Properties of the coupling constants of J/ψ→VP decays *

HUANG Jin-Shu(黄金书)[1,2;1)] SUN Jun-Feng(孙俊峰)[2] LU Gong-Ru(鲁公儒)[2] LI Hai-Bo(李海波)[3]

[1] College of Physics and Electronic Engineering, Nanyang Normal University, Nanyang 473061, China
[2] College of Physics and Electronic Engineering, Henan Normal University, Xinxiang 453007, China
[3] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract: Based on the branching fractions of J/ψ→VP from different experiments, we investigate the properties of the coupling constants of J/ψ→VP decays using a model-dependent approach. We find that the octet coupling constant, g_8, of strong interaction is about twice larger than that of the singlet coupling constant g_1; the electromagnetic breaking parameters $g_{\rm E}^{\rm i}$ are larger than the mass breaking parameters $g_{\rm M}^{\rm i}$, moreover, the three parameters of the electromagnetic effect are about equal, but the three parameters of the mass effect are obviously different and their uncertainties are also large; and the phase angle between strong and electromagnetic interactions is in the range of 70°–80°. It deepens our understanding of the coupling constant of J/ψ→VP decays.

Key words: J/ψ decay, mixing angle, coupling constant, fit

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1 Introduction

The J/ψ decays play an important role in the understanding of low-energy hadron dynamics. A particular advantage of these decays is that the initial state is a very good approximation of a flavor-$SU(3)$ singlet. In order to estimate that $SU(3)$ is an approximate symmetry of low-energy hadron interactions, we need to study in detail the information on the final-state particles of J/ψ decay.

The implication of $SU(3)$ symmetry in J/ψ decays into mesons has been studied [1]. J/ψ can decay into a vector and a pseudoscalar mainly induced by three-gluon annihilation and electromagnetic processes. However, these branching ratios are only of the order 10^{-3} since the hadronic decays of J/ψ are suppressed by the Okubo-Zweig-Iizuka (OZI) rule [2].

Based on the $SU(3)$ symmetry, three quarks may make up an octet and one singlet. Because the mass of an s quark is larger than those of u and d quarks and mixing in the mesons exists, it will cause $SU(3)$ breaking and further create a physical nonet. The octet and singlet are the eigenstates of the $SU(3)$ group, however, what we can observe in experiment is not $SU(3)$ eigenstates themselves, but their mixing. For example, for the pseudoscalar mesons, if pseudoscalar glueballs and radially excited states are ignored and only quark states are considered, then the physical states η and η' are related to η^8 and η^0, via the usual mixing formulas

$$\begin{pmatrix}\eta\\ \eta'\end{pmatrix}=\begin{pmatrix}\cos\theta_{\rm P} & -\sin\theta_{\rm P}\\ \sin\theta_{\rm P} & \cos\theta_{\rm P}\end{pmatrix}\begin{pmatrix}\eta^8\\ \eta^0\end{pmatrix}, \tag{1}$$

where $\theta_{\rm P}$ is the mixing angle between η^8 and η^0. Similarly, for the vector mesons ϕ and ω, we have

$$\begin{pmatrix}\phi\\ \omega\end{pmatrix}=\begin{pmatrix}\cos\theta_{\rm V} & -\sin\theta_{\rm V}\\ \sin\theta_{\rm V} & \cos\theta_{\rm V}\end{pmatrix}\begin{pmatrix}\omega^8\\ \omega^0\end{pmatrix}, \tag{2}$$

where $\theta_{\rm V}$ is the mixing angle between ω^8 and ω^0.

For vector mesons, the mixing between ω^8 and ω^0 is basically an idea mixing, $\sin\theta_{\rm V}=\sqrt{1/3}$, $\cos\theta_{\rm V}=\sqrt{2/3}$, i.e., $\theta_{\rm V}\approx 35.3°$, so in this case we have

$$\omega=\sqrt{\frac{1}{2}}|u\bar{u}+d\bar{d}\rangle,\ \phi=|s\bar{s}\rangle. \tag{3}$$

However, for pseudoscalar mesons, it always brings people great interest to discuss the mixing between η^8 and η^0 [3–10].

Combining the mixings in vector mesons and pseudoscalar mesons and considering the various mass effects and electromagnetic effects, in this paper we shall analyze the properties of the coupling constants of J/ψ→VP decay, which is very important to comprehend the breaking of $SU(3)$ flavor symmetry.

