Nonfactorization in $D^+ \to \bar{K}^0 K^+$ Decays *

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Abstract We analyze $D^+ \to \bar{K}^0 K^+$ decay at the leading order, α_s corrections with the QCD factorization approach and the soft-gluon effects with the light cone QCD sum rules. We find the prediction of naive factorization is far from the experimental data, and the QCD factorization result approaches to the experimental data. However, in QCD factorization method, if we consider the soft-gluon effects, then the result is in accordance with the experimental data well. Our calculation shows that the soft-gluon contributions which are firstly calculated in D meson nonleptonic decay are noticeable. So, it can't be neglected in the decay channel.

Key words QCD factorization, QCD sum rules, soft-gluon effects, Radiative correction

1 Introduction

The study of heavy meson decays is important for understanding the standard model (SM) and searching for the sources of CP violation. However, The hadronic two-body weak decays of D mesons involve nonperturbative strong interactions and spoil the simplicity of the short distance behavior of weak interactions. Therefore, a simplified approach in which the amplitudes of these processes are given by a factorizable short distance current-current effective Hamiltonian is not expected to work well. Various approaches were employed to include long distance effects. The most commonly and very frequently used prescription, motivated by $\frac{1}{N_c}$ arguments[1], is to apply generalized factorization[2—3]. This phenomenological treatment works reasonably well in Cabibbo-favored D decays[3], but it is failing in the Cabibbo-suppressed $D \to \pi\pi$, $K\pi$ and $D \to K\bar{K}$ decays[4].

It is the high time to study D meson two-body weak decays beyond the factorization approach. Recently, M. Beneke et al.[5] gave a NLO calculation of the hadronic matrix element of $B \to \pi\pi$, $K\pi$ in the heavy quark limit. They pointed out that in the heavy quark limit the radiative corrections at the order of α_s can be calculated with PQCD method. In $D^+ \to \bar{K}^0 K^+$ decay, the momentum transition square $q^2 = 1.5\text{GeV}^2$, and the radiative corrections of hard-gluon can also be calculated with PQCD approach. So, the hadronic matrix elements for $D^+ \to \bar{K}^0 K^+$ can be expanded by the powers of α_s and $\frac{\Lambda_{QCD}}{m_c}$ as follow:

$$\langle KK | O_i | D\rangle = \langle K | j_1 | D\rangle \langle K | j_2 | 0\rangle \left[1 + \sum r_n \alpha_s^n + O\left(\frac{\Lambda_{QCD}}{m_c}\right)\right], \quad (1)$$

where O_i are some local four-quark operators in the weak effective Hamiltonian and $j_{1,2}$ are bilinear quark currents. In the Eq. (1), the power correct term $O\left(\frac{\Lambda_{QCD}}{m_c}\right)$ which include soft-gluon effects, final state interaction and the soft-gluon effects can't be calculated in QCD factorization and PQCD method. For the B meson two-body decay,

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this term is small, but we find the term is large for the D meson decay. So, it can't be neglected in the $D^+ \to \bar{K}^0 K^+$ decay. Recently, A. Khodjamirian[6] has presented a new method to calculate the hadronic matrix elements of nonleptonic B meson decays within the framework of the light cone QCD sum rules, where the nonfactorizable soft-gluon contributions can effectively be dealt with. Obviously, this approach can also be applied to $D^+ \to \bar{K}^0 K^+$ decay.

2 $D^+ \to \bar{K}^0 K^+$ in QCD factorization

The low energy effective Hamiltonian for $D^+ \to \bar{K}^0 K^+$ can be expressed as follows:

$$\mathscr{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{\rm cs} V_{\rm us}^* [(C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu)] + h.c., \quad (2)$$

where $C_i(\mu)$ are Wilson coefficients which have been evaluated to next-to-leading order. The four-quark operators $O_{1,2}$ are:

$$O_1 = (\bar{\rm u}{\rm s})_{V-A} (\bar{\rm s}{\rm c})_{V-A},$$
$$O_2 = (\bar{\rm u}_\alpha {\rm s}_\beta)_{V-A} (\bar{\rm s}_\beta {\rm c}_\alpha)_{V-A}, \quad (3)$$

