Pseudospin symmetry of the Dirac equation for a Möbius square plus Mie type potential with a Coulomb-like tensor interaction via SUSYQM

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Abstract: We investigate the approximate solution of the Dirac equation for a combination of Möbius square and Mie type potentials under the pseudospin symmetry limit by using supersymmetry quantum mechanics. We obtain the bound-state energy equation and the corresponding spinor wave functions in an approximate analytical manner. We comment on the system via various useful figures and tables.

Key words: Dirac equation, Möbius potential, Mie type potential, supersymmetry quantum mechanics

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1 Introduction

In order to investigate the nuclear shell structure, the study of spin and pseudospin symmetries of the Dirac equation is in many cases a proper building block up which to proceed [1–4]. The spin symmetry in a Dirac equation occurs when the difference of the potential of the vector potential $V(r)$ and the scalar potential $S(r)$ is a constant ($\Delta(r)=V(r)-S(r)=$ constant) [5, 6]. The so-called pseudospin symmetry corresponds to the case where the sum of the scalar and vector potentials equals a constant term ($\Sigma(r)=V(r)+S(r)=$ constant) [7]. The jargon pseudospin symmetry in nuclear theory refers to a quasi-degeneracy between single-nucleon doublets with quantum numbers $\left(n,l,j=l+\frac{1}{2}\right)$ and $\left(n-1,l+2,j=l+\frac{3}{2}\right)$, where n, l and j are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively [4]. The pseudospin symmetry is exact when doublets with $j=\tilde{l}+\tilde{s}$, with $\tilde{l}=l+1$ and $\tilde{s}=\frac{1}{2}$, are degenerate [5]. These symmetries, under various phenomenological potentials, have been investigated by using various methods such as asymptotic iteration method (AIM) [8], the Nikiforov-Uvarov (NU) technique [9], supersymmetric quantum mechanics (SUSSYQM) [10], shape invariance (SI) [11], and exact quantization rule [12]. In recent years, many researchers have applied spin and pseudospin symmetry conditions on a numbers of potentials [13–30] including Pöschl-Teller [13], Hulthén [14], Pöschl Teller double-ring-shaped Coulomb [15], pseudoharmonic [16], modified deformed Hylleraas [17], harmonic oscillator [18] and Kratzer potentials [19]. The main aim of the present paper is to obtain approximate solutions of the Dirac equation with a Möbius square plus Mie type potential (MS-M) including the Coulomb-like potential for the pseudospin symmetry limit. The Möbius square potential, due to its complicated nature, has not been analyzed within the framework. The Morse, Pöschl-Teller, Manning-Rosen, Hulthén, Tietz, Eckart and Hua potentials are all special cases of this potential [31, 32]. Recently, we approximately solved the Dirac equation for the Möbius square potential under spin and pseudospin symmetry for any κ state and obtained the eigenvalue equation as well as the corresponding two-component spinors within the framework of an approximation to the term proportional to $1/r^2$ by using the NU method [32] Some new work about spin and pseudospin symmetry can be found [33–43]. In addition, the Mie type potential has the general features of a physical interaction [44]. Using the NU method, exact solutions of the Dirac equation with the Mie type potential under pseudospin and spin symmetry limits have been obtained [45]. The Dirac equation under Möbius square plus Mie type potentials be solved by using supersymmetry quantum mechanics (SUSYQM), Nikiforov-Uvarov (NU), etc. The Mie type potential has been already investigated in other cases. Agboola and Ikhdair et al., in separate work, obtained solutions of

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the N-dimensional Schrödinger equation with the potential [46, 47]. The path integral solution for a one-dimensional Mie type potential was studied in Ref. [48]. The Schrödinger equation with the potential was also analyzed via the $1/N$ expansion method [49]. The combination of the Mie type potential with the pseudoharmonic interaction within the nonrelativistic formalism was carried out via the Laplace transform approach in Ref. [50]. The term was also studied as a framework to analyze the diatomic molecules [51, 52]. We modify the Mie type potential with the Möbius square potential which is in fact equivalent to the Eckart/ Rosen–Morse potential [53]. Here, we apply the supersymmetric quantum mechanics (SUSYQM) and investigate the effects of tensor interaction on the bound states. This paper is organized as follows: In Section 2, we give a brief introduction of the SUSYQM by which we are going to solve our problem. In Section 3, the Dirac equation is written in the pseudospin limit with a Coulomb tensor term. We report the approximate analytical solution of the problem for any state in Section 4. Discussion of our numerical result is given in Section 5 and the conclusion is presented in Section 6.

2 Supersymmetry

We include this short introduction to SUSYQM to proceed on a more continuous manner. In SUSUQM we normally deal with the partner Hamiltonians [10]

$$H_{\pm}=\frac{p^2}{2m}+V_{\pm}(x), \tag{1}$$

where

$$V_{\pm}(x)=\Phi^2(x)\pm\Phi'(x). \tag{2}$$

In the case of good SUSY, i.e. $E_0=0$, the ground state of the system is obtained via

$$\phi_0^-(x)=C\mathrm{e}^{-U}, \tag{3}$$

where C is a normalization constant and

$$U(x)=\int_{x_0}^{x}\mathrm{d}z\Phi(z). \tag{4}$$

In the next step, we have to see whether the shape-invariance condition

$$V_+(a_0,x)=V_-(a_1,x)+R(a_1), \tag{5}$$

exists. In Eq. (5), a_1 is a new set of parameters uniquely determined from the old set a_0 via the mapping F: $a_0\mapsto a_1=F(a_0)$ and $R(a_1)$ does not include x. If the condition is satisfied (with the requirements), we simply have the higher states obtained via

$$E_n = \sum_{s=1}^{n}R(a_s), \tag{6a}$$

$$\phi_n^-(a_0,x) = \prod_{s=0}^{n-1}\left(\frac{A^\dagger(a_s)}{[E_n-E_s]^{1/2}}\right)\phi_0^-(a_n,x), \tag{6b}$$

$$\phi_0^-(a_n,x) = C\exp\left\{-\int_0^x\mathrm{d}z\Phi(a_n,z)\right\}, \tag{6c}$$

where

$$A_s^\dagger=-\frac{\partial}{\partial x}+\Phi(a_s,x). \tag{7}$$

Therefore, this condition determines the spectrum of the bound states of the Hamiltonian

$$H_s=-\frac{\partial^2}{\partial x^2}+V_-(a_s,x)+E_s, \tag{8}$$

and the eigenfunctions of

$$H_s\phi_{n-s}^-(a_s,x)=E_n\phi_{n-s}^-(a_s,x),\ n\geqslant s \tag{9}$$

are obtained via [1–3]

$$\phi_{n-s}^-(a_s,x)=\frac{A^\dagger}{[E_n-E_s]^{1/2}}\phi_{n-(s+1)}^-(a_{s+1},x). \tag{10}$$

