Investigation of self-affine multiplicity fluctuations of proton emission in ^{84}Kr-AgBr interactions at 1.7 A GeV *

ZHANG Dong-Hai(张东海)1) LI Hui-Ling(李惠玲)

(Institute of Modern Physics, Shanxi Normal University, Linfen 041004, China)

Abstract Self-affine multiplicity scaling is investigated in the framework of a two-dimensional factorial moment methodology using the concept of the Hurst exponent (H). Investigation of the experimental data of medium-energy knocked-out target protons in ^{84}Kr-AgBr interactions at 1.7 AGeV reveals that the best power law behavior is exhibited for $H = 0.4$, indicating a self-affine multiplicity fluctuation pattern. Multifractality among the knocked-out target protons is also observed in the data.

Key words relativistic heavy-ion collision, knocked-out target protons, nuclear emulsion

PACS 25.75.Gz, 25.70.Mn, 29.40.Rg

1 Introduction

Since Bialas and Peschanski[1] introduced a new method called intermittency for the analysis of large fluctuations, a large variety of experiments were performed to search for non-statistical fluctuations in the processes of particle production in various collisions at high energies[2—12]. For a more exhaustive review see Ref. [13]. Most of the analysis has been carried out on the produced relativistic particles (mostly pions) following the common belief that these particles are the most informative about reaction dynamics and thus could be effective in revealing the underlying physics of relativistic nucleus-nucleus collisions. However, the physics of nucleus-nucleus collisions at high energies is not yet conclusive and therefore all the available probes need to be thoroughly exploited towards a meaningful analysis of the experimental data.

In relativistic nucleus-nucleus collisions, medium-energy (30—400 MeV) knocked-out target protons (or recoiled protons), termed gray tracks according to emulsion terminology, are supposed to carry some information about the interaction dynamics because the time scale of the emission of these particles is of the same order ($\approx 10^{-22}$ s) as that of the produced particles. The general belief about these recoiled protons is that they are the low energy part of the intranuclear cascade formed in high-energy interactions. Though it is very helpful for understanding the reaction dynamics, the study of multiplicity distribution fluctuations of the recoiled target proton is limited. According to our knowledge Ghosh et al. were the first to study the intermittency of medium-energy protons in a one-dimensional phase space for ^{32}S-AgBr interactions at 200 AGeV[14]. After this work, Ghosh et al. studied self-affine multiplicity scaling of medium-energy target protons emitted in ^{32}S-AgBr interactions at 200 AGeV and ^{16}O-AgBr interactions at 60 AGeV in the framework of a two-dimensional factorial moment methodology using the concept of the Hurst exponent[15]. They also studied the fractal behavior of medium-energy protons emitted in ^{24}Mg-AgBr interactions at 4.5 AGeV[16] using the method introduced by Takagi[17] and revealed the existence of mono-fractality of proton emission in heavy ion interactions.

This paper reports an investigation on the nature of dynamical fluctuations of recoiled target protons emitted in ^{84}Kr-AgBr interactions at 1.7 AGeV in the framework of two-dimensional factorial moments

Received 19 March 2008

* Supported by National Natural Science Foundation of China (10475054, 10675077), Major Science and Technology Foundation of Ministry of Education of China (205026), Natural Foundation of Shanxi Province, China (2008011005), and Shanxi Provincial Foundation for Returned Scholars of China (20031046)

1) E-mail: zhangdh@dns.sxnu.edu.cn

considering the anisotropy of phase space. The anomalous fractal dimensions of the intermittency and non-thermal phase transition of the emission of recoiled target protons is also discussed by using information on the intermittency exponents.

