A dumbbell model with five parameters describing nuclear fusion or fission*

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Abstract: We propose a five-parameter dumbbell model to describe the fusion and fission processes of massive nuclei, where the collective variables are: the distance ρ between the center-of-mass of two fusing nuclei, the neck parameter ν, asymmetry D, two deformation variables β_1 and β_2. The present model has macroscopic qualitative expression of polarization and nuclear collision of head to head, sphere to sphere, waist to waist and so on. The conception of the "projectile eating target" based on open mouth and swallow is proposed to describe the nuclear fusion process, and our understanding of the probability of fusion and quasi-fission is in agreement with some previous work. The calculated fission barriers of a lot of compound nuclei are compared with the experimental data.

Key words: dumbbell model, nuclear fusion, nuclear fission, potential energy surface

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1 Introduction

The fusion and fission of massive nuclei have been a topic of great interest both experimentally and in theory, the first of all considerations and assumptions is a better choice of parametrical surface equation for describing nuclear deformation in the macroscopic reaction theory. Until now, there have been several ways to describe the variation of nuclear shape in the fusion and fission processes, for instance, the two-center model [1–3], the $\{c, h, \alpha\}$ parametrization [4], the dinuclear system [5], the dumbbell model [6] and so on.

In the dinuclear system, the dynamical evolution from dinucleus to mononucleus is taken into account through focusing on the exchange and transport of mass between the target and projectile nuclei [5]; the coupling between the radial and mass-asymmetry modes is weak for almost symmetric configuration but it becomes significant as the asymmetry increases [7]; the calculations of quasi-fission product and their distributions in the kinetic energy are in agreement with the recent experimental data of hot fusion reactions leading to super-heavy nuclei [8]. Forming the same heavy compound nucleus with different isotopes of the projectile and target elements allows nuclear structure effects in the dinuclear system to be disentangled [9]; the moments of inertia of hyperdeformed states formed in some reactions are calculated, and the optimal conditions for the experimental identification of such states are proposed [10]. The two-center parametrization model allows for the shape variation and continuous transition from one-center to two-center shapes with a smooth neck [11]; this model [12, 13] has been developed for describing the splitting of a deformed parent nucleus into two ellipsoidaly deformed fragments. The model of fusion by diffusion is applied to calculate the fusion probability of massive nuclei through a test particle diffusion over the saddle point [14, 15], the dynamics of nuclear neck growth are studied by a two-dimensional Langevin equation [16, 17]. However, calculations have lacked adequate exploration of the shape parameterization of sufficient dimensionality to yield features in the potential-energy surface. For this reason, Moéller used five deformation parameters of the nucleus to calculate the potential-energy landscapes in the fission study [18, 19]. The studies on nuclear shape such as the multidimensional constrained covariant density functional theory [20] and the Skyrme-Hartree-Fock (SHF) approach [21] are both well developed. A variety of results on fission barriers were discussed in terms of the two approaches.

The previous dumbbell model [6] supposed that the shape of the two fusing massive nuclei was a sphere and was successful in the exciting nuclear fission by introducing random neck rupturing. Nevertheless, it has been

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found from experimental studies that the projectile and target are deformed before fusion and may be ellipsoid. This implies that the two fusing systems should collide with various shapes and patterns. Therefore, we introduce two deformation parameters to generalize the dumbbell model to a five-parameter model, which takes into account the nutritious part for a dinuclear system and diffusion model. We will focus on the second stage [22] of fusion of massive nuclei and quasi-fission way in the five-dimensional parameter space.

In this work, we will calculate the deformation potential energy surface of fusing nuclei and discuss the ways of passing over the saddle point or quasi-fission of the system. The paper is organized as follows. In Section 2, we describe the evolution of the fusing system. In Section 3, we propose the conception of a "projectile eating target" for studying nuclear fusion and quasi-fission; and compare the numerical results of the fission barrier with experimental data [23]. Finally, the concluding remarks are given in Section 4.

2 The five-parameter dumbbell model

The five-parameter dumbbell model describing fusion and fission is an axis-symmetric shape where two ellipsoid bodies representing the approaching nuclei are joined by a cylindrical neck. The geometrical shape of the fusing system is divided into the asymmetric and symmetric cases.

