Gluon condensate in color superconductivity*

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Abstract In color superconductor the gluon condensate drops down at moderate density but goes up at high density and can even exceed its vacuum value when the density is high enough.

Key words gluon condensate, trace anomaly, chiral symmetry restoration, color superconductivity

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1 Introduction

Quarks and gluons condense in the vacuum of Quantum Chromodynamics (QCD), reflecting spontaneous chiral symmetry breaking and trace anomaly of the system [1, 2]. From lattice QCD calculations and effective QCD models in hot and dense medium, it is widely accepted that the quark condensate $\langle\bar{\psi}\psi\rangle$ which is the order parameter of the chiral symmetry restoration decreases with increasing temperature and density. The gluon condensate $\langle G^a_{\mu\nu}G^{\mu\nu}_a\rangle$ [3, 4] which describes the degree of the scale symmetry breaking is, however, not so optimistic. For lack of a direct relation to experimental data, its value in the vacuum is even not precise.

The QCD condensates in vacuum and at finite temperature are investigated in the framework of instantons [5, 6] which are semiclassical configurations of the gluon field in 4-dimensional Euclidean space. While the calculation is more feasible than starting directly from the QCD Lagrangian, the instanton mechanism is still rather complicated and encounters some problems in dealing with the gauge field. Another often used way to study the non-perturbative behavior of the QCD condensates is with effective QCD models [7, 8] at low energy. Combining with the QCD trace anomaly, the gluon condensate at finite temperature and density is calculated in various models without gauge fields. While the results are quantitatively different, depending on the mechanisms and model parameters, almost all the calculations give the same temperature behavior of the gluon condensate: it stay invariable at low temperature and starts to decrease monotonously at the critical temperature for the chiral phase transition.

At low temperature, the ground state of a QCD system is in pion superfluidity at finite isospin density and in color superconductivity at finite baryon density. A natural question is then how the gluon condensate behaviors in such a superfluidity or superconductivity. From the study of Son [9], very different from the temperature and baryon density effects which lead to the deconfinement phase transition from hadron gas to quark matter, there is no deconfinement along the isospin density axis at zero temperature. Considering a charged pion system [10], both the Lee-Huang-Yang model for a dilute Boson gas and the Nambu–Jona-Lasinio (NJL) model at finite isospin density show a surprising isospin behavior of the gluon condensate: in the pion superfluidity it drops down slightly only at very low isospin density but goes up and even exceeds its vacuum value when the density is high enough.

In this paper we study the gluon condensate in a color superconductor. We first analyze the QCD scale symmetry and then discuss the correlation between the gluon condensate and diquark condensate in the framework of the NJL model with Polyakov-loop potential (pNJL) [11].

2 Gluon condensate

The QCD Lagrangian

$$\mathcal{L}=-\frac{1}{4}G^{\mu\nu}G_{\mu\nu}+\overline{\psi}(\mathrm{i}\gamma^{\mu}D_{\mu}-m_0)\psi, \tag{1}$$

with the non-Abelian field strength tensor $G_{\mu\nu}$ is invariant under the scale transformation $x\to\lambda x,\psi(x)\to$

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$\lambda^{3/2}\psi(\lambda x)$, $A_\mu(x) \to \lambda A_\mu(\lambda x)$, if the current quark mass m_0 is neglected. The corresponding Noether current is not conserved in quantum case [1–3],

$$\partial^\mu J_\mu = T^\mu_\mu = \frac{\beta(\alpha_s)}{4\alpha_s} G^{\mu\nu} G_{\mu\nu} + m_0(1+\gamma(\alpha_s))\overline{\psi}\psi, \quad (2)$$

with the QCD β and γ functions and the effective coupling constant α_s. The first term comes from the measure of the fermionic functional integral, and the second is due to the explicit breaking of the scale symmetry by the current quark mass. Note that T^μ_ν is the divergence of the Noether current, instead of the normal energy-momentum tensor. However, if the anomaly term is taken into account, the trace of $\langle T^\mu_\nu \rangle$ is equal to the trace of the energy-momentum tensor [12], and the gluon condensate is controlled by the deviation of the system from the ideal gas,

$$\begin{aligned}\epsilon - 3p &= \langle \frac{\beta(\alpha_s)}{4\alpha_s} G^{\mu\nu} G_{\mu\nu} \rangle + m_0 \langle (1+\gamma(\alpha_s))\overline{\psi}\psi \rangle \approx \\ &-\frac{9}{8}\langle \frac{\alpha_s}{\pi} G^2 \rangle + m_0 \langle \overline{\psi}\psi \rangle, \end{aligned} \quad (3)$$

with the energy density ϵ and pressure p of the system. For the second equality, we have expanded the β function to its leading order and neglected the contribution from the function γ.