2 Experimental details

Stacks of ILFORD G-5 nuclear emulsion plates were horizontally exposed to a ^{84}Kr beam at 1.7 AGeV at Bevalac Berkeley. XSJ-1 and XSJ-2 microscopes with a 100× oil immersion objective and 16× ocular lenses were used to scan the plates. The tracks were picked up at a distance of 5 mm from the edge of the plates and were carefully followed until they either interacted with emulsion nuclei or escaped from the plates. Interactions which were within 30 μm of the top or bottom surface of the emulsion plates were not considered in the final analysis. All the primary tracks were followed back to ensure that the events chosen did not include interactions from the secondary tracks of other interactions. When they were observed to do so the corresponding events were removed from the sample. Details of the scanning and classification of events can be found in our previous paper[18—21].

According to emulsion terminology[22], the particles emitted in the interactions are classified as follows.

(1) Black particles. There are target fragments with ionization greater than or equal to $9I_0$, I_0 being the minimum ionization of a single charged particle. Their ranges are less than 3 mm, their velocities less than $0.3c$ and their energies less than 26 MeV.

(2) Gray particles. These are mostly recoil protons in the kinetic energy range $26 \leqslant E \leqslant 375$ MeV, a few kaons of kinetic energies $20 \leqslant E \leqslant 198$ MeV and pions with kinetic energies $12 \leqslant E \leqslant 56$ MeV. They have ionizations of $1.4I_0 \leqslant I \leqslant 9I_0$. Their ranges in emulsions are greater than 3 mm and they have velocities in the range of $0.3c \leqslant v \leqslant 0.7c$.

The gray and black particles together are called heavy ionizing particles.

(3) Shower particles. These are produced as single-charged relativistic particles having a velocity greater than or equal to $0.7c$. Most of them belong to pions contaminated with small proportions of fast protons and K mesons.

(4) The projectile fragments are a different class of tracks with constant ionization, long range, and small emission angle.

To ensure that the targets in the emulsion are silver or bromine nuclei, we have chosen only the events with at least eight heavy ionizing tracks of particles ($N_{\rm h} \geqslant 8$).

3 Method of study

The method used to analyze the self-affine multiplicity fluctuation has been described in many publications. Here we follow Refs. [23, 24]. Considering the two-dimensional case and denoting the two phase space variables as x_1 and x_2, the factorial moment of order q may be defined as[1]

$$F_q(\delta x_1 \delta x_2) = \frac{1}{M'} \sum_{m=1}^{M'} \frac{n_m(n_m-1)\cdots(n_m-q+1)}{\langle n_m \rangle^q} \, , \tag{1}$$

where $\delta x_1 \delta x_2$ is the size of a two-dimensional cell, n_m is the particle multiplicity in the $m^{\rm th}$ cell, $\langle n_m \rangle$ is the average multiplicity of all events in the $m^{\rm th}$ cell, and M' is the number of two-dimensional cells into which the considered phase space has been divided.

To fix δx_1, δx_2 and M', we consider a two-dimensional region $\Delta x_1 \Delta x_2$ and divide it into subcells with widths

$$\delta x_1 = \Delta x_1 / M_1 \, , \tag{2}$$

$$\delta x_2 = \Delta x_2 / M_2 \, , \tag{3}$$

in the x_1 and x_2 directions, where $M_1 \neq M_2$ and $M' = M_1 \cdot M_2$.

Here M_1 and M_2 are the scale factors that satisfy the equation

$$M_1 = M_2^H \, , \tag{4}$$

where the parameter H $(0 < H \leqslant 1)$ is called the Hurst exponent[23]. This is the parameter characterizing the self-affinity property of dynamical fluctuations. It is clear from Eq. (4) that the scale factors M_1 and M_2 cannot simultaneously be integers, so that the size of the elementary phase space cell can vary continuously.