3 Dirac equation with a tensor coupling

The Dirac equation with an attractive scalar potential $S(r)$, a repulsive vector potential $V(r)$ and a tensor potential $U(r)$ in the relativistic unit ($\hbar$=c=1) is [13]

$$\begin{aligned}&[\vec{\alpha}.\vec{p}+\beta(M+S(r))-\mathrm{i}\beta\vec{\alpha}.\hat{r}U(r)]\psi(r)\\&=[E-V(r)]\psi(r),\end{aligned} \tag{11}$$

where E is the relativistic energy of the system, $\vec{p}=-\mathrm{i}\vec{\nabla}$ is the three dimensional momentum operator and M is the mass of the fermionic particle. $\vec{\alpha}$, β are the 4×4 Dirac matrices given as

$$\vec{\alpha}=\begin{pmatrix}0 & \vec{\sigma}_i\\ \vec{\sigma}_i & 0\end{pmatrix},\ \beta=\begin{pmatrix}I & 0\\ 0 & -I\end{pmatrix}, \tag{12}$$

where I is 2×2 unitary matrix and $\vec{\sigma}_i$ are the Pauli three-vector matrices

$$\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\sigma_2=\begin{pmatrix}0 & -\mathrm{i}\\ \mathrm{i} & 0\end{pmatrix},\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}. \tag{13}$$

The eigenvalues of the spin-orbit coupling operator are $\kappa=\left(j+\frac{1}{2}\right)>0$, $\kappa=-\left(j+\frac{1}{2}\right)<0$ for unaligned spin $j=l-\frac{1}{2}$ and aligned spin $j=l+\frac{1}{2}$, respectively. The set (H, K, J^2, J_z) forms a complete set of conserved quantities. Thus, we can write the spinors as [13]

$$\psi_{n\kappa}(r)=\frac{1}{r}\begin{pmatrix}F_{n\kappa}(r)Y^{l}_{jm}(\theta,\varphi)\\ \mathrm{i}G_{n\kappa}(r)Y^{\tilde{l}}_{jm}(\theta,\varphi)\end{pmatrix}, \tag{14}$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ represent the upper and lower components of the Dirac spinors, respectively. $Y^{l}_{jm}(\theta,\varphi)$ and $Y^{\tilde{l}}_{jm}(\theta,\varphi)$ are the spin and pseudospin spherical harmonics and m is the projection on the z-axis. With other known identities [13]

$$\left(\vec{\sigma}.\vec{A}\right)\left(\vec{\sigma}.\vec{B}\right) = \vec{A}.\vec{B}+\mathrm{i}\vec{\sigma}.\left(\vec{A}x\vec{B}\right), \tag{15a}$$

$$\vec{\sigma}.\vec{p} = \vec{\sigma}.\hat{r}\left(\hat{r}.\vec{p}+\mathrm{i}\frac{\vec{\sigma}.\vec{L}}{r}\right), \tag{15b}$$

as well as

$$\left(\vec{\sigma}.\vec{L}\right)Y^{\tilde{l}}_{jm}(\theta,\varphi)=(\kappa-1)Y^{\tilde{l}}_{jm}(\theta,\varphi),$$

$$\left(\vec{\sigma}.\vec{L}\right)Y^{l}_{jm}(\theta,\varphi)=-(\kappa-1)Y^{l}_{jm}(\theta,\varphi),$$

$$(\vec{\sigma}.\hat{r})Y^{l}_{jm}(\theta,\varphi)=-Y^{\tilde{l}}_{jm}(\theta,\varphi),$$

$$(\vec{\sigma}.\hat{r})Y^{\tilde{l}}_{jm}(\theta,\varphi)=-Y^{l}_{jm}(\theta,\varphi). \tag{16}$$

We find the following two coupled first-order Dirac equations

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}-U(r)\right)F_{n\kappa}(r) = (M+E_{n\kappa}-\Delta(r))G_{n\kappa}(r), \tag{17}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}+U(r)\right)G_{n\kappa}(r) = (M-E_{n\kappa}+\Sigma(r))F_{n\kappa}(r), \tag{18}$$

where

$$\Delta(r)=V(r)-S(r), \tag{19}$$

$$\Sigma(r)=V(r)+S(r). \tag{20}$$

Eliminating $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ in Eqs. (17) and (18), we obtain

$$\left\{\begin{array}{l}\frac{\mathrm{d}^2}{\mathrm{d}r^2}-\frac{\kappa(\kappa+1)}{r^2}+\frac{2\kappa U(r)}{r}-\frac{\mathrm{d}U(r)}{\mathrm{d}r}-U^2(r)-(M+E_{n\kappa}-\Delta(r))(M-E_{n\kappa}+\Sigma(r))\\ +\frac{\frac{\mathrm{d}\Delta(r)}{\mathrm{d}r}\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}-U(r)\right)}{M+E_{n\kappa}-\Delta(r)}\end{array}\right\}F_{n\kappa}(r)=0, \tag{21}$$

$$\left\{\begin{array}{l}\frac{\mathrm{d}^2}{\mathrm{d}r^2}-\frac{\kappa(\kappa-1)}{r^2}+\frac{2\kappa U(r)}{r}+\frac{\mathrm{d}U(r)}{\mathrm{d}r}-U^2(r)-(M+E_{n\kappa}-\Delta(r))(M-E_{n\kappa}+\Sigma(r))\\ +\frac{\frac{\mathrm{d}\Sigma(r)}{\mathrm{d}r}\left(\frac{\mathrm{d}}{\mathrm{d}r}-\frac{\kappa}{r}+U(r)\right)}{M+E_{n\kappa}-\Sigma(r)}\end{array}\right\}G_{n\kappa}(r)=0, \tag{22}$$

where $\kappa(\kappa-1)=\tilde{l}(\tilde{l}+1)$, $\kappa(\kappa+1)=l(l+1)$. During this stage, we take $\Delta(r)$ or $\Sigma(r)$ as the MS-M potential. Eqs. (21) and (22) can be exactly solved for $\kappa=0,-1$ and $\kappa=0,1$, respectively, as the spin-orbit centrifugal term vanishes.