The Wilson coefficients evaluated at $\mu = m_c$ scale are[7]

$$C_1 = 1.274, \ C_2 = -0.529, \quad (4)$$

In the Eq. (2), it is useful to rewrite down the effective Hamiltonian with the help of the Fierz transformation. For example, applying the Fierz transformation to the operator $O_2 = (\bar{\rm u}\Gamma_\mu {\rm c})(\bar{\rm s}\Gamma_\mu {\rm s})$, we have the effective Hamiltonian relevant to the tree operators,

$$\mathscr{H}_{\rm eff} = \frac{G_{\rm F}}{\sqrt{2}} V_{\rm cs} V_{\rm us}^* \Big[\Big(C_1(\mu) + \frac{C_2(\mu)}{3} \Big) O_1(\mu) + 2 C_2(\mu) \tilde{O}_1(\mu) \Big] + h.c., \quad (5)$$

where

$$\tilde{O}_1 = \Big(\bar{\rm u}\Gamma_\mu \frac{\lambda^a}{2} {\rm s} \Big) \Big(\bar{\rm s}\Gamma^\mu \frac{\lambda^a}{2} {\rm c} \Big), \quad (6)$$

In the above $\Gamma_\mu = \gamma_\mu(1-\gamma_5)$, $Tr(\lambda^a \lambda^b) = 2\delta^{ab}$, and

$$O_2 = \frac{1}{3} O_1 + 2\tilde{O}_1, \quad (7)$$

In the following, we write the decay amplitudes for $D_s^+ \to K^+ \bar{K}^0$ decay, and they include factorization and nonfactorization parts.

$$M_{f+\alpha_s}(D^+ \to K^+ \bar{K}^0) = i \frac{G_F}{\sqrt{2}} f_K F^{D\to K}(0)(m_D^2 - m_K^2) V_{\rm cs} V_{\rm us}^* a_1, \quad (8)$$

$$M_{nfg}(D^+ \to K^+ \bar{K}^0) = \sqrt{2} G_F V_{\rm cs} V_{\rm us}^* A C_2, \quad (9)$$

where the amplitude $M_{f+\alpha_s} = M_f + M_{\alpha_s}$, M_f represents the amplitude of factorization M_{α_s} is the amplitude of nonfactorization which is from the hard-gluon exchanges and M_{nfg} is the amplitude of nonfactorization which is from the soft-gluon exchanges. The total amplitude M is equal to $M_{f+\alpha_s} + M_{nfg}$. In the Eq. (9), the A is the hadronic matrix element induced by the operator $\tilde{O}_1$

$$A = \langle \bar{K}^0(-q) K^+(p) | \tilde{O}_1 | D^+(p-q) \rangle \quad (10)$$

which is from the soft gluon contribution and will be calculated in section 3. Now, we calculate the nonfactorizable effects in α_s correction in the $D^+ \to \bar{K}^0 K^+$ with QCD factorization approach. The method is similar to that of the $B \to \pi\pi$, πK decay. Readers can refer to Ref. [5] for detail. As in Ref. [5], we obtain the QCD coefficients $a_i (i = 1, 2)$ at next-to-leading order (NLO) in naive dimension regularization (NDR) scheme, and the coefficients a_i(KK) is split into two terms: $a_i({\rm KK}) = a_{i,{\rm I}}({\rm KK}) + a_{i,{\rm II}}({\rm KK})$. They are given as

$$a_{1,{\rm I}} = C_1 + \frac{C_2}{N_c}\Big(1 + \frac{C_F \alpha_s}{4\pi} V_K \Big),$$
$$a_{1,{\rm II}} = \frac{C_2}{N_c} \frac{C_F \pi \alpha_s}{N_c} H_{\rm KK}, \quad (11)$$

Here $N_c = 3 (f = 4)$ is the number of colors (flavors), and $C_F = \frac{N_c^2 - 1}{2N_c}$ is the factor of color. The functions in the Eq. (11) can be found in Ref. [5], which are

$$V_K = 12 \ln \frac{m_c}{\mu} - 18 + \int_0^1 g(x) \phi_K(x) {\rm d}x,$$
$$g(x) = 3\Big(\frac{1-2x}{1-x} \ln x - {\rm i}\pi \Big) + \Big[2Li_2(x) - (\ln x)^2 + \frac{2\ln x}{1-x} - (3 + 2{\rm i}\pi)\ln x - (x \leftrightarrow 1-x) \Big],$$
$$H_{\rm KK} = \frac{f_D f_K}{m_D^2 F^{D\to K}(0)} \int_0^1 \frac{\phi_D(\xi)}{\xi} {\rm d}\xi \int_0^1 \frac{{\rm d}x}{\bar{x}} \phi_K(x) \cdot \int_0^1 \frac{{\rm d}y}{\bar{y}} \Big[\phi_K(y) + \frac{2\mu_k}{m_c} \frac{\bar{x}}{x} \Big], \quad (12)$$

