Suppression of the emittance growth induced by CSR in a DBA cell

CUI Xiao-Hao(崔小昊)[1)] JIAO Yi(焦毅) XU Gang(徐刚) HUANG Xi-Yang(黄玺洋)

Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China

Abstract: The emittance growth induced by Coherent Synchrotron Radiation (CSR) is an important issue when electron bunches with short bunch length and high peak current are transported in a bending magnet. In this paper, a single kick method is introduced that could give the same result as the R-matrix method, but is much easier to use. Then, with this method, an optics design technique is introduced that could minimize the emittance dilution within a single achromatic cell.

Key words: coherent synchrotron radiation, emittance, lattice design, DBA

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1 Introduction

The design and study of the next generation light source based on Energy Recovery Linac (ERL) and Free Electron Lasers (FEL) have been proposed worldwide. In these machines, electron bunches with short bunch length, high peak current and small emittance are generated and transported. It is very important to minimize the transverse emittance growth in order to achieve high quality electron beams. Emission of Coherent Synchrotron Radiation (CSR) is considered to be one of the most critical sources for the beam emittance dilution when bending magnets or bunch compression sections exist in the transport lines.

According to the CSR wake potential [1], the energy change of an electron due to the CSR emission is a function of its longitudinal position in the bunch. Consequently, different bunch slices are deflected with different angles in a bending magnet, and this deflecting error dilutes the projected emittance of the electron bunch [2]. This effect has been studied intensively [3, 4], and it is shown that if the longitudinal electron distribution does not change significantly, then the rms energy spread caused by CSR can be estimated by the function:

$$\Delta E_{\rm rms}=0.22\frac{eQL_{\rm b}}{4\pi\varepsilon_0\rho^{2/3}\sigma_{\rm s}^{4/3}}, \tag{1}$$

where e is electron charge, Q is the bunch charge, $L_{\rm b}$ is the length of the bending magnet, ρ is the bending radius, and $\sigma_{\rm s}$ denotes the rms bunch length. For a constant bending radius, this is a linear function of s, the longitudinal path length, and can be simplified as $\Delta E=\kappa s$, where κ denotes the coefficient of $L_{\rm b}$ in function (1). Under this linear approximation, two optics design techniques have been introduced for the suppression of the CSR induced emittance growth: the envelope matching method [5], and the cell-to-cell phase matching method [6]. However, these two methods have their shortcomings. Envelope matching can only minimize instead of completely canceling the emittance growth. In addition, the cell-to-cell phase matching method relies strongly on the symmetrical character of the lattice and on the cell-to-cell betatron phase advance. In the present paper, another method of emittance cancellation is shown, which can completely cancel the emittance growth due to linear CSR effect within a single achromatic cell.

The rest of this paper is organized as follows. In Section 2, a single kick method is introduced to describe the transverse effect of CSR. In his paper [7], R. Hajjima proposed a 5-by-5 R-matrix to calculate the emittance growth arising from CSR effect. Using this matrix method, an exact description of the linear CSR effect is acquired, which can be treated as a standard transfer matrix. But all calculations involve complex 5-by-5 R-matrix manipulations and it is hard to get an intuitive idea of what has happened. For this reason, S. Di. Mitri [8] used a single kick to describe the CSR induced transverse effect. But his kick method can only give a qualitative rather than quantitative agreement with the R-matrix method. A single kick description is given in Section 2, which gives the same results as the R-matrix method, and avoids complicated matrix manipulations. In Section 3, the CSR induced emittance in a DBA cell is studied using this single kick method, and a constraint condition on the lattice is found, which could cancel the CSR kick completely. In Section 4, different DBA lattices with various beam and optical parameters are simu-

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1) E-mail: cuixh@ihep.ac.cn

lated using code ELEGANT [9]. In addition, it is verified that our constraint condition really produces the smallest emittance growth.

