

Shape coexistence of superheavy nuclei*

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Abstract Shape coexistence appears in the region of superheavy nuclei. Calculations for the nobelium isotopes have been carried out with the cranking TRS model. It shows that normal deformed and superdeformed prolate shapes coexist. Particularly for the nuclei ^{248,250}No, the ground states are superdeformed. The kinematic moments of inertia are calculated and they agree well with available experimental results. As rotational frequency increases, the Routhians of the superdeformations decrease and the superdeformed shapes will become yrast states at high spins.

Key words superheavy nuclei, TRS, shape coexistence

PACS 21.10.-k, 21.60.-n, 25.85.Ca

1 Introduction

The nuclear synthesis of new superheavy elements in laboratory is currently a hot subject in nuclear physics. However, the heaviest nuclei, with $Z > 104$, are at the limit of Coulomb instability. They would be unstable against spontaneous fission but for a large shell-correction energy, which leads to additional binding and creates a sizable fission barrier of up to 8 MeV^[1, 2]. Experimentally, superheavy elements up to $Z = 118$ (except $Z = 117$) have been produced^[3, 4]. The rotational bands in ^{252,254}No have also been identified and extended to 20⁺ and 16⁺ respectively^[5–7]. The deduced quadrupole deformations of ground-state are 0.28 and 0.27 for ²⁵²No and ²⁵⁴No respectively, which is consistent with the theoretical calculations^[8–11]. Shape coexistences of spherical, prolate, oblate and triaxial deformations are predicated for the superheavy nuclei^[3, 12, 13]. In this paper, Deformations, shape coexistences and

collective rotational properties with increasing rotational frequency for the nobelium isotopes are discussed, which is helpful for understanding the even heavier nuclei around “superheavy island”.

2 The model

The Total-Routhian-Surface (TRS) calculations^[14] have been performed. The total Routhian $E^\omega(Z, N, \hat{\beta})$ of a nucleus (Z, N) at a rotational frequency ω and deformation $\hat{\beta}$ is calculated as follows^[14]

$$E^\omega(Z, N, \hat{\beta}) = E^{\omega=0}(Z, N, \hat{\beta}) + [\langle \Psi^\omega | \hat{H}^\omega | \Psi^\omega \rangle - \langle \Psi^\omega | \hat{H}^\omega | \Psi^\omega \rangle_{\omega=0}], \quad (1)$$

where $E^{\omega=0}(Z, N, \hat{\beta})$ is the total energy at the zero frequency, consisting of the macroscopic liquid-drop energy^[15], the microscopic shell correction^[16, 17] and pairing energy^[18]. The last two terms in the bracket represent the change in energy due to the rotation.

Received 8 July 2008

* Supported by National Natural Science Foundation of China (10735010, 10525520, 10475002) and Chinese Major State Basic Research Development Program (2007CB815000)

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The total Hamiltonian is written as^[14]

$$\hat{H}^\omega = \sum_{ij} [(\langle i|h_{ws}|j\rangle - \lambda\delta_{ij})a_i^\dagger a_j - \omega \langle i|\hat{j}_x|j\rangle a_i^\dagger a_j] - G \sum_{i,i'>0} a_i^\dagger a_i^\dagger a_{i'} a_{i'} . \quad (2)$$

For the single-particle Hamiltonian, h_{ws} , a non-axial deformed Woods-Saxon (WS) potential has been adopted.

The pairing is treated by the Lipkin-Nogami approach^[18] in which the particle number is conserved approximately and thus the spurious pairing phase transition encountered in the BCS calculation can be avoided (see Ref. [18] for the detailed formulation of the cranked Lipkin-Nogami TRS method). Both monopole and quadrupole pairings are considered^[20] with the monopole pairing strength G determined by the average gap method^[19] and quadrupole strengths obtained by restoring the Galilean invariance broken by the seniority pairing force^[20–22]. The TRS calculation is performed in the deformation space $\hat{\beta} = (\beta_2, \gamma, \beta_4)$. Pairing correlations are dependent on the rotational frequency and deformation. In order to include such dependence in the TRS, we have run the pairing-deformation-frequency self-consistent TRS calculation, i.e., for any given deformation and frequency, pairing is self-consistently treated by the Hartree-Fock-Bogolyubov-like equation^[18]. At a given frequency, the deformation of a state is determined by minimizing the calculated TRS.

3 Calculations and discussions

The TRS calculations for even-even nobelium isotopes have been performed. Fig. 1 shows the TRS of the nucleus ^{254}No . Shown in Fig. 1, it has a well-deformed prolate ground-state shape with $\beta_2 = 0.24$, which is in accordance with the experimental results. It results from the large energy gap at around $\beta_2 = 0.27$ with $N = 152$ in the single particle diagram. Also a second minimum with $\beta_2 = 0.7$ appears, which is by only 980 keV higher. Thus normal deformed and superdeformed prolate shapes coexist for ^{254}No . Calculations for other nobelium isotopes also show the shape coexistence of two prolate deformations. The energy differences between the two shapes vary with the neutron numbers as shown in Table 1. They are less than 1 MeV for the nobelium isotopes except

$^{262,264}\text{No}$. Particularly for the nuclei $^{248,250}\text{No}$, they have a superdeformed ground state, which could lead to a slightly deeper binding and give a longer lifetime for superheavy nuclei than expected as mentioned^[13].