4 Solution of the Dirac equation

In this section, we are going to solve the Dirac equation with MS-M potential in the presence of a tensor potential by using the SUSYQM.

4.1 The pseudospin symmetry limit

The exact pseudospin symmetry, proposed by Meng et al. [28], occurs in the Dirac equation when $\frac{\mathrm{d}\Sigma(r)}{\mathrm{d}r}=0$ or $\Sigma(r)=C_{\mathrm{ps}}$=const. [5]. We now take $\Delta(r)$ as the MS-M potential and consider a Coulomb tensor potential [26]:

$$\Delta(r)=V_0\left(\frac{A^{\mathrm{ps}}+B^{\mathrm{ps}}\mathrm{e}^{-\alpha r}}{C^{\mathrm{ps}}+D^{\mathrm{ps}}\mathrm{e}^{-\alpha r}}\right)^2+\left(\frac{a^{\mathrm{ps}}}{r^2}-\frac{b^{\mathrm{ps}}}{r}+d^{\mathrm{ps}}\right), \tag{23}$$

$$U(r)=-\frac{H}{r},\quad H=\frac{z_{\mathrm{a}}z_{\mathrm{b}}\mathrm{e}^2}{4\pi\varepsilon_0},\quad r\geqslant R_{\mathrm{e}} \tag{24}$$

where V_0, A^{ps}, B^{ps}, C^{ps}, D^{ps}, a^{ps}, b^{ps}, d^{ps} and H are constant coefficients, R_{e}=7.78 fm is the Coulomb radius and, z_{a} and z_{b} denotes the charges of the projectile a and target nuclei b, respectively [26]. Since the Dirac equation with the MS-M potential has no exact solution, we use an approximation for the centrifugal term as [32]

$$\begin{aligned}\frac{1}{r^2} &= \lim_{\alpha\to 0}\left[\frac{C^{\mathrm{ps}2}\alpha^{\mathrm{ps}2}}{(C^{\mathrm{ps}}+D^{\mathrm{ps}}\mathrm{e}^{-\alpha r})^2}\right]\\ &= \lim_{\alpha\to 0}\left(\frac{1}{r^2}+\frac{\alpha}{r}+\frac{5}{12}\alpha^2+\frac{1}{12}\alpha^3 r+\frac{1}{240}\alpha^4 r^2-\frac{1}{720}\alpha^5 r^3\right.\\ &\quad\left.-\frac{1}{6045}\alpha^6 r^4+O(r^5)\right),\end{aligned} \tag{25}$$

where $C=-D'$. This proposition gives a good approximation for the centrifugal term (see in Fig. 1). When performing a power series expansion and setting $\alpha\to 0$, gives the desired r^{-2} suggested by Greene and Aldrich

[54]. Substitution of Eqs. (23)–(25) into Eq. (22) gives

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{1}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}\right.$$
$$\left\{\left[\frac{-MV_0B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+\frac{V_0E^{\mathrm{ps}}_{n\kappa}B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}-\frac{V_0C_{\mathrm{ps}}B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}\right]\mathrm{e}^{-2\alpha r}\right.$$
$$+\left[-\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0M}{C^{\mathrm{ps}2}}+\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0E^{\mathrm{ps}}_{n\kappa}}{C^{\mathrm{ps}2}}-\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0C_{\mathrm{ps}}}{C^{\mathrm{ps}2}}\right.$$
$$\left.+\frac{M\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}}{C^{\mathrm{ps}}}-\frac{\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}}{C^{\mathrm{ps}}}+\frac{\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}C_{\mathrm{ps}}}{C^{\mathrm{ps}}}\right]\mathrm{e}^{-\alpha r}$$
$$+(\kappa+H)(\kappa+H-1)\alpha^2-\frac{MV_0A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+\frac{V_0E^{\mathrm{ps}}_{n\kappa}A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}$$
$$-\frac{V_0C_{\mathrm{ps}}A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+Mb^{\mathrm{ps}}\alpha-E^{\mathrm{ps}}_{n\kappa}b^{\mathrm{ps}}\alpha+C_{\mathrm{ps}}b^{\mathrm{ps}}\alpha-Ma^{\mathrm{ps}}\alpha^2$$
$$\left.+a^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}\alpha^2-a^{\mathrm{ps}}C_{\mathrm{ps}}\alpha^2\right\}G^{\mathrm{ps}}_{n\kappa}(r)=\{-M^2-MC_{\mathrm{ps}}$$
$$+(E^{\mathrm{ps}}_{n\kappa})^2-E^{\mathrm{ps}}_{n\kappa}C_{\mathrm{ps}}+Md^{\mathrm{ps}}-dE^{\mathrm{ps}}_{n\kappa}+dC_{\mathrm{ps}}\}G^{\mathrm{ps}}_{n\kappa}(r),\quad(26)$$

where $\kappa=-\tilde{\ell}$ and $\kappa=\tilde{\ell}+1$ for $\kappa<0$ and $\kappa>0$ respectively.

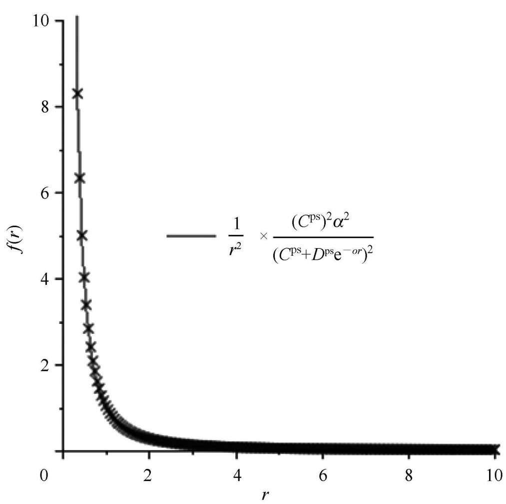

Fig. 1. The centrifugal term $(1/r^2)$ and its approximation for $\alpha=0.01$, $C^{\mathrm{ps}}=1$, $D^{\mathrm{ps}}=-1$.