2 Kick description of CSR effect

2.1 Kick description of the dispersion

The matrix description of dispersion has been widely used. Consider a particle with energy deviation δ, whose transverse coordinates are (x_0, x'_0) at the entrance of a dipole. When it goes through the dipole, its coordinates change not only by the transfer matrix of the dipole, but also by a dispersion term. If we are not interested in the particle information inside the magnet, the dispersion can be considered as a point kick at the middle point of a dipole magnet. To get the same result with the matrix description, we choose the middle point kick to get the same particle coordinates at the exit of a dipole, that is $M_{\text{half}}(x_{\text{kick}}, x'_{\text{kick}})=(D, D')\delta$, where M_{half} is the 2-by-2 transfer matrix of half a dipole, and (D, D') is the dispersion function generated by the dipole. From this equation, the dispersion kick is defined. Take a sector dipole as an example, the dispersion function at the end of the magnet is $(D, D')=(\rho(1-\cos\theta), \sin\theta)$. From the above equation, the dispersion kick can be calculated as $(x_{\text{kick}}, x'_{\text{kick}})=(0, 2\sin(\theta/2))\delta$, where θ is the bending angle of the dipole. We can see that the expression of a kick is simplified by placing the kick at the middle point. Later, when we study beam dynamics in normalized coordinates, this advantage will become more obvious.

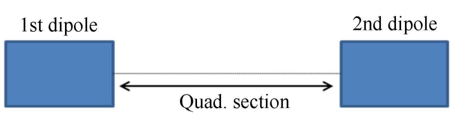

Fig. 1. Layout of a double-bending cell.

As an example, we will derive the achromatic condition of a double bending cell using this point-kick model. The double bending cell is composed of two bending dipoles at the head and end of the cell separately, and drifts and quadrupoles between, as shown in Fig. 1. Those two dipoles have the same bending directions but may have different bending radius and bending angles. In the discussion below, we will use the normalized phase space coordinates (W, W'), where $W=x/\sqrt{\beta}$, $W'=(\alpha x+\beta x')/\sqrt{\beta}$. In such a normalized phase space, particle trajectories form concentric circles of different radius as shown in Fig. 2. When the dispersion kick $(x_{\text{kick}}, x'_{\text{kick}})$ is normalized, $(W_{\text{kick}}, W'_{\text{kick}})=(0, 2\sin(\theta/2)\sqrt{\beta})\delta$, and the direction of dispersion kick in the normalized phase space always sticks upward, no matter what the Twiss parameters are before this dipole.

The motion of a particle in a double bending cell can be characterized as follows: a particle moving along a circle line gets a kick from the first dipole, it then moves along a new circle. After an angle of the phase advance between the two kick points, a kick from the second dipole moves it into the path of a third circle. If after two such kicks all of the particles go back to its original circle path, then the cell will work as if no dispersion exist and this lattice cell is achromatic. As shown Fig. 2, from elementary geometry, we can get the achromatic condition: (1) phase advance between the middle points of the two dipoles must be π or $(2n+1)\pi$; and, (2) the two dispersion kicks in normalized phase space must equal to each other, that is, $\sin(\theta_1/2)\sqrt{\beta}_1=\sin(\theta_2/2)\sqrt{\beta}_2$. These achromatic conditions can be verified by writing down the matrix of each part of the cell and solving the achromatic equation, which is straightforward but involved.

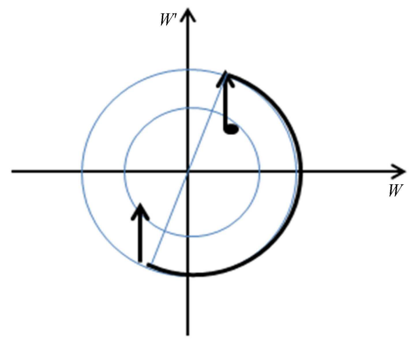

Fig. 2. A schematic figure of the particle in the normalized phase space, it is obvious that two identical kicks with p phase advance move the particle back in its original path and cancel the dispersion in the cell.