To calculate the gluon condensate in a color superconducting quark matter via the thermodynamics of the system, we choose the flavor $SU(2)$ pNJL model [11, 13, 14],

$$\begin{aligned}\mathcal{L} &= \overline{\psi}(\mathrm{i}\gamma^\mu D_\mu - m_0 + \mu\gamma_0)\psi + \\ &2G_s\left(\sigma\overline{\psi}\psi + \pi\overline{\psi}\mathrm{i}\gamma_5\tau\psi\right) + \\ &G_d\Delta\left(\overline{\psi}^c \mathrm{i}\gamma_5\epsilon_f\epsilon_c\psi + \overline{\psi}\mathrm{i}\gamma_5\epsilon_f\epsilon_c\psi^c\right) - \\ &G_s(\sigma^2+\pi^2) - G_d\Delta^2 + U(\phi) + \delta\mathcal{L}, \end{aligned} \quad (4)$$

where $\mu = \mathrm{diag}(\mu_u,\ \mu_d) = \mathrm{diag}(\mu_B/3 + \mu_I/2,\ \mu_B/3 - \mu_I/2)$ is a diagonal matrix in the flavor space with μ_B and μ_I being the baryon and isospin chemical potential, $\sigma = \langle \overline{\psi}\psi \rangle$, $\pi = \langle \psi \mathrm{i}\gamma_5\tau\psi \rangle$ and $\Delta = \langle \overline{\psi}\mathrm{i}\gamma_5\epsilon_f\epsilon_c\psi \rangle$ are the chiral, pion and diquark condensate, being respectively the order parameter of chiral restoration, pion superfluidity and color superconductivity, ϵ_f and ϵ_c are the total antisymmetric tensor in flavor and color space, $U(\phi)$ as a function of the Polyakov-loop expectation value ϕ is the effective confinement potential extracted from the lattice calculation, and $\delta\mathcal{L}$ is the contribution beyond the mean field. With the vacuum values of the pion mass, pion decay constant and the constituent quark mass $m = m_0 - 2G_s\sigma$ as input, we can determine the current quark mass $m_0 = 5.5$ MeV, the momentum cutoff $\Lambda = 0.65$ GeV and the coupling constant in the scalar and pseudoscalar channel $G_s = 5.04\ \mathrm{GeV}^{-2}$. The coupling constant in the diquark channel is taken to be $G_d/G_s = 3/4$.

Before taking numerical calculations with the thermodynamic relation (3), we first estimate the effect of QCD superfluidity and superconductivity on the gluon condensate. In the pNJL Lagrangian, the fermionic kinetic term and the mean field potential are scale invariant, while the chemical potentials and the related isospin symmetry spontaneous breaking and color symmetry spontaneous breaking destroy the scale symmetry, like the current quark mass term. Therefore, the explicit non-conservation term of the Noether current in the pNJL model is

$$\begin{aligned}T^\mu_\mu &= m_0\overline{\psi}\psi - \mu\overline{\psi}\gamma_0\psi - 2G_s\left(\sigma\overline{\psi}\psi + \pi\overline{\psi}\mathrm{i}\gamma_5\tau\psi\right) - \\ &G_d\Delta\left(\overline{\psi}^c\mathrm{i}\gamma_5\epsilon_f\epsilon_c\psi + \overline{\psi}\mathrm{i}\gamma_5\epsilon_f\epsilon_c\psi^c\right). \end{aligned} \quad (5)$$

Assuming it as the total non-conservation and substituting it into the QCD relation (2) at finite chemical potentials, the ensemble average leads to

$$\frac{9}{8}\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle \sim 2G_s(\sigma^2+\pi^2) + 2G_d\Delta^2. \quad (6)$$

For normal quark matter with only chiral condensate, the gluon condensate drops down monotonously along with the process of chiral symmetry restoration. However, when the quark system is in a superfluidity or superconductivity state, the behavior of the gluon condensate is controlled by the competition between the chiral and pion or color condensate. It decreases with isospin or baryon chemical potential in the chiral symmetry governed phase but may increase in the deep superfluidity or superconductivity phase.

We now numerically calculate the gluon condensate via the equation of state in the pNJL model. In mean field approximation, the thermodynamic potential $\Omega\ (T,\ \mu;\ \sigma,\ \pi,\ \Delta,\ \phi)$ contains two parts, one is the classical potential, and the other is the Fermi-Dirac distributions for the quasi-particles. From the minimum of the thermodynamic potential, we easily obtain the gap equations which characterize the temperature and density dependence of the order parameters of the phase transitions. Since we focus on the color superconductivity phase at finite baryon chemical potential, the order parameters σ, Δ and ϕ are determined by

$$\begin{aligned}\partial\Omega/\partial\sigma &= 0, \quad \partial^2\Omega/\partial\sigma^2 \geqslant 0, \\ \partial\Omega/\partial\Delta &= 0, \quad \partial^2\Omega/\partial\Delta^2 \geqslant 0, \\ \partial\Omega/\partial\phi &= 0, \quad \partial^2\Omega/\partial\phi^2 \geqslant 0. \end{aligned} \quad (7)$$