2.1 Asymmetric fusion system

The scheme of fusing configuration of two massive nuclei in the asymmetric case is shown in Fig. 1. The pair of nuclei are ellipsoid (including spherical), so we can describe the two nuclei colliding with each other in various ways such as head to head, sphere to sphere, waist to waist, and head to waist. The ${P_\mathrm{s}}^2(z)$ in Fig. 1 is the square of the radius of the section which is perpendicular to the z axis, where the symmetric axis is the z axis.

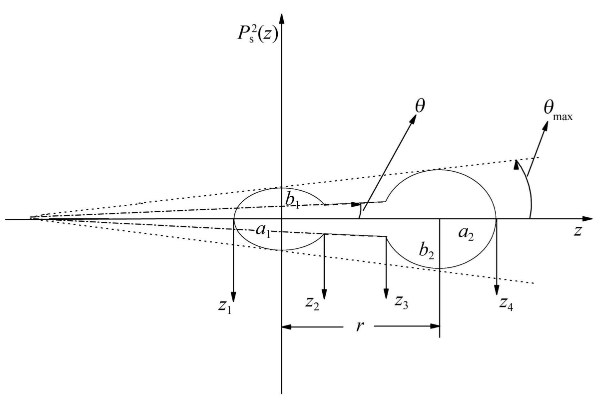

Fig. 1. The scheme of configuration of the asymmetric fusing system.

The nuclear surface equation in the asymmetric case reads

$$
{P_\mathrm{s}}^2(z)=\begin{cases} {b_1}^2-{\beta_1}^2z^2, & z_1\leqslant z\leqslant z_2 \\ \nu kz+\nu g, & z_2\leqslant z\leqslant z_3 \\ {b_2}^2-{\beta_2}^2[z-\rho(a_1+a_2)]^2, & z_3\leqslant z\leqslant z_4. \end{cases} \tag{1}
$$

where $k=\tan\theta$ is the slope of the external tangent line, g is the point of intersection between the external tangent line and the vertical axis, z_1, z_2, z_3, and z_4 are the boundaries of the definitional regions. The five dimensionless quantities in the asymmetric case are defined as follows:

(1) The distance variable $\rho=\dfrac{r}{a_1+a_2}$ describes the change in distance between the center-of-mass of the two fusing nuclei;

(2) The neck variable $\nu=\dfrac{\tan\theta}{\tan\theta_{\max}}$ describes the change of the neck size connecting two nuclei;

(3) The asymmetry variable $D=\dfrac{V_2-V_1}{V_2+V_1}$ is the ratio of the volumes of the projectile and target nuclei;

(4) The first deformation variable $\beta_1=\dfrac{b_1}{a_1}$ is the ratio of the short axis and long axis of the left ellipsoid nucleus;

(5) The second deformation variable $\beta_2=\dfrac{b_2}{a_2}$ is the ratio of the short axis and long axis of the right ellipsoid nucleus.

Among all mentioned above, r is the distance between the center-of-mass of the two nuclei; a_1 and a_2 are the half lengths of the two colliding nuclei which are parallel to the z axis; b_1 and b_2 are the half lengths of two nuclei which are perpendicular to the z axis. The above length variables are all in the unit of the radius R_0 of the compound nucleus. V_1 and V_2 are the volumes of two fusing nuclei, respectively; $\theta_{\max}$ is the semi-opening angle between the z axis and the external tangent line (the dot line in Fig. 1); θ is the angle between the z axis and the boundary line of the neck.

The condition of volume conservation must be satisfied, namely, the total volume of two fusing nuclei and the neck region is equal to the volume of the compound nucleus. We deduce the volume conservation in the asymmetric case:

$$
\frac{a_1^3[(ku+x)^3-(x+k\varphi)^3]\nu^2}{k}-a_1^3(\varphi-2)(1+\varphi)^2
$$
$$
-a_1^3\beta_2^2[t(\rho-2)+\rho-u](t-u+\rho+t\rho)^2=4. \tag{2}
$$

In Eq. (2), x, φ, and u are the transitional variables described by the present five-parameter model and a_1.