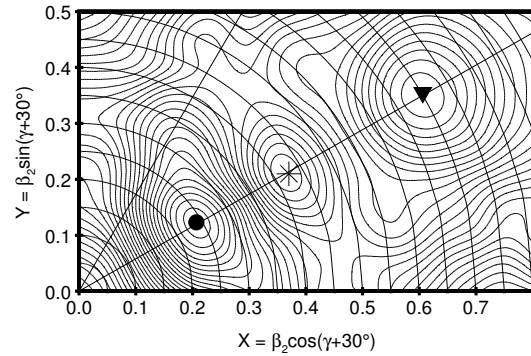


Fig. 1. The total Routhian surface for the nucleus ^{254}No . The black dot represents the first minimum (ground state) and the filled triangle represents the second minimum. The star represents the maximum. Contours are at a 200 keV interval.

Table 1. The deformations and the corresponding energy differences calculated for nobelium isotopes with experimental results included. ND represents normal deformed prolate shape. SD represents superdeformed prolate shape.

nuclei	ND		SD		$(E_{\text{SD}} - E_{\text{ND}})/$ MeV	$\beta_2^{\text{expt.}}$
	β_2	$ \gamma $	β_2	$ \gamma $		
^{248}No	0.24	0.7	0.68	0.3	-0.93	
^{250}No	0.24	0.2	0.68	0.2	-0.18	
^{252}No	0.24	0.3	0.69	0.1	0.60	0.28(2)
^{254}No	0.24	0.6	0.70	0.0	0.98	0.27(2)
^{256}No	0.24	1.1	0.70	0.1	0.96	
^{258}No	0.23	0.9	0.71	0.3	0.84	
^{260}No	0.23	0.4	0.73	0.5	0.85	
^{262}No	0.23	0.1	0.75	0.2	1.08	
^{264}No	0.23	0.2	0.78	0.1	1.17	

To investigate the rotational properties of the superheavy nuclei, particularly for the shape evolutions, the cranking TRS calculations have been carried out. $^{252,254}\text{No}$ are the heaviest nuclei whose rotational bands have been observed experimentally to date. Their kinematic moments of inertia for the rotational bands as a function of rotational frequency have been calculated and compared with the experimental results as shown in Fig. 2. As can be seen from Fig. 2, calculated kinematic moments of inertia of the nuclei $^{252,254}\text{No}$ agree rather well with the experimental results. At low rotational frequency, they show normal deformed prolate shapes. As the kinematic moments of inertia of superdeformed shapes are quite

larger than that of normal deformations, the Routhians of the superdeformations become lower with

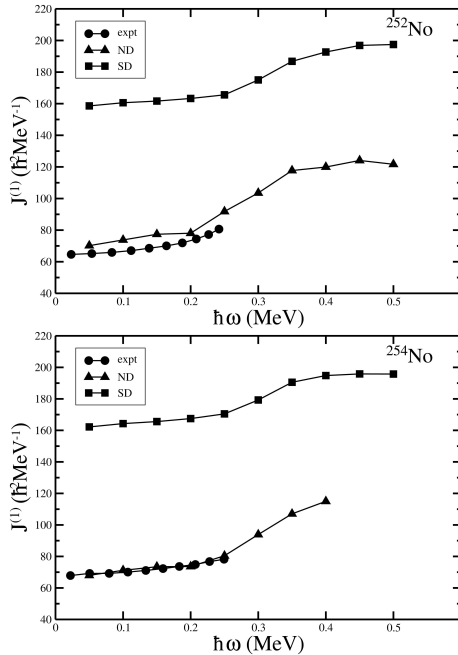


Fig. 2. Kinematic moments of inertia for ^{252}No and ^{254}No as the function of the rotational frequency. ND represents normal deformed prolate shape. SD represents superdeformed prolate shape.

frequencies increasing. At $\hbar\omega \sim 0.15$ MeV ($I \sim 24\hbar$), the superdeformation of ^{252}No becomes yrast state. For the nucleus ^{254}No , the superdeformation becomes yrast state at $\hbar\omega \sim 0.20$ MeV ($I \sim 34\hbar$). Now, such high levels have not been extended experimentally, while the superdeformed shapes are predicted to develop at high levels.

4 Summary

In conclusion, calculations for the nobelium isotopes have been performed with the cranking TRS model. Shape coexistence of normal deformed and superdeformed prolate shapes have been found appearing in the nobelium isotopes. For the even-even nuclei $^{252-264}\text{No}$, the ground states are normal deformed shapes while $^{248,250}\text{No}$ have superdeformed ground-state shapes. The kinematic moments of inertia of $^{252,254}\text{No}$ for the rotational bands as a function of rotational frequency have been calculated and they agree well with the experimental results. At high spins, the superdeformed shapes will become yrast states.

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