the ${}^{28}\text{Si}$ target, the CDCC method that considers the couplings to continuum states above the ${}^7\text{Li}$ breakup into $t + \alpha$ structure was considered, occurring at an excitation energy ($E_x = 2.467$ MeV). These couplings are significant as they generate a DPP, which in turn reduces the real potential strength. The ${}^7\text{Li} + {}^{28}\text{Si}$ ADs at energies range of 8.5 to 36 MeV [12–14] are analyzed using the CDCC method. A key requirement for performing these calculations is the determination of the core (α) + target (${}^{28}\text{Si}$) and valence (t) + target (${}^{28}\text{Si}$) potentials, denoted as $U_{\alpha+28\text{Si}}$ and $U_{t+28\text{Si}}$, at the appropriate energies, $E_\alpha = 4/7 E_{7\text{Li}}$ and $E_t = 3/7 E_{7\text{Li}}$. These potentials are generated through the standard single cluster folding (CF) procedure, leading to the formulation of central and coupling potentials as fol-

lows:

$$U_{i \rightarrow f}^{CF}(R) = \left\langle \Psi_f(r) \left| U_{\alpha+28\text{Si}} \left(\left| R - \frac{3}{7}r \right| \right) + U_{t+28\text{Si}} \left(\left| R + \frac{4}{7}r \right| \right) \right| \Psi_i(r) \right\rangle \quad (1)$$

In this context, R represents the separation between ${}^7\text{Li}$ and ${}^{28}\text{Si}$ nuclei, while r denotes the distance between (t) and (α) in the ${}^7\text{Li}$ ground state. The potential that binds α and t is modeled using a standard real Woods-Saxon shape, characterized by a radius, $R_V = 1.83$ fm, and a diffuseness $a_V = 0.65$ fm. The depth of this potential is adjusted to accurately reproduce the cluster binding energy. The $U_{\alpha+28\text{Si}}$ and $U_{t+28\text{Si}}$ potentials at $E_\alpha = 4/7 E_{7\text{Li}}$ and $E_t = 3/7 E_{7\text{Li}}$, respectively, were sourced from earlier studies [51, 52]. The **FRESCO** code [53] and **SFRESKO** search code were employed to analyze the data and determine the optimal potential parameters by minimizing the χ^2 value, which quantifies the deviation between experimental data and calculated results.

III. RESULTS AND DISCUSSION

A. Analysis of the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs utilizing non-normalized CFP in addition to a DPP

As discussed in previous work [24], the required reduction for the real DFP, SPP, and CFP strengths' as observed through calculations using the (Real DF + Imag. DF), (Real SPP + Imag. SPP), and (Real CFP + Imag. CFP) methods are due to significant coupling effects related to the ${}^7\text{Li}$ breakup channel. This coupling can be addressed either by incorporating a DPP derived from the continuum coupling or by applying the microscopic CDCC method. Both approaches are considered in this study to model the effects of coupling. According to Feshbach theory regarding microscopic optical model potentials, the nucleus-nucleus interaction that contributes to the dynamic polarization potential is represented by the following equation [54]:

$$V = \langle \varphi_{P0} \varphi_{T0} | v_{eff} | \varphi_{P0} \varphi_{T0} \rangle + \langle \varphi_{P0} \varphi_{T0} | v_{eff} Q \frac{1}{E_{c.m.} + i\varepsilon - QHQ} Q v_{eff} | \varphi_{P0} \varphi_{T0} \rangle \quad (2)$$

The first term in Eq. 2 describes the interaction between the projectile and the target nuclei when both are in their ground states, φ_{P0} and φ_{T0} , typically characterized by the DF integral. The operator Q projects onto the ground state, $E_{c.m.}$ represents the total center-of-mass energy, and ε is an infinitesimal value that ensures time retardation in

the excluded channels. The second term captures the complex contributions from all allowed inelastic channels, which is known as the DPP. Numerous studies, particularly those focused on weakly bound nuclei, have investigated this potential (see, for example, Refs. [55–58]). In this work, a straightforward semi-microscopic approach was proposed to modeling the DPP by introducing an additional repulsive (U^{DPP}) term. This potential is derived from the differentiation of the real and imaginary CFPs, as outlined below:

$$U^{DPP}(R) = N_{RDPP} \frac{d}{dR} V^{CFP}(R) + i N_{IDPP} \frac{d}{dR} W^{CFP}(R) \quad (3)$$

The implemented DPP in this work is modeled as a factor multiplied by the derivative of the real and imaginary CFPs, as shown in Eq. 3. The employed CFP and DPP are depicted in Fig. 1.

