Intra-beam scattering studies for low emittance at BAPS

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Abstract: The target parameters of modern ultra-low emittance storage ring light sources are entering into a regime where intra-beam scattering (IBS) becomes important and, in the case of the Beijing Advanced Photon Source (BAPS), which is being designed at the Institute of High Energy Physics (IHEP), even a limitation for achieving the desired emittances in both transverse planes at the diffraction limit for X-ray wavelengths (≈10 pm). Due to the low emittance, the IBS effect will be very strong. Accurate calculations are needed to check if the design goal ($\varepsilon_{\rm h}+\varepsilon_{\rm v}$=20 pm) can be reached. In this paper, we present the results of numerical simulation studies of the IBS effect on a BAPS temporary design lattice.

Key words: intra-beam scattering, coupling factor, damping wiggler, high harmonic cavity

PACS: 29.20.db, 29.27.Ac, 41.85.Ar **DOI:** 10.1088/1674-1137/39/6/067001

1 Introduction

Several example designs (PEP-X, Spring-8 upgrade) have been produced for "ultimate" storage rings (USRs) [1] with equal transverse emittances at the diffraction limit for X-rays. One possible solution for the Beijing Advanced Photon Source (BAPS) is designed to achieve diffraction-limited emittances of 10 pm·rad in both horizontal and vertical planes, which is required by "ultimate" storage rings. The total emittance, about 20 pm, is obtained first, and thus a round beam ($\varepsilon_{\rm h}=\varepsilon_{\rm v}\approx$10 pm), can be given by the locally round beam method [2].

A temporary lattice for BAPS has been designed to provide an electron beam with emittance of 51 pm by using 36 seven-bend achromat (7BA) cells, within a circumference of 1364.8 m. The main parameters of the BAPS lattice design are listed in Table 1. The optical functions and the layout of the 7BA are shown in Fig. 1.

Table 1. Main parameters for the BAPS storage ring.

parameter	symbol	value
energy/GeV	E	5
circumference/m	C	1364.8
current/mA	I_0	100
bunch number	$n_{\rm b}$	1836
number of particles per bunch	$N_{\rm b}$	1.55×10^9
natural bunch length/mm	$\sigma_{\rm s0}$	1.5
RF frequency/MHz	$f_{\rm rf}$	499.92
RF voltage/MV	$V_{\rm rf}$	6
harmonic number	h	2276
natural energy spread	$\sigma_{\rm e0}$	7×10^{-4}
momentum compaction	$\alpha_{\rm p}$	4×10^{-5}
emittance of bare lattice/pm	ε_x	51
energy loss per turn/MeV	U_0	1.07

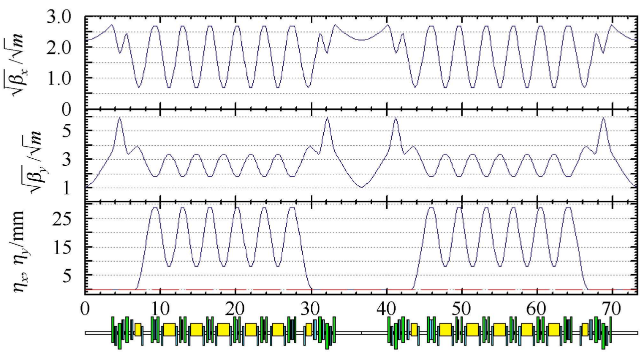

Fig. 1. Optical functions of the designed BAPS storage ring @ standard super-cell.

2 General IBS theory

Intra-beam scattering (IBS), which has implications for the smallest achievable emittance, is one of the fundamental limitations in achieving ultra-small emittances in storage rings. It leads to growth in emittance and energy spread in electron machines. Its effect depends on the lattice and beam characteristics.

