

Temperature dependence of quarks and gluon vacuum condensate in the Dyson-Schwinger Equations at finite temperature^{*}

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Abstract: Based on the Dyson-Schwinger Equations (DSEs), the two-quark vacuum condensate, the four-quark vacuum condensate, and the quark gluon mixed vacuum condensate in the non-perturbative QCD vacuum state are investigated by solving the DSEs with rainbow truncation at zero- and finite- temperature, respectively. These condensates are important input parameters in QCD sum rule with zero and finite temperature, and in studying hadron physics, as well as predicting the quark mean squared momentum m_0^2 - also called quark virtuality in the QCD vacuum state. The present calculated results show that these physical quantities are almost independent of the temperature below the critical point temperature $T_c = 131$ MeV, and above T_c the chiral symmetry is restored. For comparison we calculate the temperature dependence of the “in-hadron condensate” for pion. At the same time, we also calculate the ratio of the quark gluon mixed vacuum condensate to the two-quark vacuum condensate by using these condensates, and the unknown quark mean squared momentum in the QCD vacuum state has been obtained. The results show that the ratio $m_0^2(T)$ is almost flat in the temperature region from 0 to T_c , although there are drastic changes of the quark vacuum condensate and the quark gluon mixed vacuum condensate at the region. Our predicted ratio comes out to be $m_0^2(T) = 2.41$ GeV² at the Chiral limit, which is consistent with other theory model predictions, and strongly indicates the significance that the quark gluon mixed vacuum condensate has played in the virtuality calculations.

Key words: Dyson-Schwinger Equations at zero and finite temperature, dynamical chiral symmetry breaking, quark and gluon vacuum condensate

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1 Introduction

With the development of heavy-ion collision experiments, more attention has been turned to exploring the hot and dense QCD matter. The hot and dense matter can be studied via various approaches, such as: Lattice QCD, QCD sum rules, chiral perturbation theory as well as the Dyson-Schwinger Equations (DSEs) at finite temperature. Due to the asymptotic freedom feature of QCD, the QCD matter will take place as a phase transition from the hadronic phase, with quarks and gluons being bound states inside the hadron, to the quark gluon plasma phase where the bound clusters of quarks and gluons have been de-confined at sufficient high temperature and /or density. Studying the Chiral condensates at zero- and finite temperature is of crucial importance for nuclear and hadronic physics research, and even for

cosmological studies.

There are a lot of studies and published references about the researches. Among these works the important input ingredients are QCD vacuum condensates. The vacuum of non-perturbative QCD is densely populated by long-wave fluctuations of quark and gluon fields. The order parameters of this complicated state are described by various vacuum condensates $\langle 0 | : \bar{q}q : | 0 \rangle$, $\langle 0 | : G_{\mu\nu}^a G_{\mu\nu}^a : | 0 \rangle$, $\langle 0 | : \bar{q} \left[ig_s G_{\mu\nu}^a \sigma_{\mu\nu} \frac{\lambda^a}{2} \right] q : | 0 \rangle$, ..., which are the vacuum matrix elements of various singlet combinations of quark and gluon fields. In QCD sum rules, various condensates are input parameters so that they play an important role to reproduce various hadronic properties phenomenologically in the operator product expansion calculations (OPE) [1–3]. Contrary to the important quark condensate $\langle 0 | : \bar{q}q : | 0 \rangle$ and gluon conden-