As shown in Ref. [24], the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs were reasonably described using the CFP, with reductions of approximately 60% for the real CFP strength and 32% for the imaginary CFP strength. In the current study, by including the DPP, the data was described using non-normalized real and imaginary CFPs such that “both N_{RCF} and N_{ICF} are fixed to unity”. This same approach was adopted earlier in Ref. [59], which introduced two parameters— N_{RDPP} and N_{IDPP} —as renormalization factors for the repulsive surface DPP. Figures 2 and 3 demonstrate good agreement between the experimental ${}^7\text{Li} + {}^{28}\text{Si}$ ADs and the calculations utilizing non-normalized real and imaginary CFPs combined with the DPP, with the optimal potential parameters summarized in Table 1. At $E_{\text{lab}} = 36$ MeV, we obtained $\chi^2/N = 6.8$, which is higher than values at other studied energies. Nevertheless, our calculations successfully reproduce the experimental AD data across the full angular range. The elevated χ^2/N likely re-

flects the significantly smaller experimental uncertainties in the 36 MeV dataset.

B. Analysis of the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs within the CDCC method

The CDCC method [60–64] is an advanced technique widely used to study processes involving WB projectiles. It incorporates couplings to continuum states, allowing for a detailed examination of how projectile breakup affects scattering channels. The CDCC calculations were carried out using the **FRESCO** code, which includes coupling to resonant states ($7/2^-$, $E_x = 4.652$ MeV and $5/2^-$, $E_x = 6.604$ MeV) with width of Δk , 0.2 and 2 MeV, respectively; bound non-resonant state ($1/2^-$, $E_x = 0.4776$ MeV), and non-resonant continuum states with orbital momentum up to $L=3$, the bins with higher L are found to have a weak effect. The diagonal and coupling potentials were calculated using Eq. 1. The $\alpha + t$ continuum above the breakup threshold (2.467 MeV) was discretized into several momentum bins, based on the momentum $\hbar k$ of the $\alpha + t$ relative motion, and each bin is treated as an excited state. The model space of Kelly et al. [65] was adopted. The binding potentials for $t + {}^{28}\text{Si}$, $\alpha + {}^{28}\text{Si}$, as well as the $\alpha + t$ correspond to those used in the CF calculations (Eq. 1).

An important aspect that must be carefully tested is the convergence of the CDCC calculations. These calculations should not depend on the choice of parameters, such as the numerical parameters (e.g., matching radius, R_{match} , and integration step-size, h_{cm}) or model space parameters (e.g., momentum-bin width, Δk , maximum momentum, k_{max} , and the inclusion of pseudo-states with different orbital angular momentum, L). The test calculations performed indicate that the convergence is achieved with $R_{\text{match}} = 40$ fm, as shown in Fig. 4(a). The results are

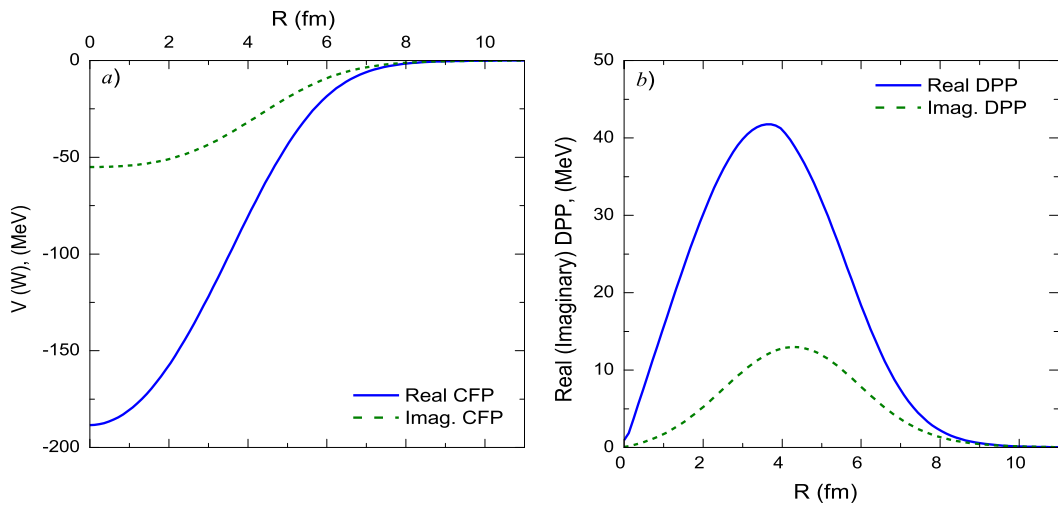


Fig. 1. (color online) Real and imaginary parts for (a) the CFPs for the ${}^7\text{Li} + {}^{28}\text{Si}$ system (b) DPPs utilized in the computation within the (non-normalized CFPs + DPP) approach.