Several theories and their approximations were developed over the years describing the effect. The Bjorken-Mtingwa formulation [3] is regarded as being the most

Received 15 July 2014, Revised 5 January 2015

general solution. For bunched beams, the growth rates for intra-beam scattering can be expressed as:

$$\frac{1}{T_{\rm i}}=4\pi A(\log)\langle F_{\rm i}\rangle, \tag{1}$$

where $T_{\rm i}$ is the growth time, i represents p, h, v for the relative energy spread and horizontal and vertical emittances respectively. The functions $F_{\rm i}$ are integrals that depend on beam parameters, such as energy and phase space density, and lattice properties, including dispersion; the angle brackets $\langle\cdots\rangle$ indicates that the integral is to be averaged around the ring lattice. The details of the function $F_{\rm i}$ you can be found in Refs. [3, 4]. (log) is the Coulomb logarithm and

$$A=\frac{r_0^2cN}{64\pi^2\beta^3\gamma^4\varepsilon_{\rm h}\varepsilon_{\rm v}\sigma_{\rm s}\sigma_{\rm p}}, \tag{2}$$

where N is the number of particles per bunch, r_0 is the classical radius of the charged particle, c is the speed of light in vacuum, β is the particle velocity divided by the speed of light, γ is the particle energy divided by the rest mass, $\varepsilon_{\rm h,v}$ is the emittance, $\sigma_{\rm s}$ is the rms bunch length, and $\sigma_{\rm p}$ is the relative energy spread.

From the expression for A, it follows that the IBS effect depends on the beam energy, energy spread, bunch length and emittances, so we can adjust these parameters to mitigate its effect.

3 IBS dependence on BAPS parameters

3.1 Influence of initial emittance

All IBS models show the same trend in the emittance evolution. In principle, any of them could be used to study the dependence of the output emittances on the other beam parameters. In low emittance machines such as BAPS, the IBS effect is determined by the initial emittance. In what follows, we will study the IBS dependence on zero current emittance. Two ways of changing the emittance at zero current are considered: phase space manipulations (changing the coupling factor) and optimized radiation integral (adding damping wigglers).

3.1.1 Changing the coupling factor

In general, there are several principal contributions to the vertical emittance of an electron beam in a storage ring [4]: a "direct" contribution, the vertical opening angle of the radiation, vertical dispersion, and betatron coupling. The direct contribution comes from the fact that two particles moving with zero transverse momentum, after collision, have some nonzero horizontal and vertical momentum. Thus transverse emittance growth will increase, even where the horizontal and vertical dispersion are both exactly zero. The transverse emittance growth from the direct contribution is rather slow.

In regions of dispersion, the dispersion could cause the synchro-betatron coupling, the emittance growth from that coupling is generally larger than the direct contribution. In Ref. [4], it is shown that 6 mm is already a large enough vertical dispersion to completely dominate the direct contribution.

Also, for high energy beams, the contribution from the vertical opening angle of the radiation is always small (magnitude of 10^{-14} m) compared to the contributions from other sources.

The vertical dispersion in the BAPS ideal design lattice is zero, but magnet misalignments will lead to vertical dispersion in the real storage ring. For simplicity, we treat the vertical emittance as generated mainly by the transverse coupling, by which we imply that the vertical dispersion can be kept sufficiently small that the contribution of vertical dispersion can be ignored. Then the vertical emittance is proportional to the horizontal emittance, and can be written as:

$$\varepsilon_{\rm h}=\frac{1}{1+\kappa}\varepsilon_{\rm nat},\ \varepsilon_{\rm v}=\frac{\kappa}{1+\kappa}\varepsilon_{\rm nat}, \tag{3}$$

with κ being the coupling constant between 0 and 1 and $\varepsilon_{\rm nau}=\varepsilon_{\rm h}+\varepsilon_{\rm v}$ being the natural emittance at zero current.

As you can see from Fig. 2, the steady-state emittance grows seriously from the natural emittance. We could say that the acceptable beam current is somewhat determined by the IBS effect.

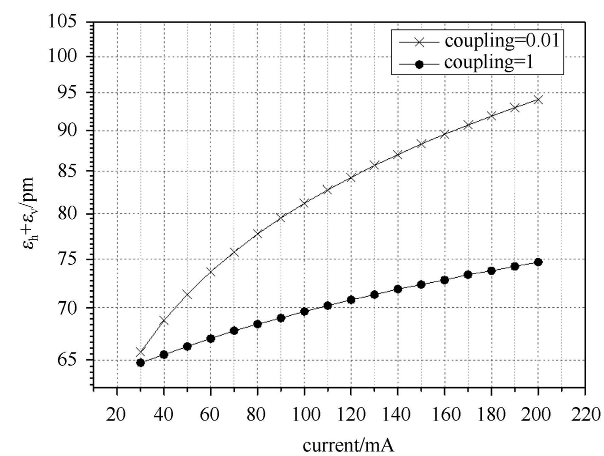

Fig. 2. The emittance as a function of bunch current for different coupling factors.