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1) E-mail: jshuang@foxmail.com

2 Phenomenological study on the decays J/ψ→VP

2.1 Effective Lagrangian of J/ψ→VP

For the two-body decays $J/\psi\to H_1H_2$, where H_1 and H_2 denote mesons, the most general interaction term in an $SU(3)$ invariant is [2]

$$L_{\rm int}=\psi(g_8O_1^{\rm a}O_2^{\rm a}+g_1S_1S_2), \tag{4}$$

where O and S denote an octet and a singlet, respectively, and a sum over $a=1,2,\cdots,8$ is implied.

The $SU(3)$ symmetry in the nonet of pseudoscalar meson is not strict, two types of $SU(3)$ breaking should be considered. The first one is induced by the different mass of quarks, and the other one is the different charge of quarks.

In our theoretical calculation, $m_{\rm u}=m_{\rm d}$ is usually assumed, but $m_{\rm s}\neq m_{\rm d}$, this difference of mass will cause the breaking effect on $SU(3)$ symmetry

$$m_{\rm d}(\bar{u}u+\bar{d}d)+m_{\rm s}\bar{s}s=m_0\bar{q}q+\frac{1}{\sqrt{3}}(m_{\rm d}-m_{\rm s})\bar{q}\lambda_8q, \tag{5}$$

where $q=(u,\ d,\ s)$, $m_0=\dfrac{1}{3}(2m_{\rm d}+m_{\rm s})$ is the average quark mass, and λ_8 is the eighth of the Gell-Mann matrices. The last term in Eq. (5) is the mass effects of violating $SU(3)$ invariance. We need to introduce a new spurion $M^{\rm a}=\delta^{\rm a8}$ to describe this $SU(3)$ breaking term.

The electromagnetic effects violate $SU(3)$ symmetry since the photon coupling to quarks is proportional to the electric charge

$$\frac{2}{3}\bar{u}\gamma_\mu u-\frac{1}{3}\bar{d}\gamma_\mu d-\frac{1}{3}\bar{s}\gamma_\mu s=\frac{1}{2}\bar{q}\gamma_\mu\left(\lambda_3+\frac{1}{\sqrt{3}}\lambda_8\right)q, \tag{6}$$

where λ_3 and λ_8 denote the third and eighth Gell-Mann matrices, respectively. It follows from Eq. (6) that the electromagnetic breaking can be simulated by a spurion $E=\delta^{\rm a3}+\dfrac{1}{\sqrt{3}}\delta^{\rm a8}$.

After the above two effects of $SU(3)$ breaking are considered, the effective Lagrangian of the $J/\psi\to H_1H_2$ process can be written [2]

$$\begin{aligned}\mathcal{L}_{\rm eff}\ =\ \psi\Bigg\{&g_8O_1^{\rm a}O_2^{\rm a}+g_1S_1S_2+g_{\rm S}d_{\rm abc}O_1^{\rm a}O_2^{\rm b}O_3^{\rm c}\\&+g_{\rm A}f_{\rm abc}O_1^{\rm a}O_2^{\rm b}O_3^{\rm c}+\sqrt{\frac{2}{3}}[C_{123}O_1^{\rm a}O_2^{\rm a}S_3\\&+C_{132}O_1^{\rm a}O_3^{\rm a}S_2+C_{231}O_2^{\rm a}O_3^{\rm a}S_1+fS_1S_2S_3]\Bigg\}.\end{aligned} \tag{7}$$

If the final states of $J/\psi\to H_1H_2$ decays are a vector and a pseudoscalar, the expression of the above effective Lagrangian can be further written as

$$\begin{aligned}\mathcal{L}_{\rm eff}\ =\ \psi\Bigg\{&g_8P_1^{\rm a}V_2^{\rm a}+g_1P_0V_0+g_{\rm M}^{88}d_{\rm abc}O_1^{\rm a}O_2^{\rm b}M_3^{\rm c}\\&+\sqrt{\frac{2}{3}}\left[g_{\rm M}^{81}O_1^{\rm a}M_3^{\rm a}S_2+g_{\rm M}^{18}O_2^{\rm a}M_3^{\rm a}S_1\right]+g_{\rm E}^{88}d_{\rm abc}O_1^{\rm a}\\&\times O_2^{\rm b}E_3^{\rm c}+\sqrt{\frac{2}{3}}\left[g_{\rm E}^{81}O_1^{\rm a}E_3^{\rm a}S_2+g_{\rm E}^{18}O_2^{\rm a}E_3^{\rm a}S_1\right]\Bigg\},\end{aligned} \tag{8}$$