where $Li_2(x)$ is the dilogarithm, $f_K (f_D)$ is the K (D meson) decay constant, m_D is the D meson mass, $F^{D\to K}(0)$ is the $D \to K$ form factor at zero momentum transfer, and ξ is the light-cone momentum fraction of the spectator in the D meson. $H_{\rm KK}$ depends on the wave function ϕ_D through the integral $\int_0^1 d\xi \phi_D(\xi)/\xi = m_D/\Lambda_{QCD} = 17.23$, $\mu_K = m_K^2/(m_d + m_s)$, $m_d = 5\text{MeV}$, $m_s = 150\text{MeV}$. We take $f_K = 160\text{MeV}$, $f_D = 235\text{MeV}$, $F^{D\to K}(0) = 0.77$[8], $\alpha_s = 0.353$, $m_D = 1.869\text{GeV}$, and the asymptotic wave function $\phi_K = 6x(1-x)$. From the Eqs. (11) and (12), we find

$$a_1 = 0.9639 + 0.0622i, \quad (13)$$

3 Soft gluon effect in $D^+ \to \bar{K}^0 K^+$

In the following, we calculate the hadronic matrix element A which is from the soft gluon corrections for $D^+ \to \bar{K}^0 K^+$ decays in the light-cone QCD sum rules (LCSR). As a starting object for the derivation of LCSR we choose the following vacuum-pion correlation function:

$$F_\alpha^{(\tilde{O}_1)}(p,q,k) = -\int d^4x e^{-i(p-q)x} \int d^4y e^{i(p-k)} \times \langle 0 | T\{ j_{\alpha 5}^{(K)}(y) \tilde{O}_1(0) j_5^D(x) \} | \bar{K}^0(q) \rangle, \quad (14)$$

where $j_{\alpha 5}^{(K)} = \bar{u}\gamma_\alpha\gamma_5 s$ and $j_5^{(D)} = m_c \bar{c} i\gamma_5 d$ are the quark currents interpolating K and D mesons, respectively. The decomposition of the correlation function (14) in independent momenta is straightforward and contains four invariant amplitudes:

$$F_\alpha^{(\tilde{O}_1)} = (p-k)_\alpha F^{(\tilde{O}_1)} + q_\alpha \tilde{F}_1^{(\tilde{O}_1)} + k_\alpha \tilde{F}_2^{(\tilde{O}_1)} + \varepsilon_{\alpha\beta\lambda\rho} q^\beta p^\lambda k^\rho \tilde{F}_3^{(\tilde{O}_1)}. \quad (15)$$

In what follows only the amplitude $F^{(\tilde{O}_1)}$ is relevant. The correlation function is calculated in QCD by expanding the T-product of three operators, two currents and $\tilde{O}_1$, near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim 0$. For this expansion to be valid, the kinematical region should be chosen as:

$$q^2 = p^2 = k^2 = 0,$$
$$|(p-k)^2| \sim |(p-q)^2| \sim |P^2| \gg \Lambda_{QCD}^2. \quad (16)$$

where $P \equiv p - k - q$. The correlation function (14) can be shown in Fig. 1, and it can be calculated employing the light-cone expansion of the quark propagator[9]:

$$S(x,0) = -i\langle 0 | Tq(x)\bar{q}(0) | 0 \rangle = \frac{\Gamma(d/2)\not{x}}{2\pi^2(-x^2)^{d/2}} + \frac{\Gamma(d/2-1)}{16\pi^2(-x^2)^{d/2-1}} \int_0^1 dv((1-v)\not{x}\sigma_{\mu\nu}G^{\mu\nu}(vx) + v\sigma_{\mu\nu}G^{\mu\nu}(vx)\not{x}) \quad (17)$$