The ranges of five deformation parameters and the corresponding nuclear shapes are as follows:

1) The variation range of ρ:

(1) ρ=0 denotes a compound or monomer nucleus;
(2) 0<ρ<1 denotes two nuclei intersecting;
(3) ρ=1 denotes two nuclei touching;
(4) ρ>1 denotes two nuclei separating.

2) The variation range of ν:

$0<\nu<1$ means that the neck size changes from zero to the maximum, where the maximum is that the blue dash line and the red line coincide.

3) The variation range of D:

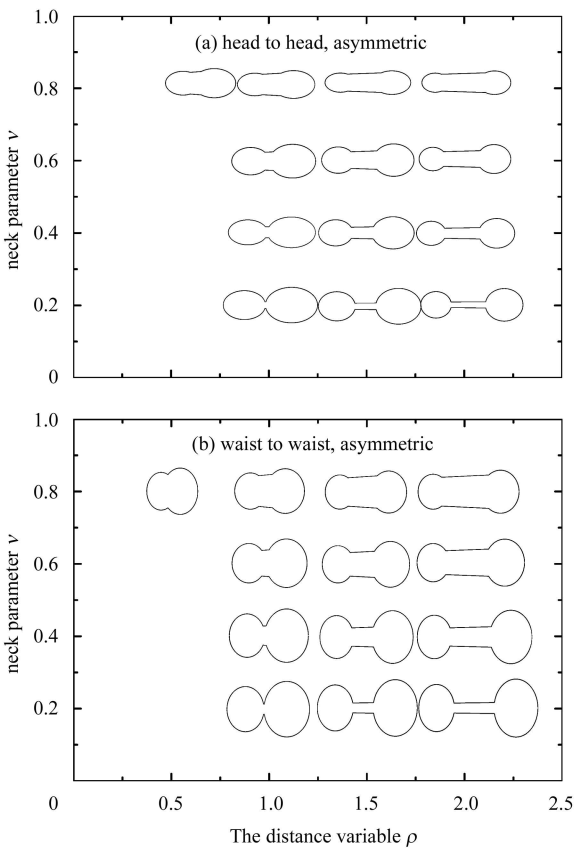

Fig. 2. A series of sectional view of nuclear fusion in the asymmetric case within various ways. (a) represents head to head, and (b) represents waist to waist.

$-1<D<1$ denotes two nuclei changing from the right nucleus larger than the left one to the right nucleus smaller than the left one.

4) The variation ranges of β_1 and β_2:

β_1 and β_2 have the same variation range, and both describe the deformation of the fusing nuclei. So we can consider nuclear fusion in various ways.

(1) $\beta_1<1$ and $\beta_2<1$ represent two nuclei colliding head to head;

(2) $\beta_1>1$ and $\beta_2>1$ represent two nuclei colliding waist to waist;

(3) $\beta_1=1$ and $\beta_2=1$ represent two nuclei colliding sphere to sphere;

(4) $\beta_1<1$ and $\beta_2>1$ or $\beta_1>1$ and $\beta_2<1$ represent two nuclei colliding head to waist or waist to head.

In Fig. 2, we show the sections of the system in the asymmetric case which is deduced by the nuclear surface equation.

2.2 Symmetric fusion system

The scheme of configuration of nuclear fusion in the completely symmetric case is shown in Fig. 3. In this case, the volumes and shapes of the two nuclei are identical ($D=0$ and $\beta_1=\beta_2=\beta$). The five dimensionless parameters reduce to three. The nuclear surface equation reads

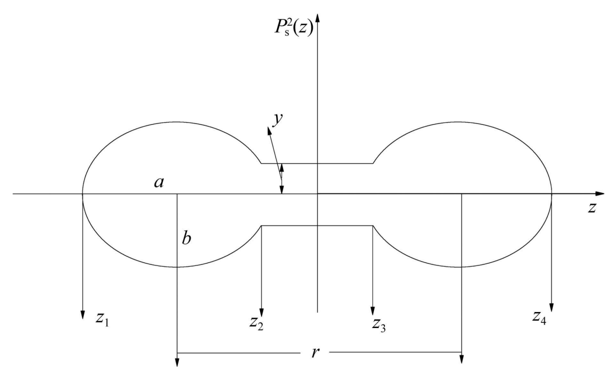

Fig. 3. The configuration of the symmetric fusing system.