in which g_8 and g_1 denote the coupling constants of the octet and singlet, $g_{\rm M}^{\rm i}$ and $g_{\rm E}^{\rm i}$ are the coupling constants of the mass breaking term and electromagnetic breaking term, respectively, $f_{\rm abc}$ and $d_{\rm abc}$ coefficients are the antisymmetrical and symmetrical structure constants of the $SU(3)$ group.

2.2 Decay amplitude and width of J/ψ → VP decays

The physical particles in corresponding to the pseudoscalar and vector mesons are

$$\begin{aligned}P_1\ &=\ \frac{1}{\sqrt{2}}(\pi^++\pi^-),\\P_2\ &=\ \frac{\rm i}{\sqrt{2}}(\pi^+-\pi^-),\\P_3\ &=\ \pi^0,\\P_4\ &=\ \frac{1}{\sqrt{2}}(K^++K^-),\\P_5\ &=\ \frac{\rm i}{\sqrt{2}}(K^+-K^-),\\P_6\ &=\ \frac{1}{\sqrt{2}}(K^0+\overline{K}^0)=K_{\rm S}^0,\\P_7\ &=\ \frac{\rm i}{\sqrt{2}}(K^0-\overline{K}^0)=iK_{\rm L}^0,\\P_8\ &=\ \eta_8^0=\eta\cos\theta_{\rm P}+\eta'\sin\theta_{\rm P},\\P_0\ &=\ \eta_1^0=-\eta\sin\theta_{\rm P}+\eta'\cos\theta_{\rm P},\end{aligned} \tag{9}$$

and

$$\begin{aligned}V_1\ &=\ \frac{1}{\sqrt{2}}(\rho^++\rho^-),\\V_2\ &=\ \frac{\rm i}{\sqrt{2}}(\rho^+-\rho^-),\\V_3\ &=\ \rho^0,\\V_4\ &=\ \frac{1}{\sqrt{2}}(K^{*+}+K^{*-}),\end{aligned}$$

$$\begin{aligned}
V_5 &= \frac{\mathrm{i}}{\sqrt{2}}(K^{*+}-K^{*-}),\\
V_6 &= \frac{1}{\sqrt{2}}(K^{*0}+\overline{K}^{*0}),\\
V_7 &= \frac{\mathrm{i}}{\sqrt{2}}(K^{*0}-\overline{K}^{*0}),\\
V_8 &= \phi\cos\theta_{\mathrm{V}}+\omega\sin\theta_{\mathrm{V}},\\
V_0 &= -\phi\sin\theta_{\mathrm{V}}+\omega\cos\theta_{\mathrm{V}}, \qquad (10)
\end{aligned}$$

then we calculate in detail the effective Lagrangian of J/ψ→VP decays

$$\begin{aligned}
P_1^{\mathrm{a}}V_2^{\mathrm{a}} &= P^1V^1+P^2V^2+P^3V^3+P^4V^4+P^5V^5+P^6V^6\\
&\quad+P^7V^7+P^8V^8\\
&= \pi^+\rho^-+\pi^-\rho^++\pi^0\rho^0+K^+K^{*-}\\
&\quad+K^-K^{*+}+K^0\overline{K}^{*0}+\overline{K}^0K^{*0}\\
&\quad+\eta\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}+\eta\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta'\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}\\
&\quad+\eta'\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}, \qquad (11)
\end{aligned}$$

$$\begin{aligned}
S_1S_2 &= P^0V^0=-\eta\omega\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta\phi\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}\\
&\quad+\eta'\omega\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta'\phi\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}, \qquad (12)
\end{aligned}$$

