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On Ground-State Correlation in Nuclear Many-Body Systems

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The ground-state correlation in model systems with different interaction strengths and particle numbers has been studied. Numerical results tell us that if the ground-state shape is quite stable, the ground-state correlation can be approximately explained by the zero-point vibration based on the static self-consistent field calculation. However, around the critical point where the monopole deformation begins to occur, behaviors of the ground-state correlation become very complicated and sensitive to the variation of controlling parameters. It seems to indicate that particular attention should be paid to ground-state correlations in further studies of light nuclei near the drip line.

Key words: ground-state correlation, ground-state shape, critical point.

1. INTRODUCTION

In recent years, nuclear many-body theory has developed greatly. Wang Shunjin *et al.* [1] have systematically developed the correlation dynamics on the basis of the self-consistent field theory. However, due to the large amount of numerical work involved, the application of the many-body correlation dynamics is for the present limited to a rather small scope of problems. Model theories are still prevalently used in nuclear physics.

The merit of the nuclear shell model is that it can deal with both spherical and deformed nuclei on the same basis of single particle states in a spherically symmetrical well. The difficulty of spherical symmetry breaking in calculations using a static deformed well is thus avoided. On the other hand, the static and dynamic correlation effects are not distinguished. When the static correlation effect is rather

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large, it is difficult to have an approximate calculation. The interacting boson model (IBM) which can be applied to both spherical and deformed nuclei seems to have retained the merit of the nuclear shell model and dispensed with its shortcomings. However, it should be noted that the IBM is a phenomenological theory. Its microscopic foundation has not been fully clarified.

The collective excitation spectrum of nuclei in the transition region from spherical to deformed nuclei is much more complicated. There may occur shape coexistence. This case is the same for nuclei near the drip line [3, 4]. Stable nuclei may become unstable as $|N - Z|$ increases. The strange features of halo nuclei like ^{11}Li have not yet been explained microscopically.

In this paper, we do not intend to investigate the above-mentioned problems concretely. However, we try to study the behaviors of the ground-state correlation with respect to different coupling strengths and quantum effects, using a simple schematic model. For spherical nuclei, the collective excitation spectrum and ground-state correlation basically come from the same origin. For deformed nuclei, the collective excitation spectrum is determined by the property closely related to the static deformation in addition to the factor which also gives the ground-state correlation. For nuclei in the transition region, if we know how to improve the calculation for ground-state correlation, then we know how to modify the calculation for the collective excitation spectrum. Therefore, the results of such a model may help us to understand the microscopic basis of the interacting boson model and to carry out microscopic investigations for light nuclei like ^{11}Li near the drip line.

The two-level Lipkin model is used in this paper. Although it is a nuclear system with N nucleons, we can consider collective motions based on a definite intrinsic state because there exists no coupling between the collective degree of freedom and a single particle degree of freedom. Since there exists only one collective particle-hole excitation mode, we need only to consider a subsystem for this collective degree of freedom. Therefore, the exact solutions can be readily obtained. In real nuclear systems, there exist couplings between various degrees of freedom, and it is very difficult to carry out exact calculations.

For this model, results of the static mean field theory can also be readily obtained. The ground-state correlation can then be easily found as the difference between the exact and approximate static results. Evidently, the ground-state correlation is a quantum effect. The inertia parameters for the collective motion of systems with different numbers of nucleons are not the same. On the other hand, the long-range interaction is related to the number of nucleons. Therefore, in this paper, we take the strength of interaction and effective Planck constant as control parameters, and regard the ground-state correlation as the order parameter to show the relationship between the ground-state correlation and the number of nucleons under different conditions.

2. THE LIPKIN MODEL AND COLLECTIVE HAMILTONIAN

The Hamiltonian of the two-level Lipkin model with monopole interaction is as follows:

$$H = \frac{\varepsilon}{2}\sum_m (a^+_{m_+} a_{m_+} - a^+_{m_-} a_{m_-} + 1) - \frac{\kappa}{2}\left[\sum_m (a^+_{m_+} a_{m_-} - a^+_{m_-} a_{m_+})\right]^2 , \tag{2.1}$$

where $a^+_{m_+}$ and $a^+_{m_-}$ indicate the fermion creation operators in the upper and lower levels with the same angular momentum. a_{m_+} and a_{m_-} are corresponding annihilation operators.

