Identical Superdeformed Bands in Tl Isotope*

HE Xiao-Tao [1,4;1)] LIU Shu-Xin [2,3] YU Shao-Ying [1,4] ZENG Jin-Yan [2,3] ZHAO En-Guang [1,2,4]

1 (Institute of Theoretical Physics, the Chinese Academy of Sciences, Beijing 100080, China)
2 (College of Physics, Peking University, Beijing 100871,China)
3 (College of Science, Huzhou University, Huzhou 313000, China)
4 (Center of Theoretical Physics, National Laboratory of Heavy Ion Accelerator, Lanzhou 730000, China)

Abstract The microscopic mechanism of the identical bands in odd-odd nucleus ^{194}Tl and its neighbor odd-A nuclei ^{193,195}Tl are investigated using the particle-number conserving (PNC) method for treating the cranked shell model with monopole and quadrupole pairing interactions. It is found that the blocking effect of the high-j intruder orbital plays an important role in the variation of moments of inertia ($J^{(1)}$ and $J^{(2)}$) with rotational frequency for the superdeformed bands and identical bands. The ω variation of the occupation probability of each cranked orbital and the contributions to moment of inertia from each cranked orbital are presented.

Key words particle-number conserving method, superdeformed band, identical bands

1 Introduction

The first observation of superdeformation (SD) in the $A\sim190$ mass region was reported[1] about ten years ago. Since then more than 80 SD bands have been observed in this region (see Ref. [2]). The superdeformation at high spin remains one of the most challenging topics of nuclear structure[3]. At present, although a general understanding of this phenomenon has been achieved, there are still many open problems. For the underlying physics of the superdeformed identical bands (IBs) (see the review [4]), some studies[5,6] showed that there is special physics or symmetry behind IBs while others[7—9] suggested the same γ-ray transition energy and the identical moment of inertia (MoI) are due to the competition among the shell effect, pairing interaction, blocking effect, rotation alignment and Coriolis anti-pairing effect. Although a lot of theoretical works has been done based on various models (e.g., particle-plus-core model, mean-field approximation, symmetry-based approach, etc.) which are successful in different aspect to study the superdeformed nuclei, they are still far from a precise quantitative description.

For most SD bands in even-even and the odd A nuclei, the dynamical moment of inertia ($J^{(2)}$) exhibits a gradual increase with the increasing rotational frequency $\hbar\omega$, which is due to the gradual alignment of nucleons occupying high-N intruder orbitals in the presence of the pair correlation while in odd-odd nuclei, quite a part of the moments of inertia for SD bands keep constant. IBs are observed both in odd-odd and even-even nuclei with their neighbor odd-A nuclei. However, for the abundant experimental date, much study of the SD bands and the identical bands have been done in even-even nuclei and their neighbor odd-A nuclei, the SD bands and the identical bands in odd-odd nuclei and their neighbor odd-A nuclei are seldom studied.

In this paper, the identical SD bands in the odd-odd nucleus ^{194}Tl and its neighbor odd-A nuclei ^{193,195}Tl are investigated by the particle-number con-

---

* Supported by National Natural Science Foundation of China (10375001, 10147205), Major State Basic Research Development Program(G2000-0774-07) and Knowledge Innovation Project of the Chinese Academy of Sciences (KJCX2-SW-N021)

1) E-mail: hext@itp.ac.cn

serving (PNC) method[10,11] for treating the cranked shell model (CSM) with monopole and quadrupole pairing interactions, in which the particle number is conserved and the blocking effects are taken into account strictly. The more detailed information on the separate contributions to MoI from each cranked orbital is also clearly exhibited.

2 Formalism

The CSM Hamiltonian with pairing interactions reads:

$$H_{\mathrm{CSM}} = H_{sp} - \omega J_x + H_p = H_0 + H_p \tag{1}$$

where $H_0 = H_{sp} - \omega J_x$ is the one-body part of H_{CSM}, H_{sp} the Nilsson Hamiltonian, and $-\omega J_x$ the Coriolis interaction. the pairing interaction including both monopole and qudrupole pairing interactions. In our calculations, $h_0(\omega)$ is firstly diagonalized to obtain the cranked Nilsson orbitals. Then, H_{CSM} is diagonalized in a sufficiently large cranked many-particle configuration (CMPC) space to extract accurate PNC solutions of the yrast and low-lying excited eigenstates.

The eigenstate of H_{CSM} is expressed as

$$|\psi\rangle = \sum_i C_i |i\rangle \tag{2}$$

where $|i\rangle$ denotes an occupation of particles in the cranked orbitals and C_i is the corresponding probability amplitude.