The following method[23] has been adopted for performing the analysis with non-integer values of the scale factor (M). For simplicity, consider a one-dimensional space (y) and let

$$M = N + a \, , \tag{5}$$

where N is an integer and $0 \leqslant a < 1$. When we use the elementary bin of width $\delta y = \Delta y / M$ as a scale to measure the region Δy, we get N of them and a smaller bin of width $a \Delta y / M$ left. Putting the smaller bin in the last (or first) place of the region and taking the average of only the first (or last) N bins, we have

$$\langle F_q(\delta y)\rangle = \frac{1}{N_{\rm ev}}\sum_{i}^{N_{\rm ev}}\frac{1}{M}\times \sum_{m=1}^{N}\frac{n_{mi}(n_{mi}-1)\cdots(n_{mi}-q+1)}{\langle n_m\rangle^q}. \quad (6)$$

where n_{mi} is the particle multiplicity in the $m^{\rm th}$ cell of the $i^{\rm th}$ event, $N_{\rm ev}$ is the number of events and M, determined by Eq. (5), can be any positive real number and can therefore vary continuously.

Self-affine multiplicity fluctuations would manifest themselves as a power-law scaling of $\langle F_q\rangle$ with the a cell size of the form

$$\langle F_q(\delta y)\rangle \propto (\delta y)^{-a_q} \quad \text{as} \quad \delta y\to 0\,.$$

or a linear relation like

$$\ln\langle F_q\rangle = -a_q \ln\delta y + b_q\,. \quad (7)$$

The invariant quantity of the scaling $a_q > 0$ is called the intermittency exponent and measures the strength of the fluctuation.

The intermittent behavior of the recoiled target protons is analyzed by using the method of factorial moments. The non-uniformity of the particle spectra influences the scaling behavior of the factorial moments. Bialas and Gazdzicki[24] introduced a "cumulative" variable which drastically reduces the distortion of intermittency due to the non-uniformity of the single-particle density distribution. According to them, the cumulative variable $X(x)$ is related to the single-particle density distribution $\rho(x)$ as

$$X(x) = \int_{x_1}^{x}\rho(x')\mathrm{d}x' \Big/ \int_{x_1}^{x_2}\rho(x')\mathrm{d}x'\,, \quad (8)$$

where x_1 and x_2 are the beginning and final point of the distribution $\rho(x)$. The variable $X(x)$ varies between 0.0 and 1.0 while $\rho(X(x))$ remains almost constant.

To probe the anisotropic structure of phase space we have calculated the factorial moments of the $q^{\rm th}$ order ($q = 2,3,4$) for various values of the Hurst exponent. The partition numbers along the $\cos\theta$ and ϕ directions are chosen as $M_\phi = 3,4,5,\cdots,20$, and $M_{\cos\theta}$ given by

$$M_{\cos\theta} = M_\phi^H\,. \quad (9)$$

We have not considered the first two data points corresponding to $M_\phi = 1,2$ in order to reduce the effect of momentum conservation[25] which tends to spread the particles in opposite directions and thus reduces the value of the factorial moments. This effect becomes weaker as M increases.

4 Experimental results

We have plotted $\ln\langle F_q\rangle$ along the Y axis and the natural logarithm of $(\delta X_{\cos\theta}\cdot\delta X_\phi)$ along the X axis for ^{84}Kr-AgBr interactions at 1.7 AGeV for different Hurst exponents (0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0). In each case linear behavior could be observed in two or three regions. In order to find the partitioning condition at which the scaling behavior is best revealed, we have performed a linear fit in the first region and have estimated the χ^2 per degrees of freedom (DOF) for each linear fit. It is interesting that the best linear behavior is revealed at $H = 0.4$ and not at $H = 1$ for each order of moment in the data set. The plots of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at $H = 0.4$ and 1.0 are shown in Figs. 1 and 2, respectively. Table 1 represents the value of χ^2 per DOF and the intermittency exponent for ^{84}Kr-AgBr

