Possible $J^{PC}=0^{+-}$ exotic states*

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Abstract: We study possible exotic $J^{PC}=0^{+-}$ states using tetraquark interpolating currents with the QCD sum rule approach. The extracted masses are around 4.85 GeV for the charmonium-like states and 11.25 GeV for the bottomonium-like states. There is no working region for the light tetraquark currents, which implies that the light 0^{+-} state may not exist below 2 GeV.

Key words: exotic state, QCD sum rule

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1 Introduction

Up until now, most of the hadrons observed experimentally could be interpreted as $q\bar{q}$/qqq states by the quark model [1, 2]. Some evidence has been accumulated of exotic states with $J^{PC}=1^{-+}$ [3–5]. Such a quantum number is not accessible for a conventional meson composed of a pair of quarks and anti-quarks in the non-relativistic quark model. Sometimes these states are named as exotic states, although all the J^{PC} quantum numbers are allowed in QCD.

For a neutral quark model $q\bar{q}$ state, $J=0$ ensures $L=S$, hence $C=(-)^{L+S}=+1$. Therefore, two possible exotic states with $J^{PC}=0^{--}$ and 0^{+-} exist. It is also interesting to note that the J^{PC} quantum number of the local operators composed of a pair of gluon field strength tensors is either 0^{++} or 0^{-+}.

On the other hand, the tetraquark operators may carry the 0^{--} and 0^{+-} quantum numbers. In fact, the 0^{--} state was investigated systematically using tetraquark currents with the QCD sum rule method [6, 7]. As a byproduct, it was noted that no tetraquark interpolating current without the derivative for the $J^{PC}=0^{+-}$ case exists.

With a similar formalism, one may construct the possible 0^{+-} tetraquark current by introducing derivatives. There are two kinds of construction, either with the qq basis or the $\bar{q}$q basis: (qq)($\bar{q}\bar{q}$) and ($\bar{q}$q)($\bar{q}$q). However, they can be related to each other by the Fierz transformation [6]. In this work, we use the first set, and it is important to note that the hybrid and three-gluon operators with $J^{PC}=0^{+-}$ exist. We focus on the tetraquark operators with derivatives in the present investigation. With these independent 0^{+-} currents, we perform QCD sum rule analysis and extract the masses of the corresponding currents.

This paper is organized as follows. In Section 2, we construct the tetraquark currents with $J^{PC}=0^{+-}$ using the diquark (qq) and antidiquark ($\bar{q}\bar{q}$) fields. In Section 3, we calculate the correlation functions and spectral densities of the interpolating currents and collect them in Appendix B. We perform the numerical analysis and extract the masses in Section 4 for the light and heavy systems, respectively, and the last section is a brief summary.

2 Tetraquark interpolating currents

It was shown that $J^{PC}=0^{+-}$ tetraquark interpolating currents without derivatives do not exist [6]. So in this work we construct the 0^{+-} currents with derivatives following similar steps to those in Ref. [6]. We first construct two independent tetraquark fields:

$$\begin{aligned} A_{abcd}&=(q_{1a}^{\mathrm{T}}C\gamma^{\mu}q_{2b})(\bar{q}_{3c}\overleftrightarrow{D}_{\mu}C\bar{q}_{4d}^{\mathrm{T}}),\\ P_{abcd}&=(q_{1a}^{\mathrm{T}}C\gamma^{\mu}\gamma_5q_{2b})(\bar{q}_{3c}\overleftrightarrow{D}_{\mu}\gamma_5C\bar{q}_{4d}^{\mathrm{T}}), \end{aligned} \quad (1)$$

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where q_{1-4} represents the flavor of the quarks, and $a-d$ stands for the color indices, $\overleftrightarrow{D}_\mu=\overrightarrow{D}_\mu-\overleftarrow{D}_\mu$, $\overrightarrow{D}_\mu=\overrightarrow{\partial}_\mu+\mathrm{i}g_sA_\mu^at^a$. It is understood that the index c is the color index of $\left(\bar{q}\overleftarrow{D}_\mu\right)_c$. In Eqs. (1) and (2) we have used the shorthand notation to simply the expression.

To compose the color singlet tetraquark currents, the diquark and antidiquark should have the same color and spin symmetries. Therefore, the color structure of the tetraquark is either $6\otimes\bar{6}$ or $\bar{3}\otimes 3$, which is denoted by labels 6 and 3, respectively. Details can be found in Ref. [6]. Considering both the color and Lorentz structures, we can obtain the currents with $J^{PC}=0^{+-}$:

$$\begin{aligned}
\eta_1(x)&=q_{1a}^{\mathrm{T}}C\gamma^\mu q_{2b}(\bar{q}_{1a}\overleftrightarrow{D}_\mu C\bar{q}_{2b}^{\mathrm{T}}+\bar{q}_{1b}\overleftrightarrow{D}_\mu C\bar{q}_{2a}^{\mathrm{T}})-q_{1a}^{\mathrm{T}}C\overleftrightarrow{D}_\mu q_{2b}(\bar{q}_{1a}\gamma^\mu C\bar{q}_{2b}^{\mathrm{T}}+\bar{q}_{1b}\gamma^\mu C\bar{q}_{2a}^{\mathrm{T}}),\\
\eta_2(x)&=q_{1a}^{\mathrm{T}}C\gamma^\mu q_{2b}(\bar{q}_{1a}\overleftrightarrow{D}_\mu C\bar{q}_{2b}^{\mathrm{T}}-\bar{q}_{1b}\overleftrightarrow{D}_\mu C\bar{q}_{2a}^{\mathrm{T}})-q_{1a}^{\mathrm{T}}C\overleftrightarrow{D}_\mu q_{2b}(\bar{q}_{1a}\gamma^\mu C\bar{q}_{2b}^{\mathrm{T}}-\bar{q}_{1b}\gamma^\mu C\bar{q}_{2a}^{\mathrm{T}}),\\
\eta_3(x)&=q_{1a}^{\mathrm{T}}C\gamma^\mu\gamma_5 q_{2b}(\bar{q}_{1a}\overleftrightarrow{D}_\mu\gamma_5 C\bar{q}_{2b}^{\mathrm{T}}+\bar{q}_{1b}\overleftrightarrow{D}_\mu\gamma_5 C\bar{q}_{2a}^{\mathrm{T}})-q_{1a}^{\mathrm{T}}C\overleftrightarrow{D}_\mu\gamma_5 q_{2b}(\bar{q}_{1a}\gamma^\mu\gamma_5 C\bar{q}_{2b}^{\mathrm{T}}+\bar{q}_{1b}\gamma^\mu\gamma_5 C\bar{q}_{2a}^{\mathrm{T}}),\\
\eta_4(x)&=q_{1a}^{\mathrm{T}}C\gamma^\mu\gamma_5 q_{2b}(\bar{q}_{1a}\overleftrightarrow{D}_\mu\gamma_5 C\bar{q}_{2b}^{\mathrm{T}}-\bar{q}_{1b}\overleftrightarrow{D}_\mu\gamma_5 C\bar{q}_{2a}^{\mathrm{T}})-q_{1a}^{\mathrm{T}}C\overleftrightarrow{D}_\mu\gamma_5 q_{2b}(\bar{q}_{1a}\gamma^\mu\gamma_5 C\bar{q}_{2b}^{\mathrm{T}}-\bar{q}_{1b}\gamma^\mu\gamma_5 C\bar{q}_{2a}^{\mathrm{T}}).
\end{aligned}\tag{2}$$

3 QCD sum rule

Consider the two-point correlation function in the framework of QCD sum rule

$$\Pi(q^2)\equiv\mathrm{i}\int\mathrm{d}^4x\mathrm{e}^{\mathrm{i}qx}\langle 0|T\eta(x)\eta^\dagger(0)|0\rangle,\tag{3}$$

where η is an interpolating current. At the hadron level, the correlation function $\Pi(p^2)$ is expressed via the dispersion relation:

$$\Pi(p^2)=(p^2)^N\int_0^\infty\frac{\rho(s)}{s^N(s-p^2-\mathrm{i}\varepsilon)}\mathrm{d}s+\sum_{n=0}^{N-1}a_n(p^2)^n,\tag{4}$$