$$P_{\rm s}{}^2(z)=\begin{cases} b^2\left[1-\dfrac{\left(z+\dfrac{r}{2}\right)^2}{a^2}\right], & -a-a\rho\leqslant z\leqslant\dfrac{a\sqrt{\beta_1^2-\nu^2}}{\beta_1}-a\rho \\ y^2, & \dfrac{a\sqrt{\beta_1^2-\nu^2}}{\beta_1}-a\rho\leqslant z\leqslant-\dfrac{a\sqrt{\beta_1^2-\nu^2}}{\beta_1}+a\rho \\ b^2\left[1-\dfrac{\left(z-\dfrac{r}{2}\right)^2}{a^2}\right], & -\dfrac{a\sqrt{\beta_1^2-\nu^2}}{\beta_1}+a\rho\leqslant z\leqslant a+a\rho. \end{cases} \tag{3}$$

The three parameters $\{\rho,\nu,\beta\}$ can be determined as follows:

(1) The variable $\rho=\dfrac{r}{2a}$ describes the variation of the distance between the center-of-mass of the two nuclei;

(2) The parameter $\nu=\dfrac{y}{a}$ describes the variation of the neck connecting the two nuclei;

(3) The parameters $\beta=\dfrac{b}{a}=\beta_1=\beta_2$ describe the shapes of the two nuclei identically.

Among the above expressions, $D=0$ means that the volumes of the two nuclei are equal (reduced into one parameter); r is the distance between the center-of-mass of two nuclei; a is the half length of each nucleus which is parallel with the z axis; b is the half length of each nucleus which is perpendicular to the z axis; y is the half width of the neck. The above length parameters are also all in a unit of the radius R_0 of the formed compound nucleus. In this case, the equal-volume condition is also satisfied:

$$\frac{4a(b^3+b^2\sqrt{b^2-y^2}-y^2\sqrt{b^2-y^2})}{b}+3ry^2=4. \tag{4}$$

The ranges of three deformation parameters and the corresponding shapes of the compound nucleus are similar to the asymmetric fusion system, however, there are two small differences:

(1) $\nu_{\max}=\beta$;

(2) There is only one deformation variable β, which means that the two nuclei change their shapes identically.

In Fig. 4, we show the sections of nuclear fusion in the completely symmetric case which is described by Eq. (3). In Fig. 5, we plot the stereopictures of nuclear fusion in both cases which are described by Eqs. (1) and (3).

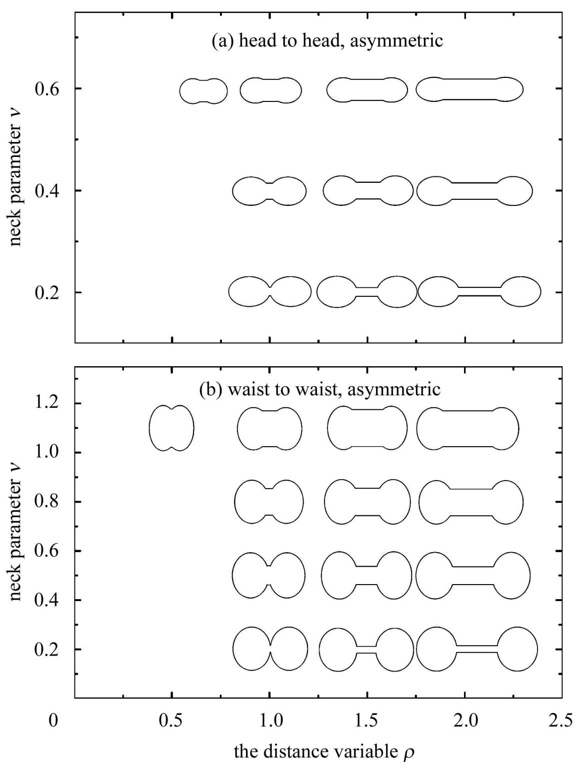

Fig. 4. A series of sectional views of nuclear fusion in the completely symmetric case in various ways. (a) represents head to head and (b) represents waist to waist.

