Intrabeam Scattering of Heavy Ions at HIRFL-CSR*

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Abstract Small-angle multiple intrabeam scattering (IBS) is an important effect for heavy-ion storage rings with electron cooling, because the cooling time is determined by the equilibrium between cooling and IBS process. All usually used numerical algorithms of IBS growth rate calculations are based on the model of the collisions proposed by A.Piwinski, but this result is a multidimensional integral. In this paper, the IBS growth rates are simulated for HIRFL-CSR using symmetric elliptic integral method, and compared with several available IBS code results.

Key words electron cooling, intrabeam scattering, emittance, momentum spread

1 Introduction

HIRFL-CSR (Heavy Ion Research Facility in Lanzhou-Cooling Storage Ring) is a multi-purpose heavy ion storage ring that consists of a main ring (CSRm), an experimental ring (CSRe) and a radioactive beam line (RIBLL2) to connect the two rings[1]. Two electron coolers located in the long straight sections of the CSRm and the CSRe, respectively, for heavy ion beam cooling. In the CSRm, e-cooler will be used for the beam accumulation at the injection energy range of 7—30MeV/u to increase the beam intensity. In the CSRe, e-cooler will be used to compensate the growth of beam emittance during internal-target experiments or to provide high quality beams for the high-resolution mass measurements of nuclei.

Electron cooling is a fast process for increasing the phase-space density of stored ion beams[2]. It is achieved by Coulomb collision between ion beam and a monochromatic electron beam is well directed over a certain distance in a section of the storage ring. Thus the IBS within the ion beam becomes important. So the underlying dynamics of this process is equilibrium between the cooling and the IBS effects. This is the reason why the IBS process in HIRFL-CSR must be simulated.

2 IBS growth rate expression

J.Bjorken and S.Mtingwa considered the Gaussian phase-space distribution for beam and gave a 3-dimensional integral expression for the total 6-dimensional IBS growth rate based on A.Piwinski's model[3],

$$\frac{1}{\tau} = \frac{\pi^2 m_{\mathrm{ion}}^3 r_{\mathrm{p}}^2 n_{\mathrm{ion}} c L_{\mathrm{C}}}{\gamma \varGamma} \times \left\langle (\lambda_1-\lambda_2)^2 \int_0^{\infty} \frac{\mathrm{d}\lambda \lambda^{1/2}}{(\lambda_1+\lambda)^{3/2}(\lambda_2+\lambda)^{3/2}(\lambda_3+\lambda)^{1/2}} + \text{two cyclic permutations} \right\rangle ,$$

where r_{p} is classical proton radius (1.53×10^{-18}m), n_{ion} is the number of particles, L_{C} is the Coulomb logarithm, usually $L_{\mathrm{C}} \approx 20$, m_{ion} is the particle mass, γ is the Lorenz factor, for bunched beam $\varGamma = (2\pi)^3\beta^3\gamma^3 m_{\mathrm{ion}}^3 \varepsilon_x \varepsilon_y \sigma_{\mathrm{p}} \sigma_{\mathrm{s}}$, for the coasting beam

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$\Gamma = (2\pi)^{5/2}\beta^3\gamma^3 m_{\rm ion}^3 \varepsilon_x \varepsilon_y \sigma_{\rm p} \sigma_{\rm s}$, in this paper we consider the bunched beam only. c is the speed of light. λ_1, λ_2, λ_3 are the three eigenvalues of the matrix L. The matrix L is

$$L=\begin{pmatrix} \dfrac{\beta_x}{\varepsilon_x} & -\dfrac{\beta_x}{\varepsilon_x}\gamma\left(D'_x-\dfrac{\beta'_x D_x}{2\beta_x}\right) & 0 \\ -\dfrac{\beta_x}{\varepsilon_x}\gamma\left(D'_x-\dfrac{\beta'_x D_x}{2\beta_x}\right) & \dfrac{\beta_x}{\varepsilon_x}\left(\dfrac{\gamma^2 D_x^2}{\beta_x^2}+\gamma^2\left(D'_x-\dfrac{\beta'_x D_x}{2\beta_x}\right)^2\right)+\dfrac{2\gamma^2}{\sigma_{\rm p}^2} & 0 \\ 0 & 0 & \dfrac{\beta_y}{\varepsilon_y} \end{pmatrix}.$$

