

## Isospin-Dependence of Nucleon Potential and Effective Mass in Relativistic Approach<sup>\*</sup>

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**Abstract** The isospin-dependence of nucleon optical potentials and the nucleon effective mass is studied in the framework of the Dirac Brueckner Hartree-Fock (DBHF) approach. A new decomposition of the Dirac structure of the nuclear self-energy in the DBHF is extended to asymmetric nuclear matter calculations. The real part of the nucleon self-energy in asymmetric nuclear matter is calculated with the  $G$ -matrix in the Hartree-Fock approach, while the imaginary part is obtained from the polarization diagram. The nucleon vector effective mass is derived from the Schroedinger equivalent potentials. It is found that the neutron vector effective mass is larger than that of proton in the neutron-rich nuclear matter once the energy dependence of the nucleon potentials is considered.

**Key words** Dirac Brueckner Hartree-Fock theory, nucleon effective mass, nucleon scattering optical potential

Recently, the radioactive beam physics has become one of the frontiers in nuclear physics. It offers the possibility to broaden our understanding of nuclear properties. In the neutron rich side, one of the most interesting questions is the isospin dependence of the nucleon effective interaction as well as the nucleon effective mass. The knowledge about the isospin dependence of nucleon potentials and the nucleon effective mass is critically important for understanding properties of neutron stars and the dynamics of nuclear collisions induced by radioactive beams. Unfortunately, up to now the knowledge about the isospin dependence of those quantities from experiments is very little. Therefore it is required to investigate the isospin dependence based on a fundamental theory. In this work we shall investigate the isospin dependence of nucleon optical potentials and the nucleon effective mass microscopically in the framework of the Dirac Brueckner Hartree-Fock(DBHF) approach.

A new decomposition of the Dirac structure of nucleon self-energies in the DBHF, which was proposed by Schiller and M uther<sup>[1, 2]</sup> is adopted. The DBHF  $G$ -matrix is separated into a bare nucleon-nucleon (NN) interaction  $V$  and a correlation term  $\Delta G$ .

$$G = V + \Delta G . \quad (1)$$

Usually, the bare NN interaction  $V$  in the DBHF is taken as an OBEP, such as Bonn potentials, which contain six meson exchanges:  $\sigma$ ,  $\omega$ ,  $\eta$ ,  $\delta$ ,  $\rho$ , and  $\pi$ . The correlation term  $\Delta G$  is parametrized by four pseudo-mesons ( $\sigma'$ ,  $\delta'$ ,  $\omega'$ ,  $\rho'$ ). These pseudo-mesons have infinite masses but finite ratios of coupling constants to the corresponding masses. Therefore, the effective NN interaction  $G$  in symmetric and asymmetric nuclear matter can be characterized in the relativistic Hartree-Fock (RHF) approach by the exchanges of those four pseudo-mesons, in addition to the OBEP.

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The DBHF nucleon self-energies in asymmetric nuclear matter have the general form:

$$\begin{aligned} \Sigma^t(k, k_F, \beta) = & \Sigma_s^t(k, k_F, \beta) - \gamma_0 \Sigma_0^t(k, k_F, \beta) + \\ & \boldsymbol{\gamma} \cdot \mathbf{k} \Sigma_v^t(k, k_F, \beta), \end{aligned} \quad (2)$$

where  $t$  stands for proton or neutron,  $\Sigma_s^t$ ,  $\Sigma_0^t$  and  $\Sigma_v^t$  are the scalar component, time-like and space-like parts of the vector component of the self-energy, respectively. They are functions of the nucleon momentum, density and asymmetry parameter  $\beta = (\rho_n - \rho_p)/\rho$ , where  $\rho_n$ ,  $\rho_p$  and  $\rho$  are neutron, proton and matter densities, respectively.