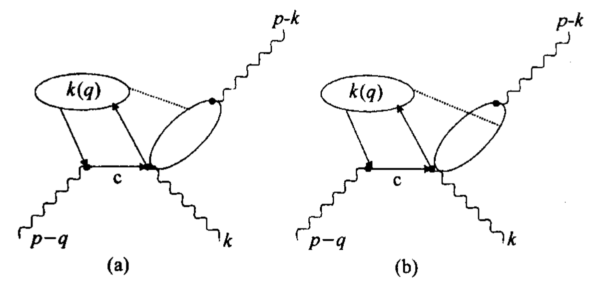

Fig. 1. Diagrams corresponding to higher-twist soft-gluon nonfactorizable Contribution for $O = \tilde{O}_1$ Soild, dashed and wavy lines represent quarks, gluons and external momenta, respectively. Thick points denote the weak interaction vertices, and ovals the Kaon distribution amplitude.

where $G_{\mu\nu} = g_s G_{\mu\nu}^a(\lambda^a/2)$, which is the gluonic field strength and induces the soft gluon effects, and d is the space-time dimension. Following the standard procedure for QCD sum rule calculation, we can obtain the hadronic matrix element of the operator $\tilde{O}_1$

$$A \equiv A^{(\tilde{O}_1)}(D^+ \to K^+ \bar{K}^0) = \langle \bar{K}^0(-q) K^+(p) | \tilde{O}_1 | D^+(p-q) \rangle = \frac{-i}{\pi^2 f_K f_D m_D^2} \int_0^{s_0^k} ds e^{\frac{-s}{M^2}} \int_{m_c^2}^{R(s,m_c^2,m_D^2,s_0^D)} ds' \times e^{\frac{m_D^2 - s'}{M'^2}} Im_{s'} Im_s F_{QCD}^{(\tilde{O}_1)}(s,s',m_D^2), \quad (18)$$

where s_0^k and s_0^D are effective threshold parameters. A straightforward calculation shows that only the twist-3 wavefunction $\varphi_{3k}(\alpha_i)$ and the twist-4 ones $\varphi_\parallel(\alpha_i)$, $\varphi_\perp(\alpha_i)$, whose definitions can be found in Ref. [6], contribute to the invariant function $F^{(\tilde{O}_1)}$. The results are:

$$F_{QCD}^{(\tilde{O}_1)} = F_{tw3}^{(\tilde{O}_1)} + F_{tw4}^{(\tilde{O}_1)}, \quad (19)$$

where $F_{tw3}^{(\tilde{O}_1)}$ and $F_{tw4}^{(\tilde{O}_1)}$ can be found in Refs. [6], [10], but it should be substituted for $m_c \to m_b$. Finally, the LCSR for the $D^+ \to K^+ \bar{K}^0$ matrix element of the operator $\tilde{O}_1$ from the soft-gluon exchange is obtained by applying the duality approximation and Borel transformation to the D channel. The result can be written as:

$$A^{(\tilde{O}_1)}(D^+ \to K^+ \bar{K}^0) = i m_D^2 \left(\frac{1}{4\pi^2 f_K} \int_0^{s_0^k} ds e^{-\frac{s}{M^2}} \right) \times \left(\frac{m_c^2}{2 f_D m_D^4} \int_{u_0^D}^1 \frac{du}{u} e^{\frac{m_D^2}{M'^2} - \frac{m_c^2}{uM'^2}} \times \left[\frac{m_c f_{3k}}{u} \int_0^u \frac{dv}{v} \varphi_{3k}(1-u, u-v, v) + f_K \int_0^u \frac{dv}{v} \left[3\tilde{\varphi}_\perp(1-u, u-v, v) - \right.\right.\right.$$

$$\left(\frac{m_c^2}{uM'^2}-1\right)\frac{\Phi_1(1-u,v)}{u}\Big]+$$

$$f_K\left(\frac{m_c^2}{uM'^2}-2\right)\frac{\Phi_2(u)}{u^2}\Big]\Big), \qquad (20)$$

where $u_0^D=m_c^2/s_0^D$ and the following definitions are introduced:

$$\frac{\partial\,\Phi_1(w,v)}{\partial\,v}=\tilde{\varphi}_\perp(w,1-w-v,v)+\tilde{\varphi}_{/\!/}(w,1-w-v,v),$$

$$\frac{\partial\,\Phi_2(v)}{\partial\,v}=\Phi_1(1-v,v). \qquad (21)$$

The asymptotic forms of the pion distribution amplitudes in Eq. (20) are given by[6]:

$$\varphi_{3k}(\alpha_i)=360\alpha_1\alpha_2\alpha_3^2,$$
$$\tilde{\varphi}_\perp(\alpha_i)=10\delta^2\alpha_3^2(1-\alpha_3),$$
$$\tilde{\varphi}_{/\!/}(\alpha_i)=-40\delta^2\alpha_1\alpha_2\alpha_3. \qquad (22)$$

