

Decay rates and electromagnetic transitions of heavy quarkonia^{*}

J. N. Pandya^{1;1)} N. R. Soni¹ N. Devlani² A. K. Rai³

¹ Applied Physics Department, Faculty of Technology & Engineering, the M S University of Baroda, Vadodara 390001, Gujarat, India

² Applied Physics Department, Polytechnic, the M S University of Baroda, Vadodara 390002, Gujarat, India

³ Department of Applied Physics, Sardar Vallabhbhai National Institute of Technology, Surat 395007, Gujarat, India

Abstract: The electromagnetic radiative transition widths for heavy quarkonia, as well as digamma and digluon decay widths, are computed in the framework of the extended harmonic confinement model (ERHM) and Coulomb plus power potential (CPP _{ν}) with varying potential index ν . The outcome is compared with the values obtained from other theoretical models and experimental results. While the mass spectra, digamma and digluon widths from ERHM as well as CPP _{$\nu=1$} are in good agreement with experimental data, the electromagnetic transition widths span over a wide range for the potential models considered here making it difficult to prefer a particular model over the others because of the lack of experimental data for most transition widths.

Key words: heavy quarkonia, radiative decays, electromagnetic transitions

PACS: 12.39.Jh, 12.39.Pn, 13.20.Gd **DOI:** 10.1088/1674-1137/39/12/123101

1 Introduction

Decay properties of mesons are of special experimental and theoretical interest because they provide us with further insights into the dynamics of these systems in addition to the knowledge we have gained from the spectra of these families. A large number of experimental facilities worldwide have provided and continue to provide enormous amounts of data which need to be interpreted using the available theoretical approaches [1]. Many phenomenological studies on numerous observables of the $c\bar{c}$ and $b\bar{b}$ bound states have established that the non-relativistic nature appears to be an essential ingredient to understand the dynamics of heavy quarkonia [2]. Thus, the heavy quarkonium spectroscopy is mostly dependent on the quark mass m , the momentum mv and the binding energy mv^2 in the non-relativistic limit. Two effective field theories, non-relativistic QCD (NRQCD) [3, 4] and potential NRQCD (pNRQCD) [5, 6], have been developed leading to a large number of new results for several observables in quarkonium physics [7].

Radiative transitions in heavy quarkonia have been a subject of interest as the CLEO-c experiment has measured the magnetic dipole (M1) transitions $J/\psi(1S) \rightarrow \gamma\eta_c(1S)$ and $J/\psi(2S) \rightarrow \gamma\eta_c(1S)$ using a combination of inclusive and exclusive techniques and reconciling with

theoretical calculations of lattice QCD and effective field theory techniques [8, 9]. M1 transition rates are normally weaker than E1 rates, but they are of more interest because they may allow access to spin-singlet states that are very difficult to produce otherwise. It is also interesting that the known M1 rates show serious disagreement between theory and experiment when it comes to potential models. This is in part due to the fact that M1 transitions between different spatial multiplets, such as $J/\psi(1S) \rightarrow \gamma\eta_c(2S \rightarrow 1S)$ are nonzero only due to small relativistic corrections to a vanishing lowest-order M1 matrix element [10].

We use the spectroscopic parameters of the extended harmonic confinement model (ERHM), which has been successful in predictions of masses of open flavour mesons from light to heavy flavour sectors [11–13]. The mass spectra of charmonia and bottomonia predicted by this model, and a Coulomb plus power potential (CPP _{ν}) with varying potential index ν (from 0.5 to 2.0), employing a non-relativistic treatment for heavy quarks [14–17], have been utilized for the present computations along with other theoretical and experimental results.

2 Theoretical framework

One of the tests for the success of any theoretical

Received 23 December 2014, Revised 26 June 2015

^{*} Supported by University Grants Commission, India for Major Research Project F. No.42-775/2013(SR) (J N Pandya) and Dept. of Science and Technology, India, under SERC fast track scheme SR/FTP/PS-152/2012 (A K Rai)

1) E-mail: jnpandya-apphy@msubaroda.ac.in



Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI. Article funded by SCOAP³ and published under licence by Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

model for mesons is the correct prediction of their decay rates. Many phenomenological models predict the masses correctly but overestimate the decay rates [14, 15, 18]. We have successfully employed a phenomenological harmonic potential scheme and CPP_ν potential with varying potential index for different confinement strengths to compute masses of bound states of heavy quarkonia, and the resulting parameters and wave functions have been used to study various decay properties [13].