The angular momentum alignment of $|\psi\rangle$ is

$$\langle\psi|J_x|\psi\rangle = \sum_i C_i^2 \langle i|J_x|i\rangle + 2\sum_{i<j} C_i C_j \langle i|J_x|j\rangle \tag{3}$$

Because J_x is a one-body operator, $\langle i|J_x|j\rangle$ $i \neq j$ may be nonzero only when $|i\rangle$ and $|j\rangle$ differ by only one particle occupation. Then the dynamical moment of inertia of $|\psi\rangle$ is

$$J^{(2)} = d\langle\psi|J_x|\psi\rangle/\mathrm{d}\omega = \sum_\mu J^{(2)}(\mu) + \sum_{\mu<\nu} J^{(2)}(\mu\nu) \tag{4}$$

with $j^{(2)}(\mu)$ being the direct contribution to $J^{(2)}$ from a particle occupying the cranked orbital μ and $j^{(2)}(\mu\nu)$ being the contribution from the interference between two particles occupying the cranked orbital μ and ν which has no counterpart in the mean-field (BCS) treatment. A similar expression for the kinematic moments of inertia $J^{(1)} = \langle\psi|J_x|\psi\rangle/\omega$ can be found in Ref. [12].

3 Results and discussion

In our calculation, the spin assignments of these SD bands are taken from Ref. [13], the Nilsson parameters (κ, μ) are taken from Ref. [14] (for neutron $N=6$ shell, their values are shifted slightly), the deformation parameters are $\varepsilon_2 = 0.46$ and $\varepsilon_4 = 0.03$. The effective pairing strengths are determined by fitting the values of $J^{(1)}$ for ^{195}Tl(2) from $\hbar\omega \approx 0.10$—0.40MeV. The effective pairing strengths also depend on the dimension of the truncated CMPC space. In the following calculation, the truncated CMPC's energy E_c is set about 0.65 $\hbar\omega_0$, (the corresponding CMPC's space dimensions are about 700) and 0.45 $\hbar\omega_0$ (the corresponding CMPC's space dimensions are about 1000) for proton and neutron respectively with $\hbar\omega_0 = 41A^{-1/3}$ MeV. In such a CMPC space, the effective pairing interaction strengths (G_0 for monopole and G_2 for quadrupole pairing interaction) in unit of MeV are given as follow: $G_{0p} = 0.3$, $G_{0n} = 0.2$, $G_{2p} = 0.01$, $G_{2n} = 0.013$.

The experimental and calculated identical bands {^{193}Tl(1), ^{194}Tl(2a)}, {^{193}Tl(1), ^{194}Tl(2b)}, {^{193}Tl(1), ^{195}Tl(1)}, {^{193}Tl(2), ^{195}Tl(2)} are shown in Fig. 1. Our calculations can reproduce quite well of the similarities of these bands in the observed frequency.

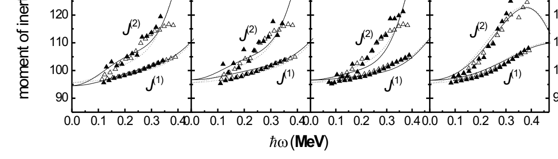

Fig. 1. Experimental and calculated $J^{(1)}$ and $J^{(2)}$ of the set of identical bands in ^{193,194,195}Tl.

The proton and neutron occupation probabilities n_μ of each cranked Nilsson orbital near the Fermi surface versus rotational frequency are given in Fig. 2 (the orbital with $n_\mu = 0$ or $n_\mu = 2$ is not shown here). The configurations (see Table 1) of all the six bands agree with the assignments of the experimental studies[15,16]. As for proton, the occupation probabilities for $^{194}\mathrm{Tl}(2a, 2b)$ and $^{193}\mathrm{Tl}(1, 2)$ are not shown here for the similarity with that for $^{195}\mathrm{Tl}(1, 2)$. As shown in the Fig. 2, the blocking of individual proton high-j intrude orbital [642]5/2 is obvious. As for the neutron, there is no blocked neutron orbitals for $^{193}\mathrm{Tl}$ and $^{195}\mathrm{Tl}$. The occupation probabilities for $^{193}\mathrm{Tl}$ are not shown here for the similarity with that for $^{195}\mathrm{Tl}$ except the exceeded two neutron in $^{195}\mathrm{Tl}$ partially occupied the [512]5/2 orbital (see Fig. 2(d)). The blocking effect is exhibited clearly by the neutron occupation probabilities. The blocked orbital is the high-Ω orbital ([624]9/2 ($\alpha = +1/2$)) in $^{194}\mathrm{Tl}(2a)$ (Fig. 2(c)), the corresponding Coriolis response is very small. Thus the occupation probabilities of this orbital keep constant ($n_\mu = 1$) up to rather high $\hbar\omega$, and the contributions to the moment of inertia coming from the blocked orbital [624]9/2 ($\alpha = \pm 1/2$) for $^{194}\mathrm{Tl}(2a)$ are negligible. From this one can understand why there are the identical MoI in $^{194}\mathrm{Tl}(2a)$ and $^{193}\mathrm{Tl}(1)$, actually, also identical with that of $^{195}\mathrm{Tl}(1)$.