The nucleon self-energy in nuclear matter can be calculated in RHF with  $V$  and  $\Delta G$  of Eq. (1). The nucleon self-energy is expressed as,

$$\begin{aligned} \Sigma(k, k_F, \beta) = & \sum_{\alpha} \int \frac{d^4 q}{(2\pi)^4} \{ \Gamma_{\alpha}^a \Delta_{\alpha}^{ab}(0) \text{Tr}[i\Gamma_{\alpha}^b \mathcal{G}(q)] - \\ & i\Gamma_{\alpha}^a \Delta_{\alpha}^{ab}(q) \Gamma_{\alpha}^b \mathcal{G}(k-q) \}, \end{aligned} \quad (3)$$

where the first and second terms correspond to direct (Hartree) and exchange (Fock) terms, respectively. The index  $\alpha$  refers to mesons,  $a$  and  $b$  are isospin components.  $\mathcal{G}$  is the single-particle Green's function, and the  $\Delta_{\alpha}^{ab}$  are meson propagators.  $\Gamma_{\alpha}^a$  are the vertices of nucleon-meson couplings.

Ma et al<sup>[3]</sup>. have used this scheme to study the properties of asymmetric nuclear matter and finite nuclei. Reasonable results are thus achieved. For scattering problems, the optical potential of a nucleon in the nuclear medium is identified with the nucleon self-energy. The real part of the relativistic microscopic optical potential (RMOP) is evaluated in the DBHF approximation by adopting the decomposition of  $G = V + \Delta G$ . The imaginary part of the  $G$ -matrix which is calculated as the  $G$ -matrix polarization diagram. Its contribution to the nucleon self-energy can be calculated as follows<sup>[4]</sup>,

$$\begin{aligned} \Sigma_{\text{pol}}(k) = & \sum_{\alpha\beta} \int \frac{d^4 q}{(2\pi)^4} \left\{ \Gamma_{\alpha}^a \Delta_{\alpha}^{ab}(q) \int \frac{d^4 p}{(2\pi)^4} \times \right. \\ & \text{Tr} \left[ \Gamma_{\alpha}^b (i\mathcal{G}(p)) \Gamma_{\beta}^c (i\mathcal{G}(p+q)) \right] \times \\ & \left. \Delta_{\beta}^{cd}(q) (i\mathcal{G}(k-q)) \Gamma_{\beta}^d \right\}. \end{aligned} \quad (4)$$

With this nucleon effective interaction we calculate

the nucleon self-energies at  $E > 0$ ,

$$\Sigma^t = \tilde{U}^t + iW^t, \quad (5)$$

where  $\tilde{U}^t$  and  $W$  are calculated in Eq. (3) and Eq. (4), respectively.

In principle the imaginary part of Eq. (4) could be calculated with the  $G$ -matrix. It has been found earlier that the pion exchange in the bare NN interaction  $V$  gives a large contribution to the imaginary part of the polarization diagram<sup>[5]</sup>. The reason of this unphysically large contribution may be due to high order correlations in the pion exchange. We also find that this large contribution to the imaginary part of the polarization diagram cannot be eliminated by the correlation term  $\Delta G$ . In order to avoid this problem and to simplify the calculation of the imaginary part of the RMOP we introduce an effective NN interaction  $G_{\text{eff}}$ . The effective nucleon interaction consists of four scalar and vector pseudo-mesons with isoscalar and isovector characters, corresponding to  $\sigma$ ,  $\omega$ ,  $\delta$  and  $\rho$  mesons. The masses of these effective mesons are fixed and the coupling constants are adjusted in the RHF to reproduce the DBHF results, namely the neutron and proton self-energies as well as the binding energy at each density in the symmetric and asymmetric nuclear matter.

