第 26 卷 第 1 期
2002年 1 月

高能物理与核物理
HIGH ENERGY PHYSICS AND NUCLEAR PHYSICS

Vol.26, No.1
Jan., 2002

Computation of ^{7}Be Electron-Capture Rate in the Solar Interior

YE Zi-Piao[1,2;1)] HU Bao-Jian[2] KUANG Hao-Huai[1]

1 (Laboratory of Cosmic Ray and Astrophysics, Institute of High Energy Physics, The Chinese Academy of Sciences, Beijing 100039, China)

2 (Department of Physics, Zhaoqing University, Zhaoqing 526061, China)

Abstract Under conditions near the center of the Sun, it shows that ^{7}Be atoms are completely ionized between $R=0$ to $R=0.1217R_\odot$. The newly calculated ^{7}Be and ^{8}B solar neutrino fluxes are about $4.00\times10^9\mathrm{cm^{-2}\cdot s^{-1}}$ and $6.18\times10^6\mathrm{cm^{-2}\cdot s^{-1}}$, while the corresponding predicted values of the standard solar model are $4.80\times10^9\mathrm{cm^{-2}\cdot s^{-1}}$ and $5.15\times10^6\mathrm{cm^{-2}\cdot s^{-1}}$, respectively. It will further increase the discrepancy between the observed and the predicted neutrino fluxes in Super Kamiokande neutrino experiment.

Key words ^{7}Be electron-capture rate, ^{7}Be neutrino, standard solar model, compound rate coefficient, compound cross section

1 Introduction

The predominant reaction by which ^{7}Be is destroyed in the pp chain, under conditions existing near the core of the Sun, is the electron-capture reaction ^{7}Be (e^-, ν_e) ^{7}Li. The ^{7}Be disappears about a thousand times more rapidly by the ^{7}Be (e^-, ν_e) ^{7}Li reaction than by the ^{7}Be(p, γ) ^{8}B reaction[1]. The equilibrium abundance of ^{7}Be is thus essentially obtained by equating the ^{7}Be (e^-, ν_e) ^{7}Li rate with the ^{3}He(^{4}He, γ) ^{7}Be. The ^{7}Be (p, γ) ^{8}B rate and, consequently, the ^{8}B (β^+ ν_e) ^{8}Be* rate are then proportional to this equilibrium abundance. J. N. Bahcall computed the ^{7}Be electron-capture only considering the capture of continuum electrons and neglecting the plasma screening by the ionized gas of the star[2]. While I. Iben thought that there is a finite probability that ^{7}Be exists as an atom with one or two bound K-shell electrons[1], they computed the capture rate of bound electrons in $^7Be^{3+}$ and $^7Be^{4+}$ using Debye-Hückel approximation to estimate the screening effect of the ionized plasma on the rate of bound-electron capture and found that the bound-electron capture increases the total capture rate in the solar interior by 17—25 percent and that plasma screening strongly affects the rate of bound-electron capture on the solar reaction of ^{7}Be (e^-, ν_e) ^{7}Li. And the bound-electron capture has an opposite effect on the solar ^{8}B neutrino flux, which is of the same magnitude as the ^{7}Be neutrino flux[1].

J. N. Bahcall[3] then renewed to calculate the ^{7}Be electron-capture rate considering the effect of plasma and bound-electron screening in the reaction ^{7}Be (e^-, ν_e) ^{7}Li and found that the total reaction rate λ_{total} (considered the effect of plasma, bound-electron screening and the free-electron capture by ^{7}Be), for ^{7}Be (e^-, ν_e) ^{7}Li is 1.2 times than λ_c, which is the reaction rate only considering the ^{7}Be capture from the continuum electrons.

Received 7 December 2000, Revised 6 June 2001

1) E-mail: yezp@zqu.edu.cn

Because the predicted solar neutrino flux from ^{7}Be electron capture and the calculated abundance of the ^{7}Be nucleus directly affect the ^{7}Be (p,γ) ^{8}B reaction rate and, the ^{8}B (β^+ ν_e) ^{8}Be* decay rate, so it is of interest to examine quantitatively the influence of the free-electron capture and of K-capture by ^{7}Be nucleus.

2 Calculation of reaction rate

The free-electron capture rate[1] may be written as

$$\lambda_c = \frac{|\psi_f(0)|^2}{2|\psi_{lab}(0)|^2}\lambda_{lab}.$$

where $|\psi_f(0)|^2$ is the free-electron density at the ^{7}Be nucleus in the star, $2|\psi_{lab}(0)|^2$ is the electron density at the ^{7}Be nucleus in a neutral, unscreened atom in the ground state, and $\lambda_{lab} = \frac{\ln 2}{53.6}\text{days}^{-1} = 1.497\times10^{-7}\text{sec}^{-1}$. Inserting the estimate of $|\psi_{lab}(0)|^2$ given by J. N. Bahcall[2] and the estimate of $|\psi_f(0)|^2$ given by I. Iben et al.[1], one has

$$\lambda_c \approx 2.31\times10^{-9}(1+X)\rho T_6^{-\frac{1}{2}}[1+0.004(T_6-16)]\text{sec}^{-1}$$

where T_6 is the temperature in units of 10^6 K, X is the mass abundance of hydrogen, and ρ is the matter density (g/cm^3).