4.2 Solution of the pseudospin symmetry limit

In the previous section, we obtained a Schrödinger-like equation of the form

$$-\frac{\mathrm{d}^2G^{\mathrm{ps}}_{n\kappa}(r)}{\mathrm{d}r^2}+V_{\mathrm{eff}}(r)G^{\mathrm{ps}}_{n\kappa}(r)=\tilde{E}^{\mathrm{ps}}_{n\kappa}G^{\mathrm{ps}}_{n\kappa}(r),\qquad(27)$$

with an effective potential

$$V_{\mathrm{eff}}=\frac{\eta_1^{\mathrm{ps}}\mathrm{e}^{-2\alpha r}+\eta_2^{\mathrm{ps}}\mathrm{e}^{-\alpha r}+\eta_3^{\mathrm{ps}}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2},\qquad(28)$$

where

$$\eta_1^{\mathrm{ps}}=\frac{-MV_0B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+\frac{V_0E^{\mathrm{ps}}_{n\kappa}B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}-\frac{V_0C_{\mathrm{ps}}B^{\mathrm{ps}2}}{C^{\mathrm{ps}2}},$$
$$\eta_2^{\mathrm{ps}}=-\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0M}{C^{\mathrm{ps}2}}+\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0E^{\mathrm{ps}}_{n\kappa}}{C^{\mathrm{ps}2}}-\frac{2A^{\mathrm{ps}}B^{\mathrm{ps}}V_0C_{\mathrm{ps}}}{C^{\mathrm{ps}2}}+\frac{M\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}}{C^{\mathrm{ps}}}-\frac{\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}}{C^{\mathrm{ps}}}+\frac{\alpha b^{\mathrm{ps}}D^{\mathrm{ps}}C_{\mathrm{ps}}}{C^{\mathrm{ps}}},$$
$$\eta_3^{\mathrm{ps}}=(\kappa+H)(\kappa+H-1)\alpha^2-\frac{MV_0A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+\frac{V_0E^{\mathrm{ps}}_{n\kappa}A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}-\frac{V_0C_{\mathrm{ps}}A^{\mathrm{ps}2}}{C^{\mathrm{ps}2}}+Mb^{\mathrm{ps}}\alpha-E^{\mathrm{ps}}_{n\kappa}b^{\mathrm{ps}}\alpha+C_{\mathrm{ps}}b^{\mathrm{ps}}\alpha-Ma^{\mathrm{ps}}\alpha^2+a^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}\alpha^2-a^{\mathrm{ps}}C_{\mathrm{ps}}\alpha^2.\qquad(29)$$

The corresponding effective energy is given by

$$\tilde{E}^{\mathrm{ps}}_{n\kappa}=-M^2-MC_{\mathrm{ps}}+(E^{\mathrm{ps}}_{n\kappa})^2-E^{\mathrm{ps}}_{n\kappa}C_{\mathrm{ps}}+Md^{\mathrm{ps}}-d^{\mathrm{ps}}E^{\mathrm{ps}}_{n\kappa}+d^{\mathrm{ps}}C_{\mathrm{ps}},\qquad(30)$$

In SUSYQM formalism, the ground-state wave function for the lower component is given as

$$G^{\mathrm{ps}}_{0\kappa}(r)=\exp\left(-\int\phi(r)\mathrm{d}r\right),\qquad(31)$$

Thus, our first step is to find the solution of the Riccati equation

$$\phi^2-\phi'=V_{\mathrm{eff}}-\tilde{E}^{\mathrm{ps}}_{0\kappa},\qquad(32)$$

for which we propose a superpotential of the form

$$\phi(r)=\frac{w^{\mathrm{ps}}\mathrm{e}^{-\alpha r}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)}+q^{\mathrm{ps}}.\qquad(33)$$

Therefore, the exact parameters of our study are obtained via

$$\frac{(w^{\mathrm{ps}})^2\mathrm{e}^{-2\alpha r}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}+(q^{\mathrm{ps}})^2+\frac{2w^{\mathrm{ps}}q^{\mathrm{ps}}\mathrm{e}^{-\alpha r}}{1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}}+\frac{w^{\mathrm{ps}}\alpha\mathrm{e}^{-\alpha r}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}=\frac{\eta_1^{\mathrm{ps}}\mathrm{e}^{-2\alpha r}+\eta_2^{\mathrm{ps}}\mathrm{e}^{-\alpha r}+\eta_3^{\mathrm{ps}}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}-\tilde{E}^{\mathrm{ps}}_{0\kappa},\qquad(34)$$

or, more explicitly

$$\tilde{E}^{\mathrm{ps}}_{0\kappa}=\eta_3^{\mathrm{ps}}-(q^{\mathrm{ps}})^2,\qquad(35\mathrm{a})$$

$$w^{\mathrm{ps}}=\frac{\alpha D^{\mathrm{ps}}}{2C^{\mathrm{ps}}}-\frac{1}{2C^{\mathrm{ps}}}\times\sqrt{\alpha^2(D^{\mathrm{ps}})^2+4\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+4\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2-4\eta_2^{\mathrm{ps}}C^{\mathrm{ps}}D^{\mathrm{ps}}},\qquad(35\mathrm{b})$$

$$q^{\mathrm{ps}}=\frac{\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2-(w^{\mathrm{ps}})^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2}{2w^{\mathrm{ps}}C^{\mathrm{ps}}D^{\mathrm{ps}}}.\qquad(35\mathrm{c})$$