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sate $\langle 0|:G_{\mu\nu}^a G_{\mu\nu}^a:|0\rangle$, the quark gluon mixed condensate $\langle 0|:\bar{q}\left[ig_s G_{\mu\nu}^a \sigma_{\mu\nu} \frac{\lambda^a}{2}\right]q:|0\rangle$ characterizes the direct correlation between quarks and gluons, and together with the nonzero two-quark condensate $\langle 0|:\bar{q}q:|0\rangle$, it is responsible for the spontaneous breakdown of chiral symmetry. In our previous works, we have studied the various non-perturbative quantities at zero temperature by use of the DSEs in the ‘‘rainbow’’ truncation, i.e. the quark condensate, the quark gluon mixed condensate, susceptibility, and so on [4]. Comparing our theoretical results with others, such as, QCD sum rules [5], Lattice QCD [6], we find our calculations are in a good agreement with theirs. Now, we want to extend the zero temperature calculations of DSEs to nonzero temperature DSEs calculation. There have been many works on the application of DSEs at finite temperature; for a recent work please see [7]. As is known to all, solving DSEs at finite temperature is quite difficult, but using separable model interactions greatly simplifies the calculations [8, 9]. In the present work, we study the DSEs at finite temperature by use of the separable model interactions. The main interest of this work focuses on the consideration of nonzero temperature, which allows us to study the QCD phase diagram along the axis of zero chemical potential, including deconfinement and the chiral symmetry restoration.

2 DSEs at zero and finite temperature

2.1 DSEs at zero temperature

To study quark and gluon vacuum condensates, we need to know the quark propagators, which determine various quark condensates and the quark gluon mixed condensates under the OPE constraints. The quark propagator in configuration space is given as the mean value of the time-ordered product formed by the quark and antiquark fields

$$S_f(x) = \langle 0|Tq_f(x)\bar{q}_f(0)|0\rangle. \quad (1)$$

For the physical vacuum, the quark propagator can be divided into a perturbative $S_f^{\text{PT}}(x)$ and a non-perturbative part $S_f^{\text{NPT}}(x)$. One can write [10, 11]

$$S_f(x) = S_f^{\text{PT}}(x) + S_f^{\text{NPT}}(x). \quad (2)$$

In momentum space, $S_f^{\text{NPT}}(p)$ is related to the quark self-energy, so the quark propagator of DSEs can be written

$$S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{4}{3}g_s^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_f(k) \times \Gamma^\nu(k,p) G_{\mu\nu}(p-k). \quad (3)$$

In Eq. (3), g_s is the strong coupling constant of QCD with the usual $\alpha_s(Q)$ by the relationship of $\alpha_s(Q) = g_s^2(Q)/4\pi$. The $G_{\mu\nu}(p-k)$ denotes the fully dressed gluon propagator which is an unknown factor, and m_f is the current

quark mass with the subscript f to stand for the quark flavor. In the Feymann gauge, the simplest separable Ansatz has the following form [8, 9]

$$g_s^2 G_{\mu\nu}(p-k) \rightarrow \delta_{\mu\nu} G(p^2, k^2, p \cdot k), \quad (4)$$

$$G(p^2, k^2, p \cdot k) = D_0 F_0(p^2) F_0(k^2) + D_1 F_1(p^2) F_1(k^2)(p \cdot k), \quad (5)$$

where D_0 and D_1 are two strength parameters, and F_0 and F_1 are corresponding form factors.

As it is impossible to solve the complete set of DSEs, one has to truncate this infinite tower in a physically acceptable way to reduce them to something that is soluble. To do it, we use a bare vertex γ^ν to replace the full one $\Gamma^\nu(k,p)$ in Eq. (3). This procedure is called a ‘‘Rainbow’’ truncation of DSEs. Thus, Eq. (3) then becomes

$$S_f^{-1}(p) = i\gamma \cdot p + m_f + \frac{4}{3}g_s^2 \int \frac{d^4k}{(2\pi)^4} \gamma^\mu S_f(k) \gamma^\nu(k,p) G_{\mu\nu}(p-k). \quad (6)$$

An important observation is that the general form of the inverse quark propagator $S_f^{-1}(p)$ can be rewritten in Euclidean space [12] as

$$S_f^{-1}(p) = i\gamma \cdot p A_f(p^2) + B_f(p^2), \quad (7)$$

with $A_f(p^2)$ and $B_f(p^2)$ are scalar functions of the p^2 .