The polynomial terms to the right of Eq. (4) are the subtraction terms, which will be removed by taking the Borel transformation to $\Pi(p^2)$ in the numerical analysis. The spectral density $\rho(s)$ is defined as:

$$\begin{aligned}\rho(s)&\equiv\sum_n\delta(s-m_n^2)\langle 0|\eta|n\rangle\langle n|\eta^\dagger|0\rangle\\ &=f_{\mathrm{X}}^2\delta(s-m_{\mathrm{X}}^2)+\text{continuum},\end{aligned}\tag{5}$$

where m_{X} is the mass of the resonance X and f_{X} is the decay constant of the meson:

$$\langle 0|\eta|\mathrm{X}\rangle=f_{\mathrm{X}}.\tag{6}$$

The correlation function can also be calculated at the quark-gluon level using the QCD operator product expansion (OPE) method. It is convenient to evaluate the Wilson coefficient in the coordinate space for the light quark systems and in the momentum space for the heavy quark systems, respectively. In our calculation we consider the first order perturbative and various condensate contributions. In order to calculate the gluonic condensate, it is convenient to work in the fixed-point gauge. The massive quark propagator $\mathrm{i}S(x,y)$ in an external field in the fixed-point gauge is listed in Appendix A. The quark lines attached by gluon contain terms proportional to y, which we can ignore in the current without derivatives. We keep these terms throughout the evaluation and let y go to zero only after finishing the derivatives. $\Pi(p^2)$ can be written as:

$$\begin{aligned}\Pi^{\mathrm{OPE}}(p^2)&=(p^2)^N\int_{4(m_1+m_2)^2}^\infty\mathrm{d}s\frac{\rho^{\mathrm{OPE}}(s)}{s^N(s-p^2-\mathrm{i}\epsilon)}\\ &+\sum_{n=0}^{N-1}a_n(p^2)^n,\end{aligned}\tag{7}$$

where m_1 and m_2 are the masses of the quark q_1 and q_2, respectively. In order to suppress the higher state contributions and remove the subtraction terms in Eqs. (4) and (7), we perform the Borel transformation to the correlation function, which is defined as:

$$L_{M_{\mathrm{B}}}\Pi(p^2)=\lim_{\substack{-p^2,n\to\infty\\ -p^2/n\equiv M_{\mathrm{B}}^2}}\frac{1}{n!}(-p^2)^{n+1}\left(\frac{d}{\mathrm{d}p^2}\right)^n\Pi(p^2).\tag{8}$$

After performing the Borel transformation and equating the two representations of the correlation function with quark-hadron duality, we obtain:

$$\Pi(M_{\mathrm{B}}^2)=f_{\mathrm{X}}^2\mathrm{e}^{-m_{\mathrm{X}}^2/M_{\mathrm{B}}^2}=\int_{4(m_1+m_2)^2}^{s_0}\mathrm{d}s\mathrm{e}^{-s/M_{\mathrm{B}}^2}\rho^{\mathrm{OPE}}(s),\tag{9}$$

where s_0 is the threshold parameter, and M_{B} is the Borel parameter. We can extract the meson mass m_{X}:

$$m_{\mathrm{X}}^2=\frac{\int_{4(m_1+m_2)^2}^{s_0}\mathrm{d}s\mathrm{e}^{-s/M_{\mathrm{B}}^2}s\rho^{\mathrm{OPE}(s)}}{\int_{4(m_1+m_2)^2}^{s_0}\mathrm{d}s\mathrm{e}^{-s/M_{\mathrm{B}}^2}\rho^{\mathrm{OPE}}(s)}.\tag{10}$$

For all the tetraquark currents in Eq. (2), we collect the spectral density $\rho^{\mathrm{OPE}}(s)$ in Appendix B. The quark condensate $\langle\bar{\mathrm{q}}\mathrm{q}\rangle$ vanishes due to the special Lorenz structures of the currents. For q=u, d, we perform the calculation in the chiral limit $m_{\rm q}=0$. Since the contribution of the three gluon condensates $\langle g_s^2 fGGG\rangle$ is very small, we consider only the power corrections from the following condensates: $\langle g_s^2 GG\rangle$, $\langle\bar{q}g_s\sigma{\cdot}Gq\rangle$, $\langle\bar{q}q\rangle^2$ and $\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle$. We list several typical Feynman diagrams in Fig. 1. According to the expressions of spectral density in Appendix B, both the perturbative and non-perturbative terms contribute to the "continuum" term in Eq. (5), except for part of the dimension eight condensate contribution $\Pi^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle 2}(M_{\rm B}^2)$ in Eq. (B3).

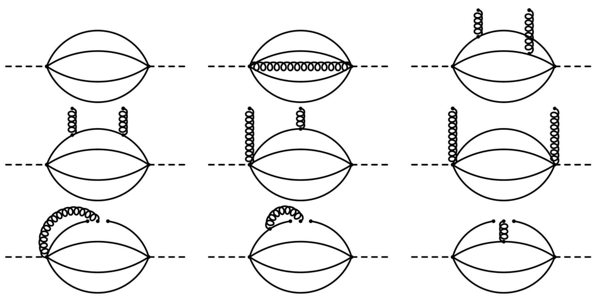

Fig. 1. Some typical Feynman diagrams of the correlation functions.

4 Numerical analysis

In the QCD sum rule analysis, we use the following values of the quark masses, coupling constant and various condensates [1, 8–10]:

$$
\begin{aligned}
&m_{\rm c}(m_{\rm c})=(1.23\pm0.09)\ \text{GeV},\\
&m_{\rm b}(m_{\rm b})=(4.20\pm0.07)\ \text{GeV},\\
&\langle\bar{q}q\rangle=-(0.23\pm0.03)^3\ \text{GeV}^3,\\
&\langle\bar{q}g_s\sigma{\cdot}Gq\rangle=-M_0^2\langle\bar{q}q\rangle,\\
&M_0^2=(0.8\pm0.2)\ \text{GeV},\\
&\langle g_s^2GG\rangle=(0.88\pm0.13)\ \text{GeV}^4,\\
&\alpha_s(1.7\ \text{GeV})=0.328\pm0.03\pm0.025.
\end{aligned}
\tag{11}
$$

The Borel mass $M_{\rm B}$ and the threshold value s_0 are two pivotal parameters. The working region of the Borel mass is determined by the convergence of the OPE and the pole contribution. The convergence requirement of the OPE determines the lower bound $M_{\rm Bmin}$ of the Borel mass, and the pole contribution determines the upper bound $M_{\rm Bmax}$.

In this work, there is no contribution from the quark condensate $\langle\bar{q}q\rangle$. We show the OPE convergence for the currents $\eta_1^{\rm q}$ and $\eta_1^{\rm c}$ in Fig. 2. One notes that the non-perturbative contributions are quite large for the low value of the Borel parameter $M_{\rm B}$ because the perturbative term has a higher power of s and $M_{\rm B}^2$. The starting point of QCD sum rule formalism is the OPE, which requires $M_{\rm B}$ to be reasonably large (at least > 1 GeV) so that OPE does not break down. On the other hand, this is highly suppressed for a high value of $M_{\rm B}$. In order to ensure the convergence of the OPE series, we should study the correlation function in the suitable value of $M_{\rm B}$. In Fig. 2, the most important non-perturbative correction is $\langle\bar{q}g_s\sigma{\cdot}Gq\rangle\langle\bar{q}q\rangle$ for both the qq$\bar{\rm q}\bar{\rm q}$ and qc$\bar{\rm q}\bar{\rm c}$ systems in the small value of $M_{\rm B}$. We require the contribution of $\langle\bar{q}g_s\sigma{\cdot}Gq\rangle\langle\bar{\rm q}{\rm q}\rangle$ to be less than the ninth of the perturbative term, which leads to the lower limits of the Borel parameter at about 1.6 GeV for the qq$\bar{\rm q}\bar{\rm q}$ system and 1.8 GeV for the qc$\bar{\rm q}\bar{\rm c}$ system of η_1. It is interesting to note that the four quark condensates $\langle\bar{\rm q}{\rm q}\rangle^2$ in the qq$\bar{\rm q}\bar{\rm q}$ system and the gluon condensate $\langle g_s^2GG\rangle$ in the qc$\bar{\rm q}\bar{\rm c}$ system become much more important for the larger value of $M_{\rm B}$.

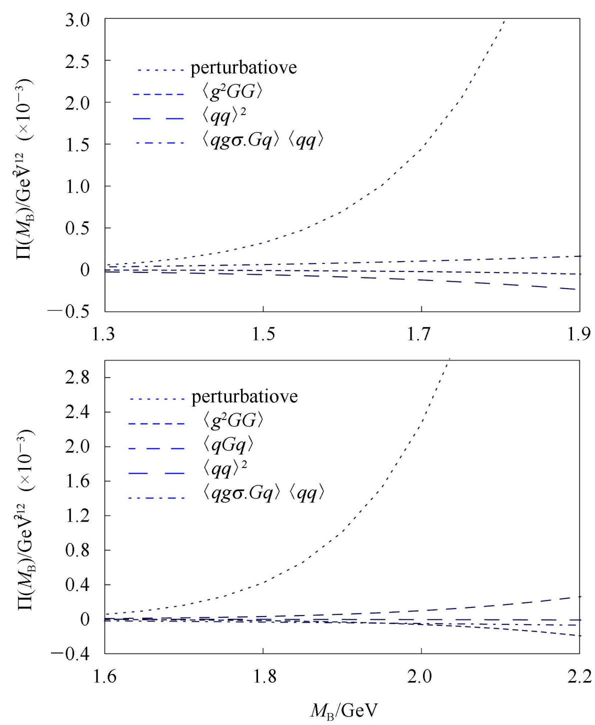

Fig. 2. The OPE convergence for the currents η_1^q and η_1^c.