From Fig. 3, we can see that the steady-state emittance can be varied by adjusting the coupling factor, with the emittance decreasing for increasing coupling factor. In the case of "ultimate" storage rings, these should be operated at full coupling with round beams, so we need a robust and efficient method to obtain round beams in electron storage rings. This is an important challenge which should be addressed in future work.

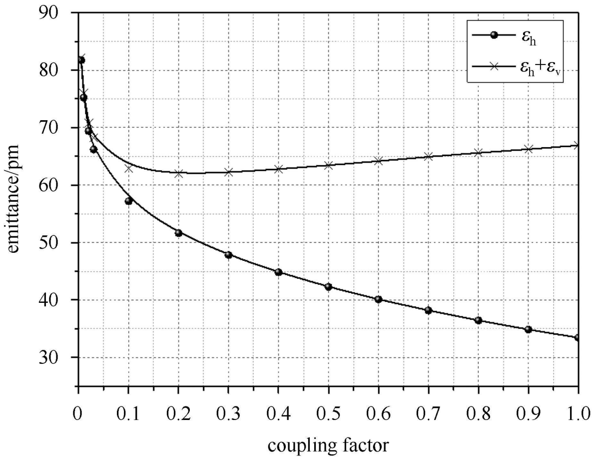

Fig. 3. Steady-state transverse emittances vs. coupling factor @ I=100 mA.

3.1.2 Adding damping wigglers

For a modern light source design, damping wigglers play an important part in reducing the emittance, and are an efficient way compared to other methods. The relative reduction in natural emittance from ε_0 to $\varepsilon_{0\rm w}$ caused by a damping wiggler in the dispersion free straight can be estimated using an approximate analytical expression [5]:

$$\frac{\varepsilon_{0\rm w}}{\varepsilon_0}=\left(\frac{J_{x0}}{J_{x\rm w}}\right)\frac{1+\dfrac{4C_{\rm q}}{15\pi J_{x0}}N_{\rm w}\dfrac{\langle\beta_x\rangle}{\varepsilon_0\rho_{\rm w}}\gamma^2\dfrac{\rho_0}{\rho_{\rm w}}\theta_{\rm w}^3}{1+\dfrac{1}{2}N_{\rm w}\dfrac{\rho_0}{\rho_{\rm w}}\theta_{\rm w}}, \qquad (4)$$

where $C_{\rm q}$=3.81×10^{-13} m, $J_{x\rm w}$, J_{x0} are damping partition numbers with and without wigglers, $N_{\rm w}$ is the number of wiggler periods, $\langle\beta_x\rangle$ is the average horizontal beta function in the wiggler, $\rho_{\rm w}$ is the bending radius at peak wiggler field, $\theta_{\rm w}=\lambda_{\rm w}/2\pi\rho_{\rm w}$ is the peak trajectory angle in the wiggler and $\lambda_{\rm w}$ is the wiggler period length, and ρ_0 is the bending radius of the ring dipole. Eq. (4) shows that the emittance reduction depends on the wiggler period length, the wiggler peak field, and the total wiggler length.

Figure 4 and Fig. 5 show the ratio of $\varepsilon_{0\rm w}/\varepsilon_0$ versus the total wiggler length and the wiggler peak field respectively for various values of wiggler period length, where the wiggler is inserted in long straight sections with $\langle\beta_x$=5.82 m$\rangle$. From the figures, we can see that most of the damping ratio occurs within a wiggler length of 70 m, and that a wiggler period below 3.1 cm does not significantly improve the damping ratio. Selecting a 62 m long wiggler with a 3.1 cm period, it follows that the optimal peak field is about 2.3 T.

For our IBS calculations, we assume the initial vertical emittance equals 1 pm [6], the new vertical emittance record established in the storage ring of the Swiss Light Source (SLS).

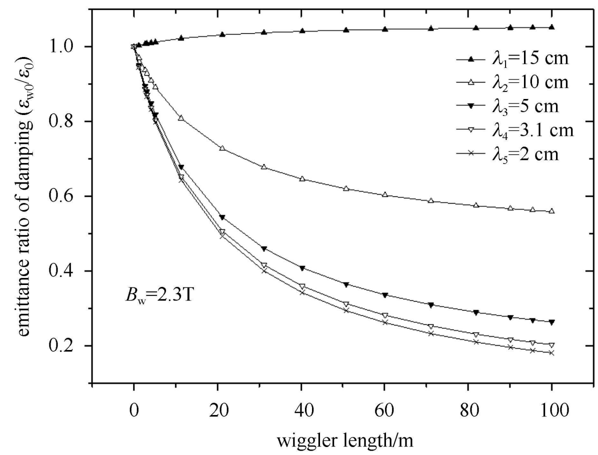

Fig. 4. Relative emittance reduction vs. wiggler length for different wiggler periods.