Obviously, one eigenvalue is $\lambda_1=\dfrac{\beta_y}{\varepsilon_y}$. Before calculating the rest eigenvalues, we will first define several parameters. Let

$$a_x=\frac{\beta_x}{\varepsilon_x},\quad a_y=\frac{\beta_y}{\varepsilon_y}\ ,$$

$$\sigma_x=\sqrt{D_x^2\sigma_{\rm s}^2+\varepsilon_x\beta_x},\quad \sigma_y=\sqrt{\varepsilon_y\beta_y}\ ,$$

$$a_{\rm s}=a_x\left(\frac{D_x^2}{\beta_x^2}+\phi^2\right)+\frac{1}{\sigma_{\rm p}^2}\ ,\quad \phi=D'_x-\frac{\beta'_x D_x}{2\beta_x}\quad ,$$

$$a_1=\frac{1}{2}\left(a_x+\gamma^2 a_{\rm s}\right),\quad a_2=\frac{1}{2}\left(a_x-\gamma^2 a_{\rm s}\right).$$

Write a new matrix L' as

$$L'=\begin{pmatrix} a_x & -a_x\gamma\phi \\ -a_x\gamma\phi & \gamma^2 a_{\rm s}\end{pmatrix}.$$

From $\left|L'-\lambda I\right|=0$, we can easily calculate the rest eigenvalues. Now we have the three eigenvalues such as

$$\lambda_1=a_y\ ,$$

$$\lambda_2=a_1+\sqrt{a_2^2+\gamma^2a_x^2\phi^2}\ ,$$

$$\lambda_3=a_1-\sqrt{a_2^2+\gamma^2a_x^2\phi^2}\ .$$

In these expressions, β_x and β_y are the horizontal and vertical betatron functions, ε_x and ε_y are the horizontal and vertical emittances, D_x is the horizontal dispersion function(here we assume that D_y and D'_y have zero value in the vertical plane), σ_x and σ_y are the rms beam size, and $\sigma_{\rm p}$ is the rms relative lab-frame momentum spread.

Now, the important thing is the fast and simple calculation of integral, but this integral expression is an abnormal form. The IBS growth rates can be presented in closed-form expressions with the help of the so-called symmetric elliptic integral. This integral can be evaluated numerically by a very efficient recursive method by employing the duplication theorem.

3 Symmetric elliptic integral

First, we introduce a symmetric elliptic integral of the second kind by Carlson's definition[4]

$$R_{\rm D}(x,y,z)=\frac{3}{2}\int_0^\infty\frac{{\rm d}t}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}\ .$$

Defining some parameters as $\lambda_1=\dfrac{1}{a}$, $\lambda_2=\dfrac{1}{b}$, $\lambda_3=\dfrac{1}{c}$, we can write the total 6-dimensional IBS growth rate as (see Appendix)

$$\frac{1}{\tau}\ =\ \frac{2\pi^2 m_{\rm ion}^3 r_{\rm p}^2 n_{\rm ion} c L_{\rm C}}{3\gamma\Gamma}\int_0^L\frac{{\rm d}s}{L\sqrt{\lambda_1\lambda_2\lambda_3}}\times \left[\lambda_1\psi\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)+\lambda_2\psi\left(\frac{1}{\lambda_2},\frac{1}{\lambda_3},\frac{1}{\lambda_1}\right)+ \lambda_3\psi\left(\frac{1}{\lambda_3},\frac{1}{\lambda_1},\frac{1}{\lambda_2}\right)\right],$$

where

$$\psi(x,y,z)=-2xR_{\rm D}(y,z,x)+yR_{\rm D}(z,x,y)+zR_{\rm D}(x,y,z).$$

Then the particle emittance growth rates can be now written as following[5]