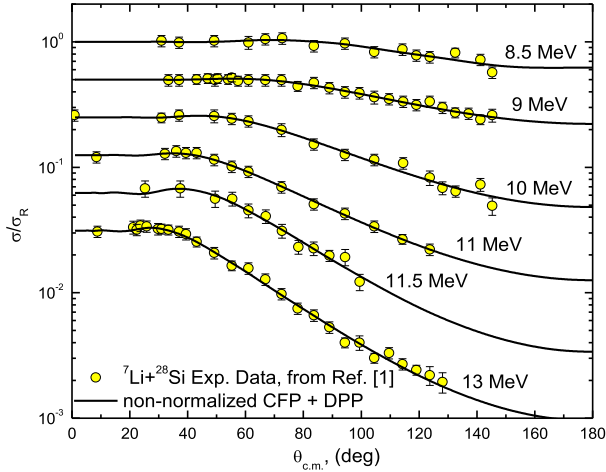


Fig. 2. (color online) Experimental ${}^{28}\text{Si}({}^7\text{Li}, {}^7\text{Li}){}^{28}\text{Si}$ elastic scattering ADs at $E_{\text{lab}} = 8.5, 9, 10, 11, 11.5,$ and 13 MeV versus calculations using both the (non-normalized CFPs + DPP) approach. Data is displaced by a factor of 0.5.

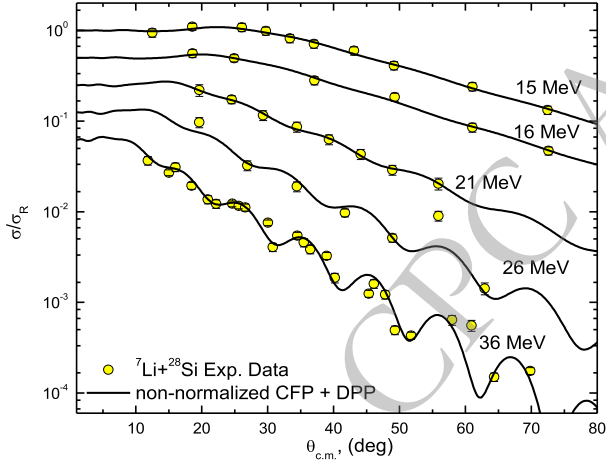


Fig. 3. (color online) Same as Fig. 2 but at $E_{\text{lab}} = 15, 16, 21, 26,$ and 36 MeV.

insensitive to the hcm parameter, with $hcm = 0.04$ fm being adopted. As shown in Fig. 4(b), the CDCC calculations include pseudo-states with $L = 0, 1, 2, 3, 4$ to examine the accumulative contributions from these states, highlighting convergence at $L = 3$. Additionally, tests were conducted with different momentum-bin widths ($\Delta k = 0.15, 0.2,$ and 0.25 fm $^{-1}$), and the results showed that convergence is achieved with $\Delta k = 0.25$ fm $^{-1}$. Generally, values of Δk between 0.1 and 0.3 fm $^{-1}$ are suitable. The current CDCC calculations utilize $hcm = 0.04$ fm, $R_{\text{match}} = 40$ fm, $\Delta k = 0.25$ fm $^{-1}$, and include the cumulative contributions from pseudo-states with $L = 0, 1, 2, 3$. It is important to note that the number of momentum bins considered varies depending on the bombarding energy. For bombarding energies between 8.5 and 16 MeV, The momentum space (k) above ${}^7\text{Li}$ breakup threshold is truncated at $k_{\text{max}} = 1.0$ fm $^{-1}$ (corresponding to $E_{\text{max}} = 12.19$

Table 1. Optimal potential parameters for ${}^7\text{Li} + {}^{28}\text{Si}$ system obtained from the calculations utilizing the (non-normalized CFPs + DPP) approach as well as the χ^2/N values. The χ^2/N values obtained from the calculations within both CDCC method and U_{eff} potential are also listed.