With the known physical order parameters as functions of temperature and baryon chemical potential, we obtain the pressure $p = -\Omega$ and energy density $\epsilon = -p + Ts + \mu_B n_B$ with the entropy density $s = -\partial\Omega/\partial T$ and baryon density $n_B = -\partial\Omega/\partial\mu_B$. Then using the thermodynamic relation (3), we can get the gluon condensate at finite temperature and density. To compare the chiral, diquark and gluon condensates in the medium with their vacuum values, we show in the following the ratios,

$$R_\sigma = \frac{\sigma_{T,\mu_B}}{\sigma_{0,0}},\ R_\Delta = \frac{\Delta_{T,\mu_B}}{\sigma_{0,0}},\ R_g = \frac{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{T,\mu_B}}{\left\langle \frac{\alpha_s}{\pi} G^2 \right\rangle_{0,0}}. \quad (8)$$

We first turn off the color superconductivity. The temperature and chemical potential dependence of R_σ and R_g are shown in Fig. 1. As we qualitatively estimated above, the behavior of the gluon condensate in normal quark matter is fully controlled by the chiral condensate. Since the chiral phase transition is of second order at finite temperature and of first order at finite density, the gluon condensate decreases continuously along the temperature axis and drops down suddenly at the critical chemical potential. Our calculation agrees with the results from other effective models [8] and instanton model [6].

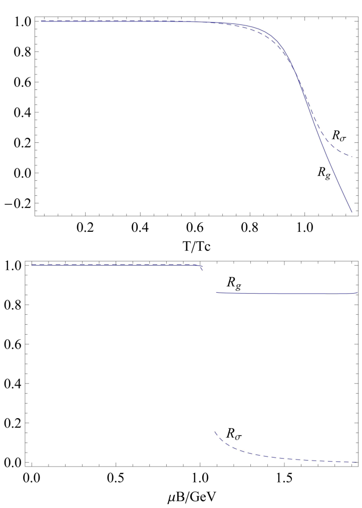

Fig. 1. The chiral (dashed lines) and gluon (solid lines) condensates as functions of temperature T ($\mu_B = 0$) and baryon chemical potential μ_B ($T = 0$) in normal quark matter.

We now move to the competition between the chiral and color condensates and see its effect on the gluon condensate. The ratios R_σ, R_Δ and R_g as functions of baryon chemical potential are demonstrated in Fig. 2 at zero temperature. In the chiral symmetry breaking phase, all the condensates are constants. At the critical chemical potential where the chiral condensate jumps down and the diquark condensate jumps up, the gluon condensate drops down, because the jump for the chiral condensate is larger than the jump for the diquark condensate and the system is controlled by the chiral symmetry at this point. In the beginning part of the color superconductor, the chiral condensate is still the dominant one and its decreasing leads to $R_g < 1$ in a wide region of baryon chemical potential. The case here is very different from the pion superfluidity where the ratio of the gluon condensate is less then unit only in a very narrow window of isospin chemical potential. Only in the region where the chiral condensate is very small and the diquark condensate becomes to control the system, the gluon condensate starts to increase and finally exceeds its vacuum value.

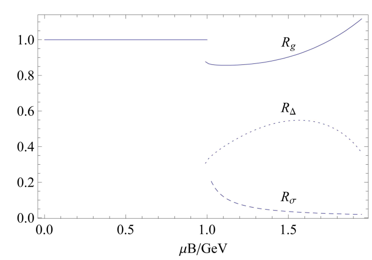

Fig. 2. The chiral (dashed line), diquark (dotted line) and gluon (solid line) condensates as functions of baryon chemical potential μ_B (T=0) in color superconductor.

3 Conclusion

The gluon condensate in color superconductor behaves very differently from that in normal quark matter. It decreases at moderate baryon density but increases later at high density and can even exceed its vacuum value when the density is high enough.

References

1 Collins J C, Duncan A, Jglekar S D. Phys. Rev. D, 1977, **16**: 438
2 Fujikawa K. Phys. Rev. D, 1980, **23**: 2262
3 Fukuda R, Kazama Y. Phys. Rev. Lett., 1980, **45**: 1142
4 Gorbar E V, Natale A A. Phys. Rev. D, 2000, **61**: 054012
5 Gross D J, Pisarski R D, Yaffe L G. Rev. Mod. Phys., 1981, **53**: 43
6 Schafer T, Shuryak E V. Rev. Mod. Phys., 1998, **70**: 323
7 Celenza L S, Shakin C M. Phys. Rev. D, 1998, **34**: 1591
8 Sollfrank J, Heinz U. Z. Phys. C, 1995, **65**: 111
9 Son D T, Stephanov M A. Phys. Rev. Lett., 2001, **86**: 592
10 He L, Jiang Y, Zhuang P F. Phys. Rev. C, 2009, **79**: 045205
11 Fukushima K. Phys. Lett. B, 2004, **591**: 277
12 Drummond I T, Horgan R R, Landshoff P V, Rebhan A. Phys. Lett. B, 1999, **460**: 197
13 FU W, ZHANG Zh, LIU Y. Phys. Rev. D, 2008, **77**: 014006
14 Dumm D G, Blaschke D B, Grunfeld A G, Scoccola N N. Phys. Rev. D, 2008, **78**: 114021