2.2 Kick description of the CSR effect

According to R. Hajima's R-Matrix, the transverse coordinate movement due to CSR effect can also be treated by a matrix method, a sector dipole can be described by a matrix:

$$R_{\text{d}}=\begin{pmatrix} \cos\theta & \rho\sin\theta & \rho(1-\cos\theta) & \rho(1-\cos\theta) & \rho^2(\theta-\sin\theta) \\ -\sin\theta/\rho & \cos\theta & \sin\theta & \sin\theta & \rho(1-\cos\theta) \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \rho\theta \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{2}$$

where the $R(1, 5)$ and $R(2, 5)$ terms are the CSR terms. Then, with a similar procedure as that in Section 2.1, this CSR induced transverse movement can also be described by a kick in the middle of a dipole. For a sector magnet, the CSR kick is calculated from equation $M_{\text{half}}(\text{kick}_{\text{CSR}}, \text{kick}'_{\text{CSR}})=(\rho^2(\theta-\sin\theta), \rho(1-\cos\theta))$.

$$\begin{aligned}&(\text{kick}_{\text{CSR}},\text{kick}'_{\text{CSR}})\\&=(\rho^2(\theta\cos(\theta/2)-2\sin(\theta/2)),\theta\rho\sin(\theta/2)).\end{aligned} \tag{3}$$

Unfortunately, the CSR kick description is not as simple as the dispersion kick because the x part of the kick is not zero. Still, this kick description is much easier than the boring 5-by-5 R-matrix manipulation.

3 Cancellation of CSR induced emittance in a DBA cell

In this section, we discuss an optics design technique for the suppression of CSR induced emittance growth within an achromatic cell. Although, for the sake of simplicity, we have considered a symmetric DBA cell, this technique can also be used on other achromatic cells. A symmetric DBA cell is considered, which is composed of two identical sector dipoles, and the drifts and quadrupoles between them are also symmetric. The Twiss parameters are chosen to satisfy the condition: $\beta_1=\beta_2, \alpha_1=-\alpha_2$, where $\beta_1,\alpha_1,\beta_2,\alpha_2$ are Twiss parameters at the middle point of the two dipoles separately, and the phase advance between the middle points of two dipoles is chosen to be π, which is required by the achromatic condition.

The normalized phase space is still adopted to analyze the particle motion in such a DBA cell. Adding the CSR into consideration, at each kick point the particle experienced two kicks: a dispersion kick and a CSR kick, and the vector sum of these two kicks is the overall effect, as shown in Fig. 3. After the first kick point, the particle moves along a circular path in the normalized phase space, until it reaches the second kick point. Similar to the procedure in the cancellation of the dispersion, the effect of the CSR can be canceled if the particle returns to its original circle path after those two kicks.

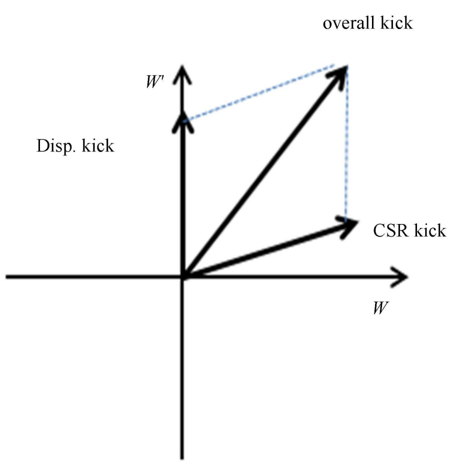

Fig. 3. Dispersion kick and CSR kick at a kick point in the normalized phase space.