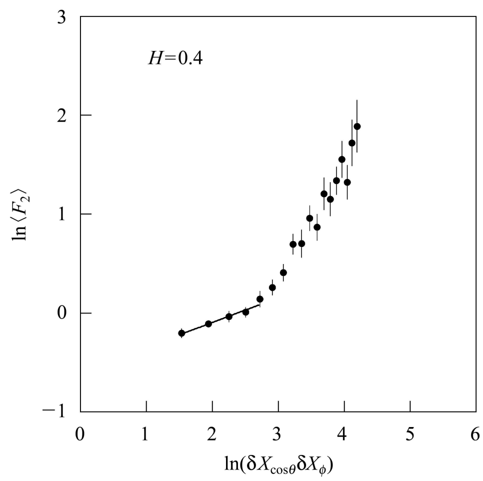

Fig. 1. Plot of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at $H = 0.4$ in the case of ^{84}Kr-AgBr interactions at 1.7 AGeV.

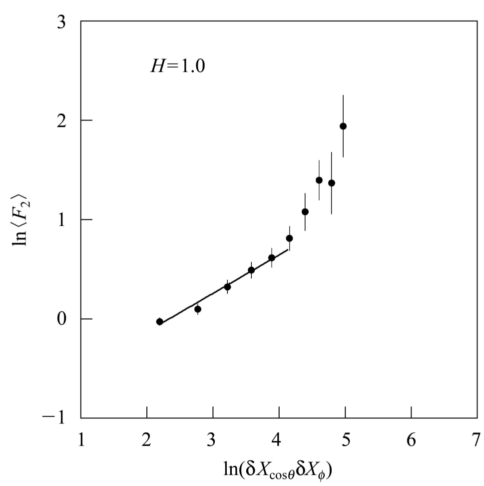

Fig. 2. Plot of $\ln\langle F_2\rangle$ against $\ln(\delta X_{\cos\theta}\delta X_\phi)$ at $H = 1.0$ in the case of ^{84}Kr-AgBr interactions at 1.7 AGeV.

Table 1. The values of χ^2 per DOF and intermittency exponents for particular values of Hurst exponent H in ^{84}Kr-AgBr interactions at 1.7 AGeV.

value of H	$q=2$		$q=3$		$q=4$	
	χ^2 per DOF	intermittency exponents (a_q)	χ^2 per DOF	intermittency exponents (a_q)	χ^2 per DOF	intermittency exponents (a_q)
0.3	0.424	0.301 ± 0.073	0.075	0.688 ± 0.208	0.110	0.987 ± 0.395
0.4	0.223	0.251 ± 0.058	0.325	0.675 ± 0.140	0.434	1.311 ± 0.488
0.5	0.552	0.340 ± 0.059	0.396	1.044 ± 0.154	0.011	1.551 ± 0.458
0.6	0.636	0.382 ± 0.048	0.950	1.090 ± 0.113	1.575	1.805 ± 0.344
0.7	1.539	0.363 ± 0.049	1.878	1.038 ± 0.161	0.486	0.934 ± 0.474
0.8	0.579	0.307 ± 0.045	0.776	0.902 ± 0.131	1.161	2.045 ± 0.442
0.9	1.116	0.362 ± 0.053	1.547	0.833 ± 0.172	1.058	1.168 ± 0.375
1.0	0.689	0.367 ± 0.045	1.221	1.060 ± 0.165	0.337	2.025 ± 0.627

interactions for different values of H and orders of moment. From the table it is seen that χ^2 per DOF is smaller at $H=0.4$ for the different orders of moment. So the dynamical fluctuation pattern in ^{84}Kr-AgBr interactions is not self-similar but self-affine.

The power-law behavior of the scaled factorial moments implies the existence of some kind of fractal pattern[26] in the dynamics of the particles produced in their final state. Therefore, it is natural to study the fractal nature of medium-energy knocked-out protons in ^{84}Kr-AgBr interactions in the self-affine scaling scenario.