$$\frac{{\rm d}\sigma_{\rm p}^2}{{\rm d}t}=\frac{n_{\rm ion}r_{\rm p}^2(\log)c}{12\pi\beta^3\gamma^5\sigma_{\rm s}}\int_0^L\frac{{\rm d}s}{L\sigma_x\sigma_y}S_{\rm p},$$

$$\frac{{\rm d}\varepsilon_y}{{\rm d}t}=\frac{n_{\rm ion}r_{\rm p}^2(\log)c}{12\pi\beta^3\gamma^5\sigma_{\rm s}}\int_0^L\frac{\beta_y{\rm d}s}{L\sigma_x\sigma_y}\psi\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right),$$

$$\frac{{\rm d}\varepsilon_x}{{\rm d}t}=\frac{n_{\rm ion}r_{\rm p}^2(\log)c}{12\pi\beta^3\gamma^5\sigma_{\rm s}}\int_0^L\frac{\beta_x{\rm d}s}{L\sigma_x\sigma_y}\left[S_x+\left(\frac{D_x^2}{\beta_x^2}+\Phi^2\right)S_{\rm p}+S_{x{\rm p}}\right],$$

where

$$S_{\rm p}\ =\ \frac{\gamma^2}{2}\left[2R_1-R_2\left(1-\frac{3a_2}{\sqrt{a_2^2+\gamma^2a_x^2\Phi^2}}\right)- R_3\left(1+\frac{3a_2}{\sqrt{a_2^2+\gamma^2a_x^2\Phi^2}}\right)\right],$$

$$S_x = \frac{1}{2}\left[2R_1 - R_2\left(1+\frac{3a_2}{\sqrt{a_2^2+\gamma^2 a_x^2 \Phi^2}}\right) - R_3\left(1-\frac{3a_2}{\sqrt{a_2^2+\gamma^2 a_x^2 \Phi^2}}\right)\right],$$

$$S_{xp} = \frac{3\gamma^2\Phi^2 a_x}{\sqrt{a_2^2+\gamma^2 a_x^2 \Phi^2}}\left[R_3 - R_2\right]$$

and

$$R_1 = \frac{1}{\lambda_1} R_D\left(\frac{1}{\lambda_2}, \frac{1}{\lambda_3}, \frac{1}{\lambda_1}\right),$$

$$R_2 = \frac{1}{\lambda_2} R_D\left(\frac{1}{\lambda_3}, \frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right),$$

$$R_3 = \frac{1}{\lambda_3} R_D\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}\right).$$

Thus, if we know the beam emittances and Twiss parameters, it's easy to calculate the IBS growth rates at a given lattice location.

4 Numerical simulation

O-Matrix Light is an integrated visual data analysis and development program that provides performance and capabilities far beyond typical integrated analysis environments. We developed a program for calculating the IBS growth rates based on this software. At the same time, we compared the results with other models and numerical methods, developed by Piwinski, Bjorken and Mtingwa, Martini in BETACOOL program[6].

The simulations described in this report will be carried out with parameters in Table 1. First we try to compare the average IBS growth rates calculated from different models, with the IBS growth rate at a given lattice location shown in Fig. 1. From the results in Table 2 we can see that the discrepancy is not too large. Because the BETACOOL program has used successfully at many accelerator laboratories, so we could believe the results calculating from symmetric elliptic integral is trust. We can also see that the longitudinal IBS growth rate is minus. The reason is that the emittance growth corresponds to a tendency of the beam rest-frame momentum space to relax to a spherical shape[3], if the momentum is small enough, the longitudinal IBS growth rate will be increased fast (see Fig. 2).

Table 1. Nominal HIRFL-CSRm ion beam parameters used in the IBS simulations.

item	parameters
ion kind	$^{12}C^{6+}$
energy	7MeV/u
number of ions per bunch	2.0×10^8
initial beam emittance (horizontal, vertical)	30, 30πmm·mrad
initial momentum spread	0.005
initial bunch length(rms)	33.24cm
average betatron (horizontal, vertical)	8.84, 13.93m
average dispersion (horizontal)	1.3497m
off momentum factor	0.9533

Table 2. IBS growth rates under parameters in Table 1 using different models.