The choice of scalar plus vector potential for quark confinement has been successful in predictions of the low lying hadronic properties in the relativistic schemes for quark confinement [19–21], which have been extended to accommodate multi-quark states from lighter to heavier flavour sectors with unequal quark masses [11, 12]. The coloured quarks are assumed to be confined through a Lorentz scalar plus a vector potential of the form

$$V(r) = \frac{1}{2}(1 + \gamma_0)A^2r^2 + B, \quad (1)$$

where A and B are the model parameters and γ_0 is the Dirac matrix.

The wave functions for quarkonia are constructed here by retaining the nature of the single particle wave function but with a two particle size parameter $\Omega_N(q_i q_j)$,

$$R_{n\ell}(r) = \left[\frac{\Omega_N^{3/2}}{2\pi} \frac{n!}{\Gamma\left(n + \ell + \frac{3}{2}\right)} \right]^{\frac{1}{2}} (\Omega_N^{1/2} r)^\ell \times \exp\left(\frac{-\Omega_N r^2}{2}\right) L_n^{\ell + \frac{1}{2}}(\Omega_N r^2). \quad (2)$$

The Coulombic part of the energy is computed using the residual Coulomb potential using the colour dielectric “coefficient”, which is found to be state dependent [11], so as to get a consistent Coulombic contribution to the excited states of the hadrons. This is a measure of

the confinement strength through the non-perturbative contributions to the confinement scale at the respective threshold energies of the quark-antiquark excitations.

The spin average (center of weight) masses of the $c\bar{c}$ and $b\bar{b}$ ground states are obtained by choosing the model parameters $m_c = 1.428$ GeV, $m_b = 4.637$ GeV, $k = 0.1925$ and the confinement parameter $A = 0.0685$ GeV^{3/2} [11, 12].

In the other approach using the CPP_ν scheme for the heavy-heavy bound state systems such as $c\bar{c}$ and $b\bar{b}$, we treat the motion of both the quarks and antiquarks non-relativistically [13]. The CPP_ν potential is given by

$$V(r) = \frac{-\alpha_c}{r} + Ar^\nu. \quad (3)$$

Here, for the study of heavy flavoured mesons, $\alpha_c = 4\alpha_s/3$, α_s being the strong running coupling constant, A is the potential parameter and ν is a general power, such that the choice $\nu = 1$ corresponds to the Coulomb plus linear potential.

We have employed the hydrogenic trial wave function here for the present calculations. For excited states we consider the wave function multiplied by an appropriate orthogonal polynomial function such that the generalized variational wave function gets orthonormalized. Thus, the trial wave function for the (n, l) state is assumed to be the form given by

$$R_{nl}(r) = \left(\frac{\mu^3 (n-l-1)!}{2n(n+l)!} \right)^{\frac{1}{2}} (\mu r)^\ell e^{-\mu r/2} L_{n-l-1}^{2l+1}(\mu r). \quad (4)$$

Here, μ is the variational parameter and $L_{n-l-1}^{2l+1}(\mu r)$ is a Laguerre polynomial.

For a chosen value of ν , the variational parameter μ is determined for each state using the virial theorem

$$\langle KE \rangle = \frac{1}{2} \left\langle \frac{rdV}{dr} \right\rangle. \quad (5)$$

The potential index ν is chosen to vary from 0.5 to 2.

Table 1. Digamma decay width of charmonia (keV).

	1^1S_0	2^1S_0	3^1S_0	4^1S_0	1^3P_0	1^3P_2	2^3P_0	2^3P_2
ERHM	8.76	5.94	3.05	1.43	69.97	73.93	6.93	6.98
ERHM(corr)	6.21	4.21	2.17	1.01	71.04	75.06	5.87	5.91
$CPP_{\nu=0.5}$	12.85	3.47	1.83	1.24	5.74	1.54	21.11	5.69
$CPP_{\nu=0.5}$ (corr)	7.32	1.98	1.04	0.71	5.84	1.19	21.59	4.40
$CPP_{\nu=1.0}$	22.79	9.88	6.73	5.28	27.29	7.45	143.30	39.41
$CPP_{\nu=1.0}$ (corr)	12.99	5.63	3.84	3.01	27.91	5.76	146.57	30.49
$CPP_{\nu=1.5}$	30.84	17.55	14.16	12.65	63.35	17.52	511.88	144.33
$CPP_{\nu=1.5}$ (corr)	17.58	10.00	8.07	7.21	64.79	13.56	523.53	111.66
$CPP_{\nu=2.0}$	37.43	25.11	22.88	22.43	108.06	30.26	1058.7	305.98
$CPP_{\nu=2.0}$ (corr)	21.34	14.31	13.04	12.79	110.52	23.41	1082.8	236.72
[29]	10.38	3.378	1.9	1.288	–	–	–	–
[30]	8.5	2.4	0.88	–	2.5	0.31	1.7	0.23
[31]	7.8	3.5	–	–	–	–	–	–
[32]	11.8	–	–	–	–	–	–	–

Table 2. Digluon decay width of charmonia (MeV).