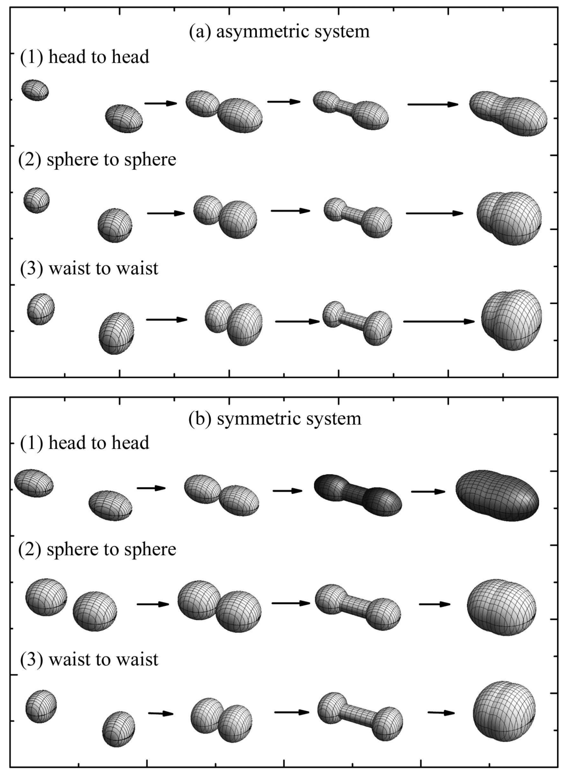

Fig. 5. A series of stereopictures of nuclear fusion in various ways. (a) represents the asymmetric system and (b) represents the symmetric system.

3 The potential energy surface

The potential energy surface is extremely important in nuclear physics. It can be determined by physical variables and further governs the variation of the physical variables by the master equations. Further study is based on the potential energy surface. The calculated results for the quasifission mass yields and the excitation function of the evaporation residue cross sections to form elements are shown to be agreeable with the measured or available data [24, 25].

The liquid-drop model [26, 27] has been successful in a series of aspects in explaining a nuclear reaction, so we use it to calculate the deformation potential energy surface of two fusing massive nuclei. For a compound nucleus, the spherical nucleus has the lowest energy, so the energy of the spherical compound nucleus is used as the basic standard. The deformation potential energy is given by

$$\begin{aligned} E(q_1,q_2,\cdots,q_n) &= E_{\rm s}-E_{\rm s0}+E_{\rm c}-E_{\rm c0} \\ &= \left[\left(\frac{E_{\rm s}}{E_{\rm s0}}-1\right)+2\chi\left(\frac{E_{\rm c}}{E_{\rm c0}}-1\right)\right]E_{\rm s0}. \end{aligned} \tag{5}$$

In Eq. (5), $q_i(i=1,2,\cdots,5)$ represent the corresponding deformation parameters; $E_{\rm s0}$ and $E_{\rm c0}$ are the surface energy and the Coulomb energy of the spherical nuclei; $E_{\rm s}$ and $E_{\rm c}$ are the surface energy and the Coulomb energy of the deformed nucleus, respectively; $\chi=\dfrac{E_{\rm c0}}{2E_{\rm s0}}$ is the fissionable parameter. According to the Myers-Swiatecki formula [28–30], we have

$$E_{\rm s0}=4\pi{R_0}^2\sigma=17.944\left[1-1.7826\left(\frac{N-Z}{A}\right)\right]A^{\frac{2}{3}}. \tag{6a}$$

$$E_{\rm c0}=\frac{3}{5}\frac{Z^2e^2}{R_0}=0.7053\frac{Z^2}{A^{\frac{1}{3}}}. \tag{6b}$$