The collective operators

$$J_0 = \frac{1}{2}\sum_m (a^+_{m_+} a_{m_+} - a^+_{m_-} a_{m_-} + 1) , \tag{2.2a}$$

$$J_+ = \sum_m a^+_{m_+} a_{m_-} ,\ J_- = \sum_m a^+_{m_-} a_{m_+} , \tag{2.2b}$$

form an $SU(2)$ algebra. J_0 indicates the number of collective particle-hole pairs. J_+, J_- denote the creation and annihilation of collective pairs, respectively. The system has $SU(2)$ dynamic symmetry. There is no coupling between collective and single particle degrees of freedom. Suppose that the system has the total number of fermions $\Omega = 2j + 1$, then the lower energy level is filled when $k = 0$. There occur the creation and annihilation of collective particle-hole pairs when $k \neq 0$. Therefore one can use the following states

$$|JM\rangle = C_M (J_+)^{M+\frac{\Omega}{2}} \left| \frac{\Omega}{2}, -\frac{\Omega}{2} \right\rangle$$
$$C_M = \left\langle \frac{\Omega}{2}, -\frac{\Omega}{2} \right| (J_-)^{M+\frac{\Omega}{2}} (J_+)^{M+\frac{\Omega}{2}} \left| \frac{\Omega}{2}, -\frac{\Omega}{2} \right\rangle^{-\frac{1}{2}} \tag{2.3}$$

to denote a set of orthonormal basis states for collective excitation. Using the generator coordinate method [7], one can obtain the Hamiltonian for collective motions in boson representation and canonical variable representation.

Let b^+, b denote the phonon creation and annihilation operators, respectively. They satisfy the commutation relation $[b, b^+] = 1$. Then the state with n phonons can be written as

$$|n) = \frac{1}{\sqrt{n!}} (b^+)^n |O) , \qquad (n = 0,1,2,\cdots) . \tag{2.4}$$

Because the $SU(2)$ group is a compact group, the dimension of its representation space is finite. In order to guarantee the one-to-one correspondence between fermion states and boson states, we truncate the phonon space at the state where the number of phonons equals to the total number of nucleons Ω. So we have the transformation

$$\overline{U} = \left\langle \frac{\Omega}{2}, -\frac{\Omega}{2} \right| \exp\{b^+ J_-\} \left| 0 \right) , \tag{2.5a}$$

$$\overline{U}^+ = (0| \exp\{bJ_+\} \left| \frac{\Omega}{2}, -\frac{\Omega}{2} \right\rangle , \tag{2.5b}$$

relating these two state spaces, where $|\Omega/2, -\Omega/2>$ and $|0)$ represent the vacuum states of phonon and boson spaces, respectively.

Using the transformation (2.5), the Dyson representations for operators J_0, J_+, J_- are

$$\overline{U} \begin{pmatrix} J_- \\ J_0 \\ J_+ \end{pmatrix} \overline{U}^+ = \begin{pmatrix} b \\ b^+ b \\ b^+(\Omega - b^+ b) \end{pmatrix} \mathcal{N} . \tag{2.6}$$

where

$$\mathcal{N} = \overline{U}\,\overline{U}^+ = \left\langle \frac{\Omega}{2}, -\frac{\Omega}{2} \right| \exp\{b^+ J_-\} \left| 0 \right) \left(0 \right| \exp\{bJ_+\} \left| \frac{\Omega}{2}, -\frac{\Omega}{2} \right\rangle ,$$

After some manipulations, we have

$$\mathcal{N} = \frac{\Omega!}{(\Omega - b^+ b)!} . \tag{2.7}$$

In order to present the Hermitian property of the Hamiltonian, we must use the Holstein-Primakoff representation. According to Eq. (2.7), we obtain the Holstein-Primakoff representation for these operators as

$$\mathcal{N}^{-\frac{1}{2}}\begin{pmatrix} b \\ b^+b \\ b^+(\Omega - b^+b) \end{pmatrix}\mathcal{N}^{\frac{1}{2}} = \begin{pmatrix} \sqrt{\Omega - b^+bb} \\ b^+b \\ b^+\sqrt{\Omega - b^+b} \end{pmatrix} . \tag{2.8}$$