4 Numerical calculation

In the numerical calculations we take $s_0^k=1.62\text{GeV}^2$[10] and $M^2=6-15\text{GeV}^2$ for the K channel, and $f_D=235\text{MeV}$, $m_c=1.3\text{GeV}$, $s_0^D=6\text{GeV}^2$, $\mu_c=\sqrt{m_D^2-m_c^2}\approx1.3\text{GeV}$, $M'^2=8-12\text{GeV}^2$, $f_{3k}(\mu_c)=0.0035\text{GeV}^2$, $\delta^2(\mu_c)=0.19\text{GeV}^2$[11], $\tau(D^+)=(1.051\pm0.013)\times10^{-12}$. In the D rest frame, the two body decay width is

$$\Gamma(D^+\to K^+\bar{K}^0)=\frac{1}{8\pi}|M(D_s^+\to K^+\bar{K}^0)|^2\frac{|P|}{m_D^2}, \qquad (23)$$

where the momentum of the K meson is given by

$$|P|=\frac{[(m_D^2-(m_K+m_{K^0})^2)(m_D^2-(m_K-m_{K^0})^2)]^{\frac{1}{2}}}{2m_D}, \qquad (24)$$

The corresponding branching ratio is given by

$$Br(D^+\to K^+\bar{K}^0)=\frac{\Gamma(D^+\to K^+\bar{K}^0)}{\Gamma_{tot}}. \qquad (25)$$

With the above parameters and formulas, we can get the branching ratios in $D^+\to K^+\bar{K}^0$ decay. In the following, we compare the branching ratios of $D^+\to K^+\bar{K}^0$ decay obtained in different approaches with that of experiment.

Table 1. The branching ratios of $D^+\to K^+\bar{K}^0$ obtained in different approaches are compared with that of experiment.

Decay mode	NF	QCDF	QCDF + SGE	Experiment
$D_s^+\to K^+\bar{K}^0$	8.79×10^{-3}	6.73×10^{-3}	5.76×10^{-3}	$(5.8\pm0.6)\times10^{-3}$

The branching ratios is presented in Table 1, where the second column is the naive factorization (NF) result, the amplitude is calculated from the Eq. (8) and $a_1=C_1+C_2/3$. The third column is QCD factorization (QCDF) result, also the amplitude is calculated from the Eq. (8) but the a_1 is in the Eq. (13). The fourth column is QCD factorization and soft-gluon exchanges (QCDF + SGE) result which is our result, the amplitude is the sum of the Eqs. (8) and (9), and the final column is the experimental data[12].

5 Summary

We have studied $D^+\to K^+\bar{K}^0$ decay channels. The prediction of naive factorization is far from the experimental data, the QCD factorization result approaches to the experimental data. However, in QCD factorization method, if we calculate the power correction term $O(\Lambda_{QCD}/m_c)$ (it includes soft-gluon exchange effects and final state interaction contributions and so on) then the result is in accordance with the experimental data. From the Table 1, we find the soft-gluon effects are noticeable and they have the same magnitude as the α_s correction's contribution, and the others power correction terms such as final state interaction can be neglected in this decay channel.

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非因子化方法研究 $D^+\to\bar{K}^0K^+$ 衰变*

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摘要　在领头阶和 α_s 修正阶,用QCD因子化方法,并对它的软胶子效应用光锥QCD求和规则分析 $D^+\to\bar{K}^0K^+$ 衰变过程,我们分析发现朴素因子化方法的结果远离实验结果,QCD因子化方法结果靠近实验结果,但是,在QCD因子化方法中,若考虑软胶子效应,其结果与实验结果相一致. 另外,计算发现,软胶子效应在该衰变道中有相当大的贡献,因此不能被忽略.

关键词　QCD因子化　光锥QCD求和规则　软胶子效应　辐射修正

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