With ‘‘Rainbow’’ truncation, we can obtain the coupling integral equations for quark amplitudes $A_f(p^2)$ and $B_f(p^2)$ and these coupling equations take the form in the Feymann gauge

$$[A_f(p^2) - 1]p^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} G(p-q) \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p \cdot q, \quad (8)$$

$$B_f(p^2) - m_f = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} G(p-q) \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)}. \quad (9)$$

2.2 DSEs at finite temperature

So far, we have only considered the quark propagator at zero temperature. An extension to the finite temperature from the zero temperature of DSEs is systematically accomplished by a transcription of the Euclidean quark four momentum via $p \rightarrow p_n = (\omega_n, \vec{p})$, where $\omega_n = (2n+1)\pi T$ for Fermion are the discrete Matsubara frequencies [13]. Therefore, a sum over the Matsubara frequencies replaces the integral over the energy.

The fully dressed quark DSEs propagator at finite temperature can now be written as

$$S_f^{-1}(p_n, T) = i\vec{\gamma} \cdot \vec{p} A_f(p_n^2, T) + i\gamma_4 \omega_n C_f(p_n^2, T) + B_f(p_n^2, T), \quad (10)$$

where $p_n^2 = \omega_n^2 + \vec{p}^2$. Due to the breaking of $O(4)$ symmetry in the four momentum space, we have three quark amplitudes A_f , B_f and C_f . The solutions have the form $A_f(p_n^2, T) = 1 + a_f(T)F_1(p_n^2)$, $B_f(p_n^2, T) = m_f + b_f(T)F_0(p_n^2)$

and $C_f(p_n^2, T) = 1 + c_f(T)F_1(p_n^2)$, are defined by the temperature dependent coefficients $a_f(T)$, $b_f(T)$ and $c_f(T)$. The explicit forms of $a_f(T)$, $b_f(T)$ and $c_f(T)$ are given by

$$a_f(T) = \frac{8D_1}{9}T \sum_n \int \frac{d^3p}{(2\pi)^3} F_1(p_n^2) \bar{p}^2 \times [1 + a_f(T)F_1(p_n^2)] d_f^{-1}(p_n^2, T), \quad (11)$$

$$c_f(T) = \frac{8D_1}{3}T \sum_n \int \frac{d^3p}{(2\pi)^3} F_1(p_n^2) \omega_n^2 \times [1 + c_f(T)F_1(p_n^2)] d_f^{-1}(p_n^2, T), \quad (12)$$

$$b_f(T) = \frac{16D_0}{3}T \sum_n \int \frac{d^3p}{(2\pi)^3} F_0(p_n^2) \times [m_f + b_f(T)F_0(p_n^2)] d_f^{-1}(p_n^2, T), \quad (13)$$

where the denominator $d_f(p_n^2, T)$ of the a_f , b_f and c_f is given by

$$d_f^{-1}(p_n^2, T) = \bar{p}^2 A_f^2(p_n^2, T) + \omega_n^2 C_f^2(p_n^2, T) + B_f^2(p_n^2, T). \quad (14)$$

As we know, solving DSEs at finite temperature is quite difficult, but using the separable model interactions greatly simplifies the calculations. For simplicity, we choose the following form for the separable interaction form factor [14]:

$$F_0(p^2) = \exp(-p^2/A_0^2), \quad (15)$$

$$F_1(p^2) = \frac{1 + \exp(-p_0^2/A_1^2)}{1 + \exp((p^2 - p_0^2)/A_1^2)}, \quad (16)$$

which is successfully used to describe the phenomenology of the light pseudoscalar mesons. Substituting Eqs. (15, 16) into Eqs. (11–13) one can solve gap equations for a given temperature T , and get the quark amplitudes A_f , B_f and C_f . However, we need to control the appropriate number of Matsubara modes in any calculation.