$$\begin{aligned}
d_{\mathrm{abc}}O_1^{\mathrm{a}}O_2^{\mathrm{b}}M_3^{\mathrm{c}} &= d_{\mathrm{ab8}}P^{\mathrm{a}}V^{\mathrm{b}}=d_{118}P^1V^1+d_{228}P^2V^2\\
&\quad+d_{338}P^3V^3+d_{448}P^4V^4+d_{558}P^5V^5\\
&\quad+d_{668}P^6V^6+d_{778}P^7V^7+d_{888}P^8V^8\\
&= \frac{1}{\sqrt{3}}[\pi^+\rho^-+\pi^-\rho^++\pi^0\rho^0]\\
&\quad+\frac{-1}{2\sqrt{3}}[K^+K^{*-}+K^-K^{*+}+K^0\overline{K}^{*0}\\
&\quad+\overline{K}^0K^{*0}]+\frac{-1}{\sqrt{3}}[\eta\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}\\
&\quad+\eta\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}+\eta'\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}\\
&\quad+\eta'\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}], \qquad (13)
\end{aligned}$$

$$\begin{aligned}
O_1^{\mathrm{a}}M_3^{\mathrm{a}}S_2 &= P^8V^0=\eta\omega\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta\phi\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}\\
&\quad+\eta'\omega\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}-\eta'\phi\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}, \qquad (14)
\end{aligned}$$

$$\begin{aligned}
O_2^{\mathrm{a}}M_3^{\mathrm{a}}S_1 &= P^0V^8=-\eta\omega\sin\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}-\eta\phi\sin\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}\\
&\quad+\eta'\omega\cos\theta_{\mathrm{P}}\sin\theta_{\mathrm{V}}+\eta'\phi\cos\theta_{\mathrm{P}}\cos\theta_{\mathrm{V}}, \qquad (15)
\end{aligned}$$

$$\begin{aligned}
d_{\mathrm{ab3}}P^{\mathrm{a}}V^{\mathrm{b}} &= d_{383}P^3V^8+d_{833}P^8V^3+d_{443}P^4V^4\\
&\quad+d_{553}P^5V^5+d_{663}P^6V^6+d_{773}P^7V^7\\
&= \pi^0\omega\sqrt{\frac{1}{3}}\sin\theta_{\mathrm{V}}+\pi^0\phi\sqrt{\frac{1}{3}}\cos\theta_{\mathrm{V}}\\
&\quad+\eta\rho^0\sqrt{\frac{1}{3}}\cos\theta_{\mathrm{P}}+\eta'\rho^0\sqrt{\frac{1}{3}}\sin\theta_{\mathrm{P}}+\frac{1}{2}K^+K^{*-}\\
&\quad+\frac{1}{2}K^-K^{*+}-\frac{1}{2}K^0\overline{K}^{*0}-\frac{1}{2}\overline{K}^0K^{*0}, \qquad (16)
\end{aligned}$$

$$d_{\mathrm{abc}}O_1^{\mathrm{a}}O_2^{\mathrm{b}}E_3^{\mathrm{c}}=d_{\mathrm{ab3}}P^{\mathrm{a}}V^{\mathrm{b}}+\frac{1}{\sqrt{3}}d_{\mathrm{ab8}}P^{\mathrm{a}}V^{\mathrm{b}}, \qquad (17)$$

$$O_1^{\mathrm{a}}E_3^{\mathrm{a}}S_2 = P^3V^0+\frac{1}{\sqrt{3}}P^8V^0, \qquad (18)$$

$$\begin{aligned}
P^3V^0 &= \pi^0(\omega\cos\theta_{\mathrm{V}}-\phi\sin\theta_{\mathrm{V}})\\
&= \sqrt{\frac{2}{3}}\pi^0\omega-\sqrt{\frac{1}{3}}\pi^0\phi, \qquad (19)
\end{aligned}$$

$$O_2^{\mathrm{a}}E_3^{\mathrm{a}}S_1 = V^3P^0+\frac{1}{\sqrt{3}}V^8P^0, \qquad (20)$$

$$\begin{aligned}
V^3P^0 &= \rho^0(-\eta\sin\theta_{\mathrm{P}}+\eta'\cos\theta_{\mathrm{P}})\\
&= -\eta\rho^0\sin\theta_{\mathrm{P}}+\eta'\rho^0\cos\theta_{\mathrm{P}}, \qquad (21)
\end{aligned}$$

therefore we obtain the coupling constants and their corresponding strengths of J/ψ→PV decays given in Table 1. It should be noted that our results are similar to Ref. [2], but we distinguish in detail the different parameters of mass effect and electromagnetic effect.