The optical potential in finite nuclei is obtained by means of the local density approximation, where the space distribution of the RMOP is directly connected with the density and asymmetry parameter of the nuclear self-energies in the asymmetric nuclear matter. The Dirac equation of a nucleon with the incident energy  $E$  in the nuclear medium has the form

$$[\boldsymbol{\alpha} \cdot \mathbf{p} + \gamma_0 (M + U_s^t) + U_0^t] \psi^t(\mathbf{r}) = \varepsilon \psi^t(\mathbf{r}), \quad (6)$$

where

$$U_s = \frac{\Sigma_s - \Sigma_v M}{1 + \Sigma_v}, \quad U_0 = \frac{-\Sigma_0 + E \Sigma_v}{1 + \Sigma_v}, \quad (7)$$

with  $\varepsilon = E + M$ ,  $U_s^t$  and  $U_0^t$  being the scalar and vector potential, respectively, which are complex.

By eliminating the lower component of the Dirac spinor in Eq. 6, a Schroedinger-type equation is obtained for the upper component:

$$\left( -\frac{\nabla^2}{2E} + V_{\text{eff}}^t(r) + V_{\text{s.o.}}^t(r)\boldsymbol{\sigma}\cdot\mathbf{L} + V_{\text{Darwin}}^t(r) \right) \varphi(\mathbf{r}) = \frac{\varepsilon^2 - M^2}{2E} \varphi(\mathbf{r}), \quad (8)$$

where  $V_{\text{eff}}^t$ ,  $V_{\text{s.o.}}^t$  and  $V_{\text{Darwin}}^t$  are the central, spin-orbit and Darwin potentials, respectively.

We have investigated elastic proton scattering off spherical nuclei and calculated the differential cross sections, analyzing powers and spin rotation functions of  $p+^{40}\text{Ca}$  and  $p+^{208}\text{Pb}$  at various values of  $E_p < 200\text{MeV}$ . It is found that the calculated cross sections as well as spin observables are in rather good agreement with the experimental data at low energies. As the energy increases, the theoretical differential cross sections are overestimated at large angles in comparison with the experimental data, but they keep the same diffraction patterns.

The isospin dependence of the optical potential is depicted by the difference between proton and neutron optical potentials<sup>[6]</sup>. In the phenomenological optical potential, the Lane potential was introduced to describe the isospin dependence<sup>[6]</sup>. The optical potential is divided into isoscalar and isovector components,

$$V = V_0 + \frac{\mathbf{t}\cdot\mathbf{T}}{A} V_1, \quad (9)$$

where  $A$  is the nuclear mass, and  $\mathbf{t}$  and  $\mathbf{T}$  are isospin operators of the incident particle and target, respectively. The Lane potential  $V_1$  is usually parametrized with a Wood-Saxon shape. We calculate the RMOP of protons and neutrons in  $^{208}\text{Pb}$  at  $E=65\text{MeV}$  and extract the corresponding Lane potential shown in Fig. 1. The phenomenological Lane potential is taken as  $V_1=50\text{MeV}$ , with Woods-Saxon parameters  $r_0=$

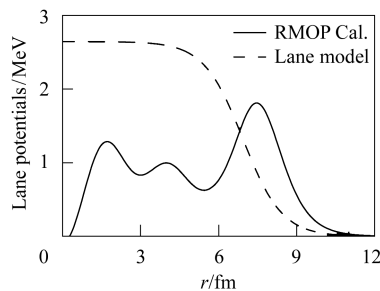


Fig. 1. The Lane potential of  $p+^{208}\text{Pb}$  at  $65\text{MeV}$ . The solid curve is calculated in the RMOP and the dashed curve is a phenomenological one.

$1.17\text{fm}$  and  $a=0.75\text{fm}$ . It is shown for comparison in Fig. 1. The agreement indicates that the RMOP obtained from the  $G$ -matrix can reasonably describe the isospin dependence of the optical potential.