The pole contribution (PC) is defined as

$$
{\rm PC}=\frac{\displaystyle\int_{4(m_1+m_2)^2}^{s_0}{\rm d}s{\rm e}^{-s/M_{\rm B}^2}\rho^{\rm OPE}(s)}{\displaystyle\int_{4(m_1+m_2)^2}^{\infty}{\rm d}s{\rm e}^{-s/M_{\rm B}^2}\rho^{\rm OPE}(s)}
\tag{12}
$$

which depends on both the Borel mass $M_{\rm B}$ and the threshold value s_0. s_0 is chosen around the region where the variation of $m_{\rm X}$ with $M_{\rm B}$ is at its minimum. Requiring the PC to be larger than 30%–50%, we get the upper bound M_{Bmax} of the Borel mass M_{B} and list the working region of the Borel parameters for the four currents with different quark compositions in Table 1. For η^{c}_{1-4}, we get the upper bound of Borel parameter M_{B} from the requirement that the PC be larger than 30%. For η^{b}_{1-4}, we need the PC to be larger than 40%. The masses are extracted using the threshold values s_0 and Borel parameters M_{B} listed in Table 1. The last column is the pole contribution with the corresponding s_0 and M_{B}.

For the light tetraquark systems, no working region for the sum rules exists. Even in the extreme case where the pole contribution is $\sim 30\%$ and the contribution of the condensate $\langle\bar{q}g_s\sigma{\cdot}Gq\rangle\langle\bar{q}q\rangle$ is around the leading order contribution, the lower bound M_{Bmin} is still much larger than the upper bound M_{Bmax}. In other words, there is no working region for light quark systems. As shown in Figs. 3 and 4, the extracted mass grows monotonically with s_0, which implies that the 0^{+-} state does not exist below 2 GeV. We note that the light $J^{PC}=0^{--}$ state does not exist either [6]. The 0^{+-} and 0^{-+} channels are in strong contrast to the 0^{++} case, where stable tetraquark QCD sum rules exist and the extracted scalar meson masses agree with the experimental scalar spectrum nicely [11].

For the heavy systems, the variation in m_{X} with s_0 and M_{B} is presented in Figs. 5–12. All the sum rules are very stable with reasonable variations of s_0 and M_{B}. The presence of the two heavy quarks reduces the kinetic energy of the tetraquark system, and hence helps to stabilize the sum rules. Numerically, the masses of the 0^{+-} states are slightly larger than those of the 0^{--} states [7].

Table 1. The threshold values, Borel window, and Borel parameters for the different tetraquark currents.

	current	s_0/GeV^2	$[M_{\text{Bmin}}, M_{\text{Bmax}}]$/GeV	M_{B}/GeV	m_{X}/GeV	PC(%)
q$_1$,q$_2$=u,d	η^{q}_{1-4}	—	—	—	—	—
q$_1$=u,d q$_2$=c	η^{c}_1	27	1.8–2.1	2.0	4.76±0.08	37.4
	η^{c}_2	28	1.8–2.1	2.0	4.85±0.09	39.9
	η^{c}_3	29	1.8–2.1	2.0	4.96±0.13	42.4
	η^{c}_4	28	1.8–2.1	2.0	4.83±0.07	40.9
q$_1$=u,d q$_2$=b	η^{b}_1	140	2.9–3.3	3.1	11.24±0.17	52.2
	η^{b}_2	142	2.9–3.3	3.1	11.27±0.14	55.6
	η^{b}_3	142	2.9–3.3	3.1	11.30±0.17	55.0
	η^{b}_4	142	2.9–3.3	3.1	11.27±0.09	55.5

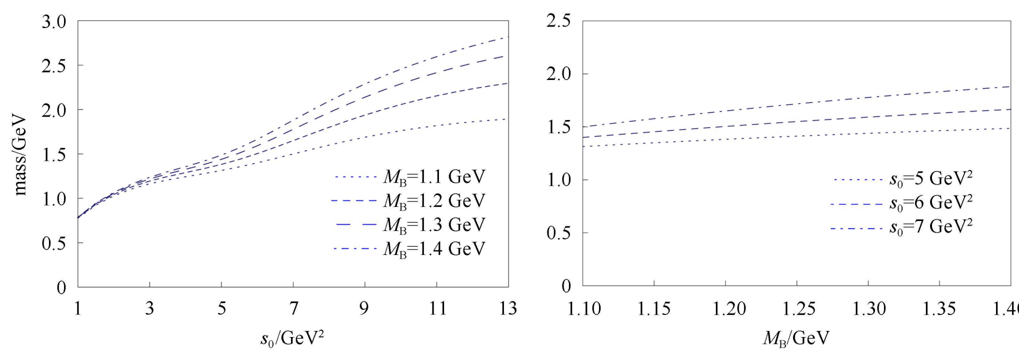

Fig. 3. The variation of m_{X} with M_{B} (left) and s_0 (right) for the current η^{q}_1.

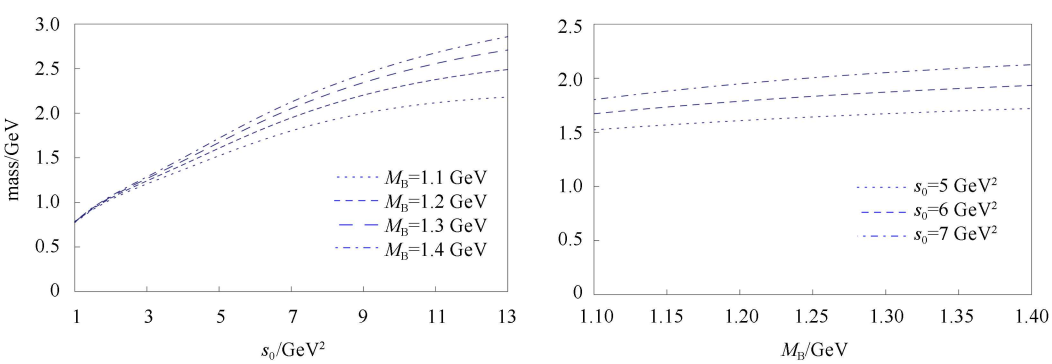

Fig. 4. The variation of m_{X} with M_{B} (left) and s_0 (right) for the current η^{q}_2.

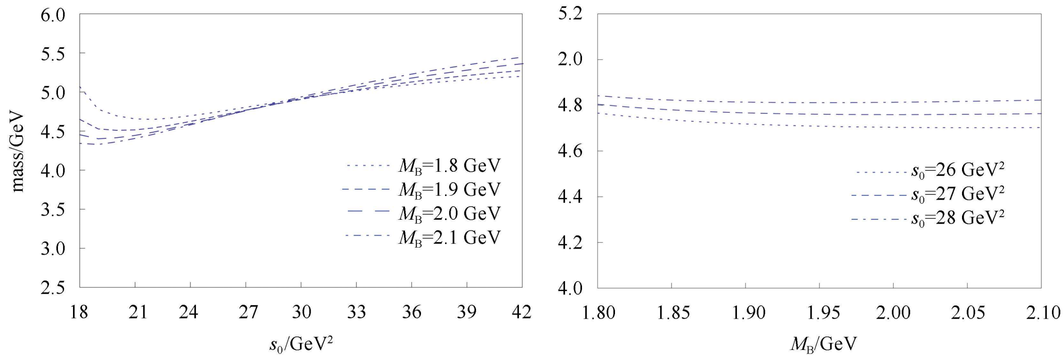

Fig. 5. The variation of m_X with M_B (left) and s_0 (right) for the current η_1^c.

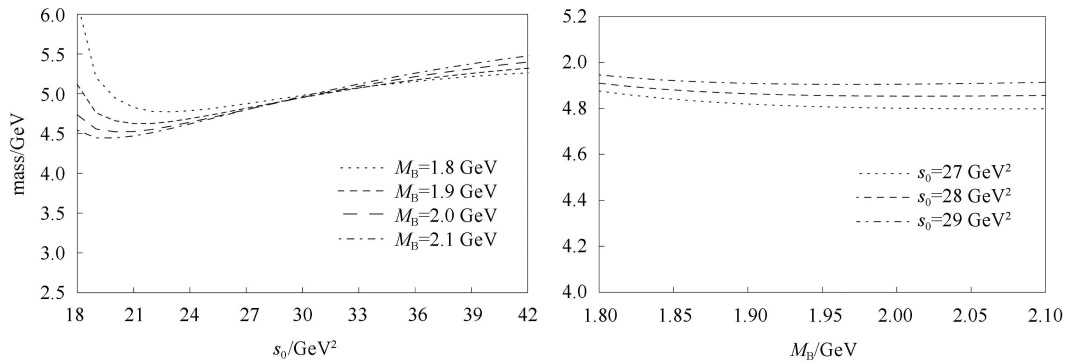

Fig. 6. The variation of m_X with M_B (left) and s_0 (right) for the current η_2^c.

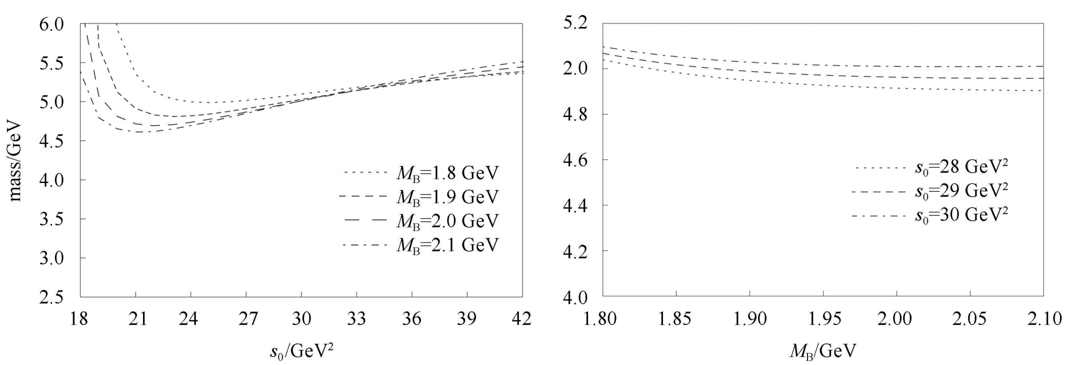

Fig. 7. The variation of m_X with M_B (left) and s_0 (right) for the current η_3^c.