After constructing the partner Hamiltonians

$$V_{\mathrm{eff}+}(r) = \phi^2+\frac{\mathrm{d}\phi}{\mathrm{d}r}=\frac{-w^{\mathrm{ps}}\frac{C^{\mathrm{ps}}}{D^{\mathrm{ps}}}\left[w^{\mathrm{ps}}+\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\right]\mathrm{e}^{-\alpha r}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}+\frac{\left[\frac{-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{C^{\mathrm{ps}}D^{\mathrm{ps}}}\right]\mathrm{e}^{-\alpha r}}{1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}}$$
$$+\left(\frac{-(w^{\mathrm{ps}})^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}w^{\mathrm{ps}}}\right)^2, \tag{36a}$$

$$V_{\mathrm{eff}-}(r) = \phi^2-\frac{\mathrm{d}\phi}{\mathrm{d}r}\frac{-w^{\mathrm{ps}}\frac{C^{\mathrm{ps}}}{D^{\mathrm{ps}}}\left[w^{\mathrm{ps}}-\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\right]\mathrm{e}^{-\alpha r}}{\left(1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}\right)^2}+\frac{\left[\frac{-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{C^{\mathrm{ps}}D^{\mathrm{ps}}}\right]\mathrm{e}^{-\alpha r}}{1+\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}\mathrm{e}^{-\alpha r}}$$
$$+\left(\frac{-(w^{\mathrm{ps}})^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}w^{\mathrm{ps}}}\right)^2, \tag{36b}$$

where $a_0=w^{\mathrm{ps}}$ and a_i is a function of a_0, i.e., $a_1=f(a_0)=a_0+\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}$. Consequently, $a_n=f(a_0)=a_0+n\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}$. We see that the shape invariance holds via a mapping of the form $w^{\mathrm{ps}}\rightarrow w^{\mathrm{ps}}+\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}$. From Eq. (5), we have [26, 27]

$$R(a_1) = \left(\frac{-a_0^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_0}\right)^2-\left(\frac{-a_1^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_1}\right)^2,$$

$$R(a_2) = \left(\frac{-a_1^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_1}\right)^2-\left(\frac{-a_2^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_2}\right)^2,$$

$$R(a_3) = \left(\frac{-a_2^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_2}\right)^2-\left(\frac{-a_3^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_3}\right)^2,$$

$$\vdots$$

$$R(a_n) = \left(\frac{-a_{n-1}^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_{n-1}}\right)^2-\left(\frac{-a_n^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_n}\right)^2, \tag{37}$$

$$\tilde{E}_{0\kappa}^{-}=0, \tag{38}$$

Therefore, from Eq. (6a), the eigenvalues can be found as

$$\tilde{E}_{n\kappa}^{\mathrm{ps}^-} = \sum_{k=1}^{n}R(a_\kappa)=\left(\frac{-a_0^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_0}\right)^2-\left(\frac{-a_n^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_n}\right)^2, \tag{39a}$$

$$\tilde{E}_{n\kappa}^{\mathrm{ps}} = \tilde{E}_{n\kappa}^{ps^-}+\tilde{E}_{0\kappa}^{\mathrm{ps}}=-\left(\frac{-a_n^2(C^{\mathrm{ps}})^2-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2}{2C^{\mathrm{ps}}D^{\mathrm{ps}}a_n}\right)^2+\eta_3^{\mathrm{ps}}. \tag{39b}$$

This completely determines the energy of the pseudospin symmetry limit. With the aid of Eqs. (29) and (35a)–(35c), we obtain the energy for any spin-orbit quantum number as

$$-M^2-MC_{\mathrm{ps}}+(E_{n\kappa}^{\mathrm{ps}})^2-E_{n\kappa}^{\mathrm{ps}}C_{\mathrm{ps}}+Md^{\mathrm{ps}}-d^{\mathrm{ps}}E_{n\kappa}^{\mathrm{ps}}+d^{\mathrm{ps}}C_{\mathrm{ps}}+\frac{1}{(C^{\mathrm{ps}}D^{\mathrm{ps}})^2}\left\{\frac{(-\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2)}{2\sigma}-\frac{\sigma(C^{\mathrm{ps}})^2}{2}\right\}^2-\eta_3^{\mathrm{ps}}=0, \tag{40}$$

where

$$\sigma=\left(\frac{\alpha D^{\mathrm{ps}}}{2C^{\mathrm{ps}}}-\frac{1}{2C^{\mathrm{ps}}}\left(\sqrt{\alpha^2(D^{\mathrm{ps}})^2+4\eta_3^{\mathrm{ps}}(D^{\mathrm{ps}})^2+4\eta_1^{\mathrm{ps}}(C^{\mathrm{ps}})^2-4\eta_2^{\mathrm{ps}}C^{\mathrm{ps}}D^{\mathrm{ps}}}\right)\right)+n\alpha\frac{D^{\mathrm{ps}}}{C^{\mathrm{ps}}}. \tag{41}$$

Thus, the lower component of the wave function is

$$G^{\rm ps}_{n\kappa}(r)=N_{n\kappa}\frac{\Gamma\left(n+1+2\sqrt{\dfrac{\eta_3^{\rm ps}-\tilde{E}^{\rm ps}_{n\kappa}}{\alpha^2}}\right)}{n!\Gamma\left(1+2\sqrt{\dfrac{\eta_3^{\rm ps}-\tilde{E}^{\rm ps}_{n\kappa}}{\alpha^2}}\right)}\left(-\frac{D^{\rm ps}}{C^{\rm ps}}{\rm e}^{-\alpha r}\right)^{\sqrt{\frac{\eta_3^{\rm ps}-\tilde{E}^{\rm ps}_{n\kappa}}{\alpha^2}}}\left(1+\frac{D^{\rm ps}}{C^{\rm ps}}{\rm e}^{-\alpha r}\right)^{\frac{1}{2}+\sqrt{\frac{\eta_1^{\rm ps}(C^{\rm ps})^2}{\alpha^2(D^{\rm ps})^2}-\frac{\eta_2^{\rm ps}C^{\rm ps}}{\alpha^2D^{\rm ps}}+\frac{\eta_3^{\rm ps}}{\alpha^2}+\frac{1}{4}}}$$

$${}_2F_1\left(-n,1+2\sqrt{\frac{\eta_3^{\rm ps}-\tilde{E}^{\rm ps}_{n\kappa}}{\alpha^2}}+2\sqrt{\frac{\eta_1^{\rm ps}(C^{\rm ps})^2}{\alpha^2(D^{\rm ps})^2}-\frac{\eta_2^{\rm ps}C^{\rm ps}}{\alpha^2D^{\rm ps}}+\frac{\eta_3^{\rm ps}}{\alpha^2}+\frac{1}{4}}+n;2\sqrt{\frac{\eta_3^{\rm ps}-\tilde{E}^{\rm ps}_{n\kappa}}{\alpha^2}}+1;-\frac{D^{\rm ps}}{C^{\rm ps}}{\rm e}^{-\alpha r}\right). \quad (42)$$