$$B_{\rm s}=\frac{E_{\rm s}}{E_{\rm s0}}=\frac{S}{4\pi{R_0}^2}=\frac{1}{4}\int_a^b\sqrt{4{P_{\rm s}}^2+\left(\frac{\partial {P_{\rm s}}^2(z)}{\partial z}\right)^2}{\rm d}z. \tag{6c}$$

$$\begin{aligned} B_{\rm c}=\frac{E_{\rm c}}{E_{\rm c0}} &= \frac{1}{2}{\rho_0}^2\iint\frac{1}{|\vec{r_1}-\vec{r_2}|}{\rm d}\vec{r_1}{\rm d}\vec{r_2} \\ &= \frac{15}{4}\int_a^b{\rm d}z\int_a^z{\rm d}z'\int_0^\pi{\rm d}\phi\frac{{P_{\rm s}}^2(z){P_{\rm s}}^2(z')\sin^2\phi}{z-z'+f}. \end{aligned} \tag{6d}$$

$$f=\sqrt{(z-z')^2+{P_{\rm s}}^2(z)+{P_{\rm s}}^2(z')-2P_{\rm s}(z)P_{\rm s}(z')\cos\phi}. \tag{6e}$$

In Eq. (6), σ is the surface energy of the unit area of the nuclear surface; R_0, A, Z, and N are the radius of the compound nucleus, mass number, protons number, and neutrons number, respectively; ρ_0 is the charge density of the nucleus, and S is the area of nuclear surface.

For nuclear fusion, when the compound nucleus is formed from heavy-ion reaction, there will be angular momentums. Therefore, rotational energy is needed in the calculation of deformation potential energy of the rotational nucleus. The potential energy can be determined as follows:

$$E(q_1,q_2,\ldots,q_n)=\left[\left(\frac{E_{\rm s}}{E_{\rm s0}}-1\right)+2\chi\left(\frac{E_{\rm c}}{E_{\rm c0}}-1\right)\right]E_{\rm s0}+E_{\rm r}. \tag{7}$$

$$E_{\rm r}=\frac{\hbar}{2}\frac{[L(L+1)-K^2]}{I_\perp}+\frac{\hbar}{2}\frac{[K^2]}{I_\parallel}. \tag{8a}$$

$$I_\parallel=\frac{1}{2}\pi\rho_{\rm m}\int_a^b{P_{\rm s}}^4(z){\rm d}z. \tag{8b}$$

$$I_\perp=\pi\rho_m\int_a^b\left(\frac{1}{4}{P_{\rm s}}^4(z)+{P_{\rm s}}^2(z)z^2\right){\rm d}z. \tag{8c}$$

In Eqs. (7) and (8), $E_{\rm r}$ is the rotational energy; $L\hbar$ is the total angular momentum of the compound nucleus and $K\hbar$ is its shadow in the z axis; $I_\parallel$ and $I_\perp$ are the rotational inertia of parallel and are perpendicular to the z axis, respectively.

We conduct a series of calculations on the deformation potential energy at zero angular momentum and mostly take ^{210}Po as a product compound nucleus for example.

3.1 The process of "open and swallow"

In our model, the spherical area in the ground state corresponds to $\rho<(a_1-a_2)$ in the asymmetric case and $\rho=0$ in the symmetric case. Without microscopic correction, the spherical state is not a point but an area, and the borderline of the forbidden area on the potential energy surface is not smooth. However, this does not affect our discussion and may be verified in our further work.

In Fig. 6, we discuss nuclear fusion due to "open mouth and swallow" from the viewpoint of the deformation potential energy surface, and hope to understand qualitatively the fusing path and energy variation of the nuclear system in three-dimensional spatial graphs. We separate nuclear fusion into three processes:

(1) Ready stage: two nuclei overcome the Bass barrier with the push of external energy along Path 1 to centrally collide without a neck and reach the touched state. It should be noticed that the potential energy of the fusing system in the touched state is higher than that of the conditional saddle point.

(2) Separate again stage: on one hand, two nuclei may separate along Path 2 without a neck.

(3) Open mouth stage: on the other hand, two nuclei slip into the saddle point along Path 3. A rapid initial growth of a neck brings the reaction system from a dinuclear regime to a mononuclear regime. The energy of the system changes from radial kinetic energy along collision direction into deformation energy of the growth of the neck.