Table 1. The configurations of the six SD bands in $^{193,194,195}\mathrm{Tl}$.

SD band	configuration
$^{193}\mathrm{T1}(1, \alpha = -1/2)$	$(\pi[642]5/2, \alpha = -1/2)$
$^{193}\mathrm{T1}(2, \alpha = +1/2)$	$(\pi[642]5/2, \alpha = +1/2)$
$^{194}\mathrm{T1}(2a, \alpha = 0)$	$(\pi[642]5/2, \alpha = -1/2)\otimes$
	$(\nu[642]9/2, \alpha = +1/2)$
$^{194}\mathrm{T1}(2b, \alpha = 1)$	$(\pi[642]5/2, \alpha = -1/2)\otimes$
	$(\nu[642]9/2, \alpha = +1/2)$
$^{195}\mathrm{T1}(1, \alpha = -1/2)$	$(\pi[642]5/2, \alpha = -1/2)$
$^{195}\mathrm{T1}(2, \alpha = +1/2)$	$(\pi[642]5/2, \alpha = +1/2)$

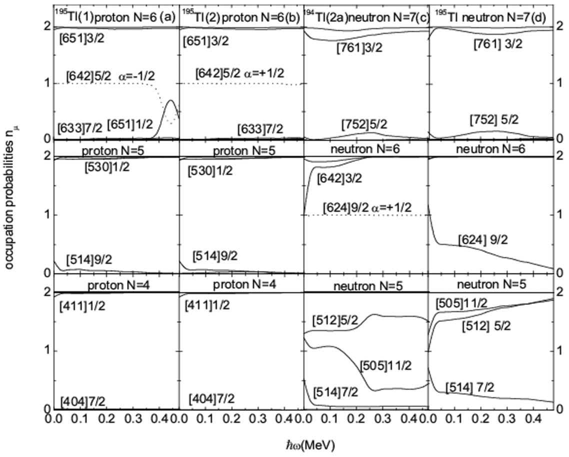

Fig. 2. Occupation probabilities n_μ of each proton and neutron cranked orbital μ near the Fermi surface. The blocked orbitals are denoted by dot line.

Our PNC calculations can provide more detailed information on the separate contributions to MoI from each cranked orbitals (see in Fig. 3), which include the direct contributions $j^{(2)}(\mu)$ from orbital μ and the interference terms $j^{(2)}(\mu\nu)$ between orbitals μ and ν [see Eq. (4)]. It is noted that (a) The contributions to MoI from each cranked orbitals depend sensitively on the location and in particular on the Coriolis response of the orbitals μ and ν. For $^{193}\mathrm{Tl}(1)$ and $^{195}\mathrm{Tl}(1)$, at $\hbar\omega > 0.35\,\mathrm{MeV}$, the sharp increase of the $J^{(2)}$ mainly comes from the direct contributions $j^{(2)}(\mu)$ of the high-j intruder orbitals [642]5/2 and [651]1/2 and their interference terms $j^{(2)}$ ([651]1/2[642]5/2, [642]5/2[633]7/2). As for $^{193}\mathrm{Tl}(2)$ and $^{195}\mathrm{Tl}(2)$, there is no band crossing occurring at $\hbar\omega > 0.35\,\mathrm{MeV}$, the contributions from the high-j intruder orbitals [642]5/2 and [651]1/2 and their interference terms $j^{(2)}(\mu\nu)$ are small so there is no such a sharp increase. (b) The contribution to $J^{(2)}$ coming from the high-Ω orbital is small and can be negligible while that coming from high-j orbital is important.