	1^1S_0	2^1S_0	3^1S_0	4^1S_0	1^3P_0	1^3P_2	2^3P_0	2^3P_2
ERHM	13.48	9.14	4.7	2.19	0.11	0.11	9.07	9.13
ERHM(corr)	19.04	12.91	6.64	3.1	0.19	0.2	5.31	5.43
CPP $_{\nu=0.5}$	43.41	11.73	6.17	4.19	0.019	3.71	0.07	13.74
CPP $_{\nu=0.5}$ (corr)	69.94	18.89	9.94	6.76	0.040	1.43	0.15	5.29
CPP $_{\nu=1.0}$	77.01	33.37	22.74	17.84	0.092	17.99	0.48	95.21
CPP $_{\nu=1.0}$ (corr)	124.08	53.77	36.64	28.74	0.195	6.93	1.02	36.69
CPP $_{\nu=1.5}$	104.18	59.28	47.85	42.73	0.214	42.33	1.73	348.66
CPP $_{\nu=1.5}$ (corr)	167.85	95.51	77.09	68.85	0.453	16.31	3.66	134.38
CPP $_{\nu=2}$	126.46	84.83	77.29	75.79	0.365	73.11	3.58	739.15
CPP $_{\nu=2.0}$ (corr)	203.75	136.67	124.53	122.12	0.773	28.18	7.57	284.88
[22]	26.7±3.0	–	–	–	10.2±0.7	2.034±0.12	–	–
[23]	48.927	–	–	–	38.574	4.396	–	–
[33]pert.	15.70	–	–	–	4.68	1.72	–	–
[33]nonpert.	10.57	–	–	–	4.88	0.69	–	–

Table 3. Digamma decay width of bottomonia (keV).

	1^1S_0	2^1S_0	3^1S_0	4^1S_0	1^3P_0	1^3P_2	2^3P_0	2^3P_2
ERHM	0.47	0.26	0.12	0.01	1.37	1.39	0.12	0.12
ERHM(corr)	0.35	0.20	0.09	0.07	1.39	1.40	0.10	0.10
CPP $_{\nu=0.5}$	0.36	0.06	0.03	0.038	0.02	0.005	0.057	0.015
CPP $_{\nu=0.5}$ (corr)	0.24	0.04	0.02	0.026	0.02	0.004	0.058	0.013
CPP $_{\nu=1.0}$	0.55	0.15	0.09	0.080	0.08	0.022	0.42	0.11
CPP $_{\nu=1.0}$ (corr)	0.37	0.10	0.06	0.054	0.08	0.018	0.43	0.09
CPP $_{\nu=1.5}$	0.71	0.27	0.18	0.123	0.20	0.055	1.34	0.36
CPP $_{\nu=1.5}$ (corr)	0.48	0.18	0.12	0.084	0.21	0.045	1.36	0.30
CPP $_{\nu=2.0}$	0.84	0.38	0.29	0.165	0.35	0.095	2.83	0.76
CPP $_{\nu=2.0}$ (corr)	0.57	0.26	0.20	0.112	0.36	0.078	2.88	0.63
[29]	0.496	0.212	0.135	0.099	–	–	–	–
[30]	0.527	0.263	0.172	–	0.037	0.0066	0.037	0.0067
[31]	0.460	0.20	–	–	–	–	–	–
[32]	0.580	–	–	–	–	–	–	–

Table 4. Digluon decay width of bottomonia (MeV).

	1^1S_0	2^1S_0	3^1S_0	4^1S_0	1^3P_0	1^3P_2	2^3P_0	2^3P_2
ERHM	7.61	4.31	1.99	1.58	22.45	22.68	1.93	1.94
ERHM(corr)	9.95	5.64	2.61	2.07	38.17	38.57	1.92	1.92
CPP $_{\nu=0.5}$	10.92	1.77	0.78	1.17	0.61	0.16	1.74	0.46
CPP $_{\nu=0.5}$ (corr)	15.51	2.51	1.11	1.66	1.20	0.16	3.40	0.46
CPP $_{\nu=1.0}$	16.71	4.65	2.72	2.43	2.51	0.67	12.81	3.42
CPP $_{\nu=1.0}$ (corr)	23.72	6.61	3.86	3.45	4.90	0.66	25.04	3.39
CPP $_{\nu=1.5}$	21.53	8.14	5.60	3.76	6.22	1.67	40.70	10.91
CPP $_{\nu=1.5}$ (corr)	30.58	11.55	7.95	5.34	12.16	1.65	79.57	10.81
CPP $_{\nu=2.0}$	25.55	11.66	8.95	5.03	10.74	2.88	86.12	23.15
CPP $_{\nu=2.0}$ (corr)	36.29	16.56	12.72	7.14	21.00	2.85	168.36	22.93
[23]	14.64	–	–	–	2.745	0.429	–	–
[33]pert.	11.49	–	–	–	0.96	0.33	–	–
[33]nonpert.	12.39	–	–	–	2.74	0.25	–	–
[39]	12.46	–	–	–	2.15	0.22	–	–