The amplitude of J/ψ→VP decays is

$$\mathcal{M}=\frac{g_{\psi\mathrm{VP}}}{m_{\psi}}\varepsilon_{\mu\nu\rho\sigma}p_{\psi}^{\mu}\varepsilon_{\psi}^{\nu}p_{\mathrm{V}}^{\rho}\varepsilon_{\mathrm{V}}^{*\sigma}, \qquad (22)$$

where $g_{\psi\mathrm{VP}}$ is the coupling constant and its various expressions are tabulated in Table 1, so we have

$$\begin{aligned}
\overline{|\mathcal{M}|^2} &= \frac{1}{2s_{\psi}+1}\sum_{s_{\psi}}\sum_{s_{\mathrm{V}}}\mathcal{M}\mathcal{M}^{\dagger}\\
&= \frac{2|g_{\psi\mathrm{VP}}|^2}{3m_{\psi}^2}\left\{\left(\frac{m_{\psi}^2+m_{\mathrm{V}}^2-m_{\mathrm{P}}^2}{2}\right)^2-m_{\psi}^2m_{\mathrm{V}}^2\right\}\\
&= \frac{2}{3}|g_{\psi\mathrm{VP}}|^2p^2, \qquad (23)
\end{aligned}$$

in which $\vec{p}$ is the momenta of vector meson in the c.m. frame, and can be denoted as

$$p=|\vec{p}|=\frac{\sqrt{\left[m_{\psi}^2-(m_{\mathrm{V}}+m_{\mathrm{P}})^2\right]\left[m_{\psi}^2-(m_{\mathrm{V}}-m_{\mathrm{P}})^2\right]}}{2m_{\psi}}. \qquad (24)$$

The differential decay rate for J/ψ→VP can be written as

$$\mathrm{d}\Gamma=\frac{1}{32\pi^2}\frac{p}{m_\psi^2}\overline{|\mathcal{M}|^2}\mathrm{d}\Omega, \quad (25)$$

so we have the total decay width

$$\Gamma=\int\frac{1}{32\pi^2}\frac{p}{m_\psi^2}\overline{|\mathcal{M}|^2}\mathrm{d}\Omega=\frac{1}{3}\frac{p^3}{m_\psi^2}\frac{|g_{\psi\mathrm{VP}}|^2}{4\pi}. \quad (26)$$

3 Numerical analysis and discussions

In Table 1, we present the free parameters involved in the decays J/ψ→PV. In these parameters, g_M^{88}, g_M^{81} and g_M^{18} denote the $SU(3)$ mass breaking terms induced by the different quark mass, g_E^{88}, g_E^{81} and g_E^{18} denote the $SU(3)$ electromagnetic terms induced by the different quark electric charge. There is a phase angle, noted by δ_E, between electromagnetic and strong interaction. We can take the coupling constants of the mass effect as real, and denote the coupling constants of electromagnetic effect with $g_\mathrm{E}^\mathrm{i}=|g_\mathrm{E}^\mathrm{i}|\mathrm{e}^{\delta_\mathrm{E}}$. Therefore there are eleven free parameters in the decays J/ψ→PV, they are g_8, g_1, g_M^{88}, g_M^{81}, g_M^{18}, $|g_\mathrm{E}^{88}|$, $|g_\mathrm{E}^{81}|$, $|g_\mathrm{E}^{18}|$, δ_E, θ_P and θ_V. For the sake of simplicity, we shall take the following four cases to analyze and discuss the properties of these parameters.

Case Ⅰ: Assuming $g_\mathrm{M}^{88}=g_\mathrm{M}^{81}=g_\mathrm{M}^{18}=g_\mathrm{M}$, $g_\mathrm{E}^{88}=g_\mathrm{E}^{81}=g_\mathrm{E}^{18}=g_\mathrm{E}$, and taking the mixing between ϕ and ω as idea mixing, i.e., $\theta_\mathrm{V}\approx 35.3°$. In this case, there are only six parameters, they are g_8, g_1, g_M, $|g_\mathrm{E}|$, δ_E and θ_P.

Case Ⅱ: Assuming $g_\mathrm{M}^{88}=g_\mathrm{M}^{81}=g_\mathrm{M}^{18}=g_\mathrm{M}$, $g_\mathrm{E}^{88}=g_\mathrm{E}^{81}=g_\mathrm{E}^{18}=g_\mathrm{E}$, but taking the mixing angle θ_V regard as free. In this case, there are seven parameters which are g_8, g_1, g_M, $|g_\mathrm{E}|$, δ_E, θ_P, θ_V respectively.