(4) Quasi-fission stage: the neck of a nuclear system may fracture randomly and split into two nuclei again along Path 4.

(5) Swallow stage: the two fusing nuclei will swallow each other and form a compound nucleus along Path 5.

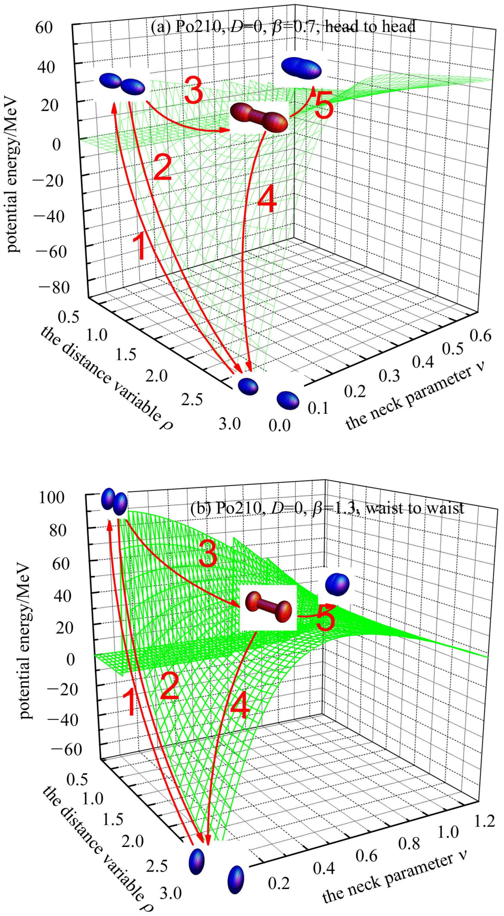

Fig. 6. (color online) Three-dimensional potential energy surface in the symmetric case. (a) represents two nuclei colliding head to head, (b) represents two nuclei colliding waist to waist.

Eventually, the excited compound nucleus will evaporate x neutrons in the cooling process or fission. In Fig. 6, we investigate fusion with "head to head" and "waist to waist" collisions of two massive nuclei. Obviously, the barrier in the "head to head" fusion is lower than that in the "waist to waist" case. In other words, the system can enter into the mononuclear regime with lower external energy in the "head to head" case but fissions again easily, because the fusing nuclei can open a mouth easily. The compound nucleus will be more stable due to swallowing all and open a mouth with difficultly when the system enters into the ground state with higher external energy. This is in agreement with the previous study [31, 32] which shows that collisions with the tips of the deformed target nuclei lead to quasi-fission, while collisions with the sides result in fusion-fission with conclusive evidence.

Figure 7 shows the potential energy surface of superheavy nucleus $Z=110$ at $D=0$, the saddle point is not very obvious and the energies of the ground state and the touched state are higher than those of the saddle point. The potential energy between the ground state and the distant state has a steep slope, so the corresponding compound nucleus can split into two equal pieces extremely easily along the red path of this figure. It states that super-heavy nucleus tends to quasi-fission. On the contrary, the small projectile can "open a big mouth to eat" the large target breezily. In other words, the probability of nuclear fusion may be higher with extremely high external energy at $D=0.9$ due to the higher barrier from the ground state to the conditional saddle point. The above discussion can explain why nuclear fusion needs large asymmetry in the experiments.