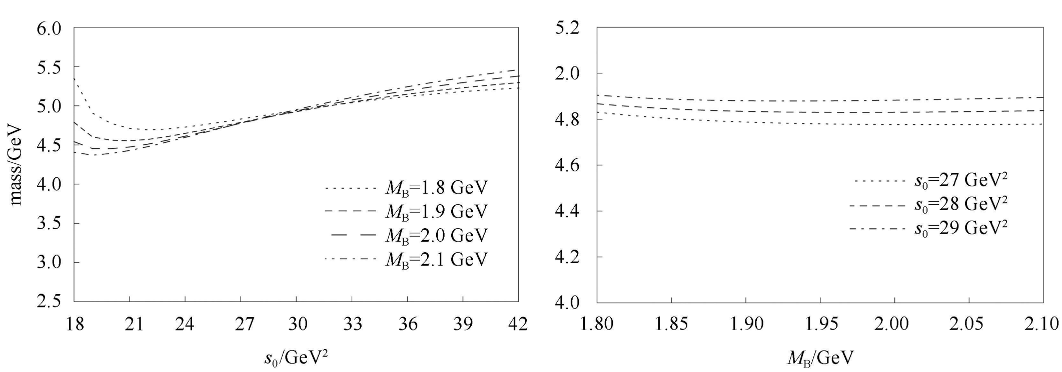

Fig. 8. The variation of m_X with M_B (left) and s_0 (right) for the current η_4^c.

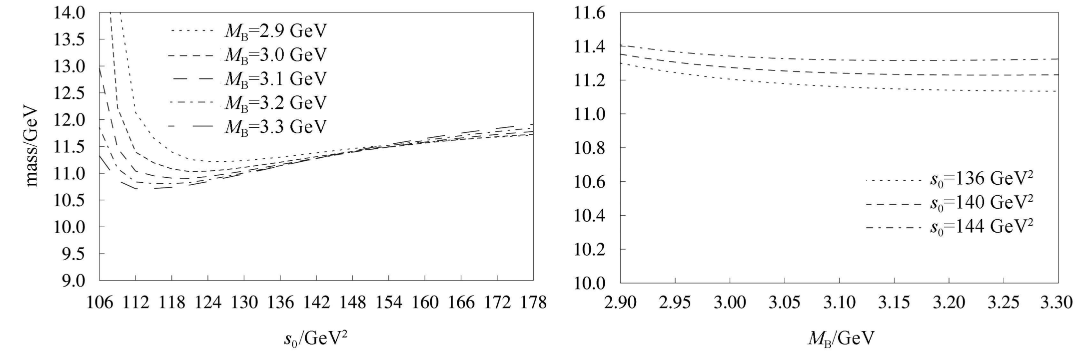

Fig. 9. The variation of m_X with M_B (left) and s_0 (right) for the current η_1^b.

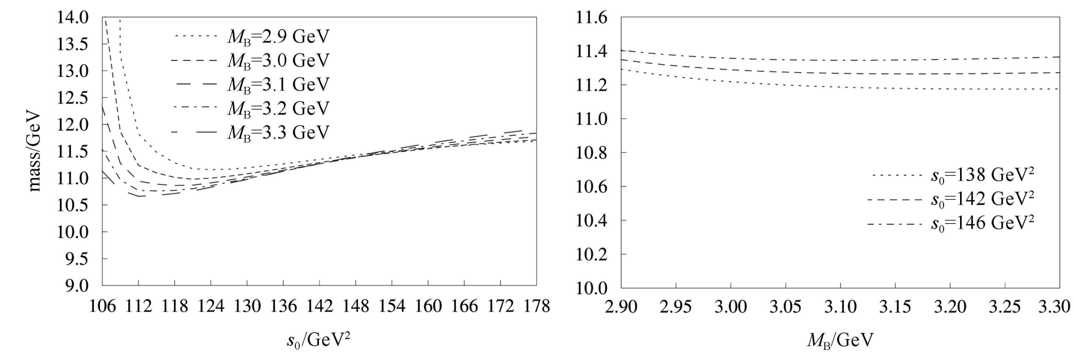

Fig. 10. The variation of m_X with M_B (left) and s_0 (right) for the current η_2^b.

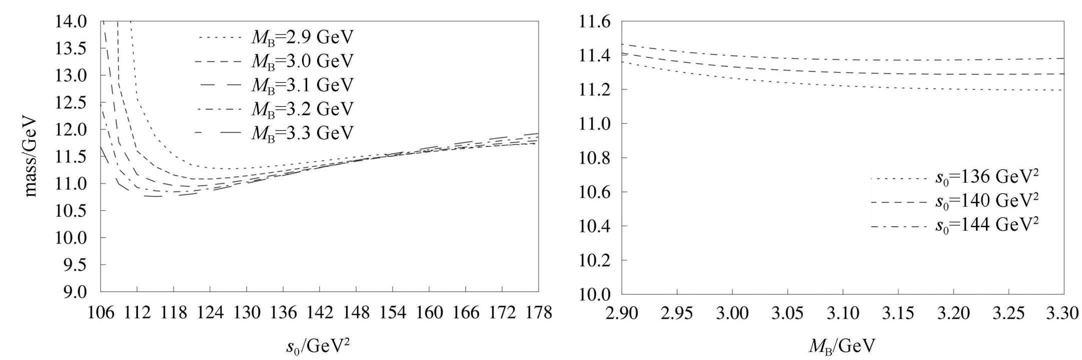

Fig. 11. The variation of m_X with M_B (left) and s_0 (right) for the current η_3^b.

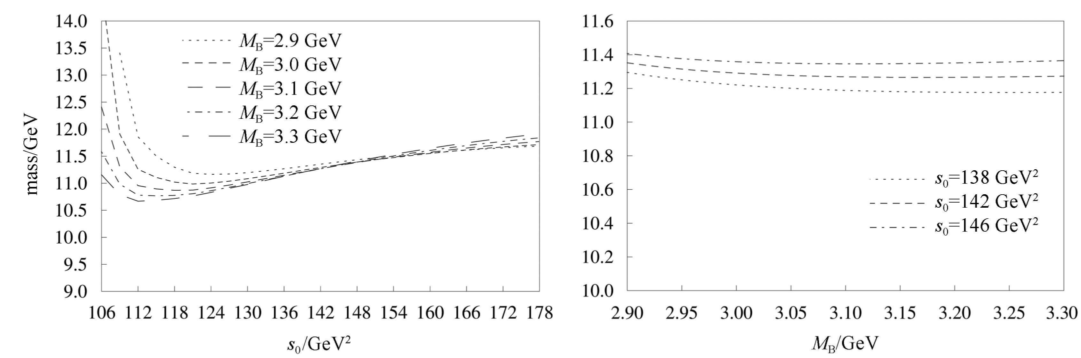

Fig. 12. The variation of m_X with M_B (left) and s_0 (right) for the current η_4^b.

5 Summary

The exotic state with $J^{PC}=0^{+-}$ cannot be composed of a pair of quarks and anti-quarks. In order to explore these exotic states, we constructed four tetraquark interpolating operators. We then created operator product expansion and extracted the spectral density. Because of the special Lorentz structures of the currents, the quark condensate $\langle\bar{q}q\rangle$ vanishes.

For the light tetraquark systems, there is no working region of the Borel parameter or threshold for all the derived sum rules. It seems that none of these independent interpolating currents supports a resonant signal below 2 GeV, which is consistent with the current experimental data [1]. The 0^{+-} hybrid state had been studied in Ref. [12], and the mass was about 2.3 GeV. In contrast, there are very stable QCD sum rules constructed from the tetraquark interpolating operators in the scalar channel. The extracted scalar spectrum agrees with the experimental data nicely [11].

For the heavy quark systems, the 0^{+-} tetraquark sum rules are quite stable. The extracted masses from the four interpolating currents η^c_{1-4} are around 4.76–4.96 GeV for the charmonium-like states. For the bottomonium-like 0^{+-} states, their masses are about 11.2–11.3 GeV. It is very interesting to note that the mass of the 0^{+-} charmonium-like state extracted from the tetraquark sum rules is numerically quite close to the mass of the 0^{+-} hybrid charmonium extracted on the lattice [13, 14]. Moreover, the extracted 0^{+-} charmonium-like tetraquark mass is also very close to the 0^{+-} glueball mass around 4.78 GeV from the quenched lattice simulation [15, 16].