Case Ⅲ: Assuming that the three parameters of mass effects are not equal, and the three parameters of electromagnetic effects are not equal, either, they are all free parameters, but taking $\theta_\mathrm{V}\approx 35.3°$, then we shall have 10 parameters: g_8, g_1, g_M^{88}, g_M^{81}, g_M^{18}, $|g_\mathrm{E}^{88}|$, $|g_\mathrm{E}^{81}|$, $|g_\mathrm{E}^{18}|$, δ_E, and θ_P.

Table 1. The coupling constants and their corresponding strengths of J/ψ→PV.

decay modes	coupling constants and their corresponding strengths							
J/ψ→PV	g_8	g_1	g_M^{88}	g_M^{81}	g_M^{18}	g_E^{88}	g_E^{81}	g_E^{18}
$\pi^+\rho^-$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^-\rho^+$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\pi^0\rho^0$	1	0	$\frac{1}{\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\mathrm{K}^+\mathrm{K}^{*-}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\mathrm{K}^-\mathrm{K}^{*+}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{1}{3}$	0	0
$\mathrm{K}^0\overline{\mathrm{K}}^{*0}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{-2}{3}$	0	0
$\overline{\mathrm{K}}^0\mathrm{K}^{*0}$	1	0	$\frac{-1}{2\sqrt{3}}$	0	0	$\frac{-2}{3}$	0	0
$\pi^0\omega$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\sin\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\cos\theta_\mathrm{V}$	0
$\pi^0\phi$	0	0	0	0	0	$-\sqrt{\frac{1}{3}}\cos\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\sin\theta_\mathrm{V}$	0
$\eta\rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\cos\theta_\mathrm{P}$	0	$-\sqrt{\frac{2}{3}}\sin\theta_\mathrm{P}$
$\eta'\rho^0$	0	0	0	0	0	$\sqrt{\frac{1}{3}}\sin\theta_\mathrm{P}$	0	$\sqrt{\frac{2}{3}}\cos\theta_\mathrm{P}$
$\eta\omega$	$\cos\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$-\sin\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\frac{-1}{\sqrt{3}}\cos\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\cos\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$-\sqrt{\frac{2}{3}}\sin\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta\phi$	$\cos\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\sin\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$-\frac{1}{\sqrt{3}}\cos\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$-\sqrt{\frac{2}{3}}\cos\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$-\sqrt{\frac{2}{3}}\sin\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta'\omega$	$\sin\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\cos\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\frac{-1}{\sqrt{3}}\sin\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\sin\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\cos\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.
$\eta'\phi$	$\sin\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$-\cos\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$-\frac{1}{\sqrt{3}}\sin\theta_\mathrm{P}\cos\theta_\mathrm{V}$	$-\sqrt{\frac{2}{3}}\sin\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\sqrt{\frac{2}{3}}\cos\theta_\mathrm{P}\sin\theta_\mathrm{V}$	$\frac{1}{\sqrt{3}}$4th colu.	$\frac{1}{\sqrt{3}}$5th colu.	$\frac{1}{\sqrt{3}}$6th colu.

Case Ⅳ: Assuming the three parameters of mass effects are not equal, and the three parameters of electromagnetic effects are not equal, either, further taking the mixing angle θ_V as free, then all of the eleven parameters are free.

For the latter two cases, we have to request more experimental information to analyze them due to too many parameters.

The experimental results for the decays J/ψ→PV are mainly from Mark-III [11], DM2 [12] and BES [13, 14], which are shown in Table 2. The last column in this table is the latest world average in 2012 [1]. To clarify the results obtained from different data sets, we divided them into three subsections to investigate the properties of the coupling constants of the decays J/ψ→PV.

3.1 Analysis of J/ψ→VP from Mark-Ⅲ, DM2 and PDG2012 data in Case Ⅰ

First, we consider the simplest case, i.e., Case Ⅰ. In Ref. [15], the pseudoscalar mixing in J/ψ and ψ(2S) decays has been analyzed, which shows η flavors only consist of light quarks and η' has a room for gluonium admixture. However, in this paper we shall only consider their quark content in order to analyze the properties of coupling constants from $SU(3)$ breaking. The fit results are given in Table 3. From this table, we can obtain the following results: (i) the coupling constant of the octet strong interaction, g_8, is about twice larger than that one of the singlet, g_1; (ii) the $SU(3)$ breaking coupling constant from the electromagnetic effect is large, about the same order of g_8 and g_1, however, the $SU(3)$ breaking coupling constant from the mass effect is rather small, about one order smaller than those of g_8 and g_1, moreover its uncertainty is also very large; (iii) the phase angle between strong and electromagnetic interactions is about $\frac{2}{5}\pi$; (iv) the mixing angle in η and η', θ_P, is about $-20°$, which is consistent with the reasonable range $-20°\sim-10°$ [16]; and (v) compared with the results of Mark-III and DM2 data, the quality of the fit to PDG2012 data is very large.