$E(\text{MeV})$	N_{RDPP}	N_{IDPP}	$\chi^2/N(\text{CFP+DPP})$	$\chi^2/N(\text{CDCC})$	$\chi^2/N(U_{\text{eff}})$
8.5	0.356	0.762	0.4	3.82	3.74
9	0.416	0.554	0.11	3.11	3.09
10	0.382	0.414	0.45	2.85	2.81
11	0.4	0.186	0.07	0.71	0.69
11.5	0.349	0.614	0.4	1.26	1.14
13	0.407	0.253	0.41	1.21	1.02
15	0.317	0.353	0.22	0.27	0.30
16	0.297	0.231	0.3	0.46	0.36
21	0.323	0.289	0.08	0.64	0.50
26	0.21	0.128	3.8	11.58	10.97
36	0.209	0.119	6.8	19.1	18.4

MeV). For higher energies ($E_{\text{lab}} = 21, 26,$ and 36 MeV), the values of k_{max} are 1.25 fm $^{-1}$ ($E_{\text{max}} = 19.05$ MeV), 1.5 fm $^{-1}$ ($E_{\text{max}} = 27.43$ MeV), and 1.75 fm $^{-1}$ ($E_{\text{max}} = 37.34$ MeV), respectively.

The outcomes of the calculations are illustrated in Figs. 5 and 6. Overall, the CDCC calculations show reasonable agreement with the experimental data across all energies examined, except for the lowest three energies ($8.5, 9,$ and 10 MeV), where the calculations tend to underestimate the measured data, particularly at angles greater than 90° . This discrepancy may be attributed to the formation of the compound nucleus, which is not accounted for in standard CDCC calculations. Similar effects have been observed in reactions with WB [66–68]), where CN contributions dominate over direct processes. Although a better fit for the angular distributions at these lower energies could be achieved by reducing the strength of the imaginary potential, we believe it is more physically accurate to present the CDCC results without making adjustments to the potential parameters.

C. Analysis of the ${}^7\text{Li} + {}^{28}\text{Si}$ ADs within an effective potential (U^{eff})

We reanalyzed the ${}^7\text{Li} + {}^{28}\text{Si}$ elastic scattering ADs using an effective potential (U^{eff}). This U^{eff} was constructed as the sum of two components: cluster folding potential (U^{CF}) and dynamic polarization potential (U^{DPP}), both derived from CDCC calculations. The U^{DPP} was obtained from the coupling to the continuum states, and was generated using the trivially equivalent local potential (TELP) approach [69]. The expression for U^{eff} is provided in Eq. 4.

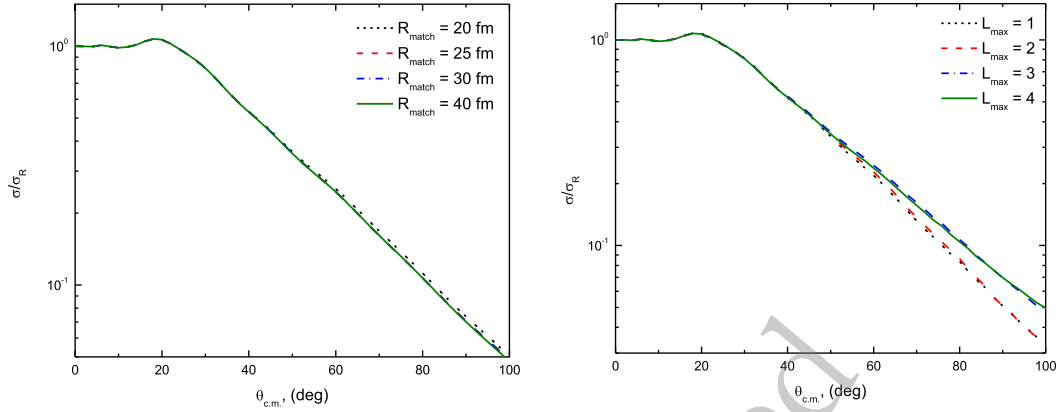


Fig. 4. (color online) Convergence test of the CDCC calculations for the ${}^7\text{Li} + {}^{28}\text{Si}$ system at $E_{\text{lab}} = 16$ MeV, showing *a*) convergence with respect to matching radius (R_{match}) and *b*) the accumulative effect of the pseudo-states with various (L_{max}).