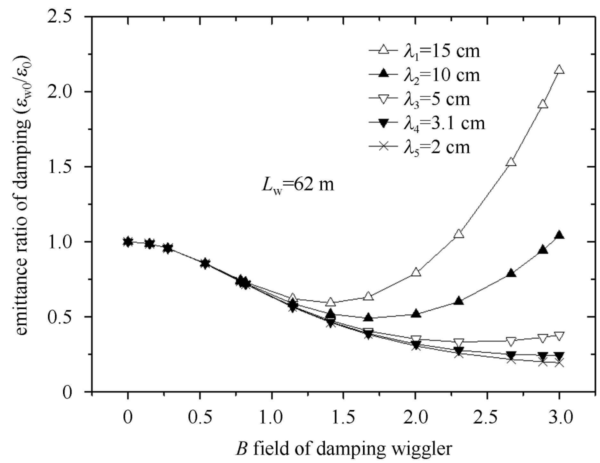

Fig. 5. Relative emittance reduction vs. wiggler field for different wiggler periods.

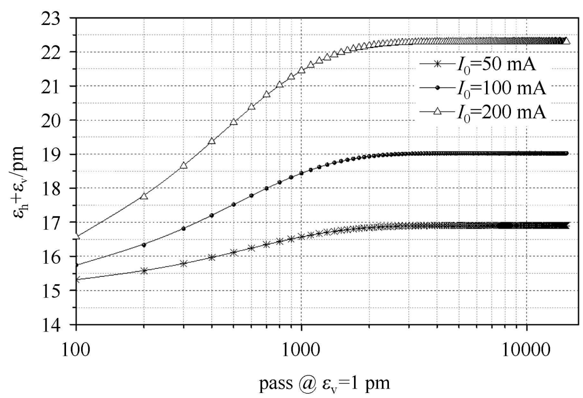

Fig. 6. Steady-state emittances for different bunch currents with long damping wigglers.

The resultant emittance with wigglers at zero current is 14 pm. The steady-state total emittances have been calculated for different currents, Fig. 6 shows that we can achieve the goal when the current is about 100 mA.

3.2 Influence of bunch length

From formulas (1) and (2), we can see that the bunch length σ_s is inversely proportional to the IBS growth rate. In order to decrease the IBS growth rate, we can lengthen the electron bunches. Bunch lengthening can be achieved by reducing the slope of the accelerating voltage in the vicinity of the electron bunch. Adding a harmonic cavity, sometimes known as a Landau cavity [7], is a good method.

In the previous section, damping wigglers were inserted into a few straights to further decrease the natural emittance. Damping wigglers, however, create various side effects. They increase the radiation loss per turn as well as the beam rms energy spread (by a factor of ~2). As a result, higher RF voltage is required to compensate for the energy loss. The damping wiggler also introduces additional nonlinearities affecting the beam dynamics. In the case of a storage ring with damping wigglers and Landau cavities, the length of the wigglers we need should be decreased.

Figures 7 and 8 show the bunch lengths in the operation by the third harmonic RF. In the operation, the bunch extends to 27 ps and becomes about 8 times longer than the natural bunch length. In our calculation, the bunch length we used is 5 mm, over half of the resulting bunch length.

Table 2. Equilibrium transverse emittances calculated with (right) and without (left) IBS.

	$\varepsilon_h+\varepsilon_v$/nm	$\varepsilon_h+\varepsilon_v$/nm
bare lattice	51+1	75+1.47
lattice with DWs	16+1	25.5+1.5
lattice with DWs and LC	16+1	18.6+1.15

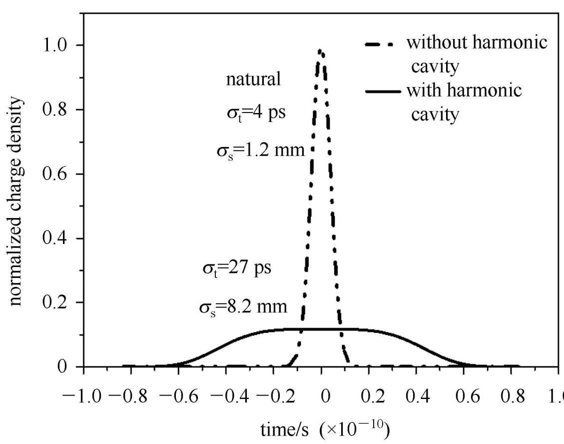

Fig. 7. Longitudinal bunch distribution for single and third harmonic system.