2.3 Formulae of various quark vacuum condensates at zero temperature

At the lowest dimension, quark and gluon vacuum condensates play an essential role in describing properties of nuclear matter and hadron physics. The nonlocal quark vacuum condensate $\langle 0 | : \bar{q}(x)q(0) : | 0 \rangle$ can be given as [15]

$$\begin{aligned} & \langle 0 | : \bar{q}(x)q(0) : | 0 \rangle \\ &= (-4N_c) \int \frac{d^4p}{(2\pi)^4} \frac{B_f(p^2) e^{ipx}}{p^2 A_f^2(p^2) + B_f^2(p^2)} \\ &= -\frac{3}{4\pi^2} \int_0^\infty p^2 dp^2 \frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \frac{2J_1(\sqrt{p^2 x^2})}{\sqrt{p^2 x^2}}, \quad (17) \end{aligned}$$

where $N_c = 3$ is the number of colors. J_1 in Eq. (17) is the Bessel function. When $x=0$, the local quark vacuum

condensate is given by

$$\langle 0 | : \bar{q}(0)q(0) : | 0 \rangle = -12 \int \frac{d^4p}{(2\pi)^4} \frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)}. \quad (18)$$

Another important physical quantity is the four-quark vacuum condensate. The factorization hypothesis for the four-quark condensate is well-known from the works of Shifman, Vainshtein, and Zakharov [1], and has been extensively used in QCD sum rules through the operator product expansion approach. According to [16–18], we have the nonlocal four-quark vacuum condensate $\langle 0 | : \bar{q}(x)\gamma_\mu \frac{\lambda_C^a}{2} q(0)\bar{q}(x)\gamma_\mu \frac{\lambda_C^a}{2} q(0) : | 0 \rangle$

$$\begin{aligned} & \langle 0 | : \bar{q}(x)\gamma_\mu \frac{\lambda_C^a}{2} q(0)\bar{q}(x)\gamma_\mu \frac{\lambda_C^a}{2} q(0) : | 0 \rangle \\ &= - \int \frac{d^4p}{(2\pi)^4} \int \frac{d^4q}{(2\pi)^4} e^{ix \cdot (p-q)} \left[4^3 \frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \right. \\ & \times \frac{B_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} + 32 \frac{A_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \\ & \left. \times \frac{A_f(q^2)}{q^2 A_f^2(q^2) + B_f^2(q^2)} p \cdot q \right]. \quad (19) \end{aligned}$$

Then, the local ($x=0$) four-quark vacuum condensate is similarly given by

$$\begin{aligned} & \langle 0 | : \bar{q}(0)\gamma_\mu \frac{\lambda_C^a}{2} q(0)\bar{q}(0)\gamma_\mu \frac{\lambda_C^a}{2} q(0) : | 0 \rangle \\ &= -4^3 \int \frac{d^4p}{(2\pi)^4} \left[\frac{B_f(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \right]^2 \\ &= -\frac{4}{9} \langle 0 | : \bar{q}(0)q(0) : | 0 \rangle^2, \quad (20) \end{aligned}$$

which is consistent with the vacuum saturation assumption of [1].

Besides the quark vacuum condensate, the quark gluon mixed vacuum condensate is another important chiral order parameter of the QCD vacuum state, which plays an important role in the application of QCD sum rules. In the framework of the global color symmetry model, the local quark gluon mixed vacuum condensates are given by [16, 19]

$$\begin{aligned} & \langle 0 | : \bar{q}(0)[g_s \sigma G(0)]q(0) : | 0 \rangle \\ &= -\frac{N_c}{16\pi^2} \left\{ 12 \int dp^2 \frac{p^4 B_f(p^2)(2 - A_f(p^2))}{p^2 A_f^2(p^2) + B_f^2(p^2)} \right. \\ & \left. + \frac{27}{4} \int dp^2 p^2 B_f(p^2) \frac{[2A_f(p^2)(A_f(p^2) - 1)]p^2 + B_f^2(p^2)}{p^2 A_f^2(p^2) + B_f^2(p^2)} \right\}. \quad (21) \end{aligned}$$