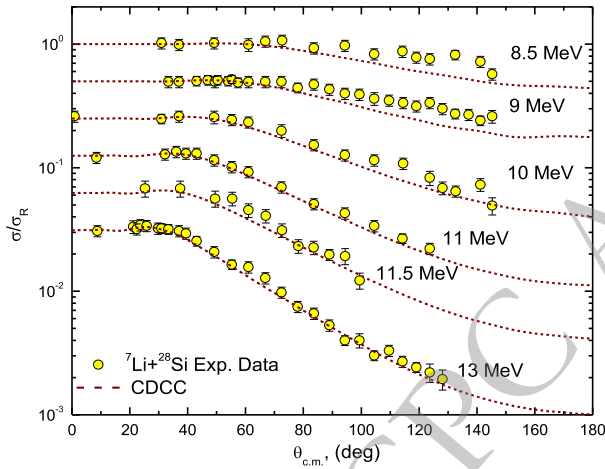


Fig. 5. (color online) Experimental ${}^{28}\text{Si}({}^7\text{Li}, {}^7\text{Li}){}^{28}\text{Si}$ elastic scattering ADs at $E_{\text{lab}} = 8.5, 9, 10, 11, 11.5,$ and 13 MeV versus calculations within the CDCC method.

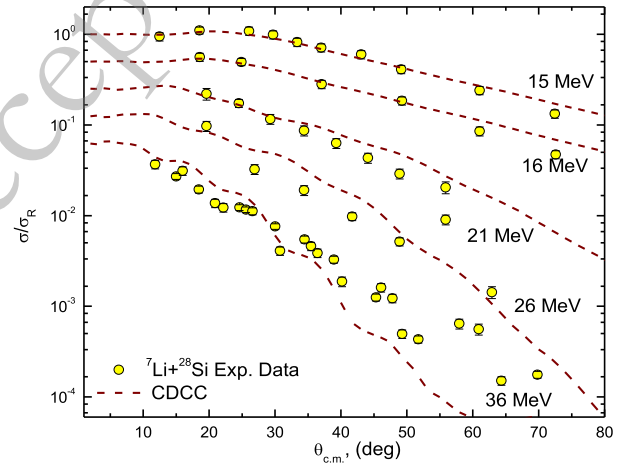


Fig. 6. (color online) Same as Fig. 5 but at $E_{\text{lab}} = 15, 16, 21, 26,$ and 36 MeV.

$$U^{\text{eff}}(R) = U^{\text{CF}}(R) + U^{\text{DPP}}(R), \quad U = V + iW \quad (4)$$

Figures 7 and 8 illustrate the generated U^{DPP} for the ${}^7\text{Li} + {}^{28}\text{Si}$ system. The characteristics of the U^{DPP} are significantly influenced by the target and the incident energy. At $E_{\text{lab}} = 16, 21, 26$ and 36 MeV, the general behavior for the real and imaginary parts of the generated U^{DPP} is close to each other, while at the lowest studied energies, $E_{\text{lab}} \leq 15$ MeV, this behavior is completely different, and the U^{DPP} exhibits a clear oscillatory nature.

The ${}^7\text{Li} + {}^{28}\text{Si}$ elastic scattering ADs were reproduced using a non-normalized effective potential, U^{eff} , characterized by two factors: N_R and N_I , both set to unity ($N_R = 1.0$ and $N_I = 1.0$) for the real and imaginary U^{eff} components, respectively. The comparison between the calculated ${}^7\text{Li} + {}^{28}\text{Si}$ ADs using the non-normalized U^{eff} and the experimental data showed good agreement, as presented in Figs. 9 and 10. The calculations demonstrated the suitability of the U^{eff} , which explicitly accounts for the ${}^7\text{Li}$ breakup effects arising from the coupling to continuum

states above the ${}^7\text{Li} \rightarrow \alpha + t$ breakup threshold, in replicating the experimental ADs data over a wide range of energies.

D. The influence of triton transfer ${}^{28}\text{Si}({}^7\text{Li}, \alpha){}^{31}\text{P}$ reaction on ${}^7\text{Li} + {}^{28}\text{Si}$ ADs

A recent study [10] investigated the breakup of ${}^7\text{Li}$ in the presence of a heavy mass target, ${}^{209}\text{Bi}$, revealing a significant population in a region accessible solely through direct triton stripping, distinct from breakup followed by triton capture. The analysis highlighted the direct cluster-stripping mechanism as a major source of alpha production. Similar results were observed in the ${}^7\text{Li} + {}^{144}\text{Sm}$ reaction [11], emphasizing the role of the triton transfer mechanism. In the ${}^7\text{Li} + {}^{28}\text{Si}$ system, both the breakup of ${}^7\text{Li}$ and the combined effects of ${}^7\text{Li}$ breakup and triton transfer in the reaction ${}^{28}\text{Si}({}^7\text{Li}, \alpha){}^{31}\text{P}$ on the elastic ${}^7\text{Li} + {}^{28}\text{Si}$ channel, as depicted in Fig. 11. The triton was modeled as being transferred as a single entity from the ${}^7\text{Li}$ ground state to a $t + {}^{28}\text{Si}$ configuration in ${}^{31}\text{P}$. For the entrance channel (${}^7\text{Li} + {}^{28}\text{Si}$), the non-normalized U^{eff} in-