simulating model	$1/\tau_{hor}/s^{-1}$ ($\times10^{-4}$)	$1/\tau_{ver}/s^{-1}$ ($\times10^{-4}$)	$1/\tau_{longitu}/s^{-1}$ ($\times10^{-5}$)
Piwinski	2.623	1.453	−2.958
Martini	1.506	2.535	−2.766
Bjorken and Mtingwa	2.725	2.652	−5.387
Jie-Wei	2.175	3.568	−6.687
symmetric elliptic integral	2.724	3.961	−4.436

Then we use our code to estimate the IBS effect at HIRFL-CSRm. As the design report[7] shows, the initial horizontal and vertical emittance will be below 5πmm·mrad and the initial momentum spread will be about 1×10^{-3}, so we calculate the emittance and momentum spread growth under these parameter (see Fig. 3). Obviously, the emittances are

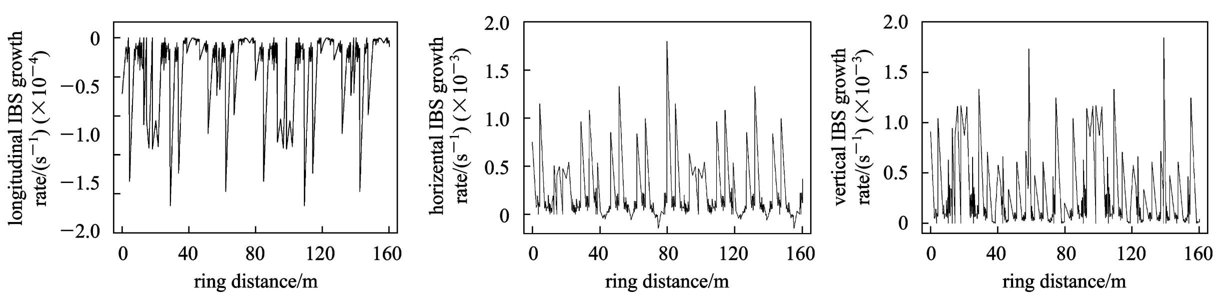

Fig. 1. IBS growth rates versus ring distance.

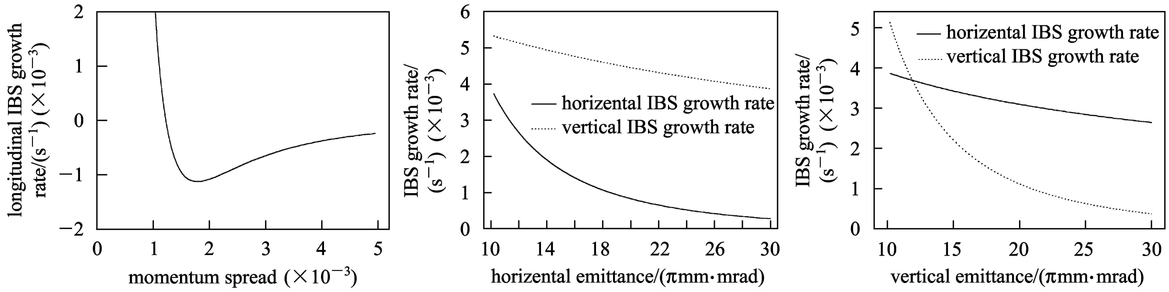

Fig. 2. IBS growth rates versus emittances and momentum spread.

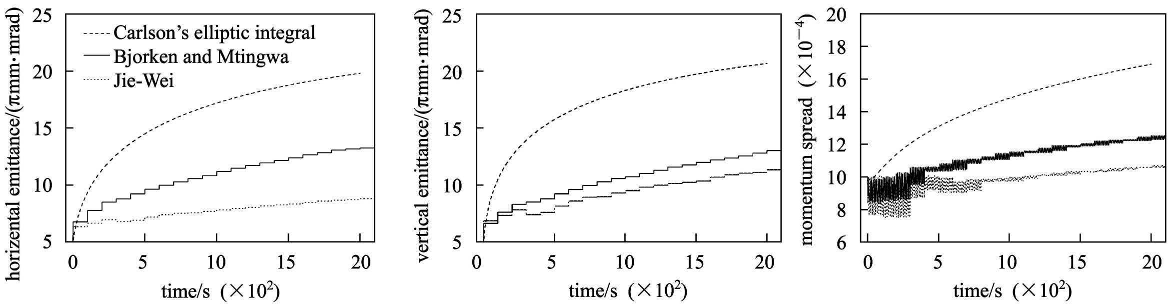

Fig. 3. Emittances and momentum spread versus time.