In order to study the dependence of the anomalous fractal dimensions d_q ($d_q = a_q/(q-1)$) on the order of moment q in the self-affine scaling scenario, the d_q values have been calculated at $H=0.4$. The variation of d_q with order q is shown in Fig. 3. From the plot it is seen that d_q is linearly dependent on order q, which suggests the presence of multifractality of emission of medium-energy knocked-out protons in ^{84}Kr-AgBr interactions. It is the same as our previous result on target residue production in ^{84}Kr-AgBr interactions at the same energy[27].

It has been suggested that self-similar cascade can occur in different phases[28], namely the normal phase populated by many relatively small fluctuations and the spin glass phase consisting of few very large fluctuations. The condition for the coexistence of the two phases of the cascade is the presence of a minimum of the intermittency parameter λ_q at a certain value q_c for order q. The value of λ_q is given by[29, 30]

$$\lambda_q = (a_q+1)/q\,. \tag{10}$$

The region $q<q_c$ and $q>q_c$ correspond to the normal and to the spin glass phases, respectively. Self-similar multiparticle systems are seen to behave differently in these two regions[29, 30]. According to the idea of Sarcevic et al.[28], we discussed the property of coexistence of the two phases of the cascade in the emission of medium-energy knocked-out protons in ^{84}Kr-AgBr interactions. Fig. 4 presents the dependence of λ_q on the order q. From the plot it is seen that a slight minimum of λ_q is present at $q=3$, which may indicate the coexistence of two different phases, i.e. the normal and spin glass phases. The same result is also observed in the target residue production in

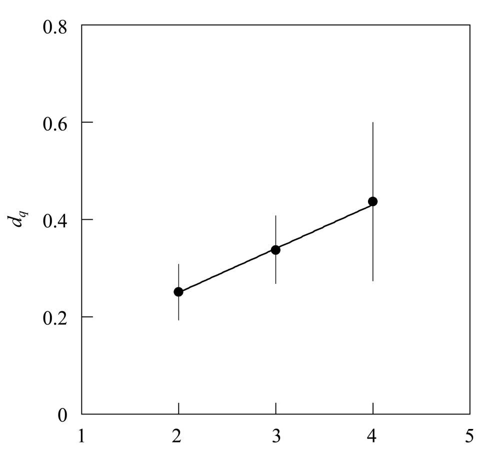

Fig. 3. Plot of d_q against q at $H=0.4$ in ^{84}Kr-AgBr interactions at 1.7 AGeV.

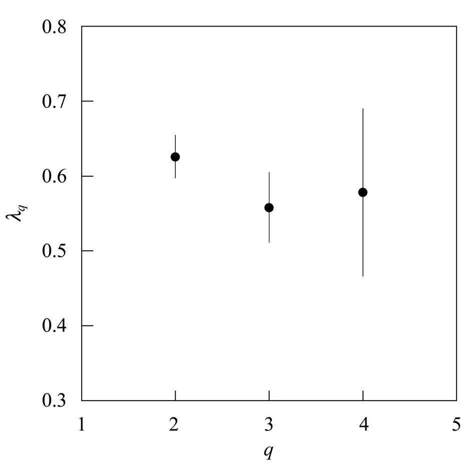

Fig. 4. Plot of λ_q against q at $H=0.4$ in ^{84}Kr-AgBr interactions at 1.7 AGeV.

^{84}Kr-AgBr interactions at 1.7 AGeV[27]. Target recoil protons (gray particles) and target residues (black particles) originate from the target, but with different energies and production mechanisms. The phenomenon of coexistence of the normal and spin glass phases may be just an indication that they come from the same source, but the theoretical explanation is still not conclusive.

5 Conclusions

From the present study of 1.7 AGeV ^{84}Kr-AgBr interactions, it may be concluded that the effect of intermittency is observed and the best power law behavior is exhibited at $H=0.4$ which suggests that the dynamical fluctuation pattern in ^{84}Kr-AgBr interactions is not self-similar but self-affine. The anomalous fractal dimensions of intermittency are found to increase with an increase in the order of moment, which suggests the presence of multifractality of the emission of medium-energy knocked-out protons in ^{84}Kr-AgBr interactions. A slight minimum value of λ_q is observed at $q=3$, which suggests that there might be a coexistence of the normal and spin glass phases.