Using Eqs. (2.7) and (2.8), we can obtain for the model system the effective Hamiltonian in the truncated phonon space as

$$\mathcal{H}^{(\mathrm{HP})} \equiv \mathcal{N}^{-\frac{1}{2}} H \mathcal{N}^{\frac{1}{2}} = \varepsilon\Omega \frac{b^+}{\sqrt{\Omega}}\frac{b}{\sqrt{\Omega}} - \frac{\kappa\Omega^2}{2} \left[\frac{b^+}{\sqrt{\Omega}}\sqrt{1 - \frac{b^+}{\sqrt{\Omega}}\frac{b}{\sqrt{\Omega}}} + \sqrt{1 - \frac{b^+}{\sqrt{\Omega}}\frac{b}{\sqrt{\Omega}}}\frac{b}{\sqrt{\Omega}}\right]^2 , \tag{2.9}$$

Let

$$q = \frac{1}{\sqrt{2\Omega}}(b^+ + b), \quad p = \frac{i}{\sqrt{2\Omega}}(b^+ - b) , \tag{2.10}$$

$$[q,p] = i\left(\frac{1}{\Omega}\right) . \tag{2.11}$$

then the collective Hamiltonian of (2.9) can be expressed with canonical variables as

$$\mathcal{H}^{(\mathrm{HP})} = \frac{\varepsilon\Omega}{2}(q - ip)(q + ip) - \frac{\kappa\Omega^2}{4} \left\{(q - ip)\left[1 - \frac{1}{2}(q - ip)(q + ip)\right]^{\frac{1}{2}} + \mathrm{H.C.}\right\}^2 . \tag{2.12}$$

Motions of the system with a certain value of Ω is determined by Eqs. (2.11) and (2.12). If we keep increases with $1/\Omega$. $C_s = q_0^2$ increases with $1/\Omega$ more quickly. ΔC is a small quantity as compared to quantities $\varepsilon\Omega$ and $\kappa\Omega^2$ fixed, $\mathcal{H}^{(\mathrm{HP})}$ is the same for systems with different fermion numbers. The difference among systems with different Ω, is only in the effective Planck constant in (2.11). Therefore, it is very easy to compare the ground-state correlations in systems with different Ω.

Because Eqs. (2.9) and (2.12) are different expressions for the same collective Hamiltonian, results given by them must be the same. However, it is easier to find its exact results using (2.9) than using (2.12). Expanding eigenstates in terms of $|n\rangle$ expressed as (2.4), we can easily obtain their exact numerical results. It is more convenient to find the approximate solutions with (2.12) for different cases, which will be discussed in next section.

3. GROUND-STATE CORRELATION, NUMERICAL RESULTS AND DISCUSSIONS

The ground-state correlation represents the difference between the exact expectation value of $b^+ b/\Omega$ for the ground state and the corresponding approximate value obtained from the static mean field theory. In order to compare the ground-state correlations of systems with different fermion numbers, we calculate the quantity $C_E = \langle b^+ b/\Omega\rangle$ and compare it with the correspondent C_S obtained approximately from the static mean field theory. The ground-state correlation is then expressed by the difference $\Delta C = C_E - C_S$. Writing b^+, b with the canonical variables as in (2.11)

$$\frac{\langle b^+ b\rangle}{\Omega} = \frac{1}{2}\left\langle q^2 + p^2 - \frac{1}{\Omega}\right\rangle . \tag{3.1}$$

and taking

$$P = p, \qquad Q = q - q_0 . \tag{3.2}$$

so Eq. (3.1) becomes

$$\frac{\langle b^+ b\rangle}{\Omega} - \frac{1}{2}q_0^2 = \frac{1}{2} \left\langle Q^2 + P^2 + 2q_0 Q - \frac{1}{\Omega}\right\rangle , \tag{3.3}$$

where $\langle\cdots\rangle$ denotes the averaging process over the ground state.

In the case that Ω is large enough, after neglecting the correction arising from noncommutative property of q, p, we obtain the approximate expression of (2.12) as follows:

$$\mathscr{H}^{(\mathrm{HP})} \doteq \frac{\varepsilon\Omega}{2}(q^2 + p^2) - \kappa\Omega^2 q^2 \left[1 - \frac{1}{2}(q^2 + p^2)\right] , \tag{3.4}$$

Then, we can obtain the result $8q_0^2$ at the deformation equilibrium point as

$$q_0^2 = \begin{cases} 0 & x < \dfrac{1}{2} \\ 1 - \dfrac{1}{2x} & x > \dfrac{1}{2} \end{cases} , \tag{3.5}$$

where $x = \dfrac{\kappa\Omega}{\varepsilon}$.