increased slowly. More than 2000 seconds are needed for the emittances grow up to 25πmm·mrad and the momentum spread to about 2×10^{-3}, because the acceptance is 200πmm·mrad (horizontal), 30πmm·mrad (vertical) and 1.25×10^{-2} (momentum spread), and the largest operating period of CSRm is about 17 seconds, so, that means the IBS effect is not the difficulty of the lattice design. In addition, the cooling time is about 2 seconds, so it's easy to achieve the aim designed.

5 Conclusions

By comparing the results from different models we see that the increase in transverse emittances and momentum spread after 2000 seconds is loss than 20%—45%. So this simple simulation method is reliable and useful for HIRFL-CSR calculation.

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Appendix

Suppose that

$$\lambda_1=\frac{1}{a},\quad \lambda_2=\frac{1}{b},\quad \lambda_3=\frac{1}{c}\quad \text{and}\quad \lambda=\frac{1}{t},$$

then

$$(\lambda_1-\lambda_2)^2\int_0^\infty\frac{\mathrm{d}\lambda\lambda^{1/2}}{(\lambda_1+\lambda)^{3/2}(\lambda_2+\lambda)^{3/2}(\lambda_3+\lambda)^{1/2}}+\text{two cyclic permutations}=$$

$$\sqrt{abc}\left(\frac{b-a}{a}+\frac{a-b}{b}\right)\int_0^\infty\frac{t\mathrm{d}t}{(a+t)^{3/2}(b+t)^{3/2}(c+t)^{1/2}}+\text{two cyclic permutations.}$$

Now combine terms with $1/a$, $1/b$, and $1/c$, the formula will be written as,

$$\sqrt{abc}\frac{1}{a}\left(\frac{-2a}{a+t}+\frac{b}{b+t}+\frac{c}{c+t}\right)\int_0^\infty\frac{\mathrm{d}t}{(a+t)^{1/2}(b+t)^{1/2}(c+t)^{1/2}}+\text{two cyclic permutations}=$$

$$\frac{2}{3}\frac{1}{\sqrt{\lambda_1\lambda_2\lambda_3}}\lambda_1\psi\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)+\text{two cyclic permutations}=$$

$$\frac{2}{3}\frac{1}{\sqrt{\lambda_1\lambda_2\lambda_3}}\left[\lambda_1\psi\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3}\right)+\lambda_2\psi\left(\frac{1}{\lambda_2},\frac{1}{\lambda_3},\frac{1}{\lambda_1}\right)+\lambda_3\psi\left(\frac{1}{\lambda_3},\frac{1}{\lambda_1},\frac{1}{\lambda_2}\right)\right],$$

where

$$\psi(x,y,z)=-2xR_{\mathrm{D}}(y,z,x)+yR_{\mathrm{D}}(z,x,y)+zR_{\mathrm{D}}(x,y,z),$$

$$R_{\mathrm{D}}(x,y,z)=\frac{3}{2}\int_0^\infty\frac{\mathrm{d}t}{(t+x)^{1/2}(t+y)^{1/2}(t+z)^{3/2}}\ .$$

HIRFL-CSR重离子束内散射效应*

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摘要　电子冷却本质上是电子冷却作用与重离子束内散射作用的动态平衡过程. 在Bjorken和Mtingwa束内散射理论的基础上, 应用对称椭圆积分的方法做束内散射增长率的数值模拟, 并应用于HIRFL-CSR的磁铁聚焦结构. 计算结果表明, 束内散射不会成为CSR磁聚焦结构设计的障碍, 并且CSR可以达到冷却设计指标.

关键词　电子冷却　束内散射　发射度　动量分散

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