We thank Professor I. Otterlund from Lund University, Sweden, for supplying the emulsion plates.

References

1 Bialas A, Peschanski R. Nucl. Phys. B, 1986, **273**: 703
2 KLM Collaboration, Holynski R et al. Phys. Rev. Lett., 1989, **62**: 733; Phys. Rev. C, 1989, **40**: R2449
3 TASSCO Collaboration, Braunschweig W et al. Phys. Lett. B, 1989, **231**: 548
4 Gustafson G et al. Phys. Lett. B, 1990, **248**: 430
5 Derado I et al. Z. Phys. C, 1990, **47**: 23
6 NA22 Collaboration, Ajinenko I V et al. Phys. Lett. B, 1989, **222**: 306; ibid. 1990, **235**: 373
7 UA1 Collaboration, Albajar C et al. Nucl. Phys. B, 1990, **345**: 1
8 Sengupta K et al. Phys. Lett. B, 1990, **236**: 219
9 EMU01 Collaboration, Adamovich M I et al. Phys. Rev. Lett., 1990, **65**: 412; Z. Phys. C, 1991, **395**: 49
10 Buschbeck B, Lipa P, Peschanski R. Phys. Lett. B, 1988, **215**: 788
11 Jain P L, Shing G. Nucl. Phys. A, 1996, **596**: 700
12 Agababyan N M et al. Phys. Lett. B, 1998, **431**: 451
13 De Wolf E A, Dremin I M, Kittel W. Phys. Rept., 1996, **270**: 1
14 Ghosh D, Deb A, Chattopadhyay R et al. Int. J. Mod. Phys. A, 1999, **14**: 2091
15 Ghosh D, Deb A, Patra K K et al. Phys. Rev. C, 2002, **66**: 047901
16 Ghosh D, Deb A, Sen I et al. Fizika B, 2003, **12**: 227
17 Takagi F. Phys. Rev. Lett., 1994, **72**: 32
18 ZHANG Dong-Hai, LIU Fang, HE Chun-Le, ZHAO Hui-Hua, JIA Hui-Ming, LI Xue-Qin, LI Zhen-Yu, LI Jun-Sheng. Chin. Phys., 2006, **15**: 2564
19 ZHANG Dong-Hai, ZHAO Hui-Hua, LIU Fang, HE Chun-Le, JIA Hui-Ming, LI Xue-Qin, LI Zhen-Yu, LI Jun-Sheng. Chin. Phys., 2006, **15**: 1987
20 SONG Fu, ZHANG Dong-Hai, LI Jun-Sheng. Chin. Phys., 2005, **14**: 942
21 ZHANG Dong-Hai, LI Zhen-Yu, LI Hui-Ling, LI Jun-Sheng. Chin. Phys., 2005, **14**: 2451
22 Powell P L, Fowler P H. The Study of Elementary Particles by Photographic Method. Oxford: Pergamon, 1959, 450
23 LIU Lian-Shou, ZHANG Yan, WU Yuan-Fang. Z. Phys. C, 1996, **69**: 323
24 Bialas A, Gozdzicki M. Phys. Lett. B, 1990, **252**: 483
25 LIU Lian-Shou, ZHANG Yan, DENG Yeu. Z. Phys. C, 1997, **73**: 535
26 Hua R C. Phys. Rev. D, 1990, **41**: 1456
27 ZHANG Dong-Hai, LI Hui-Ling. Chin. Phys. C, 2008, to be published.
28 Sarcevic I, Satz H. Phys. Lett. B, 1989, **233**: 251
29 Peschanski R. Nucl. Phys. B, 1989, **327**: 144
30 Bialas A, Zaleswki K. Phys. Lett. B, 1990, **238**: 413