2.4 Formulae of various quark vacuum condensates at finite temperature

It is a common belief that the quark vacuum condensate depends on the temperature T . In the case of finite temperature, one usually takes the same expression as the discussions above to study the temperature dependence of the quark vacuum condensate [9]. For local quark vacuum condensates we naturally have

$$\begin{aligned} & \langle 0 | : \bar{q}(0) q(0) : | 0 \rangle_T \\ &= -4N_c T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \\ & \quad \times \frac{B_f(p_n^2, T)}{p^2 A_f^2(p_n^2, T) + \omega_n^2 C_f^2(p_n^2, T) + B_f^2(p_n^2, T)}. \end{aligned} \quad (22)$$

$$\begin{aligned} \langle 0 | : \bar{q}(0) [g_s \sigma G(0)] q(0) : | 0 \rangle_T &= -36T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \frac{B_f(p_n^2, T) [(2 - A_f(p_n^2, T)) p^2 + (2 - C_f(p_n^2, T)) \omega_n^2]}{p^2 A_f^2(p_n^2, T) + C_f^2(p_n^2, T) \omega_n^2 + B_f^2(p_n^2, T)} - \frac{81}{4} T \sum_{n=-\infty}^{\infty} \int \frac{d^3 p}{(2\pi)^3} \\ & \quad \times \frac{2B_f(p_n^2, T) [A_f(p_n^2, T) (A_f(p_n^2, T) - 1) p^2 + C_f(p_n^2, T) (C_f(p_n^2, T) - 1) \omega_n^2] + B_f^3(p_n^2, T)}{p^2 A_f^2(p_n^2, T) + C_f^2(p_n^2, T) \omega_n^2 + B_f^2(p_n^2, T)}. \end{aligned} \quad (24)$$

The finite temperature DSEs provide a valuable non-perturbation tool for studying temperature dependent field theories. Phenomena, such as deconfinement, dynamical chiral symmetry breaking, temperature dependence of quark mass, and even cosmological investigation which cannot be explained by perturbation treatments, can be understood in terms of its solution of the DSEs at zero and finite temperature. Eqs. (11–13).

For completeness we also study the “in-hadron condensate”. A rigorous definition of the “in-hadron condensate” was provided by [23–25]

$$-\langle \bar{q} q \rangle_\mu^\pi = -f_\pi \langle 0 | : \bar{q}(0) \gamma_5 q(0) : | 0 \rangle = f_\pi \rho_\pi, \quad (25)$$

where ρ_π has the form

$$i\rho_\pi = Z_4 \text{tr} \int \frac{d^4 q}{(2\pi)^4} \gamma_5 S_f(q_+) \Gamma_\pi(q; P) S_f(q_-), \quad (26)$$

where $q_\pm = q \pm P/2$, $S_f(q_\pm)$ is the dressed quark propagator. The trace evaluation is routine. $\Gamma_\pi(q; P)$ is the pion’s Bethe-Salpeter (BS) amplitude, with the separable interaction. The general form of finite temperature dependence of the BS amplitude is [14]

$$\begin{aligned} \Gamma_\pi(q_n; P_m) &= \gamma_5 (iE_{\text{PS}}(P_m^2) + \gamma_4 \Omega_m \tilde{F}_{\text{PS}}(P_m^2) \\ & \quad + \vec{\gamma} \cdot \vec{P} F_{\text{PS}}(P_m^2)) F_0(q_n^2), \end{aligned} \quad (27)$$

where the various symbols are given in [14, 25].