After considering the effect of plasma and bound-electron screening in the electron-capture reaction ^{7}Be (e^-, ν_e) ^{7}Li, the rate at which ^{7}Be captures an electron in the solar interior can be written approximately in the following convenient form[3,4]

$$\lambda_{total} \approx 2.31\times10^{-9}\langle B\rangle(1+X)\rho T_6^{-\frac{1}{2}}[1+0.004(T_6-16)]\text{sec}^{-1}$$

where $\langle B\rangle$ represents the enhancement of the continuum capture rate by bound-electron capture[3], being equal to 1.2.

In Eq. (3), the numerical approximation represented by the last bracketed expression is in accuracy to 1% for $10\leqslant T_6\leqslant16$[3].

Comparing Eq. (2) with Eq. (3), we can obtain

$$\lambda_{total} = 1.2\lambda_c. \tag{4}$$

But under conditions near the center of the Sun the temperature is 15.67×10^6 K, the equilibrium energy is about 2keV. While the ^{7}Be atom completely ionized energy is about 0.45keV (the fourth ionization potential of the ^{7}Be atom is 216.6eV, the third ionization potential of the ^{7}Be atom is 153.1eV, the first ionization potential plus the second ionization potential of the ^{7}Be atom is about 30eV). The equilibrium energy is more 4 times than ^{7}Be atom completely ionized energy. Thus, even if there is a finite probability that the ^{7}Be exists as an atom with one or two bound K-shell electrons in the center of the Sun because of the Maxwell-Boltzmann distribution, the existing probability should be small.

According to the standard solar model (SSM), the abundance of ^{7}Be atom and the solar temperature are gradually decreasing nuder the solar center. From 0 to $0.1217R_\odot$ (here $R_\odot$ is the solar radius), the average abundance of ^{7}Be atom is about 94.02% ^{7}Be atom abundance of the whole solar, and the average temperature is about 13.87×10^6 K[5].

Because compound and ionization of atoms in the Sun is in heat equilibrium state, the differential compound cross section of ^{7}Be ions can be expressed as[6]

$$\sigma_R(n) = \frac{32\pi}{3\sqrt{3}}Z^2\alpha_f^3k_e^{-2}\left(\frac{Z^2I}{h\nu}\right)\left(\frac{g_R(n)}{n^3}\right), \tag{5}$$

here n is the main quantum number of electron state in atoms, Z is the atomic number, α_f is the fine structure constant $\left(\alpha_f = \frac{e^2}{4\pi\varepsilon_0\hbar c}\right)$, k_e^{-1} is $1/2\pi$ de Broglie wave length of incident electrons ($k_e^{-1} = \frac{\hbar}{mv}$,

mv is the momentum of incident electrons), I is the ionized energy, h is Planck constant ν is the frequency of radiation photons, $g_R(n)$ is Guant factor, $g_R(n) \approx 1$.

If the kinetic energy of incident electron is much less than the ionized energy of the atomic ground state, the energy of radiation photons, $h\nu \approx \frac{Z^2 I}{n^2}$. A convenient representation of Eq. (5) can be written

$$\sigma_R(n) = 1.01 \times 10^{-13} \frac{Z^2 g_R(n)}{nv^2} (\mathrm{m}^2). \qquad (6$$

By Eq. (6), we can calculate the compound rate coefficient of ^{7}Be ions, which represents its rate to compound and ionize. It can be accurately written

$$a_n(T) = \int_0^\infty \sigma_R(n) v f(v) \mathrm{d}v, \qquad (7$$

here T is the temperature of the solar core, v is the velocity of incident electrons, $f(v)$ is the Maxwell distribution function. It can be written

$$f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right).$$

Because we only consider K-shell electron of ^{7}Be, namely, $n = 1$, and $g_R(1) \approx 1$, so

$$\begin{aligned} a_1(T) &= \int_0^\infty \sigma_R(1) v f(v) \mathrm{d}v = 1.01 \times 10^{-13} \times \\ & 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty \left(\frac{Z^2 g_R(1)}{v^2}\right) v^3 \exp\left(-\frac{mv^2}{2kT}\right) \mathrm{d}v \\ &= 8.915 \times 10^{-14} (\mathrm{cm}^3 \cdot \mathrm{s}^{-1}). \end{aligned} \qquad (9)$$