3.2 Analysis of J/ψ→VP from Mark-Ⅲ, DM2 and PDG2012 data in Case Ⅱ

Next, we consider Case Ⅱ, i.e., the ω-ϕ mixing angle is left as a free parameter. The fit results are listed in Table 4. From this table, we can see that the results of this case are similar to those of Case Ⅰ only, but the fit results to MarkIII data, $\theta_P=-15.2\pm2.93$ are obviously different from those of Case Ⅰ. The table shows that the mixing in ω and ϕ is basically an idea mixing.

3.3 Analysis of the branching ratios of J/ψ→VP in Case Ⅲ and Ⅳ

Finally, we consider the latter two cases. There are 10 and 11 free parameters in Case Ⅲ and Ⅳ, respectively. Because each of the experimental results are not sufficient do a reasonable fit for 10 or 11 free parameters in these two cases, we try to figure out these free parameters by combining the data from the different collaborations. Such as for Case Ⅲ, the results of fit from Mark-Ⅲ+BES data can be written as

$$\begin{aligned} g_8&=(6.57\pm0.16)\times10^{-3},\\ g_1&=(3.14\pm0.15)\times10^{-3},\\ g_M^{88}&=(18.1\pm4.01)\times10^{-4},\\ g_M^{81}&=(5.78\pm4.20)\times10^{-4},\\ g_M^{18}&=(3.23\pm0.21)\times10^{-4},\\ |g_E^{88}|&=(1.73\pm0.22)\times10^{-3},\\ |g_E^{81}|&=(2.74\pm0.20)\times10^{-3},\\ |g_E^{18}|&=(2.18\pm0.15)\times10^{-3},\\ \delta_E&=69.5\pm12.6,\quad \theta_P=-19.0\pm3.44. \end{aligned} \tag{27}$$

We can see that (i) the coupling constants of strong interaction, g_8 and g_1, are basically consistent with those of Case Ⅰ and Case Ⅱ; (ii)the difference of three coupling constants of electromagnetic effect is not obvious, but three coupling constants of the mass effect have a rather large difference. Moreover their uncertainties are rather large; and (iii) the quality of the fit in this case is very large, it is a possible reason for dealing with it together with the data from the different collaborations.

For Case Ⅳ, the results of fit from Mark-Ⅲ and BES data are

$$\begin{aligned} g_8&=(6.57\pm0.16)\times10^{-3},\\ g_1&=(3.05\pm0.15)\times10^{-3},\\ g_M^{88}&=(18.7\pm3.30)\times10^{-4},\\ g_M^{81}&=(11.0\pm1.47)\times10^{-4},\\ g_M^{18}&=(7.00\pm0.68)\times10^{-4},\\ |g_E^{88}|&=(1.81\pm0.22)\times10^{-3},\\ |g_E^{81}|&=(2.70\pm0.20)\times10^{-3},\\ |g_E^{18}|&=(2.14\pm0.15)\times10^{-3},\\ \delta_E&=78.0\pm2.56,\quad \theta_P=-18.0\pm0.53,\\ \theta_V&=35.9\pm1.47,\quad \chi^2/\text{d.o.f}=36.1/5. \end{aligned} \tag{28}$$

This case is similar to Case Ⅲ. The difference of three coupling constants of electromagnetic effect is still not obvious, three coupling constants of the mass effect have a rather large difference, and at the same time, the phase angle between strong and electromagnetic interactions is slightly large.

The results of fit from DM2+BES combination and Mark-Ⅲ+DM2 combination, are similar to those of Mark-Ⅲ+BES. Their numerical results are not given individually.