For simplicity, we consider a particle that stays at the original point before the first kick. Its trajectory in the normalized phase space will be as follows: it gets a kick from the first kick point and its coordinates can be denoted by a vector $\boldsymbol{R}1$ in the phase space, then the vector $\boldsymbol{R}1$ turn around the original point with an angle π, which is the phase advance between the first and the second kick point, and its vector turns into $\boldsymbol{R}1'$. At the second kick point a second kick vector is added and the position vector turns to be $\boldsymbol{R}1'+\boldsymbol{R}2$. The lattice that makes the final position vector $\boldsymbol{R}'+\boldsymbol{R}2=0$ will show a cancellation of the linear CSR, as well as the dispersion in such a lattice cell, as shown in Fig. 4.

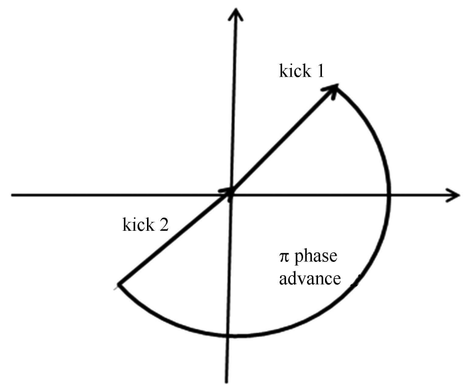

Fig. 4. The particle dynamics in normalized phase space when the CSR and dispersion kicks are considered. Where kick1 and kick2 are all sums of CSR and dispersion kicks, the phase advance is π which is required by the achromatic condition as shown in Section 2.1. From this schematic figure, it is obvious that these kick effects cancel when kick1=kick2.

After the first kick, the position vector in the normalized phase space becomes:

$$\boldsymbol{R}1=\boldsymbol{R}^{\text{norm}}_{\text{dispersion}}+\boldsymbol{R}^{\text{norm}}_{\text{CSR}}, \tag{4}$$

where,

$$\boldsymbol{R}^{\text{norm}}_{\text{dispersion}}=\left(0\quad 2\sin\frac{\theta}{2}\sqrt{\beta_1}\right)\delta_1, \tag{5}$$

$$\begin{aligned}\boldsymbol{R}^{\text{norm}}_{\text{CSR}} = &\left(\left(\rho^2\left(\theta\cos\frac{\theta}{2}-2\sin\frac{\theta}{2}\right)/\sqrt{\beta_1}\right)\right.\\ &\times\alpha_1\left(\rho^2\left(\theta\cos\frac{\theta}{2}-2\sin\frac{\theta}{2}\right)\right.\\ &\left.\left.+\beta_1\left(\theta\rho\sin\frac{\theta}{2}\right)\Big/\sqrt{\beta_1}\right)\right).\end{aligned} \tag{6}$$

Since the phase advance equals π, $\boldsymbol{R}1'$ before the second kick point turns into:

$$\boldsymbol{R}1'=-\boldsymbol{R}1. \tag{7}$$

And the kick vector at the second kick point is just one that resembles $\boldsymbol{R}1$, we get:

$$\boldsymbol{R}2=\boldsymbol{R}^{\text{norm}}_{\text{dispersion}}+\boldsymbol{R}^{\text{norm}}_{\text{CSR}} \tag{8}$$

and

$$\boldsymbol{R}^{\text{norm}}_{\text{dispersion}}=(0,2\sin(\theta/2)\sqrt{\beta_2})\delta_2, \tag{9}$$

$$\begin{aligned}\boldsymbol{R}^{\text{norm}}_{\text{CSR}} = &\left(\left(\rho^2\left(\theta\cos\frac{\theta}{2}-2\sin\frac{\theta}{2}\right)\Big/\sqrt{\beta_2}\right)\right.\\ &\times\alpha_2\rho^2\left(\theta\cos\frac{\theta}{2}-2\sin\frac{\theta}{2}\right)\\ &\left.+\beta_2\left(\theta\rho\sin\frac{\theta}{2}\right)\Big/\sqrt{\beta_2}\right).\end{aligned} \tag{10}$$