3 Calculations and theoretical predictions

We first solve the quark’s DSEs at zero temperature under rainbow truncation and separated interaction model for the gluon propagator with parameters

It is still a matter of debate for the four-quark condensate when $T \neq 0$. It was shown in [20], that factorization hypothesis implies that the four-quark condensate becomes dependent on the QCD renormalization scale. In addition, theoretical arguments from the chiral perturbation theory also do not support this approximation at the next to next leading order, except in the chiral limit [21, 22]. For simplicity, we take the form

$$\begin{aligned} & \langle 0 | : \bar{q}(0) \gamma_\mu \frac{\lambda_C^a}{2} q(0) \bar{q}(0) \gamma_\mu \frac{\lambda_C^a}{2} q(0) : | 0 \rangle_T \\ &= -\frac{4}{9} \langle 0 | : \bar{q}(0) q(0) : | 0 \rangle_T^2, \end{aligned} \quad (23)$$

as to the quark gluon mixed vacuum condensate in $T \neq 0$ region, we have

$m_{\text{ud}} = 5.5$ MeV, $m_s = 117$ MeV, $\Lambda_0 = 758$ MeV, $\Lambda_1 = 961$ MeV, $p_0 = 600$ MeV, $D_0 A_0^2 = 219$, $D_1 A_0^4 = 40$, which are completely fixed by meson calculated phenomenologically from the model as given in [9]. The present calculating results of DSEs at zero temperature are displayed in Fig. 1 and Fig. 2 as a basic test of current work.

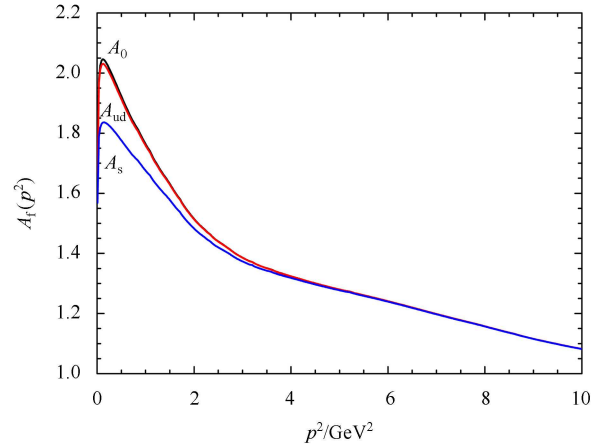


Fig. 1. p^2 -dependence of quark self-energy amplitudes $A_f(p^2)$, subscript f for the ud quark, the s quark and the chiral limit cases.

In order to demonstrate the temperature dependence of quark propagators, we use the Matsubara formula, and we then solve quark’s DSEs at nonzero temperature with the same gluon propagator and parameters. The results are given in Fig. 3 and Fig. 4. From Fig. 3, we can find that, for low temperature, the vector parts of the quark propagator $A_f(0, T)$ and $C_f(0, T)$ coincide with each other, they are almost the same. However, for the

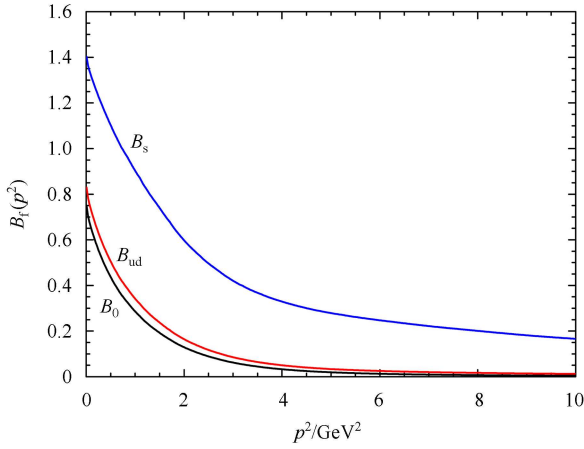


Fig. 2. p^2 -dependence of quark self-energy amplitudes $B_f(p^2)$, subscript f for the ud quark, the s quark and the chiral limit cases.