Quark mass parameters are fitted to get the experimental ground state masses of $m_c=1.31$ GeV, $m_b=4.66$ GeV, $\alpha_c=0.4$ (for $c\bar{c}$) and $\alpha_c=0.3$ (for $b\bar{b}$). The potential parameter A also varies with ν [16].

We have done a completely parameter-free computa-

tion of digamma and digluon decay widths and radiative electric and magnetic dipole transition widths using the parameters of these phenomenological models that were fixed to obtain the ground state masses of the quarkonia systems.

Table 5. E1 transition partial widths of $c\bar{c}$ (keV).

transitions	ERHM	CPP $_{\nu}$				[34]	[35]	[23]	[36]	[30]	[22]
		$\nu=0.5$	$\nu=1.0$	$\nu=1.5$	$\nu=2.0$						
$2^3S_1 \rightarrow 1^3P_0$	9.2	6.7	38.2	89.2	145.8	51.7	45	–	47	74	29.8±1.29
$2^3S_1 \rightarrow 1^3P_1$	18.6	13.8	73.6	164.6	259.7	44.9	40.9	–	42.8	62	28.2±1.47
$2^3S_1 \rightarrow 1^3P_2$	11.3	8.4	37.2	72.4	100.3	30.9	26.5	–	30.1	43	26.5±1.3
$3^3S_1 \rightarrow 2^3P_0$	16.4	5.9	51.4	164.3	349.2	–	87.3	–	–	–	–
$3^3S_1 \rightarrow 2^3P_1$	43.3	8.4	65.2	192.7	382.9	65.7	–	–	–	–	–
$3^3S_1 \rightarrow 2^3P_2$	54.2	1.6	4	4.1	3.1	–	31.6	–	–	–	–
$3^3S_1 \rightarrow 1^3P_0$	129.4	105.1	583.9	1389	2274	–	1.2	–	–	–	–
$3^3S_1 \rightarrow 1^3P_1$	336.4	281.5	1531	3607	5863	–	2.5	–	–	–	–
$3^3S_1 \rightarrow 1^3P_2$	410.1	1897	4379	6998	–	–	3.3	–	–	–	–
$1^3P_2 \rightarrow 1^3S_1$	680.7	168	421	652	828	448	390.6	250	315	424	390±26
$1^3P_1 \rightarrow 1^3S_1$	426.2	127	269	363	409	333	287	229	41	314	299±22
$1^3P_0 \rightarrow 1^3S_1$	325.9	110	209	256	264	161	142	173	120	152	133±9
$1^1P_1 \rightarrow 1^1S_0$	1076.2	401	1015	1569	2000	723	610	451	482	498	–
$2^3P_2 \rightarrow 2^3S_1$	325.3	151	701	1707	2883	–	358.6	83	–	225	–
$2^3P_1 \rightarrow 2^3S_1$	258.9	92	316	596	824	–	208.3	73.8	–	103	–
$2^3P_0 \rightarrow 2^3S_1$	231.0	68	190	291	322	–	53.6	49.4	–	61	–
$2^1P_1 \rightarrow 2^1S_0$	611.7	184	843	1961	3219	–	–	146.9	–	309	–
$2^3P_2 \rightarrow 1^3S_1$	700.1	187	1279	3510	5896	–	33	140	–	101	–
$2^3P_1 \rightarrow 1^3S_1$	661.3	160	962	2352	3590	–	28	133	–	83	–
$2^3P_0 \rightarrow 1^3S_1$	643.5	146	822	1880	2683	–	21	114	–	74	–
$2^3P_1 \rightarrow 1^1S_0$	951.6	93	549	1321	2013	–	–	227	–	134	–

 Table 6. E1 transition partial widths of $b\bar{b}$ (keV).