The nucleon effective mass characterizes the propagation of a nucleon in the nuclear medium, which is adopted to describe an independent quasi-particle model in the nuclear many-body system<sup>[7]</sup>. In the non-relativistic approach the effective mass represents the non-locality of the underlying microscopic nuclear potential. Then the nucleon effective mass can be derived by the following two equivalent expressions<sup>[8]</sup>,

$$\frac{M^*}{M} = 1 - \frac{d}{d\varepsilon} V(k(\varepsilon), \varepsilon) = \left( 1 + \frac{M}{k} \frac{d}{dk} V(k, \varepsilon(k)) \right)_{k=k(\varepsilon)}^{-1}, \quad (10)$$

where  $\varepsilon(k)$  is the function of  $k$  defined by the energy momentum relation,  $\varepsilon = k^2/2M + V(k, \varepsilon)$ . The nucleon effective mass can be determined from analysis of the experimental data performed in the framework of the nonrelativistic shell and optical model. In order to compare a same quantity in the non-relativistic and relativistic approaches one has to derive the nucleon vector effective or Lorentz mass from the Schroedinger equivalent potential<sup>[9]</sup>. Taking account of the energy dependence of the nucleon scalar and vector potentials, the vector effective mass of a nucleon is now expressed as,

$$M_v^*/M = 1 - \frac{d}{d\varepsilon} V_{\text{eff}} = 1 - U_0/M - (1 + U_s/M) dU_s/d\varepsilon - (1 + \varepsilon/M - U_0/M) dU_0/d\varepsilon. \quad (11)$$

Nucleon effective masses are then calculated with the Schroedinger equivalent potential. The results are shown in Fig. 2, where upper and lower curves correspond to nucleon scalar and vector effective masses, respectively. The solid (dashed) curves are those of neutron (proton). It is obviously that the neutron scalar mass  $\left( \frac{M_s^*}{M} = M + U_s \right)$  is smaller than that of proton in the neutron-rich nuclear matter due to the stronger scalar potential of neutron. However, the difference of neutron and proton scalar effective masses is very small in the neutron-rich nuclear matter. It is found that the energy dependence of the nucleon

self-energy largely reduces the nucleon vector effective mass. At the nucleon energy  $\varepsilon=50\text{MeV}$ , the nucleon scalar and vector effective masses in the symmetric nuclear matter are 0.66 and 0.68, respectively. Due to the stronger energy dependence of the neutron vector potential the neutron vector effective mass become larger than that of proton in the neutron-rich nuclear matter. This result is consistent with the microscopic many-body theories in the nonrelativistic approach, e.g., the Landau-Fermi liquid theory<sup>[10]</sup> and the BHF approach<sup>[11]</sup> as well as the energy and isospin dependence of the nucleon-nucleus scattering experimental

data<sup>[12]</sup>.

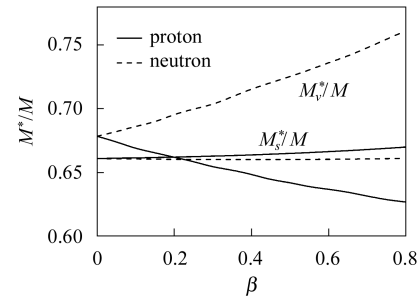


Fig. 2. Nucleon scalar and vector effective masses at  $E=50\text{MeV}$  in the asymmetric nuclear matter with  $k_F=1.36\text{fm}$ .

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# 核势和核子有效质量的同位旋相关性\*

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**摘要** 在Dirac Brueckner Hartree-Fock (DBHF)理论框架下研究了核子光学势和核子有效质量的同位旋相关性. 非对称核物质的计算采用了DBHF的核子自能的Dirac结构的新的分解方法,核子自能的实部是用 $G$ 矩阵在Hartree-Fock近似下计算得到,而虚部从极化图得到.用核子的薛定谔等价势可以得到核子矢量有效质量.研究表明考虑了核势的能量相关性在丰中子核物质情况下核子矢量有效质量比质子的.

**关键词** Dirac Brueckner Hartree-Fock理论 核子有效质量 核子光学势