The corresponding upper-spinor component of the Dirac equation can be obtained via Eq. (26) as,

$$F^{\rm ps}_{n\kappa}(r)=\frac{1}{(E^{\rm ps}_{n\kappa}-M+C_{\rm ps})}\left(\frac{\rm d}{{\rm d}r}-\frac{\kappa}{r}+U(r)\right)G^{\rm ps}_{n\kappa}(r). \quad (43)$$

Where $E^{\rm ps}_{n\kappa}\neq M-C_{\rm ps}$.

5 Numerical results

We obtain the energy eigenvalues in the absence and the presence of the Coulomb-like tensor potential for various values of the quantum numbers n and κ. The results are reported in Table 1. If we set $a^{\rm ps}=b^{\rm ps}=d^{\rm ps}=0$, we can exactly obtain the results of solution of the Dirac equation under the Möbius square potential with the NU method [32]. In Table 1, we can clearly see the degeneracy between the bound-states that these degeneracies are changed in the presence of the tensor interaction.

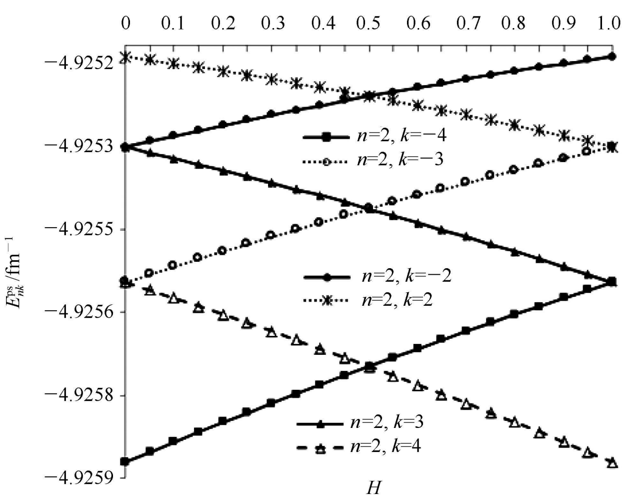

Fig. 2. Energy vs. H for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $V_0=-0.2$, $A^{\rm ps}=1$, $B^{\rm ps}=-2$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $a^{\rm ps}=0.8$, $b^{\rm ps}=0.6$, $d^{\rm ps}=0.1$.

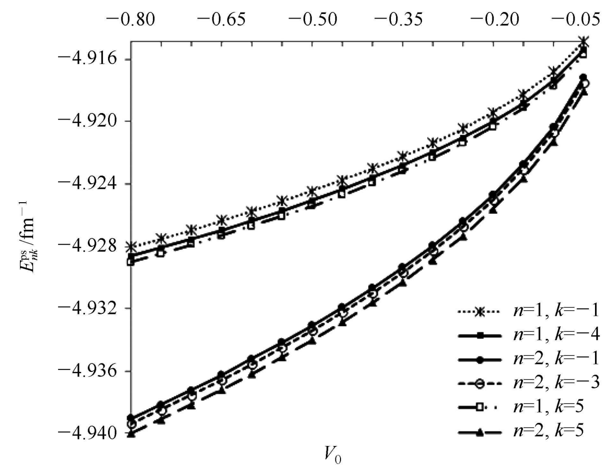

Fig. 3. Energy vs. V_0 for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $H=0.5$, $A^{\rm ps}=1$, $B^{\rm ps}=-2$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $a^{\rm ps}=0.8$, $b^{\rm ps}=0.6$, $d^{\rm ps}=0.1$.

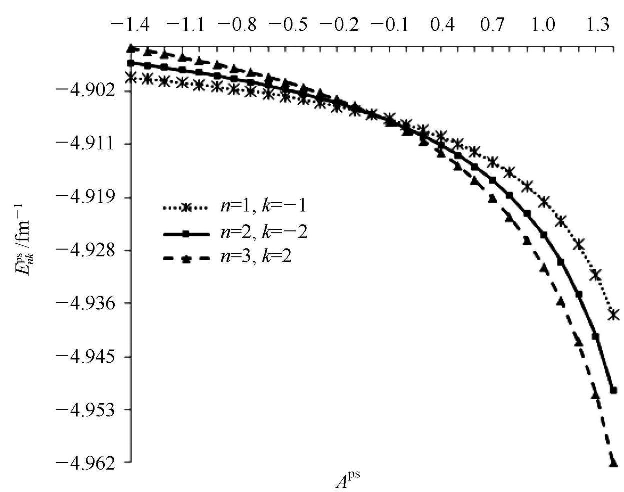

Fig. 4. Energy vs. $A^{\rm ps}$ for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $V_0=-0.2$, $H=0.5$, $B^{\rm ps}=-2$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $a^{\rm ps}=0.8$, $b^{\rm ps}=0.6$, $d^{\rm ps}=0.1$.

Table 1. Energies in the pseudospin symmetry limit for α=0.01, M=5 fm^{-1}, $C_{\rm ps}$=0, V_0=−0.2, $A^{\rm ps}$=1, $B^{\rm ps}$=−2, $C^{\rm ps}$=1, $D^{\rm ps}$=−1, $a^{\rm ps}$=0.8, $b^{\rm ps}$=0.6, $d^{\rm ps}$=0.1.