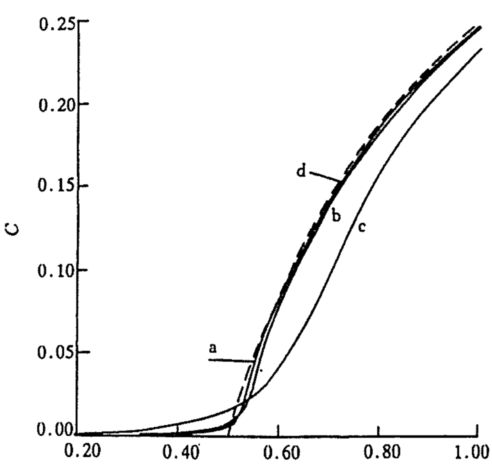

Fig. 1

Variation of $C = \langle b^+ b/\Omega\rangle$ with respect to the parameter x.

Solid curves denote the results of exact calculation for $\Omega = 20$ (curve c); 120 (curve b); and 240 (curve a). Dashed curve d stands for the approximate static result.

Substituting (3.2) into (3.4), we obtain a harmonic oscillator approximation for $\mathscr{H}^{(\mathrm{HP})}$ at equilibrium point q_0. Then we can obtain approximately the ground-state correlation as the zero-point oscillation. However, in this way, we always have $\Delta C_A > 0$. In general cases, however, there exist coupling between collective and single particle degrees of freedom. We have to consider the fluctuations of the mean field resulting from them.

Since $<b^+ b/\Omega>$ describes the main behavior of nuclear systems characterized by two factors x and $1/\Omega$, we use the former as order parameters and the latter as control parameters to show the variation of $<b^+ b/\Omega>$ with respect to x and $1/\Omega$. The results are plotted in Fig. 1. Solid curves show the variation of C_E with respect to x for $\Omega = 20$, 120, and 240, while the dashed curve shows the variation of C_S with respect to x for the case $1/\Omega \to 0$.

In order to show the intricate behaviors of ΔC which reflect directly the ground-state correlation, its variation with respects to $1/\Omega$ corresponding to different x is plotted in Fig. 2.