transitions	ERHM	CPP $_{\nu}$				[34]	[35]	[23]	[36]	[30]	[22]
		$\nu=0.5$	$\nu=1.0$	$\nu=1.5$	$\nu=2.0$						
$2^3S_1 \rightarrow 1^3P_0$	0.24	0.06	0.4	1.08	1.63	1.65	1.15	–	1.29	1.67	1.21±0.16
$2^3S_1 \rightarrow 1^3P_1$	0.40	0.12	0.74	1.75	2.71	2.57	1.87	–	2.0	2054	2.21±0.22
$2^3S_1 \rightarrow 1^3P_2$	0.12	0.04	0.38	1.39	3.03	2.53	1.88	–	2.04	2.62	2.29±0.22
$3^3S_1 \rightarrow 2^3P_0$	0.35	0.04	0.32	1.03	2.16	1.65	1.67	–	1.35	1.83	1.2±0.16
$3^3S_1 \rightarrow 2^3P_1$	0.82	0.08	0.62	1.78	3.60	2.65	2.74	–	2.20	2.96	2.56±0.34
$3^3S_1 \rightarrow 2^3P_2$	0.80	0.06	0.30	0.62	0.98	2.89	2.80	–	2.40	3.23	2.66±0.41
$3^3S_1 \rightarrow 1^3P_0$	3.91	2.38	15.4	40.4	72.0	0.124	0.03	–	0.001	0.07	0.055±0.08
$3^3S_1 \rightarrow 1^3P_1$	9.50	6.38	41.1	106.8	188.8	0.307	0.09	–	0.008	0.17	<0.018±0.001
$3^3S_1 \rightarrow 1^3P_2$	9.86	8.22	54.7	153.7	290.8	0.445	0.13	–	0.015	0.25	<0.2±0.32
$1^3P_2 \rightarrow 1^3S_1$	61.96	11.3	26.7	40.1	48.8	42.7	31	44.0	31.6	38	–
$1^3P_1 \rightarrow 1^3S_1$	39.58	09.4	21.3	33.3	43.5	37.1	27	42.0	27.8	34	–
$1^3P_0 \rightarrow 1^3S_1$	30.72	08.6	18.7	27.8	35.0	29.5	22	37.0	22.0	27	–
$1^1P_1 \rightarrow 1^3S_0$	62.70	15.7	37.7	60.4	81.6	–	38	60.0	–	56.8	–
$2^3P_2 \rightarrow 2^3S_1$	14.57	04.9	23.4	55.5	96.1	18.8	17	20.4	14.5	18.8	–
$2^3P_1 \rightarrow 2^3S_1$	10.65	04.3	18.2	39.5	63.7	15.9	14	12.5	12.4	15.9	–
$2^3P_0 \rightarrow 2^3S_1$	8.98	03.9	15.9	32.8	51.1	11.7	10	4.4	9.2	11.7	–
$2^1P_1 \rightarrow 2^1S_0$	15.67	05.4	25.4	60.0	102.1	23.6	–	25.8	–	24.7	–
$2^3P_2 \rightarrow 1^3S_1$	45.03	09.0	33.0	67.2	104.0	8.41	7.74	20.8	12.7	13	–
$2^3P_1 \rightarrow 1^3S_1$	41.71	08.6	30.2	58.9	88.0	8.01	7.31	19.9	12.7	12.4	–
$2^3P_0 \rightarrow 1^3S_1$	40.12	08.4	28.8	55.0	80.8	7.36	6.69	14.1	10.9	11.4	–
$2^1P_1 \rightarrow 1^1S_1$	49.57	0.3	01.7	04.5	08.2	9.9	–	14.1	10.9	15.9	–

3 Digamma and digluon decay widths

Using the model parameters and the radial wave functions, we compute the digamma ($\Gamma_{\gamma\gamma}(\eta_Q)$) and digluon ($\Gamma_{gg}(\chi_Q)$) decay widths. The digamma decay width of

the P -wave $Q\bar{Q}$ state χ_{Q1} is forbidden according to the Landau–Yang theorem. Most of the quark model predictions for the S -wave $\eta_Q \rightarrow \gamma\gamma$ width are comparable with the experimental result, while the theoretical predictions for the P -wave ($\chi_{Q0,2} \rightarrow \gamma\gamma$) widths differ significantly

Table 7. Radiative M1 transition widths of $c\bar{c}$ (keV).

transition	$1^3S_1 \rightarrow 1^1S_0$	$2^3S_1 \rightarrow 2^1S_0$	$3^3S_1 \rightarrow 3^1S_0$	$2^3S_1 \rightarrow 1^1S_0$
ERHM	0.703 (110)	0.151 (62)	0.023 (17)	20.521 (654)
CPP $_{\nu=0.5}$	1.86	0.03	0.004	16.52
CPP $_{\nu=1.0}$	9.68	0.55	0.135	58.13
CPP $_{\nu=1.5}$	20.45	2.60	0.942	108.44
CPP $_{\nu=2.0}$	38.35	6.92	3.241	157.23
[9]	1.5±1.0	–	–	–
[10]NR	2.90 (116)	0.21 (48)	0.046 (29)	–
[23]	1.29	0.12	0.04	–
[35]	2.7	1.2	–	–
[22]	1.21±0.37	< 0.67	–	3000±500