Once the harmonic Landau cavities and damping wigglers are added, the influence of IBS becomes small. In the case of a storage ring with damping wigglers (L_w=49.6 m), and Landau cavities (σ_s=5 mm) the equilibrium emittance (I_0=100 mA) including IBS remains well below the design goal.

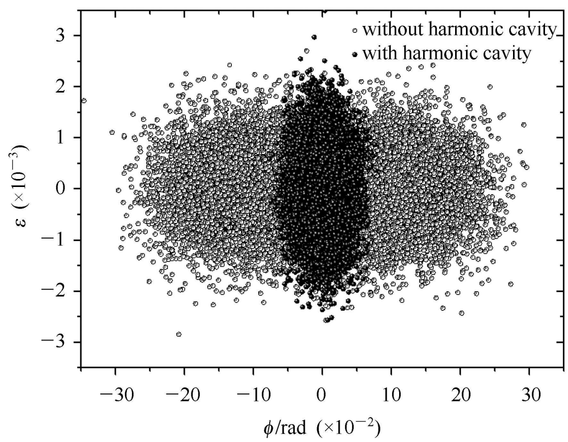

Fig. 8. Longitudinal phase space distribution for single and third harmonic system.

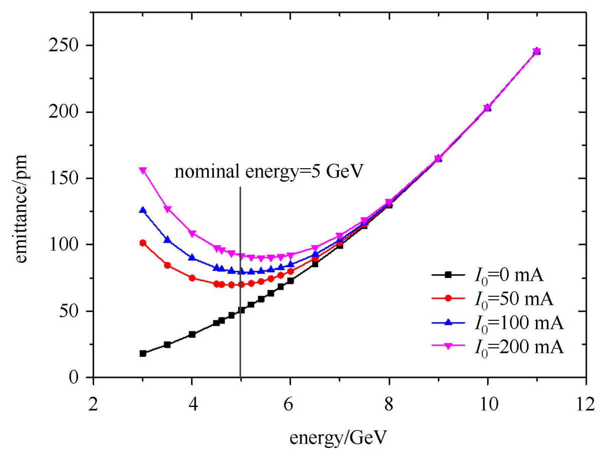

Fig. 9. Emittance vs. energy for different currents for the 51-pm design while considering IBS effect.

3.3 Influence of beam energy

As a result of the balance between quantum excitation and radiation damping, an electron beam in storage rings reaches an equilibrium distribution with horizontal emittance given by [8]:

$$\varepsilon_x = C_q \frac{\gamma^2}{J_x}\frac{I_5}{I_2}, \tag{5}$$

$$C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc}, I_2 = \oint \frac{ds}{\rho^2}, \quad I_5 = \oint \frac{\mathcal{H}_x}{\rho^3} ds, \tag{6}$$

$$\begin{aligned} \mathcal{H}_x &= \gamma_x D_x^2 + 2\alpha_x D_x D'_x + \beta_x D'^2_x \\ &= D_x^2 + \left(\beta_x D'_x - \frac{1}{2}\beta'_x D_x\right)^2 \Big/ \beta_x, \end{aligned} \tag{7}$$

J_x is the horizontal damping partition number, ρ is the bending radius, and H_x is the dispersion H-function For a given lattice that is scalable with energy, we want to know which energy is optimal energy given the IBS effect. Using the bare lattice, we performed IBS calculations for different energies.

In Fig. 9 we plot emittance vs. electron energy E, we note that the emittance minimum is near our nominal energy. It also implies that the growth rate exponentially increases as the smaller the beam energy becomes. In the design preliminary stage, the beam energy should be chosen carefully.

4 Conclusions

We calculated the emittance growth due to the IBS effect in BAPS. The steady-state emittance including IBS is much larger than the natural emittance of the bare lattice. We considered the application of damping wigglers and harmonic cavity to control the IBS effect, and found that using these devices our design goal for beam emittance can theoretically be achieved.

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