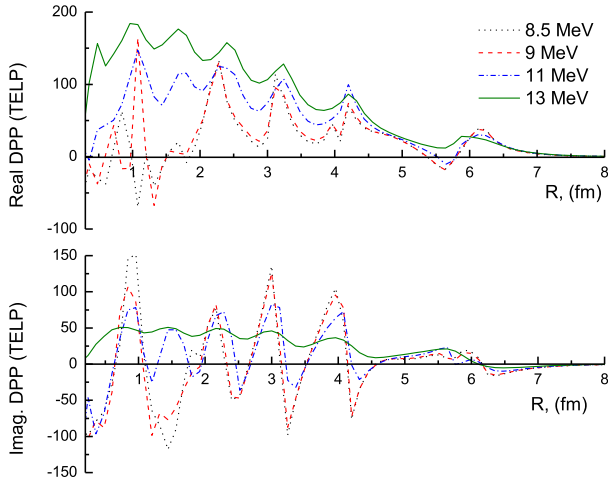


Fig. 7. (color online) The U^{DPP} obtained from the CDCC computations for the ${}^7\text{Li} + {}^{28}\text{Si}$ system at $E_{\text{lab}} = 8.5, 9, 11,$ and 13 MeV.

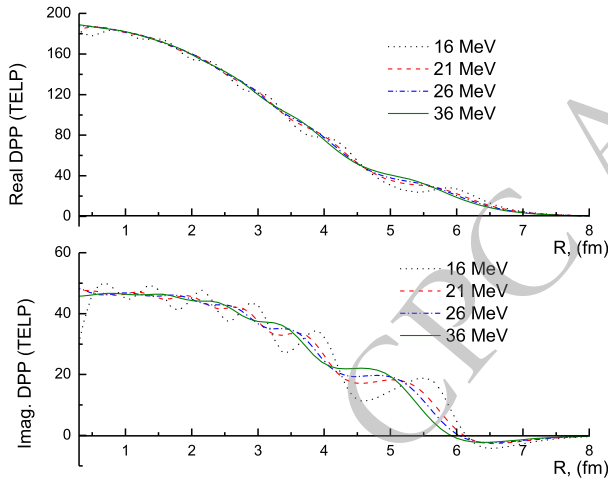


Fig. 8. (color online) Same as Fig. 7 but at $E_{\text{lab}} = 16, 21, 26,$ and 36 MeV.

corporating the ${}^7\text{Li}$ breakup was used, while for the exit channel ($\alpha + {}^{31}\text{P}$), a standard potential (SPP) [70] with standard normalization factors ($N_R = 1.0$ and $N_I = 0.78$) was employed. The bound state potentials for the ${}^7\text{Li} \rightarrow \alpha + t$ and ${}^{31}\text{P} \rightarrow {}^{28}\text{Si} + t$ configurations were modeled using the WS form, with $R_V = 1.25$, $a_V = 0.65$ fm, and the depth adjusted to reproduce the binding energies of 2.467 and 17.897 MeV, respectively, for the considered overlap $\langle {}^7\text{Li} | \alpha + t \rangle$ and $\langle {}^{31}\text{P} | {}^{28}\text{Si} + t \rangle$.

The calculations considering the combined effects of ${}^7\text{Li}$ breakup and triton transfer were conducted at various incident energies $E_{\text{lab}} = 11, 15, 16, 21, 26,$ and 36 MeV, to assess their energy dependence. As shown in Fig. 11, at all the studied energies, the dominant contributions came from breakup, while those from triton transfer were negligible. With increasing bombarding energy, especially at larger angles (Fig. 11(e) and Fig. 11(f)), the ef-