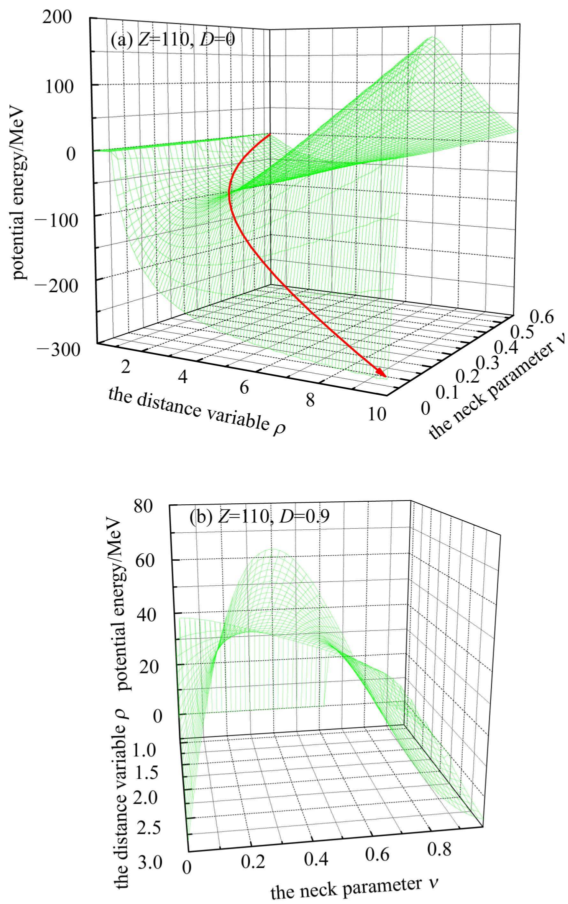

Fig. 7. (color online) The stereopictures of the potential energy of the nucleus $Z=110$ at D=0 or D=0.9.

3.2 The calculated fission barrier

In order to test the rationality of our model, we conduct a series of calculations on the potential energy surface of several compound nuclei and find the fission barriers. In Fig. 8, we compare the calculated fission barriers as a function of χ with experimental data. It is seen that the fission barrier of U isotopes increases monotonically with neutron number N. The trend of the calculated barrier is in good agreement with the experimental

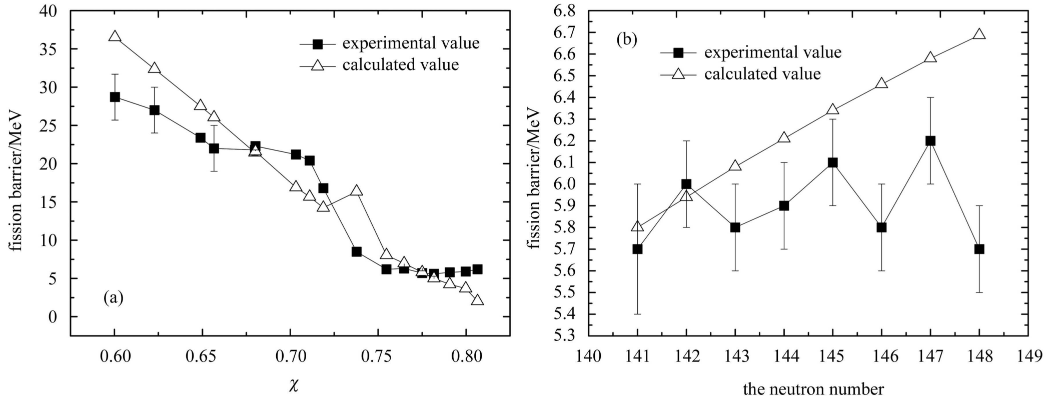

Fig. 8. Picture (a) is that the fission barriers of some nuclei vary with fissionable parameter χ. Picture (b) is that the fission barrier of U isotopes varies with the neutron number N.

results. The reason for a difference is that the present model is still a macroscopic classical one and lacks the microscopic shell correction which may be verified in the future. The calculated value of the fission barrier height becomes gradually higher from “head to head” to “waist to waist”. It is noticed that when the number of proton Z is larger than 90, the saddle point becomes not very obvious in our five-parameter dumbbell model.

4 Conclusion

We have proposed a five-parameter dumbbell model to study the fusion and fission processes of massive nuclei. With the help of two deformation variables β_1 and β_2, we can describe nuclear collisions in various ways such as head to head, sphere to sphere, waist to waist and so on. We have suggested a conception of “open mouth and swallow” and “projectile eating target” to understand alternatively the dynamics of nuclear fusion and quasi-fission, which is consistent with some previous studies to some extent. Finally, we have calculated the fission barriers of a lot of compound nuclei and compared the theoretical results with the experimental data and shown that the trends of each agree well.

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