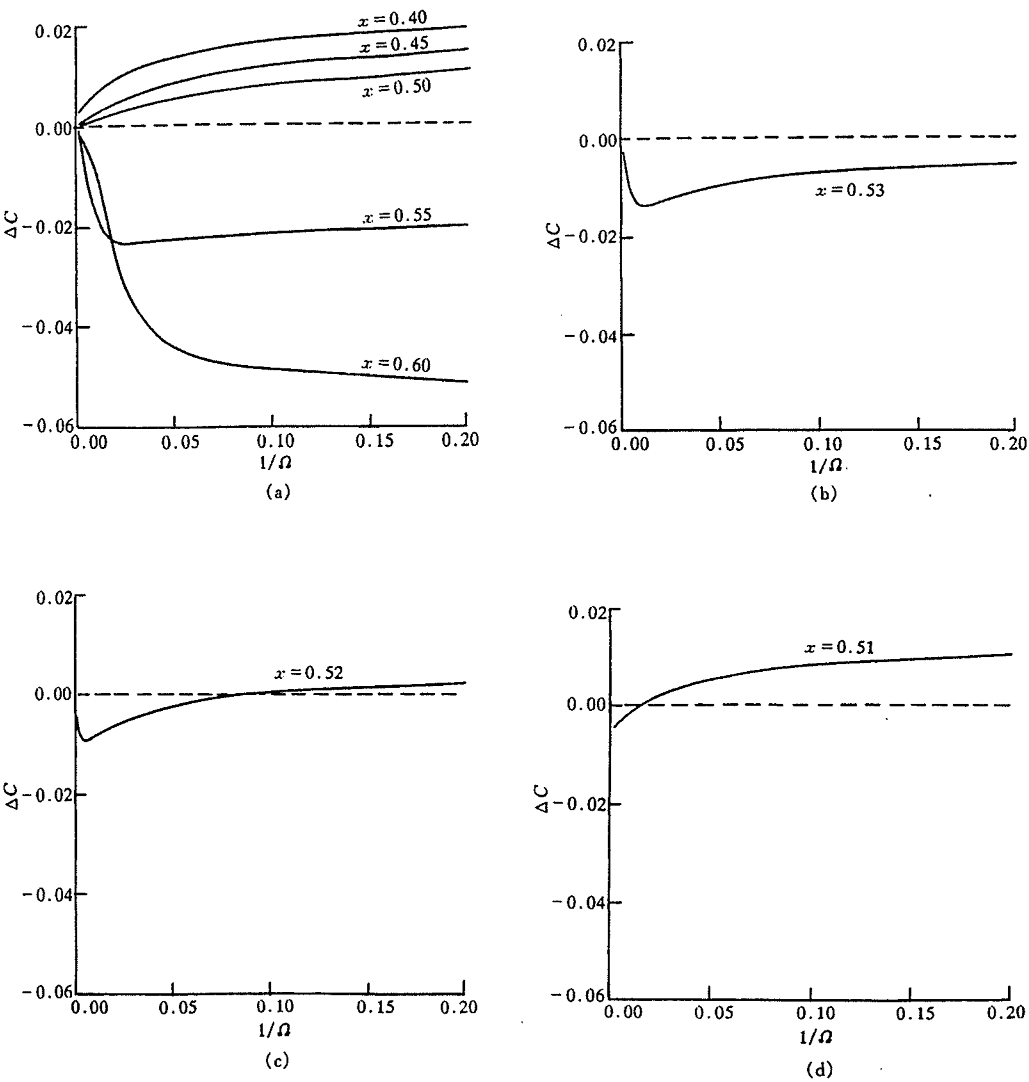

Fig. 2

Variation of ΔC with respect to $1/\Omega$ and $x = \kappa\Omega/\varepsilon$.

According to the feature of nuclear systems shown in Fig. 1, we divide it into three regions: (1) Nuclei with x much less than 1/2. They have no static monopole deformation. The value of $\Delta C > 0$ and is rather small. Its amplitude increases as $1/\Omega$ increases. (2) Nuclei with x are much larger than 1/2. They have considerable static monopole deformation. The value of $\Delta C < 0$, but its magnitude increases with $1/\Omega$. $C_s = q_0^2$ increases with $1/\Omega$ more quickly. ΔC is a small quantity as compared to C_S. The inertia parameter increases with the increase of the static deformation. The kinetic energy $\langle p^2\rangle_E/2$ decreases with the increase of the static of the static deformation. Furthermore, $\langle q^2\rangle_E$ here is less than q_0^2 obtained from the static mean field theory. Hence $\Delta C < 0$. (3) Nuclei with the value of x in the region around the point $x = 1/2$. ΔC is large as compared to C_S and varies with x and $1/\Omega$ in a rather complicated way.

We see from Fig. 2(a) that when x is less than 1/2 and very close to 1/2, ΔC becomes larger. This situation still connects with the characters of region (1). However, when x is greater than 1/2 and very close to 1/2, the variation of ΔC is particularly complicated. For very small $1/\Omega$, $\Delta C < 0$, and decreases quickly with increasing $1/\Omega$. However, after falling to a certain value, it begins to increase. From the other figures of Fig. 2 for $x = 0.51$, 0.52, and 0.53, we see that the complication of the variation of ΔC with respect to $1/\Omega$ becomes weaker and weaker. When x reaches 0.55, the characters of ΔC are the same as those of the region (2).

Therefore, according to the results of this paper, the basis set of a spherical well is not suitable to describe the main features of deformed nuclei. It is therefore fortunate that for the collective excitation spectrum of deformed nuclei, the only important factors are just the rotation moment of inertia depending mainly on the static deformation, the vibrational inertia parameter, and the corresponding recovering force. In order to show these features, one can deal with the original complicated characters using renormalization theory such that the IBM with a rather simple form can describe characteristic behaviors of nuclei in different regions. However, if doing so, the *S*- and *D*-pairs cannot be regarded simply as pairs of nucleons moving inside a spherical well. Arima himself has discussed this problem using a single-level model [8]. It seems worthwhile to carry out further studies.

However, in the region $x > 0.5$ fluctuation effects are particularly important because of the shallow deformed potential. Hence in discussing the light nuclei near the drip line, great attention must be paid not only to the ground-state correlation already discussed in this paper, but also to the fluctuations of the mean field arising from its interaction with single particle motions, a phenomenon which is absent in the model discussed here.

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