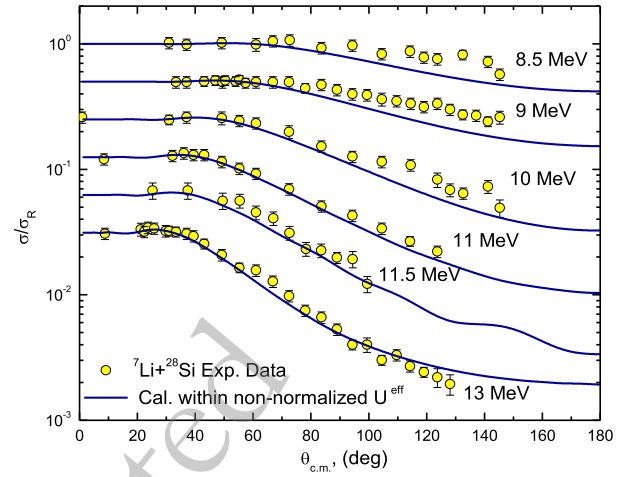


Fig. 9. (color online) Experimental ${}^{28}\text{Si}({}^7\text{Li}, {}^7\text{Li}){}^{28}\text{Si}$ elastic scattering ADs (circles) at $E_{\text{lab}} = 8.5, 9, 10, 11, 11.5,$ and 13 MeV versus calculations within the non-normalized U^{eff} (curves).

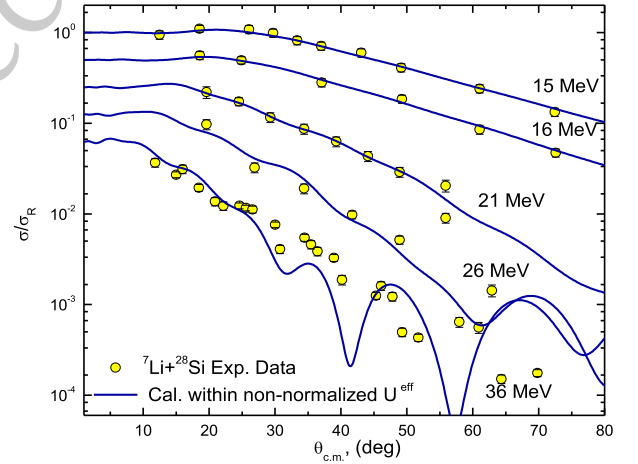


Fig. 10. (color online) Same as Fig. 9 but at $E_{\text{lab}} = 15, 16, 21, 26,$ and 36 MeV.

fects of triton transfer began to emerge. In conclusion, at energies near or below the breakup threshold, breakup is the predominant mechanism, with limited effects at higher energies. Compared to heavier systems like ${}^7\text{Li} + {}^{209}\text{Bi}$ [10] and ${}^7\text{Li} + {}^{144}\text{Sm}$ [11], the influence of triton transfer is negligible in the lighter (${}^7\text{Li} + {}^{28}\text{Si}$) system, particularly at low energies. To confirm the increasing role of triton transfer at higher energies, we compare the relative contributions of breakup and transfer processes by analyzing the ratio of the breakup cross-section (σ_{BU}) to the total breakup+transfer cross-section ($\sigma_{\text{BU+TR}}$) across the studied energies (Table 2). The results reveal a clear trend: as the energy increases, the ($\sigma_{\text{BU}}/\sigma_{\text{BU+TR}}$) ratio increases, indicating a growing contribution from triton transfer. This enhanced transfer leads to a suppression of the overall cross-section values, consistent with the observed energy-dependent behavior.