In other words, for ^{7}Be ions, its Debye screening length can be computed by following expression

$$\lambda_D = \left(\frac{\varepsilon_0 kT}{K n_{Be} Z^2 e^2}\right)^{1/2}, \qquad (10)$$

here k is Boltzmann constant, T is the average temperature in solar core, and $K = \frac{1}{4\pi\varepsilon_0}$, n_{Be} is the mean ^{7}Be atoms or ions number density, $n_{Be} = 4.886 \times 10^{20}/\mathrm{m}^3$[5]. We can calculate ^{7}Be ions' Debye length by Eq. (10), $\lambda_D = 3.07 \times 10^{-11}$ m.

While the average velocity of incident electron can be calculated by the following expression

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}} = 2.32 \times 10^7 (\mathrm{m/s}). \qquad (11)$$

So, under the heat equilibrium condition in the solar core, fraction of ^{7}Be atoms with one K-shell electron is

$$\frac{N}{N_0} = (1 - \exp(-a_1(T) N_e t)) = \left(1 - \exp\left(-a_1(T) N_e \frac{\lambda_D}{\bar{v}}\right)\right). \qquad (12)$$

here N_e is the mean electron number density between $R = 0$ to $R = 0.1217 R_\odot$, N_0 is the mean ^{7}Be ion number density at $t = 0$, $N_e = 5.121 \times 10^{25}/\mathrm{cm}^3$[5]. Thus

$$\frac{N}{N_0} \approx 0. \qquad (13)$$

It means that ^{7}Be atoms are completely ionized in the solar interior. And the rate at which ^{7}Be captures an electron in the solar interior can be computed by simple following formula

$$\lambda'_{total} = \lambda_c. \qquad (14)$$

Comparing Eq. (14) with Eq. (4), we find

$$\lambda_{total} = 1.2 \lambda'_{total}. \qquad (15)$$

3 Discussion and conclusion

From the calculation above, we may conclude that the ^{7}Be atoms completely ionized, and the predicted rate of ^{7}Be electron capture by the standard solar model is 1.2 times bigger than the newly computed electron capture rate in the solar interior. The predicted value of the ^{7}Be solar neutrino flux will decrease from $4.80\times10^9 cm^{-2}\cdot s^{-1}$[7] to about $4.00\times10^9 cm^{-2}\cdot s^{-1}$, and the predicted value of the ^{8}B solar neutrino flux will increase from $5.15\times10^6 cm^{-2}\cdot s^{-1}$[7] to $6.18\times10^6 cm^{-2}\cdot s^{-1}$. It will further increase the discrepancy between the observed and the predicted neutrino fluxes in Super Kamiokande neutrino experiment.

It is a pleasure to thank Professor DING Lin-Kai, ZHU Qing-Qi and Professor JIN Chai-Liu for helpful suggestions and discussions.

References

1 Iben I, Jr. Kenneth Kalata, Judah Schwartz. Astrophys. J., 1967, **150**:1001

2 Bahcall J N. Phys. Rev., 1962, **128**:1297

3 Bahcall J N, Charles P. Moeller. Astrophys. J., 1969, **155**:511

4 Bahcall J N et al. Rev. Mod. Phys., 1982, **54**:767

5 Bahcall J N et al. Rev. Mod. Phys., 1988, **60**:297

6 Gould, Thakur. Am. J. Phys., 1970, **38**:189

7 Bahcall J N et al. Phys. Rev. Lett., 1998, **B433**:1

太阳核心中^{7}Be电子俘获几率的计算

叶子飘[1,2;1)] 胡宝坚[2] 况浩怀[1]

1(中国科学院高能物理研究所,宇宙线与高能天体物理实验室 北京 100039)

2(肇庆学院物理系 肇庆 526061)

摘要 在太阳核心的条件下,^{7}Be原子被完全电离.所以,重新计算的^{7}Be和^{8}B太阳中微子流强分别约为$4.00\times10^9 cm^{-2}\cdot s^{-1}$和$6.18\times10^6 cm^{-2}\cdot s^{-1}$,而标准太阳模型预言的^{7}Be和^{8}B太阳中微子流强则分别是$4.80\times10^9 cm^{-2}\cdot s^{-1}$和$5.15\times10^6 cm^{-2}\cdot s^{-1}$.这将进一步增大在Super Kamiokande太阳中微子实验上中微子流强的实验测量值与理论预计值之间的差异.

关键词 ^{7}Be俘获电子几率 ^{7}Be中微子 标准太阳模型 复合速率系数 复合截面

2000-12-07收稿,2001-06-06收修改稿

1) E-mail:yezp@zqu.edu.cn