Because of the special "exotic" quantum number, the 0^{+-} charmonium-like state does not decay into a pair of particles (H) and anti-particles ($\bar{H}$). There are two types of 0^{+-} states with different isospin and G-parity: $I^G=0^-$ and $I^G=1^+$. Only a few S-wave decay modes are allowed. Some of the possible two-body decay modes are listed in Table 2. Replacing the D meson with the B meson, one gets the decay patterns of the bottomonium-like states so long as the kinematics allow. The 0^{+-} state may be searched for experimentally at facilities such as the Super-B factory, PANDE, LHC and RHIC in the future, especially at RHIC and LHC, where plenty of charm, anti-charm and light quarks are produced.

Table 2. The possible decay modes of the 0^{+-} charmonium-like state.

I^G	S-wave	P-wave
0^-	$\chi_{c1}(1P)h_1(1170)\ldots$	$D^0(1865)\bar{D}_1(2420)^0$+c.c., $D^*(2007)^0\bar{D}_0^*(2400)^0$+c.c., $\eta_c(1S)h_1(1170)$, $J/\psi(1S)f_0(600)$, $J/\psi f_0(980)$, $J/\psi f_1(1285)$, $\chi_{c0}(1P)\omega(782)$, $\chi_{c1}(1P)\omega(782)$, $\psi(2S)f_0(600)$, $\psi(3770)f_0(600)\ldots$
1^+	$J/\psi(1S)\pi_1(1400)$, $J/\psi(1S)\pi_1(1600)$, $\chi_{c1}(1P)b_1(1235)\ldots$	$D^0(1865)\bar{D}_1(2420)^0$+c.c., $D^*(2007)^0\bar{D}_0^*(2400)^0$+c.c., $D^*(2007)^0\bar{D}_1(2420)^0$+c.c., $\eta_c(1S)b_1(1235)$, $J/\psi(1S)a_0(980)$, $J/\psi(1S)a_1(1260)$, $\chi_{c0}(1P)\rho(770)$, $\chi_{c1}(1P)\rho(770)\ldots$

Appendix A

The momentum space propagator

The fixed-point gauge is defined as:

$$(x-x_0)^\mu A^a_\mu(x)=0, \tag{A1}$$

where x_0 is an arbitrary point in space which can be chosen to be the origin. Then the potential A^a_μ can be expressed in terms of the field strength tensor

$$G_{\mu\nu}\left(G_{\mu\nu}=\frac{\lambda^a}{2}G^a_{\mu\nu}\right)$$

[17, 18]:

$$A_\mu(x) = \int_0^1 t\mathrm{d}t G_{\nu\mu}(tx)x^\nu=\frac{1}{2}x^\nu G_{\nu\mu}(0) +\frac{1}{3}x^\alpha x^\nu D_\alpha G_{\nu\mu}(0)+\cdots. \tag{A2}$$

Denote the massive quark propagator between positions x and y in the coordinate space as $\mathrm{i}S(x,y)$. The massive quark propagator in the momentum space is [17]:

$$\mathrm{i}S(p)=\mathrm{i}S_0(p)+\mathrm{i}S_{g_s}(p)+\mathrm{i}S_{g_sg_s}(p)+\cdots, \tag{A3}$$

where $\mathrm{i}S_0(p)$ is the free-quark propagator:

$$\mathrm{i}S_0(p)=\frac{\mathrm{i}}{\hat{p}-m}, \tag{A4}$$

where $\hat{p}=\gamma^\mu p_\mu$, $\mathrm{i}S_{g_s}(p)$ is the quark propagator with one gluon leg attached:

$$\begin{aligned}\mathrm{i}S_{g_s}(p) \;=\; & \frac{\mathrm{i}}{4}\frac{\lambda^n}{2}g_sG^n_{\mu\nu}\frac{\sigma^{\mu\nu}(\hat{p}+m)+(\hat{p}+m)\sigma^{\mu\nu}}{(p^2-m^2)^2}\\ &+\frac{\mathrm{i}}{2}\frac{\lambda^n}{2}g_sG^n_{\mu\nu}\left[\frac{2y^\mu p^\nu(\hat{p}+m)}{(p^2-m^2)^2}-\frac{y^\mu\gamma^\nu}{p^2-m^2}\right].\end{aligned} \tag{A5}$$

$\mathrm{i}S_{g_sg_s}(p)$ is the quark propagator with two gluon legs attached:

$$\begin{aligned}\mathrm{i}S_{g_sg_s}(p) \;=\; & -\frac{\mathrm{i}}{4}\frac{\lambda^a}{2}\frac{\lambda^b}{2}g_s^2G^a_{\mu\rho}G^b_{\nu\sigma}\frac{\hat{p}+m}{(p^2-m^2)^5}(f^{\mu\rho\nu\sigma}+f^{\mu\nu\rho\sigma}+f^{\mu\nu\sigma\rho})\\ &-\frac{1}{4}\frac{\lambda^a}{2}\frac{\lambda^b}{2}g_s^2G^a_{\mu\rho}G^b_{\nu\sigma}\frac{\hat{p}+m}{(p^2-m^2)^4}[y^\sigma(f^{\mu\rho\nu}+f^{\mu\nu\rho})+y^\rho f^{\mu\nu\sigma}],\end{aligned} \tag{A6}$$

where $f^{\mu\nu\dots\alpha\beta}=\gamma^\mu(\hat{p}+m)\gamma^\nu(\hat{p}+m)\dots\gamma^\alpha(\hat{p}+m)\gamma^\beta(\hat{p}+m)$.

Appendix B

The spectral densities

In this appendix, we list the spectral densities of the tetraquark interpolating currents. For the light quark systems (q_1,q_2=u,d), the spectral densities are:

$$\begin{aligned}\rho_1^{\mathrm{OPE}}(s)&=\frac{s^5}{51200\pi^6}\left(1+\frac{17}{108}\frac{\alpha_s}{\pi}\right)-\frac{\langle g_s^2GG\rangle s^3}{18432\pi^6}-\frac{\langle\bar{q}q\rangle^2s^2}{6\pi^2}-\frac{799\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle s}{768\pi^2},\\ \rho_2^{\mathrm{OPE}}(s)&=\frac{s^5}{102400\pi^6}\left(1+\frac{7}{54}\frac{\alpha_s}{\pi}\right)+\frac{\langle g_s^2GG\rangle s^3}{18432\pi^6}-\frac{\langle\bar{q}q\rangle^2s^2}{12\pi^2}-\frac{245\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle s}{768\pi^2},\\ \rho_3^{\mathrm{OPE}}(s)&=\frac{s^5}{51200\pi^6}\left(1+\frac{17}{108}\frac{\alpha_s}{\pi}\right)-\frac{\langle g_s^2GG\rangle s^3}{18432\pi^6}-\frac{\langle\bar{q}q\rangle^2s^2}{6\pi^2}-\frac{7\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle s}{12\pi^2},\\ \rho_4^{\mathrm{OPE}}(s)&=\frac{s^5}{102400\pi^6}\left(1+\frac{7}{54}\frac{\alpha_s}{\pi}\right)+\frac{\langle g_s^2GG\rangle s^3}{18432\pi^6}-\frac{\langle\bar{q}q\rangle^2s^2}{12\pi^2}-\frac{7\langle\bar{q}g_s\sigma\cdot Gq\rangle\langle\bar{q}q\rangle s}{24\pi^2}.\end{aligned} \tag{B1}$$

For the heavy systems (q_1=u, d, q_2=c, d), the spectral densities are:

$$\rho^{\mathrm{OPE}}(s)=\rho^{\mathrm{pert}}(s)+\rho^{\langle GG\rangle}(s)+\rho^{\langle\bar{q}Gq\rangle}(s)+\rho^{\langle\bar{q}q\rangle^2}(s)+\rho^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(s). \tag{B2}$$

For the condensate $\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle$, this contains two parts: one part could be written as ρ, and the other part couldn't. We perform the Borel transformation directly on this. Therefore

$$\Pi^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle}(M_{\mathrm{B}}^2)=\int_{4m^2}^{\infty}\mathrm{d}s\mathrm{e}^{-s/M_{\mathrm{B}}^2}\rho^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle1}(s)+\Pi^{\langle\bar{q}Gq\rangle\langle\bar{q}q\rangle2}(M_{\mathrm{B}}^2). \tag{B3}$$