Table 2. The branching fractions of J/ψ→VP from Mark-III, DM2, BES and PDG2012 ($\times10^{-3}$).

decay modes	Mark-III	DM2	BES	PDG2012
ρπ	14.2±0.1±1.9	13.2±2.0	21.8±0.05±2.01	16.9±1.5
ρη	0.193±0.013±0.029	0.194±0.017±0.029		0.193±0.023
ρη′	0.114±0.014±0.016	0.083±0.030±0.012		0.105±0.018
ϕπ	<0.0068		<0.0064	<0.0064
ϕη	0.661±0.045±0.078	0.64±0.04±0.11	0.898±0.024±0.089	0.75±0.08
ϕη′	0.308±0.034±0.036	0.41±0.03±0.08	0.546±0.031±0.056	0.40±0.07
ωπ	0.482±0.019±0.064	0.360±0.028±0.054	0.538±0.012±0.065	0.45±0.05
ωη	1.71±0.08±0.20	1.43±0.10±0.21	2.352±0.273	1.74±0.20
ωη′	0.166±0.017±0.019	$0.18^{+0.10}_{-0.08}\pm0.03$	0.226±0.043	0.182±0.021
$K^{*-}K^{+}$+c.c.	5.26±0.13±0.53	4.57±0.17±0.70		5.12±0.30
$K^{*0}\bar{K}^{0}$+c.c.	4.33±0.12±0.45	3.96±0.15±0.60		4.39±0.31

Table 3. The fit results to MarkIII, DM2 and PDG2012 data in Case Ⅰ.

parameter	Mark-III	DM2	PDG2012
g_8 ($\times10^{-3}$)	5.89±0.18	5.57±0.22	5.98±0.13
g_1 ($\times10^{-3}$)	2.72±0.20	3.12±0.40	2.77±0.26
g_M ($\times10^{-4}$)	8.08±3.21	5.84±4.22	9.44±2.72
$\lvert g_E\rvert$ ($\times10^{-3}$)	2.11±0.10	1.95±0.11	2.06±0.08
δ_E	71.4±11.5	74.9±17.0	75.8±6.87
θ_P	−19.5±1.52	−20.0±0.68	−19.0±1.59
χ^2/d.o.f	7.19/4	3.63/4	19.4/4

Table 4. The fit results to Mark-III, DM2 and PDG2012 data in Case Ⅱ.

parameter	Mark-III	DM2	PDG2012
g_8 ($\times10^{-3}$)	5.87±0.18	5.57±0.22	5.99±0.13
g_1 ($\times10^{-3}$)	2.84±0.21	3.12±0.42	2.84±0.29
g_M ($\times10^{-4}$)	5.75±3.40	5.88±4.84	8.42±3.32
$\lvert g_E\rvert$ ($\times10^{-3}$)	2.15±0.10	1.94±0.12	2.07±0.08
δ_E	71.7±11.1	74.9±16.9	76.0±6.81
θ_P	−15.2±2.93	−20.1±5.06	−17.8±2.68
θ_V	41.7±3.65	35.4±7.27	37.5±3.42
χ^2/d.o.f	4.95/3	3.63/3	19.2/3

4 Conclusions

Based on the general phenomenological model, we have studied the properties of the coupling constants of the decays J/ψ→VP. Considering the experimental data of Mark-III, DM2 and BES Collaborations and the world average in 2012 for J/ψ→VP decays, we can find that (i) the octet coupling constant of strong interaction g_8 is about twice larger than that of the singlet coupling constant g_1; (ii) the electromagnetic breaking terms g_E^i are larger, about the same order of g_8 and g_1, the difference of three coupling constants g_E^i isn't obvious, then it is appropriate that we take them as equal in Case Ⅰ and Ⅱ; (iii) the $SU(3)$ breaking coupling constant from the mass effect is rather small, about one order smaller than those of g_8 and g_1. Moreover, its uncertainty is quite large. It is a very strong assumption that we take them as equal in Case Ⅰ and Ⅱ; (iv) the phase angle between strong and electromagnetic interactions is about the range of 70°–80°; (v) the mixing angle in η and η', θ_P, is about the range of −15°–−20°, which is consistent with the reasonable range usually considered; (vi) the mixing angle θ_V is about the rang of 35°–37°. It means the ϕ−ω mixing is basically an idea mixing. Therefore in the breaking of $SU(3)$ favor symmetry, the electromagnetic effects are large, moreover the mass effects are comparatively small, which provides useful information for comprehending the breaking of $SU(3)$ flavor symmetry.

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