 Table 8. Radiative M1 transition widths of $b\bar{b}$ (eV).

transition	$1^3S_1 \rightarrow 1^1S_0$	$2^3S_1 \rightarrow 2^1S_0$	$3^3S_1 \rightarrow 3^1S_0$	$2^3S_1 \rightarrow 1^1S_0$
ERHM	2.33 (36)	0.169 (15)	0.050 (10)	1395.9 (580)
CPP $_{\nu=0.5}$	2.51	0.01	0.001	223.23
CPP $_{\nu=1.0}$	9.13	0.17	0.036	799.45
CPP $_{\nu=1.5}$	19.12	0.98	0.244	1629.06
CPP $_{\nu=2.0}$	31.20	2.51	1.088	2514.04
[23]	7.28	0.67	0.19	–
[34]	5.8 (60)	1.40 (33)	0.80 (27)	–
[35]	4.0	0.5	–	–
[36]	8.95	1.51	0.826	–
[37]	9.2	0.6	0.6	–
[38]	7.7 (59)	0.53 (25)	0.13 (16)	–

from the experimental observations [22]. The contribution from QCD corrections takes care of this discrepancy. The one-loop QCD radiative corrections in the digamma decay widths of $1^1S_0(\eta_Q)$, $3^1P_0(\chi_{Q0})$ and $3^1P_2(\chi_{Q2})$ are computed using the non relativistic expressions given by [23, 24]:

$$\Gamma_{\gamma\gamma}(\eta_Q) = \frac{3e_Q^4 \alpha_{\text{em}}^2 M_{\eta_Q} |R_0(0)|^2}{2m_Q^3} \left[1 - \frac{\alpha_s}{\pi} \frac{(20 - \pi^2)}{3} \right], \quad (6)$$

$$\Gamma_{\gamma\gamma}(\chi_{Q0}) = \frac{27e_Q^4 \alpha_{\text{em}}^2 M_{\chi_{Q0}} |R'_1(0)|^2}{2m_Q^5} \left[1 + B_0 \frac{\alpha_s}{\pi} \right], \quad (7)$$

$$\Gamma_{\gamma\gamma}(\chi_{Q2}) = \frac{4}{15} \frac{27e_Q^4 \alpha_{\text{em}}^2 M_{\chi_{Q2}} |R'_1(0)|^2}{2m_Q^5} \left[1 + B_2 \frac{\alpha_s}{\pi} \right], \quad (8)$$

where $B_0 = \pi^2/3 - 28/9$ and $B_2 = -16/3$ are the next-to-leading-order (NLO) QCD radiative corrections [25–27].

Similarly, the digluon decay widths of the η_Q , χ_{Q0} and χ_{Q2} states are given by [28]:

$$\Gamma_{\text{gg}}(\eta_Q) = \frac{\alpha_s^2 M_{\eta_Q} |R_0(0)|^2}{3m_Q^3} [1 + C_Q(\alpha_s/\pi)], \quad (9)$$

$$\Gamma_{\text{gg}}(\chi_{Q0}) = \frac{3\alpha_s^2 M_{\chi_{Q0}} |R'_1(0)|^2}{m_Q^5} [1 + C_{0Q}(\alpha_s/\pi)]. \quad (10)$$

$$\Gamma_{\text{gg}}(\chi_{Q2}) = \left(\frac{4}{15} \right) \frac{3\alpha_s^2 M_{\chi_{Q2}} |R'_1(0)|^2}{m_Q^5} [1 + C_{2Q}(\alpha_s/\pi)]. \quad (11)$$

Here, the quantities in the brackets are the NLO QCD radiative corrections [27] and the coefficients have values of $C_Q = 4.4$, $C_{0Q} = 10.0$ and $C_{2Q} = -0.1$ for the bottom quark.

4 Radiative E1 and M1 transitions

In the non-relativistic limit, the M1 transition width between two S -wave states is given by [9]

$$\Gamma_{n^3S_1 \rightarrow n'^1S_0 \gamma} = \frac{4}{3} \alpha e_Q^2 \frac{k_\gamma^3}{m^2} \left| \int_0^\infty r^2 dr R_{n'0}(r) R_{n0}(r) j_0\left(\frac{k_\gamma r}{2}\right) \right|^2, \quad (12)$$

where e_Q is the fraction of electrical charge of the heavy quark ($e_b = -1/3$, $e_c = 2/3$), α is the fine structure constant and $R_{nl}(r)$ are the radial Schrödinger wave functions. The photon energy k_γ is nearly equal to the mass difference of the two quarkonia, so it is of order mv^2 or smaller. This is unlike radiative transitions from a heavy quarkonium to a light meson, such as $J/\psi \rightarrow \eta\gamma$, where a hard photon is emitted. Since $r \sim 1/(mv)$, the spherical Bessel function is expanded as $j_0(k_\gamma r/2) = 1 - (k_\gamma r)^2/24 + \dots$ [9]. While the overlap integral in (12) is unity at leading order for $n=n'$ (allowed transitions), it vanishes for $n \neq n'$ (hindered transitions). The widths of hindered transitions are determined by higher-order and relativistic corrections only.