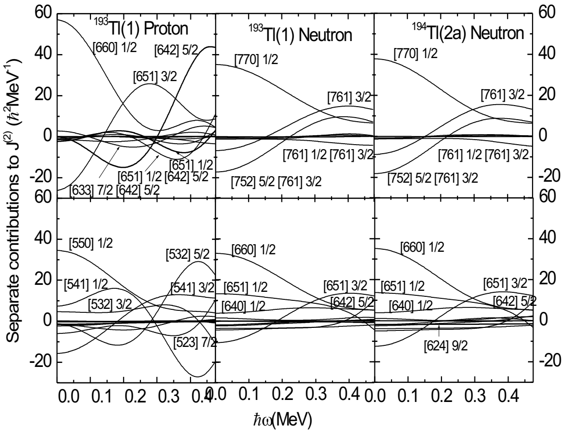

Fig. 3. The calculated contributions to $J^{(2)}$ from each cranked proton and neutron orbitals near the Fermi surface, $j^{(2)}(\mu)$ and $j^{(2)}(\mu\nu)$. The blocked orbitals are denoted by dot line.

4 Summary

In summary, the PNC method for treating the cranked shell model with monopole and quadrupole pairing interactions has been used to investigate the microscopic mechanism of the identical bands in the typical odd-odd nucleus ^{194}Tl and their neighbor odd-A nuclei ^{193,195}Tl. It is found that the blocking effect is very important. Moreover, the influence of blocking effect on MoI depends on the orbital location and in particular on the Coriolis response of the blocked levels. The blocked proton orbital $[642]5/2\,(\alpha=+1/2)$ plays a very important role in IBs while the blocked neutron orbital $[624]9/2\,(\alpha=+1/2)$ contributes little to the MoI. The contribution from the interference terms $(j^{(2)}(\mu\nu))$, which has no counterpart in the mean-field (BCS) treatment, is very important and can not be negligible.

References

1 Moore E F et al. Phys. Rev. Lett., 1989, **63**: 360

2 Nuclear data sheet 97, 2002, 241

3 Afanasjev A V, Ring P, König J. Nucl. Phys., 2000, **A676**: 196

4 Baktash C, Hass B, Nazarewicz W. Annu. Rev. Nucl. Part. Sci., 1995, **45**: 485

5 Szymanski Z, Nazarewicz W. Phys. Lett., 1998, **B433**: 229

6 LIU Y X et al. Phys. Rev., 1999, **C59**: 2511; Phys. Rev., 2001, **C63**: 054314

7 LIU S X, ZENG J Y, ZHAO E G. Phys. Rev., 2002, **C66**: 024320

8 SUN Y, ZHANG J Y, Guidry M. Phys. Rev. Lett., 1997, **78**: 2321; Phys. Rev., 2001, **C63**: 047306

9 Lennart B Karlsson, Ingemar Kagnarsson, Sven Aberg, Phys. Lett., 1998, **B416**: 16

10 ZENG J Y et al. Phys. Rev., 2001, **C63**: 024305

11 ZENG J Y et al. Phys. Rev., 1994, **C50**: 1388. 024305

12 ZENG J Y et al. Phys. Rev., 2002, **C65**: 044307

13 LIU S X, ZENG J Y. Phys. Rev., 1998, **C58**: 3266

14 Bengtsson T, Ragnarsson I. Nucl. Phys., 1985, **A436**: 14

15 Bouneau S et al. Phys. Rev., 1996, **C53**: R9

16 Duprat J et al. Phys. Lett., 1994, **B341**: 6

Tl同位素中全同带的研究*

贺晓涛 1,4;1)　刘树新 2,3　于少英 1,4　曾谨言 2,3　赵恩广 1,2,4

1 (中国科学院理论物理研究所　北京　100080)
2 (北京大学物理系　北京　100871)
3 (湖州大学理学院　湖州　313000)
4 (兰州重离子加速器国家实验室原子核理论中心　兰州　730000)

摘要　采用推转壳模型下处理对力的粒子数守恒方法对奇奇核^{194}Tl与奇-$A^{193,195}$Tl中全同带存在的微观机制进行了研究. 结果表明, 高-j闯入轨道的堵塞效应在全同带的形成中起了重要作用. 文中对单粒子轨道占有几率和各推转单粒子能级上的粒子对转动惯量的贡献做了详细分析.

关键词　粒子数守恒方法　超形变带　全同带

---

* 国家自然科学基金(10375001,10147205), 国家重点基础研究发展规划项目(G2000-0774-07)和中国科学院知识创新工程(KJCX2-SW-N021)资助

1) E-mail: hext@itp.ac.cn