For the interpolating current η_1:

$$\begin{aligned}\rho_1^{\mathrm{pert}}(s) \;=\; & \frac{384}{\pi^4}\Bigg[\big(16\rho^{\mathrm{L}}_{115}(s)+m^2\rho^{\mathrm{L}}_{114}(s)-2m^2\rho^{\mathrm{K}}_{114}(s)+6m^2\rho^{\mathrm{I}}_{114}(s)-\rho^{\mathrm{O}}_{114}(s)+2\rho^{\mathrm{N}}_{114}(s)-4\rho^{\mathrm{J}}_{114}(s)\big)\\ &+\frac{\alpha}{\pi}\left(\frac{17}{6}\rho^{L}_{115}(s)-\frac{25}{24}m^2\rho^{I}_{114}(s)\right)\Bigg],\end{aligned} \tag{B4}$$

$$\rho_1^{\langle g^2GG\rangle}(s) = \frac{\langle g^2GG\rangle}{\pi^4}\Bigg[5\rho^{\rm J}_{123}(s)+\frac{23}{8}\rho^{\rm N}_{123}(s)-\frac{2}{3}\rho^{\rm N}_{213}(s)-\frac{10}{3}\rho^{\rm N}_{224}(s)+\frac{1}{6}\rho^{\rm O}_{123}(s)+\frac{1}{3}\rho^{\rm O}_{213}(s)-\frac{10}{3}\rho^{\rm O}_{224}(s)$$

$$-\frac{39}{8}\rho^{\rm I}_{123}(s)+1152m^2\rho^{\rm I}_{134}(s)-20m^2\rho^{\rm I}_{224}(s)-16m^2\rho^{\rm J}_{133}(s)-768m^2\rho^{\rm J}_{144}(s)-\frac{5}{3}\rho^{\rm J}_{223}(s)$$

$$+192m^2\big(\rho^{\rm N}_{144}(s)+\rho^{\rm N}_{414}(s)\big)-96m^2\big(\rho^{\rm O}_{144}(s)+\rho^{\rm O}_{414}(s)\big)+1152m^4\rho^{\rm I}_{144}(s)-\frac{95}{48}\rho^{\rm K}_{113}(s)+\frac{5}{6}m^2\rho^{\rm K}_{123}(s)$$

$$-192m^4\big(\rho^{\rm K}_{144}(s)+\rho^{\rm K}_{414}(s)\big)+\frac{10}{3}m^2\rho^{\rm K}_{213}(s)-\frac{5}{3}\rho^{\rm K}_{214}(s)-\frac{5}{3}m^2\rho^{\rm K}_{224}(s)-240m^2\rho^{\rm K}_{314}(s)+\frac{91}{48}\rho^{\rm L}_{113}(s)$$

$$-\frac{m^2}{2}\rho^{\rm L}_{123}(s)-\frac{29}{3}\rho^{\rm L}_{124}(s)+160m^2\rho^{\rm L}_{134}(s)+3072m^2\rho^{\rm L}_{145}(s)+10m^2\rho^{\rm L}_{224}(s)-\frac{31}{12}\rho^{\rm O}_{124}(s)-\frac{8}{3}\rho^{\rm O}_{214}(s)$$

$$-192m^2\rho^{\rm K}_{134}(s)+192m^4\rho^{\rm L}_{144}(s)\Bigg], \tag{B5}$$

$$\rho_1^{\langle\bar{q}Gq\rangle}(s) = -\frac{m\langle\bar{q}g\sigma\cdot Gq\rangle}{3\pi^2}\big(29\rho^{\rm I}_{112}(s)+39m^2\rho^{\rm I}_{122}(s)-5\rho^{\rm N}_{212}(s)-116\rho^{\rm M}_{123}(s)-40\rho^{\rm M}_{213}(s)+10m^2\rho^{\rm K}_{122}(s)$$

$$+5m^2\rho^{K}_{212}(s)+\frac{521}{8}\rho^{\rm K}_{112}(s)\big), \tag{B6}$$

$$\rho_1^{\langle\bar{q}q\rangle^2}(s)=\frac{16}{3}\langle\bar{q}q\rangle^2\big(\rho^{\rm Q}_{110}(s)-m^4\rho^{\rm I}_{110}(s)\big), \tag{B7}$$

$$\rho_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle1}(s)=\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(\frac{1}{18}\rho^{\rm Q}_{120}(s)-\frac{119}{36}m^2\rho^{\rm I}_{110}(s)-\frac{1}{18}m^4\rho^{\rm I}_{120}(s)-\frac{2}{3}\rho^{P}_{110}(s)+\frac{61}{48}\rho^{\rm N}_{110}(s)+\frac{61}{48}m^2\rho^{\rm K}_{110}(s)\right), \tag{B8}$$

$$\Pi_1^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle2}(M_{\rm B}^2)=\frac{2}{3}\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\big(m^4\Pi^{\rm I}(M_{\rm B}^2)-\Pi^{\rm II}(M_{\rm B}^2)\big). \tag{B9}$$

For the interpolating current η_2:

$$\rho_2^{\rm pert}(s) = \frac{192}{\pi^4}\Bigg[\big(16\rho^{\rm L}_{115}(s)+m^2\rho^{\rm L}_{114}(s)-2m^2\rho^{\rm K}_{114}(s)+6m^2\rho^{\rm I}_{114}(s)-\rho^{\rm O}_{114}(s)+2\rho^{\rm N}_{114}(s)-4\rho^{\rm J}_{114}(s)\big)$$

$$+\frac{\alpha}{\pi}\left(\frac{7}{6}\rho^{\rm L}_{115}(s)-\frac{5}{24}m^2\rho^{\rm I}_{114}(s)\right)\Bigg], \tag{B10}$$

$$\rho_2^{\langle g^2GG\rangle}(s) = \frac{\langle g^2GG\rangle}{\pi^4}\Bigg[-3\rho^{\rm J}_{123}(s)+\frac{7}{8}\rho^{\rm N}_{123}(s)+\frac{2}{3}\rho^{\rm N}_{213}(s)-\frac{2}{3}\rho^{\rm N}_{224}(s)-\frac{7}{6}\rho^{\rm O}_{123}(s)-\frac{1}{3}\rho^{\rm O}_{213}(s)-\frac{1}{3}\rho^{\rm O}_{224}(s)$$

$$+\frac{57}{8}m^2\rho^{\rm I}_{123}(s)+576m^2\rho^{\rm I}_{134}(s)-4m^2\rho^{\rm I}_{224}(s)-8m^2\rho^{\rm J}_{133}(s)-384m^2\rho^{\rm J}_{144}(s)-\frac{1}{3}m^2\rho^{\rm J}_{223}(s)$$

$$+96m^2\big(\rho^{\rm N}_{144}(s)+\rho^{\rm N}_{414}(s)\big)-48m^2\big(\rho^{\rm O}_{144}(s)+\rho^{\rm O}_{144}(s)\big)+576m^4\rho^{\rm I}_{144}(s)+\frac{17}{48}\rho^{\rm K}_{113}(s)+\frac{1}{6}m^2\rho^{\rm K}_{123}(s)$$

$$-96m^2\rho^{\rm K}_{134}(s)-96m^4\big(\rho^{\rm K}_{144}(s)+\rho^{\rm K}_{414}(s)\big)-\frac{10}{3}m^2\rho^{\rm K}_{213}(s)-\frac{1}{3}\rho^{\rm K}_{214}(s)-\frac{1}{3}m^2\rho^{\rm K}_{224}(s)$$

$$-120m^2\rho^{\rm K}_{314}(s)+\frac{47}{48}\rho^{\rm L}_{113}(s)+\frac{3}{2}m^2\rho^{\rm L}_{123}(s)+\frac{35}{3}\rho^{\rm L}_{124}(s)+80m^2\rho^{\rm L}_{134}(s)+96m^4\rho^{\rm L}_{144}(s)$$

$$+1536m^2\rho^{\rm L}_{145}(s)+2m^2\rho^{\rm L}_{224}(s)+\frac{1}{12}\rho^{\rm H}_{124}(s)+\frac{8}{3}\rho^{\rm H}_{214}(s)\Bigg], \tag{B11}$$

$$\rho_2^{\langle\bar{q}Gq\rangle}(s)=\frac{m\langle\bar{q}g\sigma\cdot Gq\rangle}{3\pi^2}\left(11\rho^{\rm I}_{112}(s)+\rho^{\rm N}_{212}(s)-44\rho^{\rm M}_{123}(s)+8\rho^{\rm M}_{213}(s)+9m^2\rho^{\rm I}_{122}(s)-\frac{73}{8}\rho^{\rm K}_{112}(s)+10m^2\rho^{\rm K}_{122}(s)-m^2\rho^{\rm K}_{212}(s)\right), \quad \text{(B12)}$$

$$\rho_2^{\langle\bar{q}q\rangle^2}(s)=\frac{8}{3}\langle\bar{q}q\rangle^2\left(\rho^{\rm Q}_{110}(s)-m^4\rho^{\rm I}_{110}(s)\right), \quad \text{(B13)}$$

$$\rho_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 1}(s)=\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(\frac{17}{18}\rho^{\rm Q}_{120}(s)-\frac{79}{36}m^2\rho^{\rm I}_{110}(s)-\frac{17}{18}m^4\rho^{\rm I}_{120}(s)-\frac{1}{3}\rho^{\rm P}_{110}(s)+\frac{29}{48}\rho^{\rm N}_{110}(s)+\frac{29}{48}m^2\rho^{\rm K}_{110}(s)\right), \quad \text{(B14)}$$

$$\Pi_2^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 2}(M_{\rm B}^2)=\frac{1}{3}\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(m^4\Pi^{\rm I}(M_{\rm B}^2)-\Pi^{\rm II}(M_{\rm B}^2)\right). \quad \text{(B15)}$$