In the non-relativistic limit, radiative E1 and M1 transition partial widths are given by [9]

$$\begin{aligned} & \Gamma_{n^2S+1L_i J_i \rightarrow n'^2S+1L_f J_f \gamma} \\ &= \frac{4\alpha e_Q^2 k_\gamma^3}{3} (2J'+1) \max(L_i, L_f) \\ & \times \left\{ \begin{matrix} J_i & 1 & J_f \\ L_f & S & L_i \end{matrix} \right\} \times |\langle f|r|i \rangle|^2, \end{aligned} \quad (13)$$

$$\begin{aligned} \Gamma_{n^3S_1 \rightarrow n'^1S_0 \gamma} &= \frac{4}{3} \frac{2J'+1}{2L+1} \delta_{LL'} \delta_{S,S'+1} \alpha e_Q^2 \frac{k_\gamma^3}{m^2} \\ & \times \left| \int_0^\infty r^2 dr R_{n'0}(r) R_{n0}(r) j_0\left(\frac{k_\gamma r}{2}\right) \right|^2. \end{aligned} \quad (14)$$

The CLEO-c experiment has measured the magnetic dipole (M1) transitions $J/\psi(1S) \rightarrow \gamma\eta_c(1S)$ and $\psi(2S) \rightarrow \gamma\eta_c(1S)$ using a combination of inclusive and exclusive techniques reconciling with the theoretical calculations of lattice QCD and effective field theory techniques [8, 9]. M1 transition rates are normally weaker than E1 rates, but they are of more interest because they may allow access to spin-singlet states that are very difficult to produce otherwise. The spectroscopic parameters of ERHM and CPP $_{\nu}$ are utilized for the present computations.

5 Results and conclusions

In this paper, we have employed the masses of the pseudoscalar and vector mesons, their wave functions, and other input parameters from our earlier work [13] for the calculations of the digamma, digluon decay widths as well as E1 & M1 transitions. E1 and M1 radiative transitions of the $c\bar{c}$ and $b\bar{b}$ mesons in the ERHM and Coulomb plus power potential CPP_ν models and computed numerical results are tabulated in Tables 1–8. The digamma and digluon decay widths of the $c\bar{c}$ and $b\bar{b}$ mesons are computed with and without QCD corrections. The ERHM predictions of digamma decay widths of charmonia for the ground state are found to be comparable to the other theoretical results. In case of the CPP_ν model these values are fairly close around $\nu < 1$. A similar trend is found in the case of digluon decay rates of charmonia. The digamma and digluon decay widths predicted by the ERHM and CPP_ν models are very close to the other theoretical predictions.

The computations of E1 transition widths are done without any relativistic correction terms. This indicates the possible inclusion of the same in the wave function with a single center size parameter. The E1 and M1

transitions of the $c\bar{c}$ and $b\bar{b}$ mesons have been calculated by several groups (See Tables 5–8) but their predictions are not in mutual agreement. The predictions from References [34, 35] and the CPP_ν model (at $\nu \simeq 1$ for $c\bar{c}$ and at $\nu \simeq 1.5$ for $b\bar{b}$ mesons) are in fair agreement with experimental values. One of the limitations of the CPP_ν model is the inability to obtain the mass spectra of the $c\bar{c}$ and $b\bar{b}$ mesons at the same potential index ν . The computed magnetic radiative transition rates are tabulated along with other theoretical predictions and available experimental values in Tables 7 and 8. The values in the parentheses are the energy of the photon in MeV. The transition widths obtained by the potential models show a large deviation from the experimental data; however, the values computed using effective mean field theories ($\Gamma_{J/\psi \rightarrow \eta_c \gamma} = 1.5 \pm 1.0$ keV and $\Gamma_{\Upsilon(1S) \rightarrow \eta_b \gamma} = 3.6 \pm 2.9$ eV [9]), are found to be nearly the same as the potential model results. The photon energies in all the models are found to be nearly the same as the mass splitting. The wide variation in predicted hyperfine splitting leads to considerable uncertainty in the predicted rates for these transitions. Differences in the theoretical assumptions of the potential models make it difficult to draw sharp conclusions about the validity of a particular model because of the lack of experimental data.