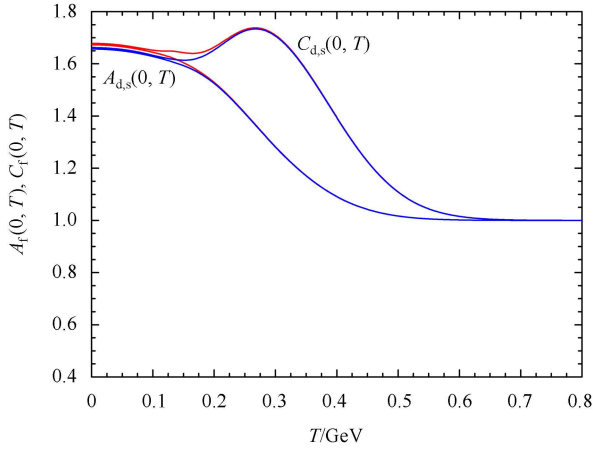


Fig. 3. T -dependence of quark self-energy amplitudes $A_f(0, T)$, and $C_f(0, T)$, subscript f for the ud quark, the s quark and the chiral limit cases.

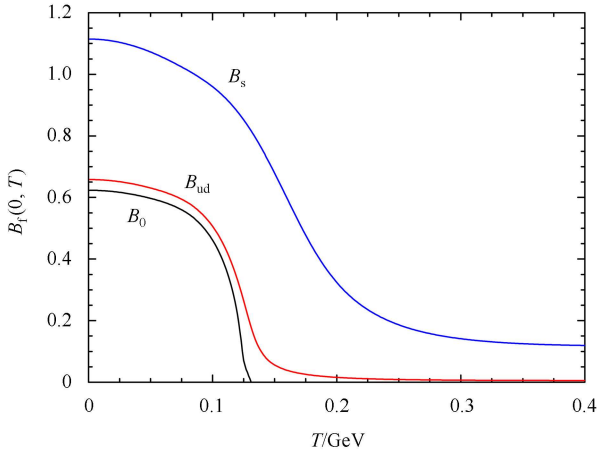


Fig. 4. T -dependence of quark self-energy amplitudes $B_f(0, T)$, subscript f for the ud quark, the s quark and the chiral limit cases.

temperature higher than about $T = 131$ MeV, they become dramatically different. The reason is that the $O(4)$ symmetry has been broken.

Using the individual solutions of the quark's DSEs at zero- and finite temperature, A_f , B_f and C_f we obtain the properties of the QCD vacuum at zero- and nonzero temperature and in the chiral limit. The quark vacuum condensate $\langle \bar{q}q \rangle$, the four-quark vacuum condensate $\langle \bar{q}\Gamma q\bar{q}\Gamma q \rangle$ and the quark gluon mixed vacuum condensate $\langle \bar{q}[g\sigma G]q \rangle$ are important condensates of the lowest dimension, which reflect the non-perturbative structure of the QCD vacuum state, and can be the chiral order parameter of QCD. In Fig. 5, the temperature dependence of the two-quark condensate, the four-quark condensate and the quark gluon mixed condensate in the chiral limit and in the separable model are plotted respectively. We predicted the critical temperature for the chiral symmetry restoration to be $T_c = 131$ MeV. These order parameters give the same critical temperature and the same critical behavior. From Fig. 5, we find these three condensates are almost independent of the temperature below T_c , while a clear signal of chiral symmetry restoration is shown at T_c . For comparison we calculate the temperature dependence of the “in-hadron condensate” for pion shown in Fig. 5, when $T = 0$ we have $\langle \bar{q}q \rangle_{\mu=1 \text{ GeV}}^\pi = (0.229 \text{ GeV})^3$, which is approximate to $\langle \bar{q}q \rangle_{\mu=1 \text{ GeV}}^{\text{ud}}$ as expected. We also calculate the ratio of the quark gluon mixed vacuum condensate to the two-quark vacuum condensate. The results are shown in Fig. 6. We can see from the figure, although there are dramatic changes of the quark condensate and the quark gluon mixed condensate near T_c , the ratio $m_0^2(T)$ is almost flat when the temperature at