For the interpolating current η_3:

$$\rho_3^{\rm pert}(s)=\frac{384}{\pi^4}\left[\left(16\rho^{\rm L}_{115}(s)+m^2\rho^{\rm L}_{114}(s)-2m^2\rho^{\rm K}_{114}(s)+6m^2\rho^{\rm I}_{114}(s)-\rho^{\rm O}_{114}(s)+2\rho^{\rm N}_{114}(s)-4\rho^{\rm J}_{114}(s)\right)\right.$$
$$\left.+\frac{\alpha}{\pi}\left(\frac{17}{6}\rho^{\rm L}_{115}(s)-\frac{25}{24}m^2\rho^{\rm I}_{114}(s)\right)\right], \quad \text{(B16)}$$

$$\rho_3^{\langle g^2GG\rangle}(s)=\frac{\langle g^2GG\rangle}{\pi^4}\left[5\rho^{\rm J}_{123}(s)+\frac{23}{8}\rho^{\rm N}_{123}(s)-\frac{2}{3}\rho^{\rm N}_{213}(s)-\frac{10}{3}\rho^{\rm N}_{224}(s)+\frac{1}{6}\rho^{\rm O}_{123}(s)+\frac{1}{3}\rho^{\rm O}_{213}(s)-\frac{10}{3}\rho^{\rm O}_{224}(s)\right.$$
$$-\frac{39}{8}\rho^{\rm I}_{123}(s)+1152m^2\rho^{\rm I}_{134}(s)-20m^2\rho^{\rm I}_{224}(s)-16m^2\rho^{\rm J}_{133}(s)-768m^2\rho^{\rm J}_{144}(s)-\frac{5}{3}\rho^{\rm J}_{223}(s)$$
$$+192m^2\left(\rho^{\rm N}_{144}(s)+\rho^{\rm N}_{414}(s)\right)-96m^2\left(\rho^{\rm O}_{144}(s)+\rho^{\rm O}_{414}(s)\right)+1152m^4\rho^{\rm I}_{144}(s)-\frac{95}{48}\rho^{\rm K}_{113}(s)+\frac{5}{6}m^2\rho^{\rm K}_{123}(s)$$
$$-192m^4\left(\rho^{\rm K}_{144}(s)+\rho^{\rm K}_{414}(s)\right)+\frac{10}{3}m^2\rho^{\rm K}_{213}(s)-\frac{5}{3}\rho^{\rm K}_{214}(s)-\frac{5}{3}m^2\rho^{\rm K}_{224}(s)-240m^2\rho^{\rm K}_{314}(s)+\frac{91}{48}\rho^{\rm L}_{113}(s)$$
$$-\frac{m^2}{2}\rho^{\rm L}_{123}(s)-\frac{29}{3}\rho^{\rm L}_{124}(s)+160m^2\rho^{\rm L}_{134}(s)+3072m^2\rho^{\rm L}_{145}(s)+10m^2\rho^{\rm L}_{224}(s)-\frac{31}{12}\rho^{\rm O}_{124}(s)-\frac{8}{3}\rho^{\rm O}_{214}(s)$$
$$\left.-192m^2\rho^{\rm K}_{134}(s)+192m^4\rho^{\rm L}_{144}(s)\right], \quad \text{(B17)}$$

$$\rho_3^{\langle\bar{q}Gq\rangle}(s)=\frac{m\langle\bar{q}g\sigma\cdot Gq\rangle}{3\pi^2}\left(29\rho^{\rm I}_{112}(s)+39m^2\rho^{\rm I}_{122}(s)-5\rho^{\rm N}_{212}(s)-116\rho^{\rm M}_{123}(s)\right.$$
$$\left.-40\rho^{\rm M}_{213}(s)+10m^2\rho^{\rm K}_{122}(s)+5m^2\rho^{\rm K}_{212}(s)+\frac{521}{8}\rho^{\rm K}_{112}(s)\right), \quad \text{(B18)}$$

$$\rho_3^{\langle\bar{q}q\rangle^2}(s)=\frac{16}{3}\langle\bar{q}q\rangle^2\left(\rho^{\rm Q}_{110}(s)-m^4\rho^{\rm I}_{110}(s)\right), \quad \text{(B19)}$$

$$\rho_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 1}(s)=\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(\frac{1}{18}\rho^{\rm Q}_{120}(s)-\frac{119}{36}m^2\rho^{\rm I}_{110}(s)-\frac{1}{18}m^4\rho^{\rm I}_{120}(s)-\frac{2}{3}\rho^{\rm P}_{110}(s)+\frac{61}{48}\rho^{\rm N}_{110}(s)+\frac{61}{48}m^2\rho^{\rm K}_{110}(s)\right), \quad \text{(B20)}$$

$$\Pi_3^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 2}(M_{\rm B}^2)=\frac{2}{3}\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(m^4\Pi^{\rm I}(M_{\rm B}^2)-\Pi^{\rm II}(M_{\rm B}^2)\right). \quad \text{(B21)}$$

For the interpolating current η_4:

$$\rho_4^{\rm pert}(s)=\frac{192}{\pi^4}\left[\left(16\rho^{\rm L}_{115}(s)+m^2\rho^{\rm L}_{114}(s)-2m^2\rho^{\rm K}_{114}(s)+6m^2\rho^{\rm I}_{114}(s)-\rho^{\rm O}_{114}(s)+2\rho^{\rm N}_{114}(s)-4\rho^{\rm J}_{114}(s)\right)\right.$$
$$\left.+\frac{\alpha}{\pi}\left(\frac{7}{6}\rho^{\rm L}_{115}(s)-\frac{5}{24}m^2\rho^{\rm I}_{114}(s)\right)\right], \quad \text{(B22)}$$

$$\rho_4^{\langle g^2GG\rangle}(s)=\frac{\langle g^2GG\rangle}{\pi^4}\Bigg[-3\rho^{\rm J}_{123}(s)+\frac{7}{8}\rho^{\rm N}_{123}(s)+\frac{2}{3}\rho^{\rm N}_{213}(s)-\frac{2}{3}\rho^{\rm N}_{224}(s)-\frac{7}{6}\rho^{\rm O}_{123}(s)-\frac{1}{3}\rho^{\rm O}_{213}(s)-\frac{1}{3}\rho^{\rm O}_{224}(s)$$
$$+\frac{57}{8}m^2\rho^{\rm I}_{123}(s)+576m^2\rho^{\rm I}_{134}(s)-4m^2\rho^{\rm I}_{224}(s)-8m^2\rho^{\rm J}_{133}(s)-384m^2\rho^{\rm J}_{144}(s)-\frac{1}{3}m^2\rho^{\rm J}_{223}(s)$$
$$+96m^2\left(\rho^{\rm N}_{144}(s)+\rho^{\rm N}_{414}(s)\right)-48m^2\left(\rho^{\rm O}_{144}(s)+\rho^{\rm O}_{144}(s)\right)+576m^4\rho^{\rm I}_{144}(s)+\frac{17}{48}\rho^{\rm K}_{113}(s)+\frac{1}{6}m^2\rho^{\rm K}_{123}(s)$$
$$-96m^2\rho^{\rm K}_{134}(s)-96m^4\left(\rho^{\rm K}_{144}(s)+\rho^{\rm K}_{414}(s)\right)-\frac{10}{3}m^2\rho^{\rm K}_{213}(s)-\frac{1}{3}\rho^{\rm K}_{214}(s)-\frac{1}{3}m^2\rho^{\rm K}_{224}(s)$$
$$-120m^2\rho^{\rm K}_{314}(s)+\frac{47}{48}\rho^{\rm L}_{113}(s)+\frac{3}{2}m^2\rho^{\rm L}_{123}(s)+\frac{35}{3}\rho^{\rm L}_{124}(s)+80m^2\rho^{\rm L}_{134}(s)+96m^4\rho^{\rm L}_{144}(s)$$
$$+1536m^2\rho^{\rm L}_{145}(s)+2m^2\rho^{\rm L}_{224}(s)+\frac{1}{12}\rho^{\rm H}_{124}(s)+\frac{8}{3}\rho^{\rm H}_{214}(s)\Bigg], \tag{B23}$$

$$\rho_4^{\langle\bar{q}Gq\rangle}(s)=-\frac{m\langle\bar{q}g\sigma\cdot Gq\rangle}{3\pi^2}\Bigg(11\rho^{\rm I}_{112}(s)+\rho^{\rm N}_{212}(s)-44\rho^{\rm M}_{123}(s)+8\rho^{\rm M}_{213}(s)+9m^2\rho^{\rm I}_{122}(s)-\frac{73}{8}\rho^{\rm K}_{112}(s)$$
$$+10m^2\rho^{\rm K}_{122}(s)-m^2\rho^{\rm K}_{212}(s)\Bigg), \tag{B24}$$

$$\rho_4^{\langle\bar{q}q\rangle^2}(s)=\frac{8}{3}\langle\bar{q}q\rangle^2\left(\rho^{\rm Q}_{110}(s)-m^4\rho^{\rm I}_{110}(s)\right), \tag{B25}$$

$$\rho_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 1}(s)=\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(\frac{17}{18}\rho^{\rm Q}_{120}(s)-\frac{79}{36}m^2\rho^{\rm I}_{110}(s)-\frac{17}{18}m^4\rho^{\rm I}_{120}(s)-\frac{1}{3}\rho^{\rm P}_{110}(s)+\frac{29}{48}\rho^{\rm N}_{110}(s)+\frac{29}{48}m^2\rho^{\rm K}_{110}(s)\right), \tag{B26}$$

$$\Pi_4^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle 2}(M_{\rm B}^2)=\frac{1}{3}\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\left(m^4\Pi^{\rm I}(M_{\rm B}^2)-\Pi^{\rm II}(M_{\rm B}^2)\right). \tag{B27}$$