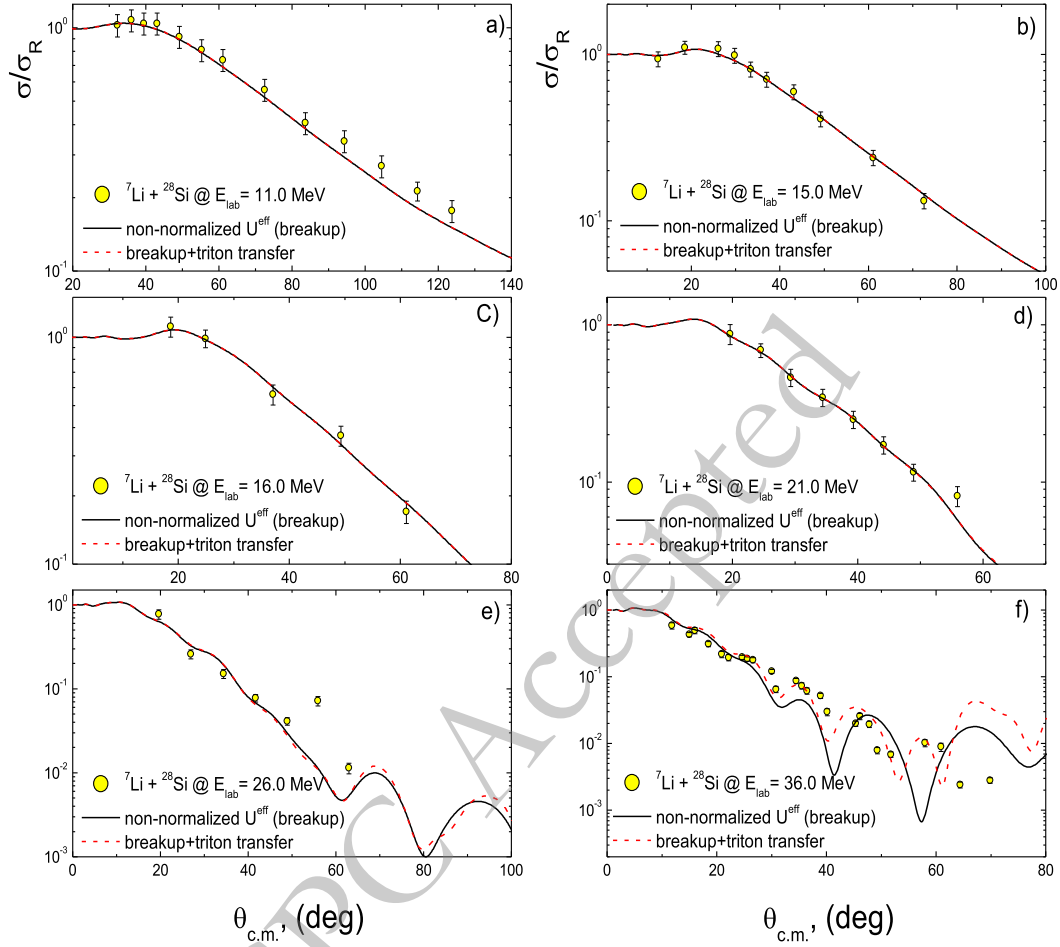


Fig. 11. (color online) The $^{28}\text{Si}(^7\text{Li},^7\text{Li})^{28}\text{Si}$ experimental ADs at $E_{\text{lab}} = 11, 15, 16, 21, 26$ and 36 MeV versus the calculations within non-normalized U^{eff} that explicitly considers the ^7Li breakup effect (solid curves) in comparison with the combined effects of breakup and triton transfer (dashed curves).

Table 2. Breakup cross-sections (σ_{BU}), total breakup+transfer cross-sections ($\sigma_{\text{BU+TR}}$), and their ratios at $E_{\text{lab}} = 11$ – 36 MeV

$E(\text{MeV})$	$\sigma_{\text{BU}}(\text{mb})$	$\sigma_{\text{BU+TR}}(\text{mb})$	$(\sigma_{\text{BU}}/\sigma_{\text{BU+TR}})$
11	711.0	711.6	0.999
15	1140	1123	1.015
16	1208	1192	1.013
21	1442	1424	1.013
26	1558	1531	1.018
36	1677	1538	1.090

IV. SUMMARY

This study provides a detailed analysis of the elastic scattering ADs for the $^7\text{Li} + ^{28}\text{Si}$ system at near barrier energies. The focus is on examining the influences of the breakup of ^7Li in the field of the ^{28}Si target and the triton stripping reaction $^{28}\text{Si}(^7\text{Li}, \alpha)^{31}\text{P}$, on the $^7\text{Li} + ^{28}\text{Si}$ elastic ADs. Based on previous research [24], it was determined

that the strengths of the real DFP, SPP, and CFP needed reductions of 63%, 72%, and 55%, respectively. This reduction in potential strength is a characteristic feature of systems involving WB projectiles like ^7Li , arising from coupling to the continuum that generates a DPP. In this analysis, two methodologies were employed to simulate these effects. The first approach involved reproducing the data using a non-normalized CFP “ N_{RCF} and $N_{\text{ICF}} = 1$ ”. Additionally, surface-shaped real and imaginary DPPs were included, modeled as factors multiplied by the derivative of the real and imaginary CFPs. The second approach utilized the continuum discretized coupled channels (CDCC) method with freely adjustable parameters to reproduce the elastic ADs. The analysis revealed that the characteristics of the real and imaginary DPPs exhibit a strong dependence on the bombarding energy. Moreover, the study demonstrates that the U^{eff} potential—without any adjustments—successfully reproduces the $^7\text{Li} + ^{28}\text{Si}$ ADs data, supporting its suitability for this nuclear system. Furthermore, the analysis included an investigation of the influence of the triton transfer reaction

$^{28}\text{Si}(^7\text{Li},\alpha)^{31}\text{P}$ reaction on the $^7\text{Li} + ^{28}\text{Si}$ elastic scattering channel. Results indicate that below and near the barrier, breakup is the dominant mechanism, while the effects of triton transfer become increasingly significant with higher bombarding energy.

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