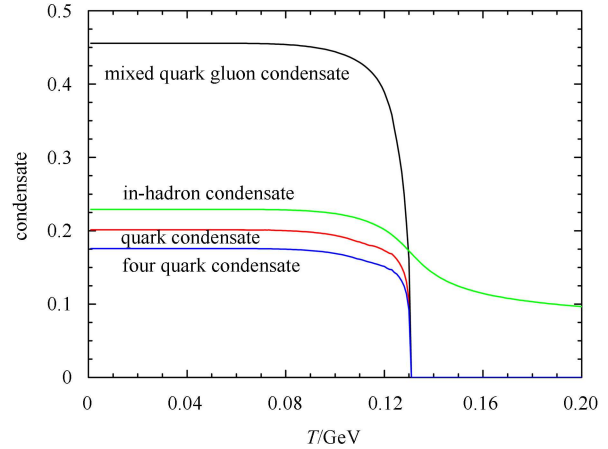


Fig. 5. T -dependence of the quark condensate, the four-quark condensate and the mixed quark gluon condensate in the chiral limit cases, T -dependence of “in-hadron condensate” for pion is given for comparison.

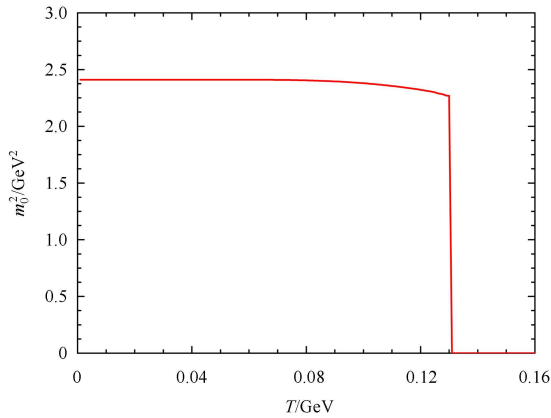


Fig. 6. T -dependence of the ratio of the mixed quark gluon condensate to the quark condensate, $m_0^2 = \frac{\langle 0 | \bar{q}(0) g \sigma \cdot G(0) q(0) | 0 \rangle_T}{\langle \bar{q} q \rangle_T}$, in the chiral limit case.

the region from 0 to T_c . At the chiral limit, the ratio $m_0^2(0) = 2.41 \text{ GeV}^2$, which is larger in comparison with the results from Lattice QCD which is about 1 GeV^2 [26]. This fact shows the great significance that the quark gluon mixed vacuum condensate has played in OPE calculations.

4 Summary and concluding remarks

In summary, we study temperature dependence of fully dressed quark propagator $S_f(p^2, T)$ in QCD by use

of the DSEs at zero- and finite- temperature under the “rainbow” truncation, $\Gamma^\nu = \gamma^\nu$ and the separated interaction model for the gluon propagator. We solve the DSEs numerically and get quark propagator functions, $A_f(p^2, T)$, $B_f(p^2, T)$ and $C_f(p^2, T)$ in Eq. (10) at two cases of $T = 0$ and $T \neq 0$, and then we obtained the temperature dependence of the quark propagator $S_f(p^2, T)$. The resulting quark propagator at the finite temperature has no Lehmann representation and hence there are no quark production thresholds in any observable calculations. The absence of such thresholds admits the interpretation that $S_f(p^2, T)$ describes the propagator of a confined quark. With the solutions of the quark’s DSEs A_f , B_f and C_f , the temperature dependence of the two-quark vacuum condensate, the four-quark vacuum condensate, and the quark gluon mixed vacuum condensate in the chiral limit are obtained. We find these condensates have the same critical temperature $T_c = 131 \text{ MeV}$ for the chiral symmetry restoration and the same critical behavior for QCD phase transition, which characterize different aspects of the QCD vacuum. At the same time, using calculation condensates we also obtain the quark virtuality which is the ratio between the quark gluon mixed vacuum condensate and the two-quark vacuum condensate. The calculation result shows that the quark virtuality is insensitive to temperature below the critical point $T_c = 131 \text{ MeV}$. The quark virtuality is an unknown physical quantum and has to be predicted theoretically in an acceptable way.

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