$\tilde{\ell}$	n	κ	(ℓ,j)	$E^{\rm ps}_{n\kappa}(H=0)$	$E^{\rm ps}_{n\kappa}(H=0)$=Ref. [32] $a^{\rm ps}=b^{\rm ps}=d^{\rm ps}=0$	$E^{\rm ps}_{n\kappa}(H=0.5)$	$E^{\rm ps}_{n\kappa}(H=0.5)$=Ref. [32] $a^{\rm ps}=b^{\rm ps}=d^{\rm ps}=0$	$E^{\rm ps}_{n\kappa}(H=1)$	$E^{\rm ps}_{n\kappa}(H=1)$=Ref. [32] $a^{\rm ps}=b^{\rm ps}=d^{\rm ps}=0$
1	1	−1	$1S_{\frac{1}{2}}$	−4.919905172	−5.00840912	−4.919856805	−5.00836049	−4.919827782	−5.00833131
2	1	2	$1P_{\frac{3}{2}}$	−4.920059901	−5.00856470	−4.919972874	−5.00847720	−4.919905172	−5.00840912
3	1	−3	$1d_{\frac{5}{2}}$	−4.920291870	−5.00879794	−4.920166239	−5.00867162	−4.920059901	−5.00856470
4	1	−4	$1f_{\frac{7}{2}}$	−4.920600929	−5.00910869	−4.920436775	−5.00894364	−4.920291870	−5.00879794
1	2	−1	$2S_{\frac{1}{2}}$	−4.925188706	−5.01378636	−4.925141213	−5.01373861	−4.925112715	−5.01370995
2	2	−2	$2P_{\frac{3}{2}}$	−4.925340641	−5.01393914	−4.925255186	−5.01385321	−4.925188706	−5.01378636
3	2	3	$2d_{\frac{5}{2}}$	−4.925568422	−5.01416818	−4.925445059	−5.01404413	−4.925340641	−5.01393914
4	2	−4	$2f_{\frac{7}{2}}$	−4.925871903	−5.01447334	−4.925710711	−5.01431125	−4.925568422	−5.01416818
1	1	2	$0d_{\frac{3}{2}}$	−4.919905172	−5.00840912	−4.919972874	−5.00847720	−4.920059901	−5.00856470
2	1	3	$0f_{\frac{5}{2}}$	−4.920059901	−5.00856470	−4.920166239	−5.00867162	−4.92029187	−5.00879794
3	1	4	$0g_{\frac{7}{2}}$	−4.920291870	−5.00879794	−4.920436775	−5.00894364	−4.920600929	−5.00910869
4	1	5	$0h_{\frac{9}{2}}$	−4.920600929	−5.00910869	−4.920784308	−5.00929308	−4.92098688	−5.00949676
1	2	2	$1d_{\frac{3}{2}}$	−4.925188706	−5.01378636	−4.925255186	−5.01385321	−4.925340641	−5.01393914
2	2	3	$1f_{\frac{5}{2}}$	−4.925340641	−5.01393914	−4.925445059	−5.01404413	−4.925568422	−5.01416818
3	2	4	$1g_{\frac{7}{2}}$	−4.925568422	−5.01416818	−4.925710711	−5.01431125	−4.925871903	−5.01447334
4	2	5	$1h_{\frac{9}{2}}$	−4.925871903	−5.01447334	−4.926051972	−5.01465440	−4.926250890	−5.01485442

Table 2. Energies in the pseudospin symmetry limit for $\alpha=0.01$, $H=0.5$, $C_{\rm ps}=0$, $V_0=-0.2$, $A^{\rm ps}=1$, $B^{\rm ps}=-2$, $C^{\rm ps}$=1, $D^{\rm ps}$=−1, $a^{\rm ps}$=0.8, $b^{\rm ps}$=0.6, $d^{\rm ps}$=0.1.

M	$E^{\rm ps}_{n\kappa}$/fm^{-1}				
	$1S_{\frac{1}{2}}$	$1d_{\frac{5}{2}}$	$0h_{\frac{9}{2}}$	$1g_{\frac{7}{2}}$	$2f_{\frac{7}{2}}$
0	−0.029634765	−0.049535678	−0.074996700	−0.074660915	−0.074660915
1	−0.929931804	−0.931459972	−0.934490948	−0.943835375	−0.943835375
2	−1.924592824	−1.925361338	−1.926892874	−1.934131840	−1.934131840
3	−2.922229368	−2.922743251	−2.923768700	−2.929904018	−2.929904018
4	−3.920819617	−3.921205818	−3.921976951	−3.927405930	−3.927405930
5	−4.919856805	−4.920166239	−4.920784308	−4.925710711	−4.925710711
6	−5.919145550	−5.919403724	−5.919919522	−5.924464292	−5.924464292

Table 3. Energies in the pseudospin symmetry limit for α=0.01, M=5 fm^{-1}, H=0.5, V_0=−0.2, $A^{\rm ps}$=1, $B^{\rm ps}$=−2, $C^{\rm ps}$=1, $D^{\rm ps}$=−1, $a^{\rm ps}$=0.8, $b^{\rm ps}$=0.6, $d^{\rm ps}$=0.1.

$C_{\rm ps}$	$E^{\rm ps}_{n\kappa}$/fm^{-1}				
	$1S_{\frac{1}{2}}$	$1d_{\frac{5}{2}}$	$0h_{\frac{9}{2}}$	$1g_{\frac{7}{2}}$	$2f_{\frac{7}{2}}$
−4	−4.922229368	−4.922743251	−4.923768700	−4.929904018	−4.929904018
−3	−4.921448940	−4.921889898	−4.922770137	−4.928518801	−4.928518801
−2	−4.920819617	−4.921205818	−4.921976951	−4.927405930	−4.927405930
−1	−4.920298135	−4.920641704	−4.921327849	−4.926486636	−4.926486636
0	−4.919856805	−4.920166239	−4.920784308	−4.925710711	−4.925710711
1	−4.919476973	−4.919758456	−4.920320765	−4.925044454	−4.925044454
2	−4.919145550	−4.919403724	−4.919919522	−4.924464292	−4.924464292
3	−4.918853045	−4.919091482	−4.919567892	−4.923953180	−4.923953180
4	−4.918592383	−4.918813894	−4.919256516	−4.923498446	−4.923498446