References

- 1 Wiedner U. Progress in Particle and Nucl. Phys., 2011, **66**: 477
- 2 Brambilla N et al. hep-ph/0412158
- 3 Caswell W E, Lepage G P. Phys. Lett. B, 1986, **167**: 437
- 4 Bodwin G T, Braaten E, Lepage G P. Phys. Rev. D, 1995, **51**: 1125; 1997, **55**: 5853
- 5 Pineda A, Soto J. Nucl. Phys. Proc. Suppl., 1998, **64**: 428. [arXiv:hep-ph/9707481]
- 6 Brambilla N, Pineda A, Soto J, Vairo A. Nucl. Phys. B, 2000, **566**: 275. [arXiv:hep-ph/9907240]
- 7 Brambilla N et al. Eur. Phys. J. C, 2011, **71**: 1534
- 8 Mitchell R E et al. (CLEO collaboration). Phys. Rev. Lett., 2009, **102**: 011801; 2011, **106**: 159903
- 9 Brambilla N, JIA Y, Vairo A. Phys. Rev. D, 2006, **73**: 054005
- 10 Barnes T, Godfrey S, Swanon E S. Phys. Rev. D, 2005, **72**: 054026
- 11 Vinodkumar P C, Pandya J N, Bannur V M, Khadkikar S B. Eur. Phys. J., A, 1999, **4**: 83
- 12 Pandya J N, Vinodkumar P C. Pramana J. Phys., 2001, **57**: 821
- 13 Rai A K, Pandya J N, Vinodkumar P C. Eur. Phys. Jnl. A, 2008, **38**: 77; Jnl. Phys. G, 2005, **31**: 1453
- 14 Rai A K, Parmar R H, Vinodkumar P C. Jnl. Phys. G, 2002, **28**: 2275
- 15 Rai A K, Vinodkumar P C. Pramana J. Phys., 2006, **66**: 953
- 16 Rai A K, Patel B, Vinodkumar P C. Phys. Rev. C, 2008, **78**: 055202
- 17 Rai A K, Pandya J N, Vinodkumar P C. Nucl. Phys. A, 2007, **782**: 406; Indian J. Phys. A, 2006, **80**: 387
- 18 Buchmüller W, Tye S H H. Phys. Rev. D, 1981, **24**: 132
- 19 Khadkikar S B, Gupta S K. Phys. Lett. B, 1983, **124**: 523
- 20 Gupta S K, Khadkikar S B. Phys. Rev. D, 1987, **36**: 307
- 21 Jena S N, Behera M R, Panda S. Phys. Rev. D, 1996, **54**: 11
- 22 Olive K A et al. (Particle Data Group). Chin. Phys. C, 2014, **38**: 090001
- 23 Parmar A et al. Nucl. Phys. A, 2010, **848**: 299
- 24 Patel B, Vinodkumar P C. J. Phys. G: Nucl. Part. Phys., 2009 **36**: 035003
- 25 Kwong Waikwok et al. Phys. Rev. D, 1988, **37**: 3210
- 26 Barbieri R, Caffo M, Gatto R, Remiddi E. Nucl. Phys. B, 1981, **192**: 61
- 27 Mangano M, Petrelli A. Phys. Lett. B, 1945, **352**: 445
- 28 Lansberg J P, Pham T N. Phys. Rev. D, 2009, **79**: 094016
- 29 Shah M, Parmar A, Vinodkumar P C. Phys. Rev. D, 2012, **86**: 034015
- 30 B. Quing Li, K. Ta Chao. Phys. Rev. D, 2009, **79**: 094004; Bai Quing Li, Kung- Ta Chao. Commun. Theor. Phys., 2009, **52**: 653
- 31 Schuler G A, Berends F A, van Gulik R. Nucl. Phys. B, 1998, **523**: 423
- 32 Ahmady M R, Mendel R R. Phys. Rev. D, 1995, **51**: 141
- 33 Lavery J T, Radford S F, Repko W W. arXiv:0901.3917 [hep-ph]
- 34 Ebert D, Faustov N, Galkin V O. Phys. Rev. D, 2003, **67**: 014027
- 35 Radford S F, Repko W W. Phys. Rev. D, 2007, **75**: 074031
- 36 Brambilla N et al. CERN Yellow Report 2005-005. arxiv://hep-ph/0412158
- 37 Anisovich V V, Dakhno L G, Matveev M A, Nikonov V A, Sarantsev A V. Phys. Atom. Nucl., 2007, **70**: 63
- 38 Lähde T A. Nucl. Phys. A, 2003, **714**: 183
- 39 Gupta Suraj N, Johnson James M, Repko Wayne W. Phys. Rev. D, 1996, **54**: 2075. arXiv:hep-ph/9606349