The functions $\rho^{\rm I,J,K\cdots}_{hjk}(s)$ and $\Pi^{\rm I,II}(M_{\rm B}^2)$ in the above expressions are defined as:

$$\rho^{\rm I}_{hjk}(s)=\frac{(-1)^k4^{-k-2}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(3-h-j+k)}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}{\rm d}\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{2-h-j+k}}{\alpha^{1+k-h}\beta^{1+k-j}}, \tag{B28}$$

$$\rho^{\rm J}_{hjk}(s)=\frac{(-1)^k4^{-k-2}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(4-h-j+k)}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}{\rm d}\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}{\rm d}\beta$$
$$\times\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{2-h-j+k}\left(2m^2(\alpha+\beta)+\alpha\beta s(h+j-k-5)\right)}{\alpha^{1+k-h}\ \beta^{1+k-j}}, \tag{B29}$$

where $h,j,k>0$, $h+j-k\leqslant 2$.

$$\rho^{\rm K}_{hjk}(s)=\frac{(-1)^k2^{-2k-3}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(3-h-j+k)}$$
$$\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}{\rm d}\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{1-h-j+k}\left(2m^2(\alpha+\beta)+\alpha\beta s(h+j-k-4)\right)}{\alpha^{1+k-h}\beta^{k-j}}, \tag{B30}$$

where $h,j,k>0$, and $h+j-k\leqslant 1$.

$$\rho^{\rm L}_{hjk}(s)=\frac{(-1)^k4^{-k-1}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(3-h-j+k)}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}{\rm d}\alpha\int_{\beta_{\rm min}}^{\beta_{\rm max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}(m^2(\alpha+\beta)-\alpha\beta s)^{-h-j+k}}{\alpha^{k-h}\beta^{k-j}}$$
$$\times\left[6(m^2(\alpha+\beta)-\alpha\beta s)^2-\alpha\beta s\left(6(m^2(\alpha+\beta)-\alpha\beta s)(2+k-h-j)-\alpha\beta s(2+k-h-j)(1+k-h-j)\right)\right], \tag{B31}$$

where, $h,j,k>0$, and $h+j-k\leqslant 0$.

$$\rho^{\rm L}_{144}(s)=\frac{3}{512\pi^2\Gamma(4)^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^3\left(m^2(\alpha+\beta)-2\alpha\beta s\right)}{\alpha^3}-\frac{m^4}{\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\frac{\alpha s^2\left(m^2-(1-\alpha)\alpha s\right)^3}{36864(m^2-\alpha s)^6}, \quad \text{(B32)}$$

$$\rho^{\rm N}_{hjk}(s)=\frac{(-1)^{k+1}4^{-k-2}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(4-h-j+k)}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{-h-j+k+1}}{\alpha^{k+1-h}\beta^{k+1-j}}$$

$$\times\Big(2(6\alpha-1)\left(\alpha\beta s-m^2(\alpha+\beta)\right)^2-\alpha\beta s(3-h-j+k)\left(\alpha\beta s(2\alpha(h+j-k-8)+1)+(12\alpha-1)m^2(\alpha+\beta)\right)\Big), \quad \text{(B33)}$$

where $h,j,k>0$, $h+j-k\leqslant 1$.

$$\rho^{\rm O}_{hjk}(s)=\frac{(-1)^{k+1}2^{-2k-3}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(4-h-j+k)}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{-h-j+k}}{\alpha^{k-h}\beta k-j+1}$$

$$\times\big[6(8\alpha-3)\left(m^2(\alpha+\beta)-\alpha\beta s\right)^3-18(4\alpha-1)\alpha\beta s(3-h-j+k)\left(m^2(\alpha+\beta)-\alpha\beta s\right)^2$$

$$+3(8\alpha-1)\alpha^2\beta^2s^2(2-h-j+k)(3-h-j+k)\left(m^2(\alpha+\beta)-\alpha\beta s\right)$$

$$-2\alpha^4s^3(1-h-j+k)(2-h-j+k)(1-h-j+k)\big], \quad \text{(B34)}$$

where $h,j,k>0$, $h+j-k\leqslant 1$.

$$\rho^{\rm O}_{144}(s)=-\frac{m^4}{36864\pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\frac{\alpha^3s^3\left(m^2-(1-\alpha)\alpha s\right)^3}{(m^2-\alpha s)^6}$$

$$-\frac{1}{2048\pi^2\Gamma(4)^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^3\left((8\alpha-3)m^4(\alpha+\beta)^2-4\alpha(10\alpha-3)\beta m^2s(\alpha+\beta)+10\alpha^2(4\alpha-1)\beta^2s^2\right)}{\alpha^3\beta}, \quad \text{(B35)}$$

$$\rho^{\rm O}_{414}(s)=\frac{1}{36864\ \pi^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\frac{\alpha^3s^3\left(m^2-(1-\alpha)\alpha s\right)^3}{m^2\ (m^2-\alpha s)^3}$$

$$-\frac{1}{2048\pi^2\Gamma(4)^2}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^3\left((8\alpha-3)m^4(\alpha+\beta)^2-4\alpha(10\alpha-3)\beta m^2s(\alpha+\beta)+10\alpha^2(4\alpha-1)\beta^2s^2\right)}{\beta^4}. \quad \text{(B36)}$$

$$\rho^{\rm H}_{hjk}(s)=\frac{(-1)^{k+1}2^{-2k-3}}{\pi^2\Gamma(h)\Gamma(j)\Gamma(k)\Gamma(3-h-j+k)}\int_{\alpha_{\min}}^{\alpha_{\max}}{\rm d}\alpha\int_{\beta_{\min}}^{\beta_{\max}}{\rm d}\beta\frac{(\alpha+\beta-1)^{k-1}\left(m^2(\alpha+\beta)-\alpha\beta s\right)^{-h-j+k-1}}{\alpha^{k-h}\beta^{k-j}(1-\alpha)^2}$$

$$\times\Big(12(1-\alpha)^3(8\alpha-1)\left(m^2(\alpha+\beta)-\alpha\beta s\right)^3-3\left(48\alpha^3-100\alpha^2+56\alpha-3\right)\alpha\beta s(-h-j+k+2)\left(\alpha\beta s-m^2(\alpha+\beta)\right)^2$$

$$-2(\alpha-1)^2(24\alpha-1)\alpha^2\beta^2s^2(h+j-k-2)(h+j-k-1)\left(\alpha\beta s-m^2(\alpha+\beta)\right)$$

$$+4(\alpha-1)^2\alpha^4\beta^3s^3(h+j-k-2)(h+j-k-1)(h+j-k)\Big), \quad \text{(B37)}$$

where $h,j,k>0$, $h+j-k\leqslant -1$.

$$\rho^{\rm I}_{110}(s)=-\frac{1}{16\pi^2}\sqrt{1-\frac{4m^2}{s}}, \quad \text{(B38)}$$

$$\rho^{\rm I}_{120}(s)=\frac{1}{16\pi^2}\frac{1}{\sqrt{s(s-4m^2)}}, \quad \text{(B39)}$$

$$\rho^{\rm Q}_{110}(s)=-\frac{1}{16\pi^2}\left(\frac{s}{2}-m^2\right)\sqrt{1-\frac{4m^2}{s}}, \quad \text{(B40)}$$

$$\rho^{\mathrm{Q}}_{120}(s)=-\frac{1}{16\pi^2}\left(\frac{s}{2}-m^2\right)^2\sqrt{1-\frac{4m^2}{s}}, \tag{B41}$$

$$\rho^{\mathrm{K}}_{110}(s)=-\frac{1}{8\pi^2}\left(\frac{1}{4(1-s/4m^2)}+\sqrt{1-\frac{4m^2}{s}}\right), \tag{B42}$$

$$\rho^{\mathrm{N}}_{110}(s)=-\frac{2}{32\pi^2}\left((3s-2m^2)\sqrt{1-\frac{4m^2}{s}}-\frac{4m^4}{\sqrt{s(s-4m^2)}}\right), \tag{B43}$$

$$\rho^{\mathrm{P}}_{110}(s)=\frac{-83m^6-559m^4s+326m^2s^2-53s^3}{120\pi^2s\sqrt{s(s-4m^2)}}, \tag{B44}$$

$$\Pi^{\mathrm{I}}(M_{\mathrm{B}}^2)=\frac{1}{4\pi^2}\int_0^1\mathrm{d}x\frac{m^2}{x^2M_{\mathrm{B}}^2}\exp\left[-\frac{m^2}{x(1-x)M_{\mathrm{B}}^2}\right], \tag{B45}$$

$$\Pi^{\mathrm{II}}(M_{\mathrm{B}}^2)=-\frac{1}{8\pi^2}\int_0^1\mathrm{d}x\frac{m^6}{x(1-x)M_{\mathrm{B}}^2}\exp\left[-\frac{m^2}{x(1-x)M_{\mathrm{B}}^2}\right]. \tag{B46}$$

The integration limits in the above expressions are:

$$\alpha_{\max}=\frac{1+\sqrt{1-4m^2/s}}{2}, \qquad \alpha_{\min}=\frac{1-\sqrt{1-4m^2/s}}{2}, \tag{B47}$$

$$\beta_{\max}=1-\alpha, \qquad \beta_{\min}=\frac{\alpha m^2}{\alpha s-m^2}. \tag{B48}$$

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