But some caution is still needed because of the CSR effect the energy spread δ_2 at kick point 2 is larger than that at kick point 1:

$$\delta_2=\delta_1+\kappa\rho\theta. \tag{11}$$

The CSR cancellation condition requires that:

$$\boldsymbol{R}2+\boldsymbol{R}1'=\boldsymbol{R}2-\boldsymbol{R}1=0. \tag{12}$$

Substituting Eqs. (4), (8) into the equation above, and considering that $\beta_1=\beta_2$, $\alpha_1=-\alpha_2$, the CSR cancellation condition in a DBA cell can be found as:

$$\beta_1=\frac{\alpha_1\rho\left(-2+\theta\cot\dfrac{\theta}{2}\right)}{\theta}, \tag{13}$$

for $\theta\ll 1$, a condition that most transport dipoles fulfill, this condition can be simplified as:

$$\beta_1\sim\alpha_1\rho\theta/6. \tag{14}$$

4 Particle numerical simulations to verify the emittance suppression

This method of emittance compensation is confirmed by a simulation using the particle tracking code ELEGANT. The initial condition of the electron bunch is assumed to be: central energy 1 GeV, bunch charge $Q=$ 500 pC, normalized emittance $\varepsilon_{\text{n}}=0.2$ mm·mrad, bunch length $\sigma_{\text{s}}=100$ fs.

For our purpose, symmetric DBA cells with different initial Twiss parameters are constructed and simulated using ELEGANT, the bending angle of dipoles are 3°. From our emittance cancellation condition (14), we can find the requirement on the Twiss parameters at the entrance of the DBA cell:

$$\begin{aligned}&2\beta_0^2\rho\cot(\theta/2)-2\beta_0\alpha_0[\rho^2-\cot^2(\theta/2)]\\ &-2\rho\cot(\theta/2)(1+\alpha_0^2)=0.\end{aligned} \tag{15}$$

Where β_0, α_0 are the twiss parameters at the DBA entrance, and θ, ρ are bending angle and radius of the dipoles. Simulation results are shown in Fig. 5, where a redder color means a larger emittance growth. The theoretical condition of (15) and that of envelope matching are also shown in the figure. It is shown that the DBA cells that satisfy our constraint really give the smallest emittance growth, and our optic's condition is better than the envelope matching method.

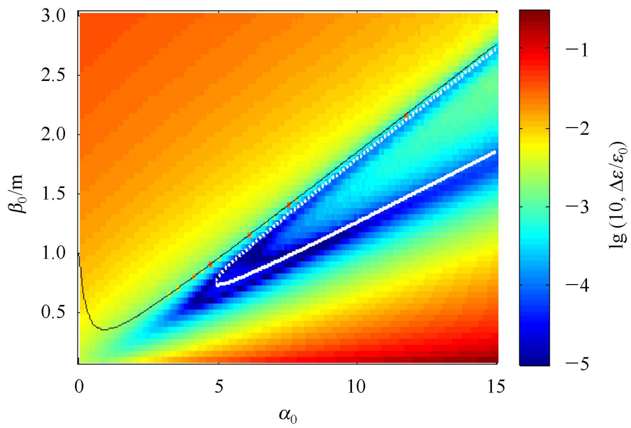

Fig. 5. Simulation results of DBA lattices with different α_0, and β_0. The white line represents DBA lattices that meet our condition (14), and the black line represents lattices satisfy the envelope matching condition.

5 Conclusion

In conclusion, we have derived an optics design technique which can significantly suppress the emittance growth induced by CSR within a DBA cell, which has been verified by simulation results. We believe that using similar derivations shown above, this technique can be expanded to be used in the design of other types of achromatic cells, such as asymmetric double bending cells or TBAs, etc. However, in our study we have assumed a constant bunch length and linear approximation of the CSR induced energy spread, so that in beam transport systems where the bunch length changes significantly, or nonlinear effects of CSR takes an important role, further studies are still needed.

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