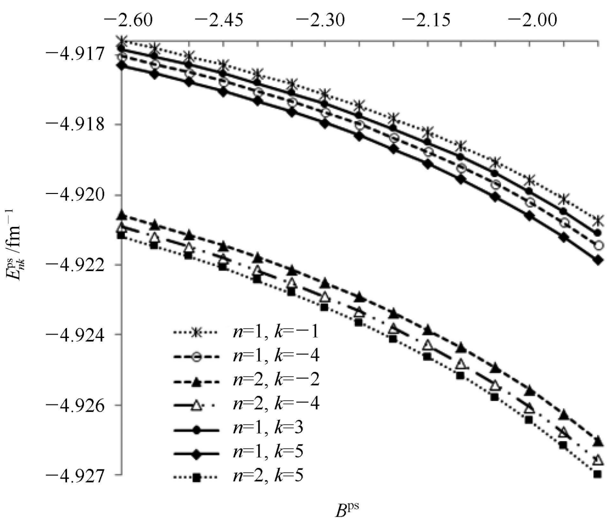

Fig. 5. Energy vs. $B^{\rm ps}$ for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $V_0=0.2$, $H=0.5$, $A^{\rm ps}=1$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $a^{\rm ps}=0.8$, $b^{\rm ps}=0.6$, $d^{\rm ps}=0.1$.

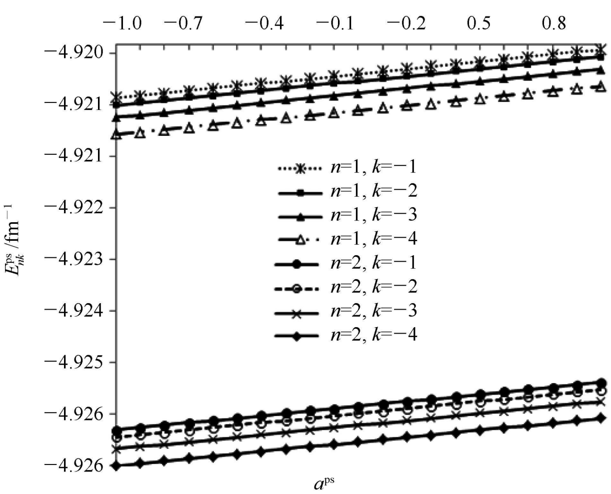

Fig. 6. Energy vs. $a^{\rm ps}$ for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $V_0=-0.2$, $H=0.5$, $A^{\rm ps}=1$, $B^{\rm ps}=-2$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $b^{\rm ps}=0.6$, $d^{\rm ps}=0.1$.

In this case, for $H=0$, the degenerate states are $ns_{1/2}$,$(n-1)d_{3/2}$ for $\tilde{\ell}=1(\ell=0)$, $np_{3/2}$,$(n-1)f_{5/2}$ for $\tilde{\ell}=2(\ell=1)$, $nd_{5/2}$,$(n-1)g_{7/2}$ for $\tilde{\ell}=3(\ell=2)$, and $nf_{7/2}$,$(n-1)h_{9/2}$ for $\tilde{\ell}=4(\ell=3)$, etc. For $H=0.5$, the degeneracy occurs for $(1P_{\frac{3}{2}}=0d_{\frac{3}{2}})$, $(1d_{\frac{5}{2}}=0f_{\frac{5}{2}})$, $(1f_{\frac{7}{2}}=0g_{\frac{7}{2}})$, $(2P_{\frac{3}{2}}=1d_{\frac{3}{2}})$, $(2d_{\frac{5}{2}}=1f_{\frac{5}{2}})$, $(2f_{\frac{7}{2}}=1g_{\frac{7}{2}})$. For $H=1$, the behavior is repeated for $(1d_{\frac{5}{2}}=0d_{\frac{3}{2}})$, $(1f_{\frac{7}{2}}=0f_{\frac{5}{2}})$, $(2d_{\frac{5}{2}}=1d_{\frac{3}{2}})$, $(2f_{\frac{7}{2}}=1f_{\frac{5}{2}})$. We see that the difference of degenerate states increases as H increases. Tables 2 and 3 reported the energy eigenvalues for different values of M and $C_{\rm ps}$, respectively. Fig. 2 shows the behavior of energy vs. the coefficient of the tensor interaction. In Figs. 3–7, it is seen that if the $A^{\rm ps}$,$B^{\rm ps}$ and $b^{\rm ps}$ parameters increase, the bound-states become more bounded and with increasing V_0 and $a^{\rm ps}$, they become less bounded in the pseudospin symmetry limit.

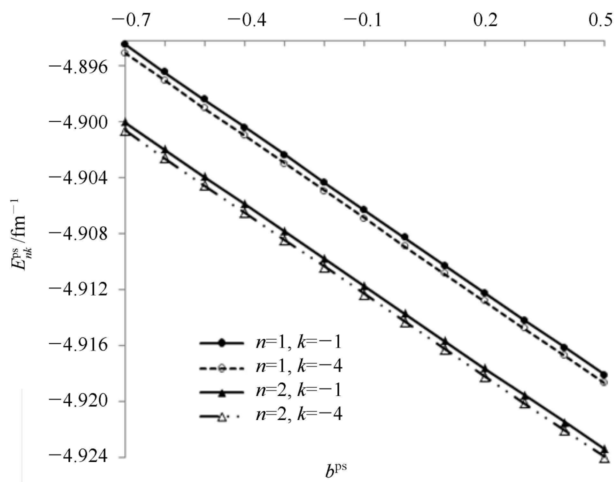

Fig. 7. Energy vs. $b^{\rm ps}$ for pseudospin symmetry limit for $\alpha=0.01$, $M=5$ fm^{-1}, $C_{\rm ps}=0$, $V_0=-0.2$, $H=0.5$, $A^{\rm ps}=1$, $B^{\rm ps}=-2$, $C^{\rm ps}=1$, $D^{\rm ps}=-1$, $a^{\rm ps}=0.8$, $d^{\rm ps}=0.1$.

6 Conclusions

In this paper, we obtained any state approximate solution of the Dirac equation with a combination of Möbius square and Mie type potential in the presence of the Coulomb tensor interaction in the pseudospin symmetry limit. We observed that the SUSYQM can solve the problem in an approximate analytical manner without having to deal with the cumbersome numerical approaches.

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